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3 On the sensitivity of computed higher tidal harmonics

4 to mesh size in a finite element model

5 by

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11 ABSTRACT

12     A finite element model of the Irish and Celtic Sea regions with a range of grid

13 resolutions is used to examine the influence of resolution upon the higher harmonics of the

14 tide in the region. Comparisons are also made with published results from finite difference

15 models of the area, and observations. Calculations using fine near-shore elements with non-

16 zero water depths in coastal regions were found to be more accurate and less time consuming

17 than those using a zero coastal water depth. A detailed examination of the spatial variability

18 of the higher harmonics in near-shore regions of the eastern Irish Sea particularly the Solway

19 and Morecambe Bay showed significant small scale variability. This together with the

20 variation in higher harmonics in the eastern Irish Sea and adjacent estuaries, clearly shows

21 the need for an unstructured grid model of the region that can include the estuaries. To

22 match the high resolution of the model in nearshore regions accurate high resolution

23 topography is required.

24 1. INTRODUCTION

25     The generation of higher harmonics of the tide by non-linear effects has been well

26 established in the literature, since the early work of Proudman in the 1960's. Numerous

27 numerical simulations of their spatial distributions in a range of shallow water regions have

28 been performed using finite difference models (e.g. for the Irish Sea region, Young et al

29 2000, Davies and Jones 1992, hereafter referenced as DJ92, Davies 1986, Davies et al. 2001,

30 Jones 1983), although to date less work has been done with finite element models. Recent

31 examples of finite element work are Fortunato et al 1997, 1999, Henich et al 2000, Legrand

32 et al (2006, 2007), Greenberg (2007) and references therein. In addition Blanton et al (2004)

33 used a finite element model to examine the principal although not the higher harmonics of

34 the tide in the South Atlantic Bight. They found that even for the principal tides it was

essential to have high resolution in the coastal region. Since the higher harmonics are largest and show greatest spatial variability in shallow water regions then maximum resolution in these areas is desirable. Consequently an unstructured grid model with enhanced resolution in the coastal region would appear to be the ideal tool to simulate these harmonics. In the finite difference model, the need to use a uniform grid means that the model domain is generally limited in order to have a fine mesh in the coastal region without excessive computational effort. A limited area coastal model therefore requires some description of the higher harmonics along its open boundary which is rather restrictive. Consequently in a small domain model, part of the solution has been specified through the open boundary (e.g. Jones and Davies 1996, hereafter referenced as JD96, Greenberg (2007)). The alternative is to use a large area model (e.g. the shelf wide model of Kwong et al 1997). In this case there is no input of higher harmonics through the open boundary, since these are in deep water. However, the coarse grid nature of the model means that an accurate representation of the nearshore region is not possible.

The finite element model with its ability to vary the mesh (e.g. Henry and Walters (1993), Luettich and Westerink (1995), Lynch et al (1995, 2004), Werner 1995, Malcherek 2000, Hervouet 2002, Jones 2002) , is able to cover a significant offshore region while retaining a fine mesh near the coast. By this means a computationally efficient solution is obtained. Although this means of grading the mesh, based on local water depth is popular in many shallow sea models (see Greenberg (2007) for a recent review of methods in oceanic and coastal situations) it is not the only approach. Recently Legrande et al (2006) developed a method in which both the bathymetric field and approximate distance to offshore features such as islands and reefs could be included in the mesh generation algorithm. This approach was very successful in generating an optimal mesh that could resolve tidal features and re-circulating eddies in regions such as the Australian Great Barrier Reef. A comparable, in the sense of requiring a fine grid in the region of rapid change whilst minimizing computational effort was given by Hagen et al (2002). This Local Truncation Error Analysis (LTEA) approach seeks to design a mesh to make the truncation error uniform across the whole domain. Recent papers (Legrande et al 2007, Greenberg 2007) present a range of parameters that must be considered in the design of an optimal grid. The importance of these parameters varies depending upon grid location (e.g. shelf edge, shallow sea) and processes to be modelled. In the case of tidal processes Legrande et al (2007), suggest that a grid aligned with the major axis of the $M_2$ tidal ellipse with anisotropy related to the major and minor axis of the ellipse could be optimal. This approach is very attractive, although in the

Irish Sea example given in Legrand et al (2007) it did reduce grid resolution in near coastal regions where higher harmonics are important. Although other parameters could possibly also be included to enhance local resolution.

Despite this significant recent progress in grid design (see review of Greenberg (2007)) it is clear that a major problem still remains, namely how should the grid be graded from offshore to nearshore in order to obtain an accurate solution for both the fundamental and higher harmonics. In a more complex region such as the west coast of Britain where there are a number of channels (Fig. 1a, see Fig. 1b for details of topography in eastern Irish Sea) where "wetting and drying" can occur, with associated small scale changes in the higher tidal harmonics, and deep water regions adjacent to shallow (e.g. Irish Sea) the choice of grid is complex. Also measurements and coarse grid shelf wide models show that the $M_4$ tide along the west coast of Britain increases from the shelf edge (where models (e.g. Davies 1986) assume it is zero) through the Celtic Sea with a subsequent decrease at the southern end of the Irish Sea and increase in the eastern Irish Sea. This spatial distribution is very different to that in a near coastal region or estuary where a uniform increase towards the coast or head of the estuary occurs. The simulation of the complex $M_4$ distribution that occurs off the west coast of Britain therefore represents a good test of the ability of an irregular grid model to adequately describe the tide and its higher harmonics. Such a test is performed here. In addition the existence of a solution from a high resolution finite difference eastern Irish Sea model, Fig. 2 (JD96) and a comprehensive data set in terms of tidal observations and bottom topography in this nearshore region, means that the sensitivity to grid resolution in the coastal domain can also be examined. For this reason the area off the west coast of Britain with particular emphasis on the eastern Irish Sea is used here to examine the accuracy of a finite element code (TELEMAC) with a range of irregular grids, to represent the fundamental constituent $M_2$ and its higher harmonics.

The primary objective of the paper is to develop an optimal irregular mesh finite element model of the tides along the west coast of Britain, with particular emphasis on the eastern Irish sea where a detailed (horizontal resolution of 1 km) topographic data set exists (JD96). The finite element distribution will be such that it can adequately describe the fundamental tide, to such an extent that the higher harmonics can be accurately reproduced. Since these are largest in near coastal regions then the mesh will be refined locally, particularly in the eastern Irish Sea, to allow for their correct reproduction. The focus of the calculation will be the eastern Irish Sea, where the higher harmonics are significant and an

1 accurate finite difference solution and associated data (JD96) exists for comparison
2 purposes.

3       The present paper complements and extends an earlier paper (Jones and Davies 2007
4 hereafter JD07) where the focus was the influence of grid resolution upon tidal residual
5 currents.  In that paper by comparison with a published finite difference solution, it was
6 shown that the removal by using finite elements of artificial vorticity associated with the
7 "staircase" representation of the coast in a finite difference model (Greenberg 2007),
8 significantly improved the representation of tidal residuals in the near coastal region.  In
9 essence in a finite difference model the poor representation of the coastline corrupted the
10 tidal residual within five grid boxes of the coast.  In addition JD07 examined the detailed
11 distribution and convergence of the tidal residual within the Mersey estuary for a range of
12 locally refined meshes in the estuary.  Calculations showed that there were significant gyres
13 at the entrance and within the Mersey, in the tidal residual current.  These gyres could only
14 be reproduced by using a finite element grid that could resolve coastal and topographic
15 variations within the Mersey.  Although these convergence studies gave significant insight
16 into the role of mesh refinement, there were no tidal residual currents or elevations with
17 which comparisons could be made.  Also the focus of JD07 was a convergence study within
18 the Mersey and the influence of coastal discretization method on the solution.  Due to a lack
19 of observations no model data comparisons were possible.

20       In this paper the focus of the study is the influence of finite element resolution upon
21 higher harmonics of the tide that are generated by non-linear effects and "wetting and
22 drying" in the coastal region of the eastern Irish Sea.  Since extensive observational data sets
23 are available over the whole region, as are published results from a finite difference model, a
24 detailed comparison with observations and comparable finite difference solutions are
25 possible. The focus here is no longer limited to the Mersey estuary as in JD07, but rather to
26 spatial variability in a range of nearshore regions where observations are available.  This
27 spatial variability is illustrated in a number of figures and quantified in various tables.

28       The basic hydrodynamic equations are described in the next section.  In subsequent
29 sections the influence of the nearshore coastal boundary, and finite element resolution, upon
30 the higher harmonics are considered.  Conclusions as to an optimal form of the mesh and
31 sensitivity of the solution are presented in the final part of the paper.

32 2.      HYDRODYNAMIC EQUATIONS

33       To examine the sensitivity of the tidal distribution to variations in horizontal grid
34 resolution, it is sufficient to solve the two dimensional vertically integrated hydrodynamic

equations. Since details are given in DJ92 and JD96 only the main features are presented here. As the region spans a range of latitude, the hydrodynamic equations in spherical coordinates were used. The model contains all the non-linear terms, including momentum advection and quadratic bottom friction with a friction coefficient $k = 0.0025$, an appropriate value to use in a two dimensional model and consistent with DJ92.

Along the open boundaries (Fig. 1a) the $M_2$ tidal elevation was specified, with higher harmonics set to zero. Consequently the higher harmonics were generated by the model dynamics. The input was identical to that used by DJ92. In all calculations (Table 1a) the same tidal input was used although the finite element grid varied (Table 1b). In the finite element calculations in which the grid was refined to give enhanced resolution in near coastal regions with coarser resolution offshore, the tidal input was interpolated to the new open boundary nodes. Solutions were determined in all cases by integrating forward in time over seven tidal cycles and harmonically analysing the final cycle to yield tidal amplitude and phase for the $M_2$, $M_4$ and $M_6$ constituents.

All solutions were generated from initial conditions of zero elevation and motion at $t = 0$. At a closed boundary the normal component of velocity was set to zero. In shallow regions "wetting and drying" occurred over the tidal cycles. As details of this are given in Jones and Davies (2005), (hereafter referenced as JD05) they will not be presented here. The approach is consistent with that used by JD96 in their finite difference model, and hence a comparison with that model is valid. However the nearshore solution will be sensitive to the algorithm used to represent "wetting and drying" (Balzano 1998) and the nearshore element resolution (see later discussion).The horizontal gradient normal to the coast of alongshore velocity was taken as zero, corresponding to perfect slip.

3.    FINITE ELEMENT SOLUTION

*3.1    Choice of grid*

In a previous paper (JD05), the $M_2$ tidal distribution over the west coast of Britain was computed with the finite element model using a range of meshes. Calculations (JD05) showed that the optimal $M_2$ tidal solution was obtained using a mesh refinement based on water depth such that the ratio between element size and $(gh)^{\frac{1}{2}}$ was constant. This condition has been used by a number of authors (e.g. Henry and Walters (1993), see also references in the comprehensive review of Greenberg et al (2007) for other criteria and recent work by Hagan et al (2002) on the LTEA approach). Calculations with both a zero and non-zero water depth at the coast were performed, with the non-zero depth yielding the most accurate $M_2$ solution at minimum computational cost (JD05). Here we examine to what extent the

1     higher harmonics $M_4$ and $M_6$ are influenced by the use of a zero or non-zero coastal water

2     depth. Taking topography and eastern Irish Sea coastline from a 1 km model of this region

3     (Fig.2) (JD96) gave an optimal $M_2$ tidal solution (JD05).

4     *3.2     Calculation using a non-zero coastal water depth (Calc 1)*

5     In an initial calculation (Calc 1, Table 1a), the optimal grid (Grid G3AX (Fig. 3))

6     used by JD05 was applied, with linear triangular elements and an advection scheme based on

7     characteristics. Since a detailed comparison of the computed $M_2$ cotidal chart (not shown

8     but given in JD05) with measurements is given by JD05, only the major features will be

9     examined here with comparisons at coastal and offshore gauges together with current meters

10     that illustrate the major changes (Tables 2a,b). A more detailed comparison in the eastern

11     Irish Sea is given in Table 3a. The $M_2$ cotidal chart is characterized by a rapid increase in

12     tidal amplitude in the eastern Irish Sea, and in the shallow channels such as the Bristol

13     Channel. Within the North Channel the amphidromic point found in measurements (George

14     1980) and high resolution models of the region (Davies et al. 2001) has been displaced

15     slightly to the north. However as discussed in Davies et al. (2004), tides in this region are

16     sensitive to small changes in the tidal distribution to the north and south of the North

17     Channel. The amphidromic point in the western Irish Sea and the overall distribution of co-

18     amplitude and co-phase lines are in good agreement with co-tidal charts based on

19     observations (e.g. Robinson 1979) and computed distributions derived from both two

20     dimensional (Jones 1983) and three dimensional (DJ92) models.

21     A detailed comparison (Table 3a) of computed and observed $M_2$ tidal amplitude and

22     phase at identical points (Fig 2) in the eastern Irish Sea to those used in JD96 with their high

23     resolution (of order 1 km) eastern Irish Sea model, showed that the $M_2$ tidal amplitude

24     determined with grid G3AX (Table 3a) was of the order of 15 cm too high. The phase error

25     was on average about 10° too high. This is an appreciably larger error than that found in

26     JD96 but can be understood in terms of differences in open boundary input to the models. In

27     the present model the open boundary forcing was located well away from the Irish Sea (Fig.

28     1a) in regions where the tidal distribution was not well known. Consequently errors in

29     boundary forcing and tidal propagation into the Irish Sea due to a lack of detailed

30     topography appear at offshore gauges S, T, U, V within the Eastern Irish Sea. On average

31     these errors were of the order of 25 cm in elevation and -15° in phase (Table 3a). On the

32     other hand the limited area model of the eastern Irish Sea (JD96) had very small errors at

33     these offshore gauges, since the open boundary of this model was of limited extent and ran

north-south through the Isle of Man, where the $M_2$ tidal distribution was well known at gauges S, T, U, V. The errors in amplitude increase as the water depth shallows giving errors of order 30 cm at gauges Q, R in Liverpool Bay.

At some coastal gauges for example Barrow (locations Y, Z and AA) there is significantly more spatial variability in amplitude in the present model (Table 3a) than found in the observations and in JD96. Similarly at Morecambe and Fleetwood (locations BB, CC) amplitudes on grid G3AX tend to be below observed despite the fact that in offshore regions the tidal amplitude is too large. To examine the reason for this in more detail it is necessary to consider tidal spatial variability in the region of these locations. This was accomplished, using values at nodal points close to the observation (Table 4a), where $\Delta$ is the distance between the nodal point and observation.

Consider initially Hilbre Island, close to the entrance to the Mersey estuary. At the nearest nodal point located 1.1 km away in a water depth of 9.6 m, $h_c$, $g_c$ are 321 cm and 309°, of order 30 cm too high and 10° error in phase. However, at an equivalent point in shallower water (h = 2.0 m), $h_c$, $g_c$ are 275 (cm) and 307°, of order 20 cm too low in amplitude, although with little change in phase. At two other locations namely $\Delta$ = 1.3 km and 1.8 km, similar variations in amplitude occur depending upon water depth. These results clearly show that at a near coastal location such as Hilbre there are significant variations in tidal amplitude, of order 50 cm, over short distances comparable with the element resolution of the model. In addition these variations are linked to water depth suggesting that a detailed description of nearshore topography is required.

In coastal regions of more complex topographic variation where water depths are below one metre, for example Barrow RI, the variation of tidal amplitude in locations close to the observation point are more drastic. At locations in close proximity ($\Delta$ = 1.0 km) to the observation point, there is significant "wetting and drying" over the tidal cycle, which together with bottom frictional effects, appreciably reduces tidal magnitude ($h_c$ = 164 cm) compared with observations ($h_o$ = 306 cm) and nearby points which remain wet throughout the tidal cycle ($h_c$ = 324 cm). At the majority of points there is little change in phase with change in $\Delta$. At nearby locations, Barrow HP and Barrow HS the tidal amplitude shows similar variations with water depth to that at Barrow RI with a major decrease where drying occurs (Table 4a).

At Morecambe, although drying does not occur at nearby points, the tidal amplitude at locations adjacent to the observation point, namely points $\Delta$ = 0.6 km and $\Delta$ = 1.5 km from

it show a variation in tidal amplitude from 298 cm to 322 cm, bracketing the observed value of 308 cm. This variation is large considering that water depth only varies from 3.0 m to 3.7 m at these two locations. Slightly farther away, $\Delta = 2.0$ km, amplitude decreases to 271 cm, although again there is little change in water depth. At a comparable point ($\Delta = 2.0$ km) in shallow water where drying can occur tidal amplitude is reduced to 214 cm. It is interesting that there is little phase variation with the computed value of 325° being in excellent agreement with the observed 326°.

At Fleetwood where the three nearest locations are in shallow water and drying occurs, the tidal amplitude is below the observed value while slightly farther away ($\Delta = 2.3$ km) in deeper water (h = 9.6 m) the computed value exceeds the observed. These calculations clearly show significant spatial variability in computed $M_2$ tidal amplitude in the nearshore region, over quite short distances, comparable with the finite element resolution.

The computed $M_4$ co-tidal chart (Fig. 4a), shows the $M_4$ tidal elevation amplitude increasing over the Celtic Sea region, from an input value of zero along the open boundary to a maximum of 12 cm in the southern part of the Irish Sea. A subsequent decrease in the western Irish Sea with a minimum to the west of the Isle of Man is evident in Fig. 4a. This increase through the Celtic Sea and subsequent decrease is found in observations (Table 2b) and in shelf wide numerical models (Davies 1986, Kwong et al. 1997). A rapid increase in elevation within the eastern Irish Sea as the water shallows is apparent in Fig. 4a and found in observations (Table 3b). At Douglas and Ramsay (locations C and J) the model slightly overpredicts the $M_4$ tidal amplitude (Table 3b). However this overprediction is less than that found by JD96 in a high resolution model of the eastern Irish Sea. In the nearshore region particularly around Barrow and Morecambe, the model shows a rapid increase in $M_4$ tidal amplitude that is also found in the observations (Table 3b). A more gradual change occurs in the Liverpool Bay region with computed values in good agreement with observations. These comparisons clearly show that the irregular grid has sufficient resolution to accurately represent the spatial variability of the tide in the offshore Celtic and Irish Sea regions. In addition the finer grid in the coastal region of the eastern Irish Sea can reproduce the rapid increase in the observed $M_4$ tide in this area.

To understand the reasons for the spatial variability of the $M_4$ tidal elevation in the nearshore region it is useful to examine its variation at a number of coastal gauges (Table 4b). It is evident that at Hilbre the $M_4$ tidal amplitude varies from 26 cm to 35 cm within 1.3 km of the gauge, with an associated variation of water depth from 2.0 m to 16.2 m. Also,

there is a significant variation in phase from 228° to 146°, reflecting the shorter wavelength of this component. This variation was not found in the $M_2$ tide where the wavelength was much larger.

At Barrow RI in the region where drying occurred (h less than 1.0 m) the tidal amplitude was over twice that observed, although there was good agreement in phase. However, at deeper water locations, the amplitude was in good agreement with observations, although not the phase (Table 4b). At Barrow HP at locations close to the gauge where drying did not occur amplitude and phase were in good agreement with observations. As previously when drying occurred the amplitude was overestimated. Similarly at Barrow HS, amplitude agreed well with observations, although phase was underpredicted. At Morecambe and Fleetwood there was significant spatial variability within 2 km of the tide gauge with amplitude increasing rapidly as the water shallowed.

The computed $M_6$ co-tidal chart, shows (Fig. 4b) a rapid increase in tidal amplitude in shallow water regions due to an increase in bottom friction. In the eastern Irish Sea there is a gradual increase in the Liverpool Bay region as the water shallows. However, farther north off the coast of Cumbria the $M_6$ tidal amplitude rapidly increases as the coast is approached. These spatial changes in $M_6$ reflect the spatial variability in the bottom topography in these regions (Fig. 1b). The spatial variability of computed $M_6$ tidal elevation amplitude in the eastern Irish Sea is also found in the observations (Table 3c).

It is evident from Table 3c, that at a number of shallow water locations the model tends to overpredict the value of the $M_6$ tidal elevation amplitude. To understand the reason for this it is useful to examine its spatial variability (Table 4c). As found previously for the $M_4$ component at Hilbre, the $M_6$ tidal amplitude increases rapidly as the water shallows giving rise to a major overprediction of amplitude, although a reasonable phase value. In deeper water the amplitude is still overpredicted with a large phase error. However, at Barrow RI in drying regions, the amplitude is only slightly overpredicted, whereas in deeper water the error increases, with a rapid change in phase over distances of the order of 1 km. Similar rapid changes in phase occur at Barrow HP, again with amplitude decreasing in shallow water. At Barrow HS the water depth in the region of the gauge varies from 5.4 m to 8.0 m with no drying area. Consequently amplitude and phase change gradually over the region (Table 4c).

1    At Morecambe and Fleetwood the $M_6$ tidal amplitude is overpredicted at all locations

2    in the region of the gauge. Also, the phase changes rapidly over short distances. This

3    suggests that the spatial variability of the $M_6$ tide is under-resolved in the model.

4        The computed $M_2$ tidal ellipse distribution over the whole region (not presented)

5    shows near circular ellipses at the southern end of the Celtic Sea, changing to a rectilinear

6    flow aligned with the coastline in St. George's Channel. Away from the coastal boundary, at

7    the entrance to the Bristol Channel there is a region of near circular current ellipses. A

8    rectilinear flow aligned with the channel axis is evident in the North Channel. Within the

9    central Irish Sea to the west of the Isle of Man, a region of near zero tidal currents is

10   apparent. This distribution of tidal currents is in good agreement with measurements (Table

11   5a, also see JD05) and results from a uniform grid finite difference model of the west coast

12   of Britain (DJ92). Although the tidal currents in the region exhibit some vertical variation,

13   this is primarily due to frictional effects in the near bed region (JD96). Consequently

14   currents above this region (i.e. those in Tables 5a-c and 6) are not appreciably different from

15   the depth mean derived by averaging observed current in the vertical. The $M_2$ tidal current

16   ellipses within the eastern Irish Sea are aligned in a west-east direction, with an area of

17   reduced current strength to the east of the Isle of Man (Fig. 5a(i)). The distribution of $M_2$

18   current ellipses in the eastern Irish Sea is in good agreement with a limited area high

19   resolution finite difference model of the region given by JD96 and observations (Table 5a).

20   The enhanced mesh resolution that can be achieved with the finite element model in the

21   eastern Irish Sea is evident in the distribution of current ellipses in the expanded plot of this

22   region shown in Fig. 5a(i). The enhanced resolution in the nearshore and estuarine regions

23   (Fig. 5a(i)) which is possible with the finite element mesh gives significantly more detail

24   than that presented in JD96. For example in the Solway (Fig. 5a(i)) the rapid decrease in

25   current magnitude and change of orientation of the ellipse is resolved significantly better

26   than in JD96. Similarly the higher resolution in Morecambe Bay enables the spatial

27   variability in magnitude and orientation of the $M_2$ current ellipses to be resolved to a greater

28   accuracy than in JD96. A significant improvement in resolution in the Liverpool Bay region

29   (Fig. 5a(ii)) above that used by JD96, enables the rapid increase in magnitude and orientation

30   of tidal currents in the Mersey entrance to be resolved in the model. In JD96 the Liverpool

31   Bay region was significantly under-resolved even with a 1 km horizontal grid. In terms of

32   recent measurements at a coastal observatory in the region (Proctor and Howarth 2003) high

33   accuracy in this region is essential to understand the dynamic interactions between Liverpool

34   Bay and the Mersey River.

The spatial distribution of the major and minor axis of the $M_4$ tidal current ellipse over the whole region (not presented) shows that in deep water regions these tidal currents are small. However, in coastal regions particularly in the vicinity of headlands, the non-linear advective terms lead to a significant generation of $M_4$ currents. The rapid increase in $M_4$ tidal current magnitude and enhanced spatial variability in the near coastal region of the eastern Irish Sea is clearly evident in Fig. 5b(i). This spatial variability in amplitude of $M_4$ tidal currents and ellipse orientation is comparable to that found by JD96 in the region and in good agreement with observations (Table 5b). Since the $M_4$ generation is primarily in the nearshore and estuarine regions then the enhanced resolution which is possible in these areas with the finite element grid is more important for $M_4$ than $M_2$.

An expanded plot of the Solway region (not presented) clearly shows an increase in $M_4$ currents in the region where the orientation of the estuary changes from north-south to west-east. In this area the $M_2$ tidal currents are strong and there is a rapid change in their orientation which through the non-linear momentum advection term leads to a local maximum in the $M_4$ ellipse. To the south of this region and near the head of the estuary where $M_2$ tidal currents are weaker and do not change direction, the $M_4$ tidal currents are reduced.

In Morecambe Bay it is evident (Fig. 5a(i)) that the $M_2$ tidal currents increase towards the head of the Bay, and then rapidly decrease due to bottom friction in the shallow near coastal regions at the top of the bay. Associated with this variation in tidal current magnitude is a change in orientation of the current ellipse, which gives rise to the $M_4$ current distribution shown in Fig. 5b(i). The level of detail presented here is a significant improvement on that given in JD96. Similarly in Liverpool Bay (Fig. 5b(ii)), the enhanced resolution in the near coast region which is present in the finite element mesh enables the near coastal generation of the $M_4$ tide to be accurately resolved, whereas in JD96, the Liverpool Bay region adjacent to the Mersey River was barely resolved.

The spatial distribution of $M_6$ current ellipses over the whole domain (not presented) clearly shows that these currents are negligible in deep water offshore regions, and only become important close to the coast. The plot of $M_6$ tidal currents in the eastern Irish Sea shows (Fig. 5c(i)) significant spatial variability in current magnitude and orientation in this region comparable to that found by JD96, and in reasonable agreement with observations in the region (Table 5c). The higher resolution made possible by the use of a finer mesh (compared to JD96) in the near coastal region allows nearshore details to be resolved to a greater degree than previously. The significant improvement compared to JD96 in nearshore

resolution resulting from using the finite element mesh is clearly evident in the Solway estuary and Morecambe Bay (Fig. 5c(i)). This figure shows that as for the $M_4$ tide, the $M_6$ tidal current is also a maximum in shallow water. However, the region of maximum $M_6$ tidal currents does not correspond to that found for the $M_4$ tide. This is because bottom frictional effects are a major source of the $M_6$ component of the tide, rather than the non-linear momentum term. Consequently the $M_6$ tidal current increases as water depth decreases and $M_2$ tidal velocity increases giving rise to enhanced bottom friction. This explains the increase in $M_6$ currents as water depth decreases, with a subsequent decrease as $M_2$ tidal currents are reduced in very shallow water. Although the calculations of JD96 indicated this behaviour they could not resolve $M_6$ tidal currents to the level of detail shown in Figs. 5c(i). Similarly in the Liverpool Bay/Mersey region the rapid increase in $M_6$ tidal currents at the mouth of the river Mersey (Fig. 5c(ii)) could not be resolved by JD96.

It is apparent from Table 5a (Calc 1), that at some locations, for example Rig Nos. 15, 16 and 23, the computed $M_2$ tidal current amplitude is significantly overestimated. The reason for this can be understood in terms of the significant spatial variability of $M_2$ currents in the vicinity of these observations (Table 6). For example at Rig 15, $A_c$ changes from 44 cm s$^{-1}$ to 30 cm s$^{-1}$ (in essence bracketing the observed value of 37 cm s$^{-1}$) as $\Delta$ varies from 2.2 km to 4.0 km (Table 6). Similarly at Rig 16, as $\Delta$ increases from 1.0 km to 3.7 km, $A_c$ decreases from 40 cm s$^{-1}$ to 37 cm s$^{-1}$ (a value in good agreement with observations). A significantly larger change occurs at Rig 23, with $A_c$ decreasing from 68 cm s$^{-1}$ to 41 cm s$^{-1}$ as $\Delta$ changes from 0.9 km to 2.6 km.

The computed $M_4$ tidal currents are appreciable (exceed 10 cm s$^{-1}$) at locations 18, 24 and 32 (Table 5b). A similar increase in $M_4$ tidal currents in these regions was found in JD96. This suggests that despite differences in grid size and approach (finite element, compared with finite difference JD96) the offshore variability of $M_4$ tidal currents is being reproduced in both finite element and finite difference models. To examine to what extent local spatial variability influences the $M_4$ tidal current amplitude, both a deep water (location 6) and a nearshore (location 18) point are examined (Table 6). It is evident that at Posn. 6, in deep water (h = 53 m) there is only limited spatial variability in the magnitude ($A_c$) and orientation ($T_c$) of the semi-major axis. However, at Posn. 18 in shallow water $h_o$ = 3 m, $A_c$ varies from 25.2 cm s$^{-1}$ to 11.8 cm s$^{-1}$ bracketing the observed value of 21.9 cm s$^{-1}$, with $T_c$ varying from 56° to 100° ($T_o$ = 64°). As with the elevation comparisons this suggests a lack of resolution in the near coastal region.

1    The $M_6$ tidal currents (Table 5c) are on average small (of order 3 cm s$^{-1}$).  The local

2    intensification at positions 24 and 32 was also found by JD96 and DJ92.  On average values

3    given in Table 5c were larger than those found by JD96 and more comparable to the $M_2$

4    calculation of DJ92.  As pointed out in JD96 the use of a larger set of tidal constituents,

5    namely $M_2$, $S_2$, $N_2$, $K_1$, $O_1$ as in JD96 and DJ92 tended to reduce the $M_6$ current magnitude

6    and probably explains its overprediction in the present calculation.

7    A detailed study (Table 6) of the spatial variability of $M_6$ currents in the Morecambe

8    Bay region from deeper water (Posn. 23, $h_o = 20$ m) (Fig. 1b) to nearshore Posns. 24 and 32,

9    showed significant spatial variability in the nearshore region for example at Posn. 24 as $\Delta$

10   changed from 1.3 km to 1.5 km, $A_c$ decreased from 9.1 cm s$^{-1}$ to 3.6 cm s$^{-1}$, although the

11   ellipse orientation $T_c$ and water depth varied very little.  Similarly at Posn. 32, as $\Delta$ changed

12   from 1.6 km to 1.7 km, $A_c$ increased from 3.5 cm s$^{-1}$ to 14.2 cm s$^{-1}$ with $T_c$ varying from 59°

13   to 122°, associated with a reduction in water depth from 11 m to 5 m.  This again suggests

14   rapid spatial variability in the nearshore region with associated implications for model

15   element resolution and the need for detailed topography.  Results from this calculation

16   clearly show the importance of high resolution in the near coastal regions in order to

17   accurately resolve the higher harmonics of the tide.

18   *3.3.    Calculation using a zero coastal water depth (Calc 2, Grid G3A)*

19   In a subsequent calculation (Calc 2, Table 1a) the same criterion used for mesh

20   generation as in Calc 1 was applied.  However, in order to generate a fine nearshore grid (Fig

21   6, Grid G3A) a zero water depth condition was applied at the coast.  As shown by JD05 this

22   approach lead to a less accurate $M_2$ tidal solution than previously, and a significantly longer

23   run time than in Calc 1 (Table 1b) due to increased "wetting and drying" occurring in the

24   coastal region.  Although the overall solution was less accurate, in offshore regions the $M_2$

25   tide was not substantially different to that found in Calc 1.  Major differences that occurred

26   were principally confined to the near coastal regions, in particular in the eastern Irish Sea

27   and to higher harmonics of the tide.  Consequently in order to determine differences in the

28   higher harmonics between Calcs 1 and 2 we will concentrate on the eastern Irish Sea.  This

29   region has the added advantage that a high resolution finite difference solution already exists

30   (JD96) with associated observations that can be used in the comparisons.  In addition an

31   optimal graded mesh is required in this region to complement a coastal observatory.

32   It is evident from Table 3a, that $M_2$ tidal amplitudes computed with grid G3A (Calc

33   2) are on average lower than those derived with grid G3AX (Calc 1) reflecting increased

energy loss in the shallow water regions that are resolved on this grid. This not only affects coastal gauges, but offshore locations Q, R, S, T, U, V within the eastern Irish Sea. Although the amplitude changes significantly, there is a minor phase change, of order one or two degrees (Table 3a).

As in Calc 1, there is an appreciable change in amplitude, although not phase of the $M_2$ tide in shallow water locations such as Hilbre, Barrow, Morecambe and Fleetwood associated with small changes in $\Delta$ (Table 4a). Since grid G3A resolves the near coastal region, at a number of locations water depths are below 1 m, and extensive "wetting and drying" occurs with an associated decrease in tidal amplitude (Table 4a).

At offshore locations Q, R, S, T, U, V the $M_4$ tidal amplitude is reduced slightly from that found previously, with a 10° reduction in phase (compare Calcs 1 and 2) in Table 3b. For tidal amplitude this improves the agreement with observations, particularly at Q and R, although not the phase. At coastal locations the $M_4$ tidal amplitude is in general significantly overpredicted as a result of "wetting and drying" occurring at the nearest nodal point to the gauge due to the increased resolution in the nearshore region. The effect of "wetting and drying" upon $M_4$ tidal amplitude, and how this influences spatial variability is clearly evident in Table 4b. For example at Hilbre, $M_4$ amplitude changes from 17 cm to 22 cm as $\Delta$ increases from 0.6 km to 1.0 km, bracketing the observed value of 20 cm. With $\Delta = 1.2$ km, water depth drops below 1 m, and "wetting and drying" occurs with an associated increase in amplitude to 66 cm (Table 4b). Similarly at Barrow RI, HP and HS in shallow wet/dry regions $M_4$ amplitude exceeds 60 cm, whereas in water depths of order 10 m, its value is about 14 cm. This value is considerably lower than the observed and computed (Calc 1) values of order 30 cm (28 cm computed) (Barrow RI) and 26 cm (28 cm computed) (Barrow HP) found in Calc 1 in comparable water depths. This suggests, as found for the $M_2$ tide, that the inclusion of extensive wetting/drying regions has removed significant tidal energy. A similar picture emerges at Morecambe and Fleetwood with large $M_4$ tidal amplitudes in wet/dry regions and underpredicted values in deeper water. As previously (Calc 1) in non-wet/dry regions the $M_6$ amplitude is larger than observed (Table 3c) with on average a significant reduction in areas where drying occurs. The effect of the finer grid is to introduce appreciably larger spatial variability over shorter distances than found in Calc 1 (Table 4c).

The "wetting/drying" algorithm used here has been specifically chosen to be consistent with that used by JD96 in their finite element calculations. However a range of

methods exists in the literature (see the review of Balzano 1998). Naturally different approaches combined with different methods of choosing mesh refinement in near coastal regions will affect the solution in these regions. Such a detailed study is however outside the scope of this paper.

The main features of the distribution of $M_2$ tidal current ellipses in the eastern Irish Sea (not shown) are comparable to those found previously (Fig. 5a(i)). However, it is evident that in the near coastal region the use of a zero water depth condition gives rise to increased resolution close to the coast. In this region the water is shallow, and the resulting increase in bottom friction leads to a reduced $M_2$ tidal current at nodes adjacent to a coastal boundary (not shown). This effect could be clearly seen in a comparison of expanded plots of the Solway Firth, Morecambe Bay (not presented) and Liverpool Bay/Mersey River (Fig. 7a) compared with $M_2$ tidal ellipses computed with Grid G3AX. However, away from the nearshore region there were no substantial differences in the $M_2$ tidal currents, as is evident from Table 4a.

Similarly for the $M_4$ component of the tide, differences between solutions using Grids G3AX and G3A (Calcs 1 and 2) were only evident in near coastal regions. From a comparison of solutions in the Solway (not shown) and Morecambe Bay (not shown) it is evident that in near coastal areas $M_4$ tidal currents computed with grid G3AX (non-zero water depth at the coast) were slightly larger than those computed with grid G3A (zero water depth at the coast). This arises because in G3A, the nearshore $M_2$ tidal current has been reduced and hence the generation of an $M_4$ current is also reduced.

This effect is clearly evident in the Liverpool Bay region, where the nearshore $M_4$ current ellipses computed with mesh G3A (Fig. 7b) in the near coastal region (e.g. along the Welsh coast to the west of the Dee estuary) are appreciably smaller than those computed with mesh G3AX (Fig. 5b(ii)). Also slightly farther offshore (e.g. at 53.4°, -3.35°W) there are some differences in current magnitudes and ellipse orientation. This suggests that for the $M_4$ tide the influence of coastal water depth is not just confined to nodal points in the nearshore region, but does spread some way offshore.

As previously the $M_6$ tidal current is mainly affected by differences in the G3AX and G3A mesh in the near coast region. This is clearly evident in the upper reaches of the Solway (not shown) where the region of $M_6$ currents exceeding 25 cm s$^{-1}$ has been reduced compared to that computed with grid G3AX, due to a decrease in $M_2$ tidal currents through friction effects. However farther south in a deeper central channel at about 54.9°N, -3.5°W,

1 and in the deeper water regions to the south of this, there is a slight increase in $M_6$ current

2 between grid G3A and G3AX. The reason for this is not obvious, but is in part related to the

3 fact that changes in grid representation in the neashore region as in the $M_4$ case do influence

4 nodal solutions farther offshore.

5      The increase in $M_6$ currents within the Moricambe Bay (approximate location

6 54.9°N, -3.3°W) region of the Solway (not shown) compared to that computed with grid

7 G3AX, reflects the increase in "wetting and drying" in this region due to the shallower near

8 coast water depths in G3A compared to G3AX . The harmonic analysis of a time series in

9 which "step like transitions" have occurred due to "wetting and drying" gives rise to a

10 "Gibbs type" phenomenon (Hall and Davies 2004) which leads to an increase in amplitude

11 of the higher harmonics.

12      Differences in $M_6$ tidal currents are also evident in Morecambe Bay (not shown),

13 particularly in the shallow northern part where the reduction in near coastal water depths in

14 grid G3A, leads to a reduction in $M_6$ tidal currents compared to G3AX. However, as in the

15 Solway, in some shallow embayments (e.g. at location -54°N, -2.9°W), $M_6$ currents

16 computed with grid G3A are larger than G3AX due to wetting and drying.

17      Differences in the near coastal and adjacent offshore $M_6$ currents in the Liverpool

18 Bay region between grids G3A and G3AX are evident from a comparison between Figs 7c

19 and 5c(ii). This is consistent with the differences in the $M_4$ solutions and confirms that the

20 influence of changes in representing coastal water depths affects the nearshore distribution of

21 the higher harmonics.

22      The influence of differences in grid G3A compared with grid G3AX upon spatial

23 variability of $M_2$ currents in the region of offshore current meters (Table 6) for example Rigs

24 15 and 16, is small, and less than the differences found between nearest nodes, namely

25 within the model's resolution. At shallower near coastal locations, for example Rig 23,

26 (Table 6) at the nearest point to the observation ($\Delta = 0.7$ km) the computed value using G3A

27 is in good agreement with the observed and significantly more accurate than found with

28 G3AX. Similarly at $\Delta = 2.5$ km the magnitude although not the orientation is in better

29 agreement on grid G3A than G3AX. This suggests that for the $M_2$ tidal currents the

30 enhanced resolution in the near coastal region due to using G3A improves the solution.

31      For the $M_4$ tidal current in deeper water, namely location 6 (Table 6), the improved

32 grid resolution using G3A leads to enhanced accuracy. In shallow regions namely Rig 18

1 the solution shows similar spatial variability to that found with grid G3AX, suggesting that

2 the grid is still not sufficiently fine to resolve small scale variations in $M_4$ tidal currents.

3      For $M_6$, as for the elevation, the current amplitude is on average significantly higher

4 than observed (Table 6) and shows appreciable nearshore variability as indicated at current

5 meters at the entrance to Morecambe Bay (Rig 23) and within the Bay (Rigs 24 and 32).

6 The spatial variability in this region using grid G3A is comparable to that using G3AX and

7 suggests that neither has sufficient resolution to represent the small scale variability of $M_6$ in

8 this region.

9 4.      CONCLUDING REMARKS

10      A west coast unstructured grid model covering the same region as the finite

11 difference model of DJ92 with enhanced resolution in the eastern Irish Sea, comparable to

12 that of JD96, has been used to examine the sensitivity of the $M_2$, $M_4$ and $M_6$ tides to

13 variations in the finite element mesh.  Comparisons with observational data particularly in

14 the eastern Irish Sea and with results from JD96 using a uniform mesh eastern Irish Sea

15 model have been made.

16      Initial results using grid G3AX in which the nearshore water depth was taken as non-

17 zero, showed that an accurate distribution of the $M_2$, $M_4$, and $M_6$ tide could be obtained

18 throughout the region.  By refining the mesh within the eastern Irish Sea, a solution

19 comparable to the limited area high resolution model of JD96 could be obtained even though

20 a coarse mesh was used outside this region in deeper water.  Despite the coarser nature of the

21 mesh in the deeper Celtic Sea region the model could reproduce the observed spatial

22 variability of the tide in this region.

23      Calculations using grid G3A, where water depths were zero at the coast, showed that

24 in deeper water the tide changed very little, although in coastal regions there were some

25 slight modifications particularly to the distribution of the higher harmonics.  However as

26 shown by JD05 on average for the $M_2$ tide the solution was not as accurate as using grid

27 G3AX and the computational effort was significantly larger.  For this reason a non-zero

28 coastal water depth was the optimum choice when computing the fundamental and higher

29 harmonics.  The calculations presented here using grid G3AX are presently being extended

30 to the simulation of storm surges and comparison with corresponding finite difference

31 solutions.  Results of this intercomparison will be reported in due course.

32

1        The origin of the TELEMAC SYSTEM is EDF-LNHE and is therefore © EDF-

2    LNHE.  Thanks are also due to Mrs L. Parry for typing the paper.

3

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1    Captions for Figures

2    Fig. 1:    (a) Topography and place names over the whole region, (b) expanded plot of

3              topography in eastern Irish Sea and local place names.

4    Fig. 2:    Locations of eastern Irish Sea gauges (with sample of finite difference grid of

5              JD96) used in the comparisons.

6    Fig. 3:    Finite element Grid 1 (G3AX) used in Calc. 1.

7    Fig. 4:    Computed (Calc. 1, Grid G3AX) (a) $M_4$ (i) whole region, (ii) Eastern Irish Sea,

8              (b) $M_6$ (i) whole region, (ii) Eastern Irish Sea, interpolated from nodal values,

9              with co-amplitude lines in cms.

10   Fig 5:     Distribution of the major and minor axis of (a) $M_2$, (b) $M_4$, (c) $M_6$ current ellipse

11             for (i) the eastern Irish Sea, (ii) Liverpool Bay (Calc. 1, Grid G3AX).

12   Fig. 6:    Finite element Grid G3A (zero water depth at the coast) as used in Calc. 2.

13   Fig. 7:    Distribution of major and minor axis of (a) $M_2$, (b) $M_4$, (c) $M_6$ current ellipse in

14             Liverpool Bay, derived from Calc 2 (Grid G3A).

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1    Table 1a:  Summary of Calculations

| Calc | Nature of Grid |
|---|---|
| 1 | Grid G3AX, element size determined by $(gh)^{\frac{1}{2}}$, with non-zero water depth at coast. |
| 2 | As Calc 1, but element size determined by $(gh)^{\frac{1}{2}}$, with zero water depth at coast.  Grid G3A. |

2
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4    Table 1b:  Summary of grid, nodes, run times and main features of each calculation

| Calc | Grid | Nodes | Run time | Notes |
|---|---|---|---|---|
| 1 | G3AX | 6842 | 1h 36m | h non-zero at coast. |
| 2 | G3A | 11702 | 10h 02m | h zero at coast. |

5
6

1    Table 2a:  Latitude and longitude of tide gauges used in comparison tables

| Point | Port | Latitude (N) | Longitude(W) |
|---|---|---|---|
| Eastern Irish Sea: | | | |
| A | Barrow | 54.0944 | 3.2236 |
| B | Birkenhead | 53.4000 | 3.0167 |
| C | Douglas | 54.1461 | 4.4667 |
| D | Heysham | 54.0333 | 2.9167 |
| E | Hilbre | 53.3833 | 3.2167 |
| F | Liverpool | 53.4083 | 2.9983 |
| G | Formby | 53.5667 | 3.1167 |
| H | Hestan | 54.8333 | 3.8000 |
| I | Liverpool Bay | 53.4833 | 3.2500 |
| J | Ramsay | 54.3167 | 4.3667 |
| K | Workington | 54.6500 | 3.5667 |
| L | Wylfa Head | 53.4167 | 4.4667 |
| M | Liverpool (GD) | 53.4494 | 3.0167 |
| N | Llandudno | 53.3315 | 3.8237 |
| O | New Brighton | 53.4500 | 3.0333 |
| P | Amlwch | 53.4167 | 4.3333 |
| Q | Ostg | 53.5000 | 3.2167 |
| R | Queens Channel | 53.5167 | 3.2000 |
| S | STD Irish Sea | 53.7667 | 4.1167 |
| T | STN10 | 53.7667 | 3.7167 |
| U | STN34 | 54.1500 | 3.6667 |
| V | STN35 | 54.6500 | 3.9167 |
| W | Creetown | 54.8667 | 4.4000 |
| X | Conwy | 53.2833 | 3.8333 |
| Y | Barrow RI | 54.0833 | 3.1667 |
| Z | Barrow HP | 54.0667 | 3.1667 |
| AA | Barrow HS | 54.0167 | 3.1833 |
| BB | Morecambe | 54.0833 | 2.8833 |
| CC | Fleetwood | 53.9236 | 3.0056 |
| Offshore Gauges External To Eastern  Irish Sea: | | | |
|  | G | 53.4333 | 5.3667 |
|  | 33J | 52.0667 | 5.7833 |
|  | B78 | 51.7500 | 6.6000 |
|  | C78 | 51.3333 | 6.5000 |
|  | F78 | 50.5500 | 7.5333 |
|  | F80 | 50.5333 | 7.6167 |
|  | D78 | 50.5833 | 6.1667 |
|  | G78 | 49.6000 | 8.6167 |
|  | G80 | 49.6667 | 8.5333 |

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3 Table 2a: Latitude and longitude of current meters used in comparison tables

| Point | Latitude (N) | Longitude(W) |
|---|---|---|
| 2 | 54.6208 | 4.4750 |
| 3 | 54.5792 | 4.3583 |
| 6 | 54.5375 | 4.3417 |
| 8 | 54.5042 | 4.4083 |
| 9 | 54.4042 | 3.5583 |
| 12 | 54.2375 | 4.1917 |
| 15 | 54.2208 | 3.8750 |
| 16 | 54.1042 | 3.5583 |
| 18 | 54.0375 | 2.9417 |
| 23 | 54.0208 | 3.3417 |
| 24 | 54.0208 | 2.9917 |
| 32 | 54.0042 | 2.9583 |
| 33 | 53.9042 | 4.4083 |
| 34 | 53.8875 | 3.5083 |
| 35 | 53.8708 | 3.5250 |
| 38 | 53.7708 | 3.9250 |
| 44 | 53.7542 | 4.1250 |
| 50 | 53.6542 | 4.3750 |
| 56 | 53.5875 | 4.0917 |
| 57 | 53.5375 | 3.5583 |
| 58 | 53.4875 | 3.3083 |
| 59 | 53.4708 | 3.4917 |
| 60 | 53.4042 | 3.9417 |
| 61 | 53.3875 | 3.7583 |

4
5 Table 2b: Comparison of observed amplitude ($h_o$) (cms) and phase ($g_o$) (deg) and computed
6 $h_c$, $g_c$ from Calc 1, at offshore gauges

| Gauge | $M_2$ | | | | $M_4$ | | | |
|---|---|---|---|---|---|---|---|---|
| | Obs | | Calc | | Obs | | Calc | |
| | $h_o$ | $g_o$ | $h_c$ | $g_c$ | $h_o$ | $g_o$ | $h_c$ | $g_c$ |
| STN 10 | 262 | 318 | 287 | 306 | 16.0 | 199 | 13.5 | 145 |
| 34 | 263 | 324 | 284 | 312 | 11.0 | 217 | 13.8 | 151 |
| 35 | 255 | 332 | 276 | 319 | 11.0 | 248 | 11.9 | 162 |
| G | 138 | 309 | 156 | 295 | 7.2 | 54 | 5.8 | 350 |
| 33J | 111 | 183 | 97 | 180 | 3.8 | 19 | 5.6 | 287 |
| B78 | 143 | 154 | 128 | 148 | 2.5 | 210 | 1.7 | 141 |
| C78 | 164 | 151 | 152 | 149 | 3.5 | 223 | 3.2 | 138 |
| F78 | 156 | 136 | 156 | 132 | 5.2 | 219 | 3.2 | 116 |
| F80 | 151 | 136 | 153 | 132 | 4.4 | 218 | 3.1 | 116 |
| D78 | 189 | 142 | 183 | 139 | 5.5 | 220 | 4.6 | 122 |
| G78 | 138 | 121 | 142 | 120 | 2.9 | 231 | 0.9 | 105 |
| G80 | 136 | 123 | 144 | 121 | 2.9 | 232 | 1.1 | 103 |

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8

1 Table 3a: Comparison of observed amplitude (cm) and phase (degrees) ($h_o$, $g_o$) and
2 computed ($h_c$, $g_c$) in the eastern Irish Sea (JD96) for the $M_2$ tidal elevation
3

| Point | Port | Observed | | Calc 1 | | Calc 2 | |
|---|---|---|---|---|---|---|---|
| | | | | G3AX | | G3A | |
| | | $h_o$ | $g_o$ | $h_c$ | $g_c$ | $h_c$ | $g_c$ |
| A | Barrow | 308 | 331 | 327 | 327 | 152 | 327 |
| B | Birkenhead | 311 | 323 | 323 | 312 | 301 | 310 |
| C | Douglas | 230 | 326 | 246 | 313 | 237 | 313 |
| D | Heysham | 315 | 325 | 329 | 316 | 319 | 314 |
| E | Hilbre | 292 | 317 | 321 | 308 | 313 | 305 |
| F | Liverpool | 312 | 323 | 322 | 313 | 296 | 312 |
| G | Formby | 312 | 315 | 316 | 307 | 305 | 305 |
| H | Hestan | 275 | 339 | 287 | 321 | 277 | 321 |
| I | Liverpool Bay | 262 | 315 | 321 | 305 | 302 | 305 |
| J | Ramsay | 262 | 328 | 261 | 315 | 251 | 314 |
| K | Workington | 273 | 332 | 293 | 318 | 280 | 317 |
| L | Wylfa Head | 206 | 300 | 235 | 290 | 221 | 290 |
| M | Liverpool (GD) | 307 | 321 | 323 | 311 | 303 | 309 |
| N | Llandudno | 267 | 308 | 293 | 300 | 280 | 299 |
| O | New Brighton | 306 | 318 | 323 | 311 | 307 | 309 |
| P | Amlwch | 235 | 305 | 256 | 294 | 244 | 294 |
| Q | Ostg | 290 | 315 | 322 | 306 | 304 | 304 |
| R | Queens Channel | 296 | 316 | 322 | 306 | 303 | 305 |
| S | STD Irish Sea | 235 | 317 | 263 | 305 | 246 | 304 |
| T | STN10 | 262 | 318 | 290 | 306 | 275 | 305 |
| U | STN34 | 263 | 324 | 287 | 312 | 271 | 311 |
| V | STN35 | 255 | 332 | 279 | 318 | 266 | 317 |
| W | Creetown | 233 | 342 | 265 | 326 | 257 | 331 |
| X | Conwy | 241 | 318 | 224 | 299 | 136 | 299 |
| Y | Barrow RI | 306 | 329 | 163 | 321 | 312 | 319 |
| Z | Barrow HP | 292 | 327 | 324 | 321 | 312 | 319 |
| AA | Barrow HS | 297 | 325 | 317 | 315 | 300 | 311 |
| BB | Morecambe | 308 | 326 | 298 | 325 | 267 | 322 |
| CC | Fleetwood | 305 | 326 | 202 | 323 | 316 | 315 |

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Table 3b: Comparison of observed amplitude (cm) and phase (degrees) ($h_o$, $g_o$) and computed ($h_c$, $g_c$) in the eastern Irish Sea (JD96) for the $M_4$ tidal elevation

| Point | Port | Observed | | Calc 1 G3AX | | Calc 2 G3A | |
|---|---|---|---|---|---|---|---|
| | | $h_o$ | $g_o$ | $h_c$ | $g_c$ | $h_c$ | $g_c$ |
| A | Barrow | 19 | 252 | 38 | 203 | 61 | 283 |
| B | Birkenhead | 23 | 217 | 27 | 150 | 56 | 252 |
| C | Douglas | 6 | 233 | 7 | 170 | 49 | 262 |
| D | Heysham | 20 | 243 | 9 | 216 | 68 | 269 |
| E | Hilbre | 20 | 203 | 28 | 160 | 17 | 128 |
| F | Liverpool | 23 | 214 | 30 | 150 | 53 | 254 |
| G | Formby | 25 | 235 | 18 | 158 | 58 | 250 |
| H | Hestan | 12 | 280 | 12 | 167 | 11 | 152 |
| I | Liverpool Bay | 21 | 196 | 20 | 142 | 17 | 132 |
| J | Ramsay | 7 | 237 | 10 | 154 | 51 | 263 |
| K | Workington | 13 | 253 | 13 | 166 | 58 | 270 |
| L | Wylfa Head | 4 | 182 | 2 | 195 | 47 | 218 |
| M | Liverpool (GD) | 22 | 202 | 33 | 152 | 56 | 248 |
| N | Llandudno | 12 | 181 | 13 | 130 | 56 | 233 |
| O | New Brighton | 23 | 198 | 32 | 154 | 27 | 142 |
| P | Amlwch | 6 | 185 | 6 | 143 | 50 | 225 |
| Q | Ostg | 17 | 196 | 22 | 144 | 16 | 133 |
| R | Queens Channel | 17 | 197 | 22 | 147 | 18 | 136 |
| S | STD Irish Sea | 6 | 201 | 8 | 142 | 7 | 132 |
| T | STN10 | 16 | 199 | 13 | 143 | 12 | 132 |
| U | STN34 | 11 | 217 | 13 | 149 | 12 | 136 |
| V | STN35 | 11 | 248 | 11 | 160 | 11 | 140 |
| W | Creetown | 30 | 274 | 27 | 206 | 30 | 208 |
| X | Conwy | 26 | 216 | 40 | 223 | 54 | 231 |
| Y | Barrow RI | 30 | 274 | 70 | 281 | 62 | 274 |
| z | Barrow HP | 26 | 216 | 28 | 195 | 14 | 152 |
| AA | Barrow HS | 16 | 200 | 16 | 176 | 12 | 147 |
| BB | Morecambe | 11 | 217 | 33 | 246 | 34 | 239 |
| CC | Fleetwood | 11 | 248 | 68 | 254 | 62 | 249 |

1   Table 3c:  Comparison of observed amplitude (cm) and phase (degrees) ($h_o$, $g_o$) and
2   computed ($h_c$, $g_c$) in the eastern Irish Sea (JD96) for the $M_6$ tidal elevation
3

| Point | Port | Observed | | Calc 1 | | Calc 2 | |
|---|---|---|---|---|---|---|---|
| | | | | G3AX | | G3A | |
| | | $h_o$ | $g_o$ | $h_c$ | $g_c$ | $h_c$ | $g_c$ |
| A | Barrow | 3 | 49 | 14 | 35 | 14 | 176 |
| B | Birkenhead | 5 | 321 | 16 | 323 | 9 | 43 |
| C | Douglas | 1 | 354 | 1 | 67 | 1 | 55 |
| D | Heysham | 2 | 11 | 5 | 315 | 2 | 25 |
| E | Hilbre | 2 | 33 | 7 | 332 | 6 | 339 |
| F | Liverpool | 5 | 322 | 15 | 325 | 11 | 53 |
| G | Formby | 5 | 11 | 7 | 359 | 8 | 98 |
| H | Hestan | - | - | 1 | 72 | 3 | 250 |
| I | Liverpool Bay | - | - | 6 | 311 | 6 | 341 |
| J | Ramsay | - | - | 1 | 101 | 3 | 100 |
| K | Workington | 2 | 325 | 1 | 126 | 4 | 148 |
| L | Wylfa Head | - | - | 0 | 178 | 2 | 152 |
| M | Liverpool (GD) | 5 | 349 | 14 | 324 | 9 | 50 |
| N | Llandudno | 2 | 356 | 3 | 289 | 4 | 30 |
| O | New Brighton | 5 | 329 | 14 | 324 | 11 | 339 |
| P | Amlwch | - | - | 1 | 271 | 2 | 52 |
| Q | Ostg | 4 | 14 | 6 | 318 | 6 | 337 |
| R | Queens Channel | 3 | 18 | 7 | 321 | 7 | 347 |
| S | STD Irish Sea | 1 | 354 | 1 | 296 | 2 | 322 |
| T | STN10 | 3 | 335 | 3 | 306 | 4 | 328 |
| U | STN34 | 1 | 7 | 2 | 315 | 2 | 305 |
| V | STN35 | 1 | 234 | 1 | 93 | 2 | 235 |
| W | Creetown | 5 | 117 | 13 | 65 | 4 | 123 |
| X | Conwy | 6 | 22 | 22 | 8 | 11 | 68 |
| Y | Barrow RI | 5 | 117 | 6 | 164 | 1 | 13 |
| Z | Barrow HP | 6 | 22 | 14 | 0 | 11 | 327 |
| AA | Barrow HS | 3 | 355 | 6 | 315 | 6 | 318 |
| BB | Morecambe | 1 | 7 | 16 | 15 | 16 | 5 |
| CC | Fleetwood | 1 | 234 | 8 | 108 | 9 | 323 |

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1  Table 4a:  Spatial variability of computed $M_2$ tidal elevation $h_c$(m) and phase $g_c$ (degrees)
2  with distance $\Delta$(km) between nearest nodal points and observational point
3

| Port | Obs | | Calc 1 (G3AX) | | | | Calc 2 (G3A) | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $h_o$ | $g_o$ | $\Delta$ | $h_c$ | $g_c$ | h(m) | $\Delta$ | $h_c$ | $g_c$ | h(m) |
| Hilbre | 293 | 317 | 1.1 | 321 | 309 | 9.6 | 0.6 | 313 | 305 | 3.8 |
| | | | 1.1 | 275 | 307 | 2.0 | 1.0 | 304 | 306 | 17.4 |
| | | | 1.3 | 330 | 306 | 16.2 | 1.2 | 132 | 308 | 0.0 |
| | | | 1.8 | 296 | 307 | 2.3 | 1.5 | 305 | 304 | 8.6 |
| | | | | | | | | | | |
| Barrow RI | 306 | 329 | 1.0 | 164 | 321 | 0.0 | 0.9 | 155 | 317 | 0.0 |
| | | | 1.0 | 161 | 312 | 0.0 | 1.6 | 312 | 319 | 9.9 |
| | | | 1.9 | 324 | 321 | 9.1 | 2.0 | 312 | 317 | 8.1 |
| | | | 2.2 | 327 | 326 | 6.4 | 2.2 | 198 | 318 | 0.7 |
| | | | | | | | | | | |
| Barrow HP | 292 | 327 | 1.0 | 324 | 321 | 9.0 | 0.5 | 312 | 319 | 9.9 |
| | | | 1.1 | 323 | 322 | 6.0 | 1.6 | 312 | 317 | 8.1 |
| | | | 1.6 | 326 | 326 | 3.0 | 1.6 | 153 | 321 | 0.0 |
| | | | 2.0 | 164 | 321 | 0.0 | 2.6 | 198 | 318 | 0.7 |
| | | | | | | | | | | |
| Barrow HS | 297 | 325 | 0.6 | 317 | 315 | 6.2 | 0.8 | 300 | 311 | 7.1 |
| | | | 1.9 | 325 | 315 | 5.4 | 1.6 | 306 | 312 | 5.5 |
| | | | 2.1 | 319 | 311 | 8.0 | 1.7 | 302 | 312 | 4.2 |
| | | | | | | | | | | |
| Morecambe | 308 | 326 | 0.6 | 298 | 325 | 3.0 | 1.3 | 267 | 322 | 2.8 |
| | | | 1.5 | 322 | 323 | 3.7 | 1.5 | 158 | 319 | 0.0 |
| | | | 2.0 | 271 | 329 | 2.9 | 1.6 | 327 | 318 | 4.0 |
| | | | 2.0 | 214 | 326 | 0.9 | 2.0 | 231 | 329 | 1.3 |
| | | | | | | | | | | |
| Fleetwood | 305 | 326 | 0.3 | 202 | 323 | 0.9 | 0.2 | 158 | 302 | 0.0 |
| | | | 1.0 | 221 | 319 | 1.0 | 1.3 | 136 | 319 | 0.0 |
| | | | 1.0 | 260 | 311 | 1.5 | 1.8 | 316 | 315 | 8.2 |
| | | | 2.3 | 331 | 315 | 9.6 | 1.9 | 130 | 329 | 0.0 |
| | | | | | | | | | | |

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Table 4b: Spatial variability of computed $M_4$ tidal elevation $h_c$(m) and phase $g_c$ (degrees) with distance $\Delta$(km) between nearest nodal points and observational point

| Port | Obs | | Calc 1 (G3AX) | | | | Calc 2 (G3A) | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $h_o$ | $g_o$ | $\Delta$ | $h_c$ | $g_c$ | h(m) | $\Delta$ | $h_c$ | $g_c$ | h(m) |
| Hilbre | 20 | 203 | 1.1 | 28 | 160 | 9.6 | 0.6 | 17 | 129 | 3.8 |
| | | | 1.1 | 35 | 228 | 2.0 | 1.0 | 22 | 147 | 17.4 |
| | | | 1.3 | 26 | 146 | 16.2 | 1.2 | 66 | 238 | 0.0 |
| | | | 1.8 | 31 | 212 | 2.3 | 1.5 | 15 | 120 | 8.6 |
| | | | | | | | | | | |
| Barrow RI | 30 | 274 | 1.0 | 70 | 281 | 0.0 | 1.0 | 62 | 274 | 0.0 |
| | | | 1.0 | 64 | 276 | 0.0 | 1.1 | 64 | 278 | 0.0 |
| | | | 1.9 | 28 | 195 | 9.1 | 1.6 | 14 | 152 | 9.9 |
| | | | 2.2 | 33 | 203 | 6.4 | 2.0 | 14 | 170 | 8.1 |
| | | | | | | | | | | |
| Barrow HP | 26 | 216 | 1.0 | 28 | 195 | 9.1 | 0.5 | 14 | 152 | 9.9 |
| | | | 1.1 | 28 | 200 | 5.9 | 1.6 | 14 | 170 | 8.1 |
| | | | 1.6 | 27 | 205 | 3.3 | 1.6 | 62 | 282 | 0.0 |
| | | | 2.0 | 70 | 280 | 0.0 | 2.3 | 63 | 278 | 0.0 |
| | | | | | | | | | | |
| Barrow HS | 16 | 200 | 0.6 | 16 | 176 | 6.2 | 0.8 | 12 | 147 | 7.1 |
| | | | 1.9 | 15 | 173 | 5.4 | 1.6 | 9 | 148 | 5.5 |
| | | | 2.1 | 14 | 158 | 8.0 | 1.7 | 12 | 164 | 4.2 |
| | | | | | | | | | | |
| Morecambe | 11 | 217 | 0.6 | 33 | 246 | 3.0 | 1.3 | 34 | 239 | 2.8 |
| | | | 1.5 | 19 | 181 | 3.7 | 1.5 | 60 | 286 | 0.0 |
| | | | 2.0 | 30 | 249 | 2.9 | 1.6 | 8 | 142 | 4.0 |
| | | | 2.0 | 55 | 286 | 0.9 | 2.0 | 46 | 272 | 1.3 |
| | | | | | | | | | | |
| Fleetwood | 11 | 248 | 0.3 | 68 | 254 | 0.9 | 0.2 | 63 | 249 | 0.0 |
| | | | 1.0 | 63 | 256 | 1.0 | 0.5 | 56 | 273 | 0.0 |
| | | | 1.0 | 48 | 262 | 1.5 | 0.7 | 66 | 265 | 0.0 |
| | | | 2.3 | 13 | 162 | 9.6 | 1.8 | 5 | 140 | 8.2 |
| | | | | | | | | | | |

1  Table 4c  Spatial variability of computed $M_6$ tidal elevation $h_c$(m) and phase $g_c$ (degrees)
2  with distance $\Delta$(km) between nearest nodal points and observational point
3

| Port | Obs | | Calc 1 (G3AX) | | | | Calc 2 (G3A) | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $h_o$ | $g_o$ | $\Delta$ | $h_c$ | $g_c$ | h(m) | $\Delta$ | $h_c$ | $g_c$ | h(m) |
| Hilbre | 2 | 33 | 1.1 | 7 | 332 | 9.6 | 0.6 | 6 | 339 | 3.8 |
| | | | 1.1 | 24 | 30 | 2.0 | 1.0 | 9 | 347 | 17.4 |
| | | | 1.3 | 7 | 303 | 16.2 | 1.2 | 4 | 64 | 0.0 |
| | | | 1.8 | 24 | 25 | 2.3 | 1.5 | 4 | 309 | 8.6 |
| | | | | | | | | | | |
| Barrow RI | 5 | 117 | 1.0 | 6 | 164 | 0.0 | 1.0 | 1 | 12 | 0.0 |
| | | | 1.0 | 8 | 264 | 0.0 | 1.1 | 2 | 206 | 0.0 |
| | | | 1.9 | 14 | 0 | 9.1 | 1.6 | 11 | 327 | 9.9 |
| | | | 2.2 | 11 | 19 | 6.4 | 2.0 | 13 | 324 | 8.1 |
| | | | | | | | | | | |
| Barrow HP | 6 | 22 | 1.0 | 13 | 0 | 9.1 | 0.5 | 11 | 327 | 9.9 |
| | | | 1.1 | 13 | 350 | 5.9 | 1.6 | 13 | 324 | 8.1 |
| | | | 1.6 | 9 | 2 | 3.3 | 1.6 | 6 | 190 | 0.0 |
| | | | 2.0 | 6 | 164 | 0.0 | 2.3 | 2 | 206 | 0.0 |
| | | | | | | | | | | |
| Barrow HS | 3 | 335 | 0.6 | 6 | 315 | 6.2 | 0.8 | 6 | 318 | 7.1 |
| | | | 2.0 | 10 | 321 | 5.4 | 1.6 | 7 | 314 | 5.5 |
| | | | 2.1 | 7 | 298 | 8.0 | 1.7 | 8 | 317 | 4.2 |
| | | | | | | | | | | |
| Morecambe | 1 | 7 | 0.6 | 16 | 15 | 3.0 | 1.3 | 16 | 5 | 2.8 |
| | | | 1.5 | 14 | 280 | 3.7 | 1.5 | 8 | 289 | 0.0 |
| | | | 2.0 | 10 | 27 | 2.9 | 1.6 | 10 | 296 | 4.0 |
| | | | 2.0 | 9 | 105 | 0.9 | 2.0 | 8 | 111 | 1.3 |
| | | | | | | | | | | |
| Fleetwood | 1 | 234 | 0.3 | 8 | 108 | 0.9 | 0.2 | 9 | 323 | 0.0 |
| | | | 1.0 | 12 | 82 | 1.0 | 0.5 | 17 | 172 | 0.0 |
| | | | 1.0 | 24 | 27 | 1.5 | 0.7 | 1 | 274 | 0.0 |
| | | | 2.3 | 6 | 291 | 9.6 | 1.8 | 5 | 307 | 8.2 |
| | | | | | | | | | | |

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Table 5a: Comparison of observed semi-major axis $A_o$ (cm s$^{-1}$) and orientation $T_o$ (degrees) of the $M_2$ current ellipse at locations in the eastern Irish Sea used in JD96

| Rig No. | Observed | | | Calc 1 G3AX | | | Calc 2 G3A | | |
|---|---|---|---|---|---|---|---|---|---|
| | $A_o$ | $T_o$ | $h_o$ | $A_c$ | $T_c$ | $h_c$ | $A_c$ | $T_c$ | $h_c$ |
| 2 | 110 | 9 | 33 | 106 | 3 | 42 | 107 | 6 | 47 |
| 3 | 97 | 7 | 48 | 109 | 11 | 51 | 96 | 11 | 51 |
| 6 | 86 | 171 | 53 | 92 | 5 | 49 | 83 | 5 | 49 |
| 12 | 32 | 46 | 30 | 42 | 25 | 25 | 41 | 46 | 28 |
| 15 | 37 | 38 | 25 | 44 | 25 | 26 | 40 | 30 | 24 |
| 16 | 35 | 168 | 25 | 40 | 179 | 26 | 34 | 172 | 25 |
| 23 | 49 | 148 | 20 | 67 | 165 | 18 | 49 | 157 | 20 |
| 33 | 79 | 18 | 60 | 89 | 20 | 58 | 88 | 58 | 58 |
| 44 | 75 | 5 | 46 | 86 | 5 | 45 | 80 | 5 | 45 |
| 50 | 89 | 7 | 57 | 101 | 7 | 54 | 101 | 8 | 56 |
| 56 | 79 | 175 | 46 | 84 | 176 | 46 | 87 | 176 | 47 |
| 60 | 58 | 169 | 28 | 63 | 169 | 25 | 56 | 168 | 24 |
| 61 | 48 | 166 | 20 | 54 | 163 | 19 | 54 | 160 | 15 |

Table 5b: Comparison of observed semi-major axis $A_o$ (cm s$^{-1}$) and orientation $T_o$ (degrees) of the $M_4$ current ellipse at locations in the eastern Irish Sea used in JD96

| Rig No. | Observed | | | Calc 1 G3AX | | | Calc 2 G3A | | |
|---|---|---|---|---|---|---|---|---|---|
| | $A_o$ | $T_o$ | $h_o$ | $A_c$ | $T_c$ | $h_c$ | $A_c$ | $T_c$ | $h_c$ |
| 3 | 5.0 | 34 | 48 | 7.0 | 7 | 51 | 5.2 | 7 | 51 |
| 6 | 4.3 | 20 | 53 | 5.7 | 4 | 49 | 3.7 | 6 | 49 |
| 12 | 10.7 | 42 | 30 | 4.3 | 28 | 25 | 3.8 | 45 | 28 |
| 18 | 21.9 | 64 | 3 | 25.3 | 56 | 6 | 23.9 | 65 | 8 |
| 23 | 4.8 | 7 | 20 | 2.6 | 59 | 18 | 2.0 | 44 | 20 |
| 24 | 9.8 | 52 | 7 | 18.6 | 62 | 10 | 19.6 | 10 | 10 |
| 32 | 11.9 | 54 | 11 | 15.5 | 55 | 9 | 15.4 | 59 | 9 |
| 33 | 6.3 | 29 | 60 | 6.4 | 20 | 58 | 5.7 | 20 | 58 |
| 44 | 5.8 | 11 | 46 | 6.3 | 9 | 45 | 5.2 | 11 | 45 |
| 50 | 7.0 | 19 | 57 | 6.6 | 8 | 54 | 5.9 | 6 | 55 |
| 56 | 6.0 | 10 | 46 | 7.1 | 5 | 45 | 6.2 | 5 | 47 |
| 57 | 5.1 | 165 | 33 | 5.6 | 167 | 30 | 4.2 | 158 | 29 |
| 58 | 3.4 | 172 | 16 | 6.8 | 172 | 13 | 4.6 | 160 | 12 |
| 59 | 4.5 | 180 | 20 | 5.7 | 169 | 17 | 4.0 | 153 | 21 |
| 60 | 5.5 | 171 | 28 | 5.0 | 175 | 25 | 4.4 | 163 | 24 |
| 61 | 4.7 | 186 | 20 | 3.6 | 161 | 19 | 3.8 | 131 | 15 |

1 Table 5c  Comparison of observed semi-major axis $A_o$ (cm s$^{-1}$) and orientation $T_o$ (degrees)
2 of the $M_6$ current ellipse at locations in the eastern Irish Sea used in JD96
3

| Rig No. | Observed | | | Calc 1 G3AX | | | Calc 2 G3A | | |
|---|---|---|---|---|---|---|---|---|---|
| | $A_o$ | $T_o$ | $h_o$ | $A_c$ | $T_c$ | $h_c$ | $A_c$ | $T_c$ | $h_c$ |
| 2 | 3.1 | 2 | 33 | 2.9 | 179 | 42 | 3.0 | 177 | 46 |
| 8 | 2.5 | 110 | 40 | 3.3 | 160 | 41 | 3.5 | 0 | 42 |
| 9 | 0.7 | 146 | 19 | 1.9 | 164 | 19 | 2.5 | 134 | 17 |
| 15 | 1.0 | 139 | 25 | 1.9 | 145 | 26 | 1.9 | 134 | 24 |
| 23 | 1.0 | 159 | 20 | 6.7 | 160 | 18 | 3.3 | 136 | 20 |
| 24 | 2.3 | 64 | 7 | 8.7 | 41 | 10 | 12.0 | 108 | 10 |
| 32 | 1.9 | 3 | 11 | 5.5 | 60 | 8 | 7.2 | 39 | 9 |
| 33 | 1.3 | 4 | 60 | 1.8 | 6 | 58 | 2.3 | 3 | 60 |
| 34 | 2.5 | 161 | 25 | 3.3 | 4 | 24 | 3.3 | 178 | 25 |
| 35 | 1.6 | 6 | 25 | 4.2 | 179 | 17 | 3.2 | 159 | 21 |
| 38 | 1.5 | 173 | 42 | 2.6 | 177 | 42 | 2.6 | 169 | 42 |
| 44 | 1.2 | 167 | 46 | 2.6 | 0 | 45 | 2.8 | 178 | 45 |
| 50 | 1.6 | 184 | 57 | 2.7 | 3 | 54 | 3.0 | 179 | 55 |
| 56 | 1.4 | 163 | 46 | 2.8 | 173 | 46 | 3.2 | 176 | 46 |
| 57 | 1.8 | 162 | 33 | 2.9 | 170 | 30 | 2.9 | 160 | 29 |
| 58 | 2.1 | 153 | 16 | 4.1 | 149 | 13 | 3.1 | 170 | 12 |
| 59 | 1.6 | 185 | 20 | 4.2 | 179 | 17 | 3.2 | 159 | 21 |
| 60 | 2.0 | 165 | 28 | 2.8 | 168 | 25 | 2.4 | 167 | 24 |
| 61 | 1.0 | 174 | 20 | 2.3 | 158 | 19 | 2.6 | 157 | 15 |

4
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1 Table 6: Spatial variability of computed $M_2$, $M_4$ and $M_6$ tidal ellipse semi-major axis $A_c$ (cm
2 $s^{-1}$) and orientation $T_c$ (degrees) with distance $\Delta$ (km) between nearest nodal points and
3 observation point
4

| Rig No | Observed | | | Calc 1 G3AX | | | | Calc 2 G3A | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $A_o$ | $T_o$ | $h_o$ | $\Delta$ | $A_c$ | $T_c$ | $h_c$ | $\Delta$ | $A_c$ | $T_c$ | $h_c$ |
| **$M_2$ Tide** | | | | | | | | | | | |
| 15 | 37 | 38 | 25 | 2.2 | 44 | 24 | 26 | 0.6 | 40 | 30 | 24 |
| | | | | 3.0 | 40 | 23 | 23 | 4.2 | 39 | 28 | 26 |
| | | | | 4.0 | 30 | 7 | 32 | 4.5 | 30 | 25 | 28 |
| | | | | | | | | | | | |
| 16 | 36 | 168 | 25 | 1.0 | 40 | 180 | 26 | 1.4 | 34 | 172 | 25 |
| | | | | 3.3 | 38 | 165 | 25 | 2.2 | 38 | 1 | 26 |
| | | | | 3.7 | 37 | 158 | 25 | 2.2 | 39 | 177 | 25 |
| | | | | | | | | | | | |
| 23 | 49 | 148 | 20 | 0.9 | 68 | 165 | 18 | 0.7 | 49 | 157 | 20 |
| | | | | 2.6 | 41 | 148 | 17 | 2.5 | 52 | 154 | 15 |
| | | | | | | | | | | | |
| **$M_4$ Tide** | | | | | | | | | | | |
| 6 | 4.3 | 20 | 53 | 2.4 | 5.7 | 4 | 48 | 1.8 | 3.7 | 6 | 49 |
| | | | | 3.7 | 5.9 | 7 | 53 | 1.8 | 5.3 | 6 | 48 |
| | | | | 4.8 | 6.7 | 6 | 47 | 2.0 | 4.3 | 1 | 50 |
| | | | | 4.8 | 5.3 | 19 | 50 | 3.2 | 5.3 | 16 | 47 |
| | | | | | | | | | | | |
| 18 | 21.9 | 64 | 3 | 0.3 | 25.2 | 56 | 6 | 0.8 | 24.0 | 65 | 8 |
| | | | | 1.5 | 23.3 | 100 | 3 | 1.4 | 29.9 | 72 | 0 |
| | | | | 1.6 | 11.8 | 100 | 8 | 1.8 | 22.2 | 68 | 0 |
| | | | | 1.9 | 28.6 | 63 | 3 | 1.9 | 35.1 | 58 | 11 |
| | | | | | | | | | | | |
| **$M_6$ Tide** | | | | | | | | | | | |
| 23 | 1.1 | 159 | 20 | 0.9 | 55.5 | 157 | 18 | 0.7 | 3.3 | 136 | 20 |
| | | | | 2.6 | 3.0 | 152 | 17 | 2.5 | 5.2 | 152 | 15 |
| | | | | 2.8 | 3.4 | 154 | 23 | 2.7 | 3.4 | 152 | 16 |
| | | | | | | | | | | | |
| 24 | 2.3 | 64 | 7 | 1.3 | 9.1 | 54 | 10 | 1.2 | 12.0 | 108 | 10 |
| | | | | 1.5 | 3.6 | 59 | 11 | 1.4 | 6.7 | 54 | 10 |
| | | | | 1.7 | 9.7 | 79 | 9 | 2.0 | 7.3 | 2 | 13 |
| | | | | | | | | | | | |
| 32 | 1.9 | 3 | 11 | 1.0 | 4.1 | 166 | 8 | 0.8 | 7.3 | 39 | 9 |
| | | | | 1.6 | 3.5 | 59 | 11 | 1.5 | 8.0 | 54 | 10 |
| | | | | 1.7 | 14.2 | 122 | 5 | 1.6 | 9.5 | 137 | 1 |
| | | | | | | | | | | | |

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7

8

1    FIG1(a)

2

1    FIG1(b)



2

3

4
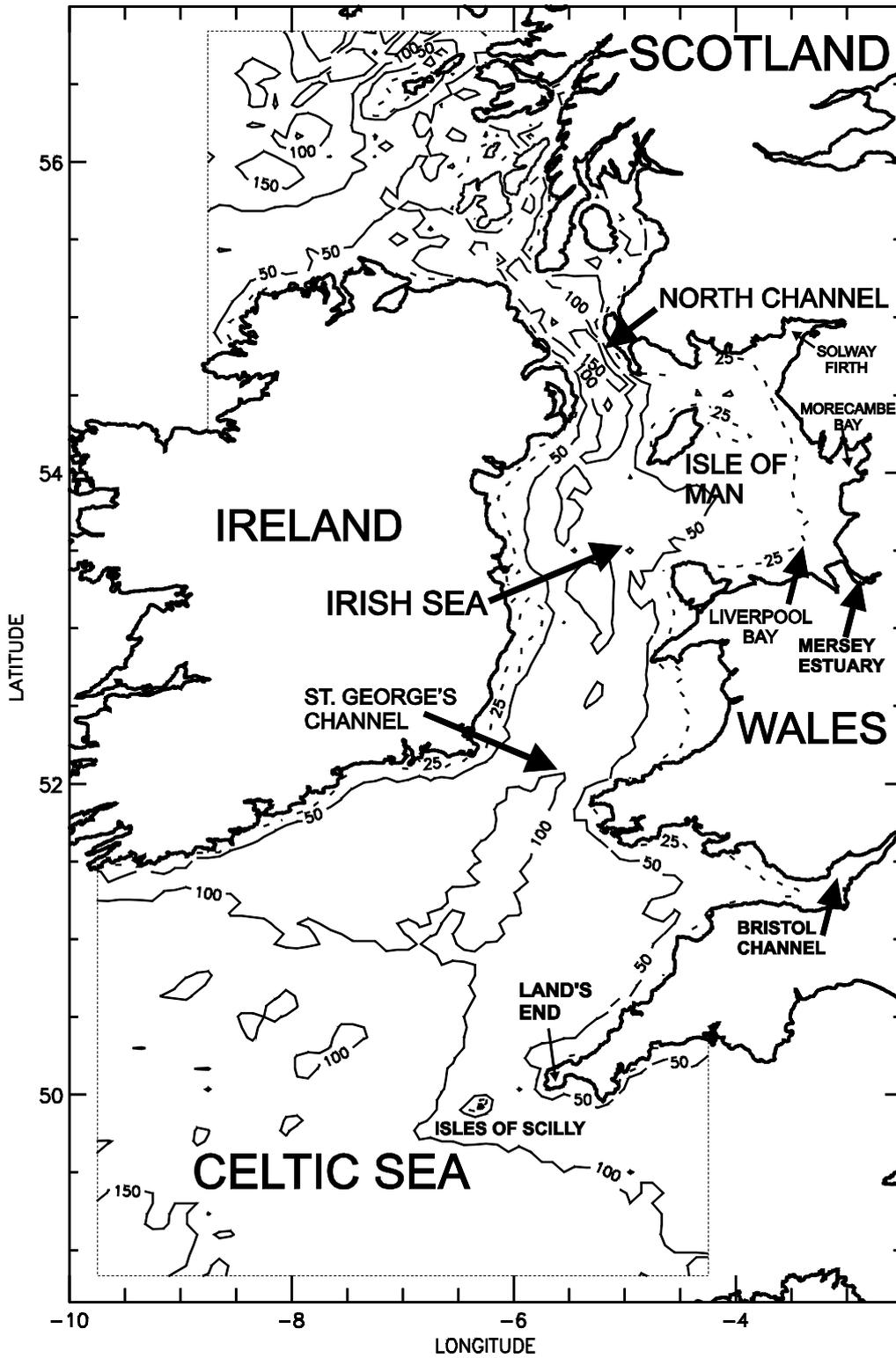
1    FIG2
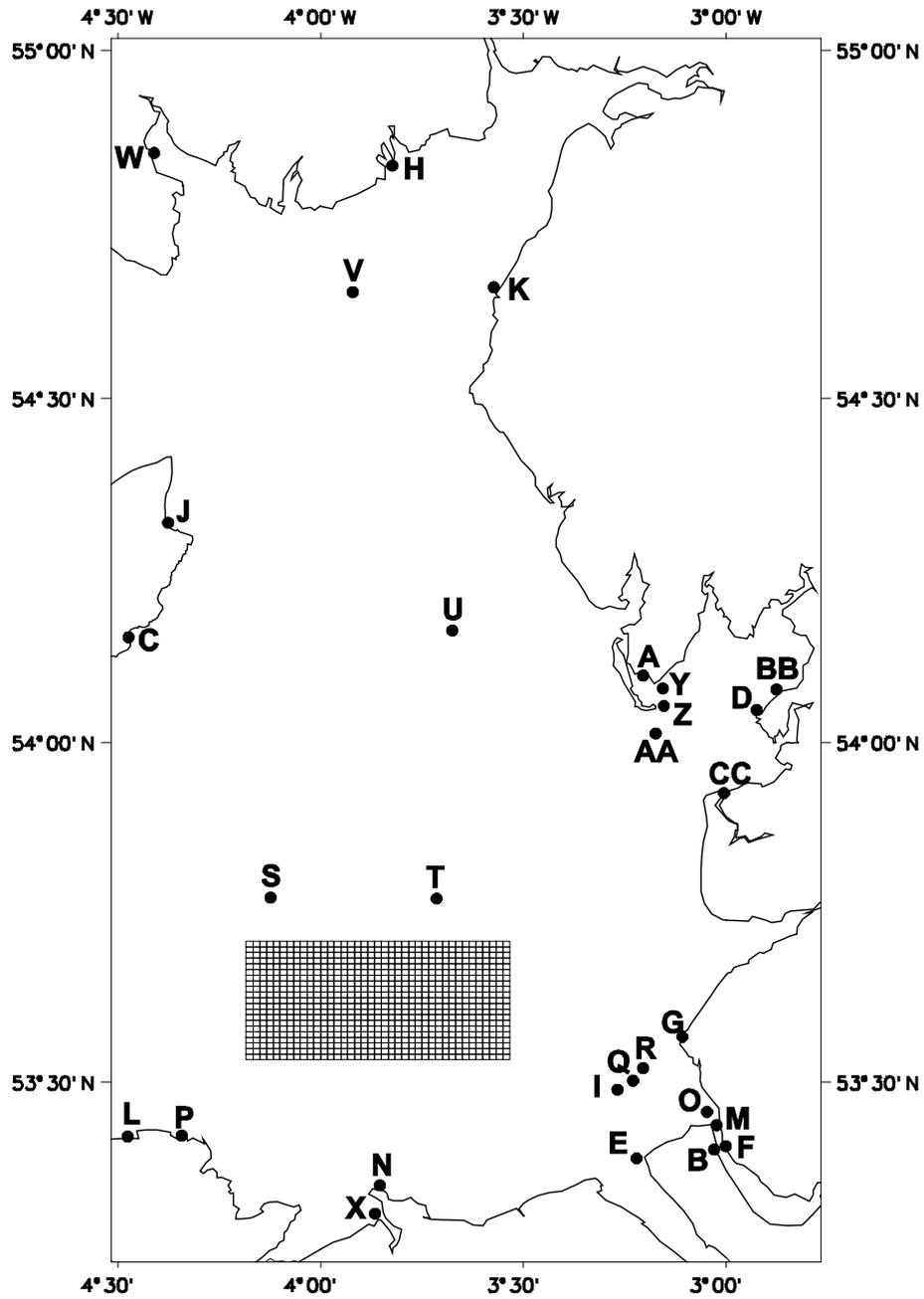


2
3
4
5

1    FIG3

## GRID G3AX
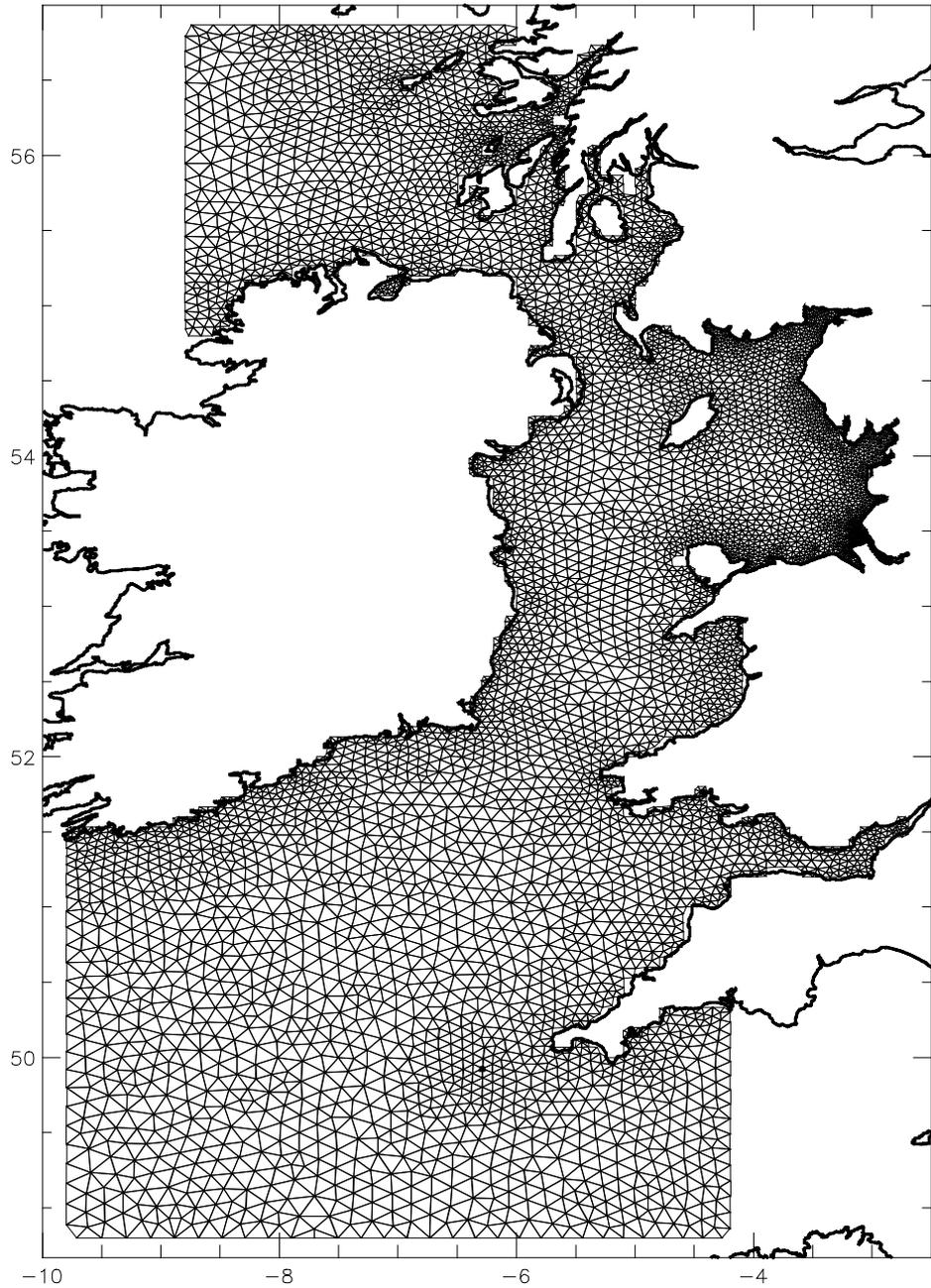


2
3
4

1    FIG4a(i)



M4 RUN G3AX

AMPLITUDE IN CM

2
3
4

1    FIG4a(ii)



M4 RUN G3AX

AMPLITUDE IN CM

2
3
4

1    FIG4b(i)

2

M6 RUN G3AX

AMPLITUDE IN CM

1    FIG4b(ii)



M6 RUN G3AX

AMPLITUDE IN CM

2
3
4

1    FIG5a(i)



M2 TIDAL ELLIPSES— G3AX

CM/SEC

— 50
— 100
≡ 150
≡ 200
▬ 250
▭ over 250

2

3

4

5

6

1    FIG5a(ii)

M2 TIDAL ELLIPSES— G3AX

1    FIG5b(i)



M4 TIDAL ELLIPSES— G3AX

CM/SEC
10
20
30
40
50
over  50

2
3
4
5
6

1    FIG5b(ii)

M4 TIDAL ELLIPSES— G3AX
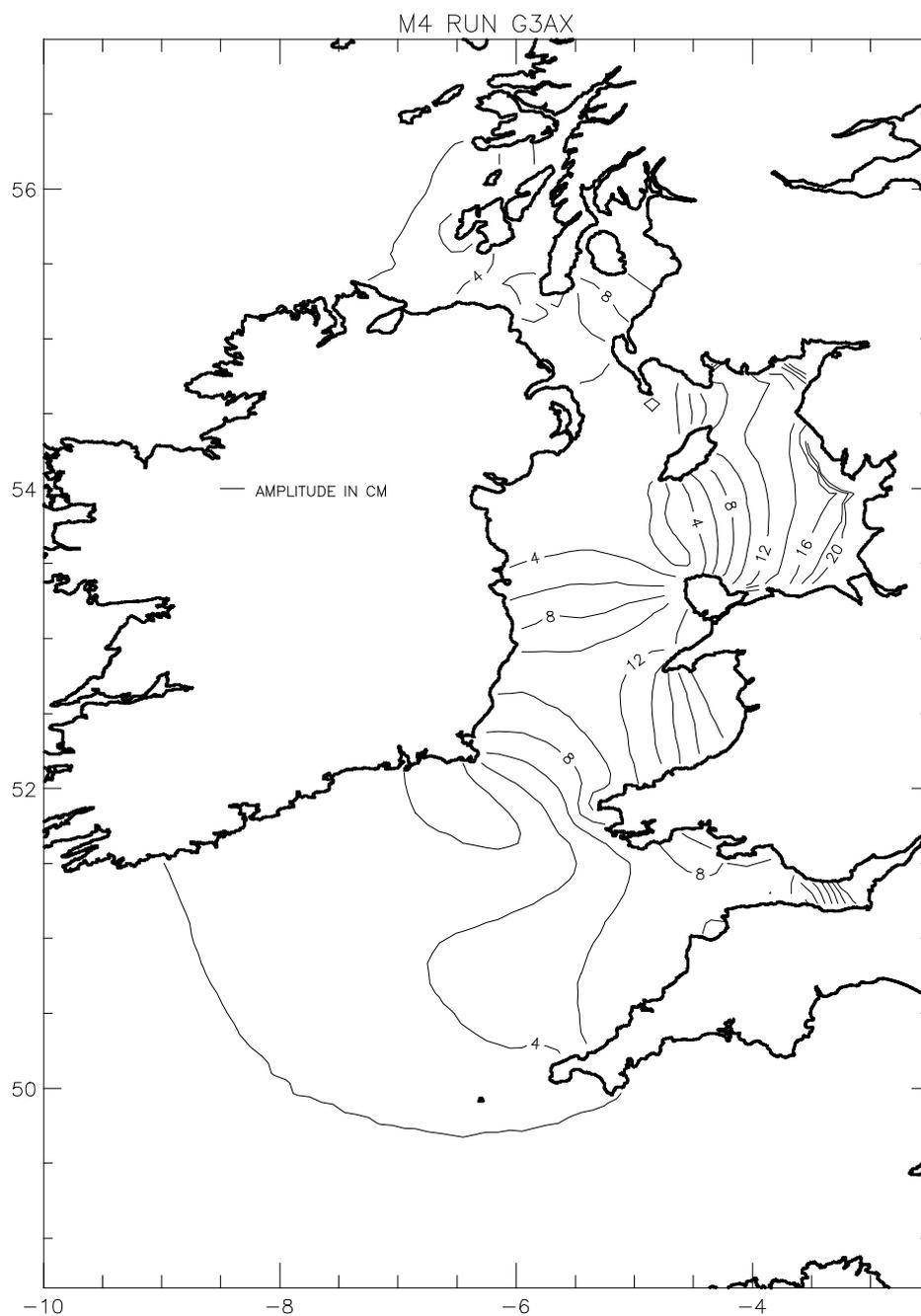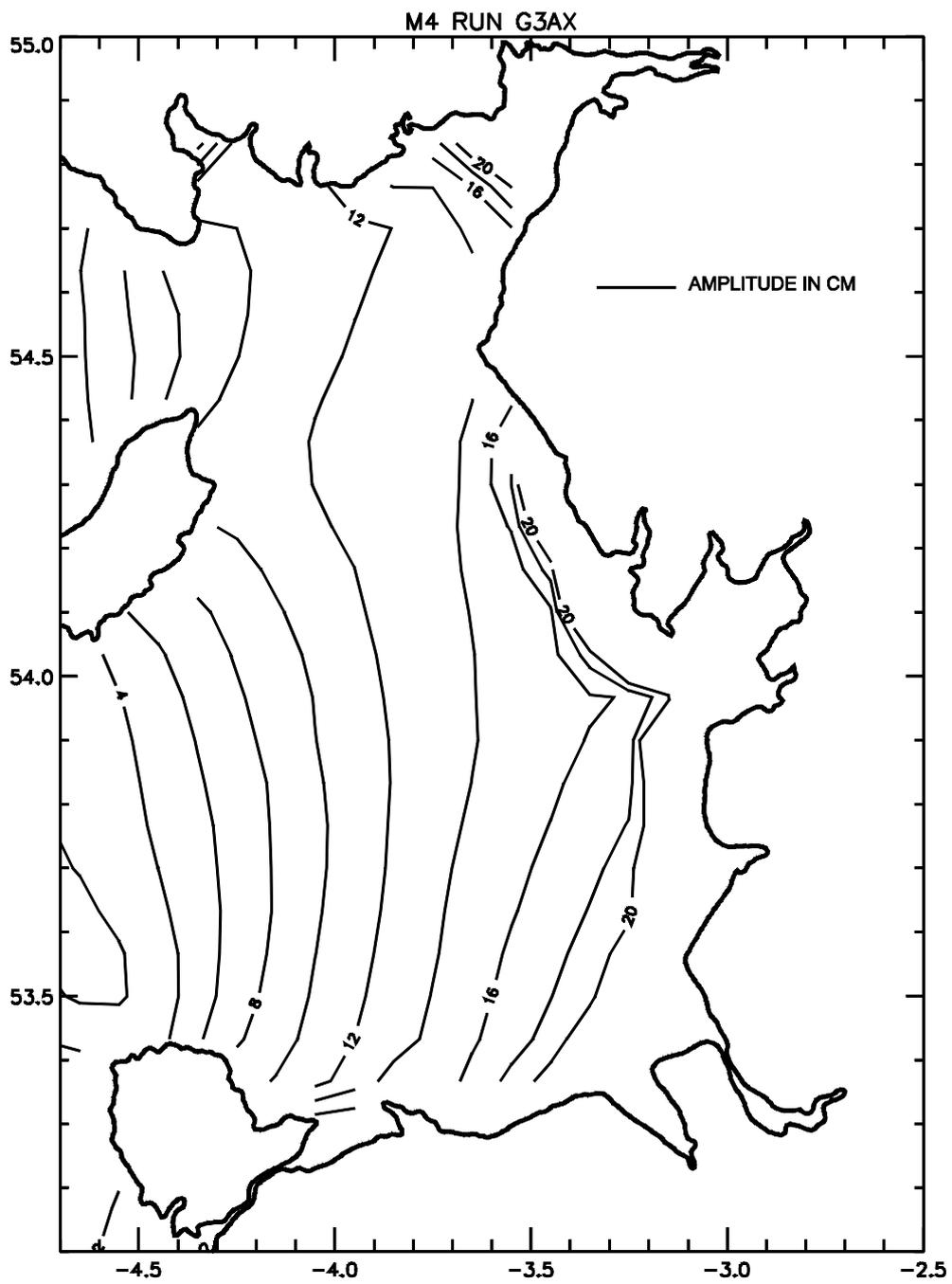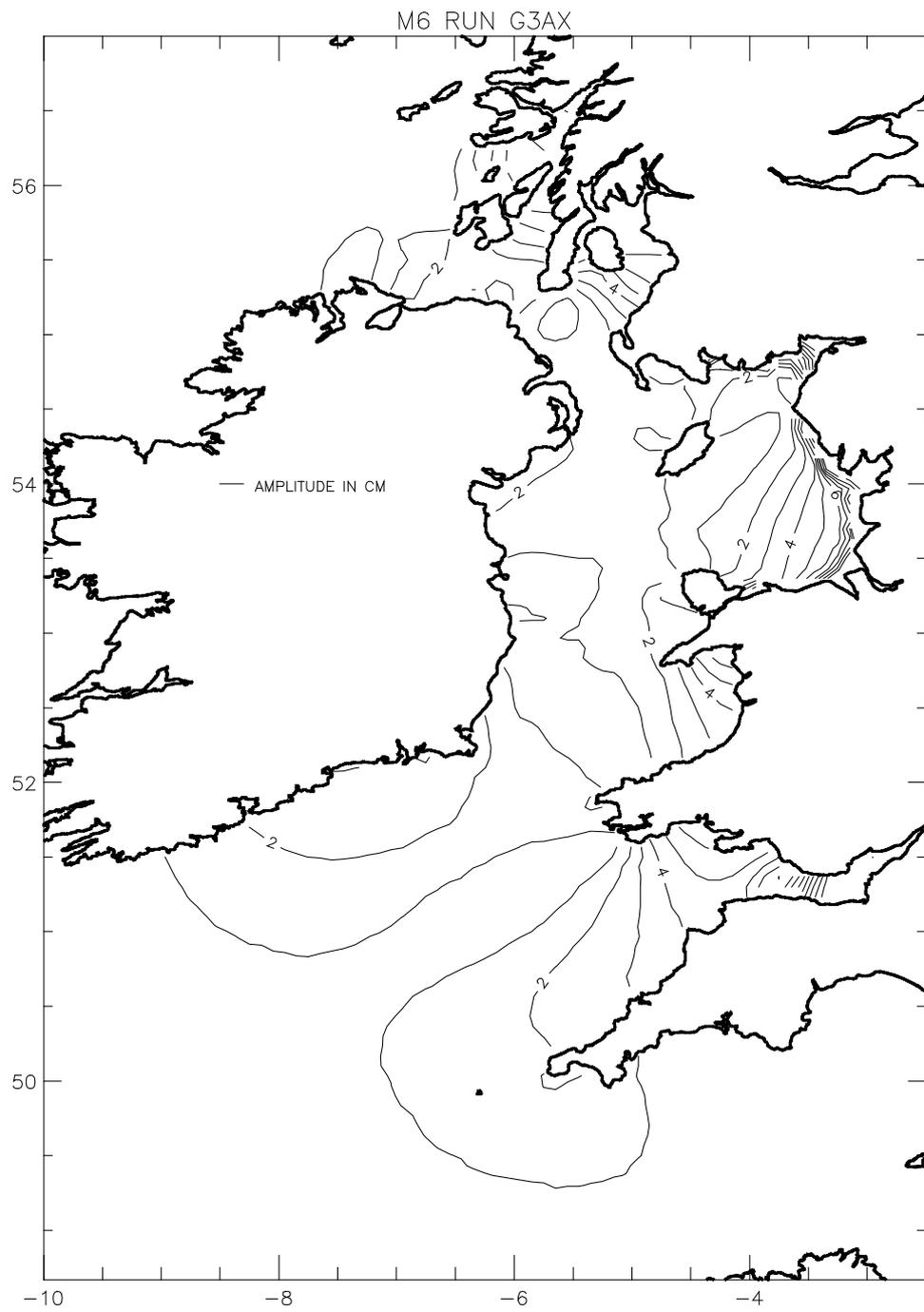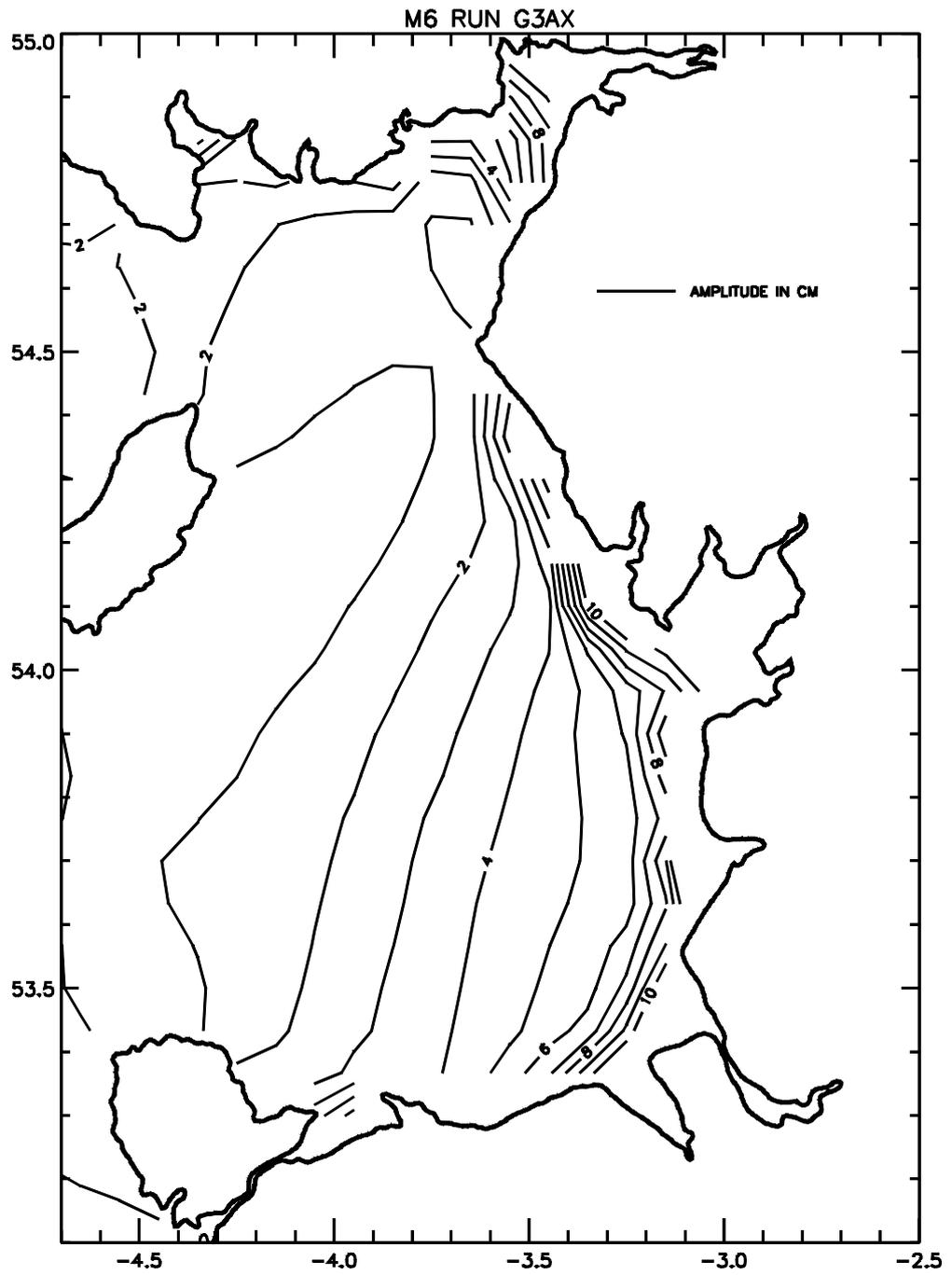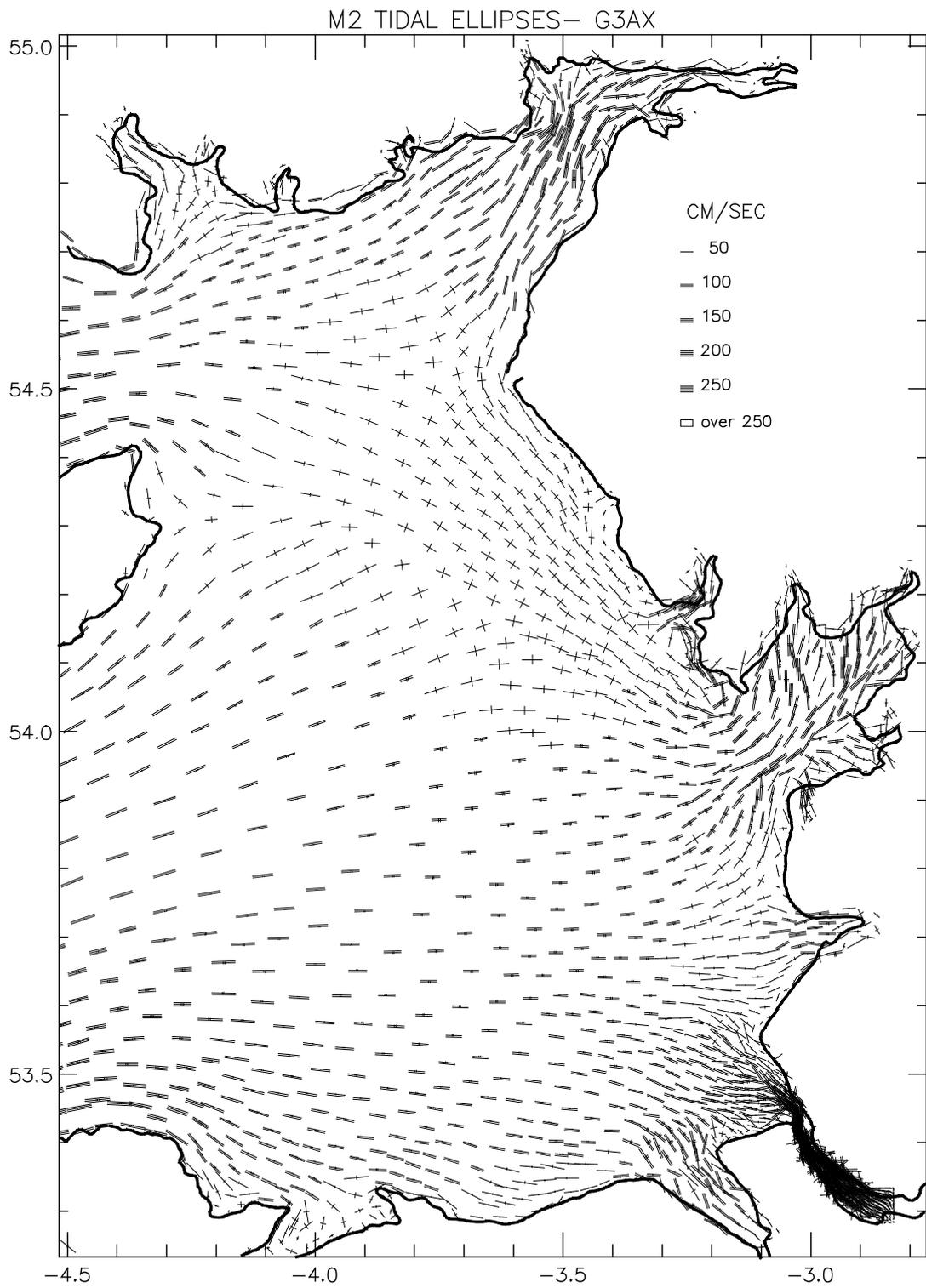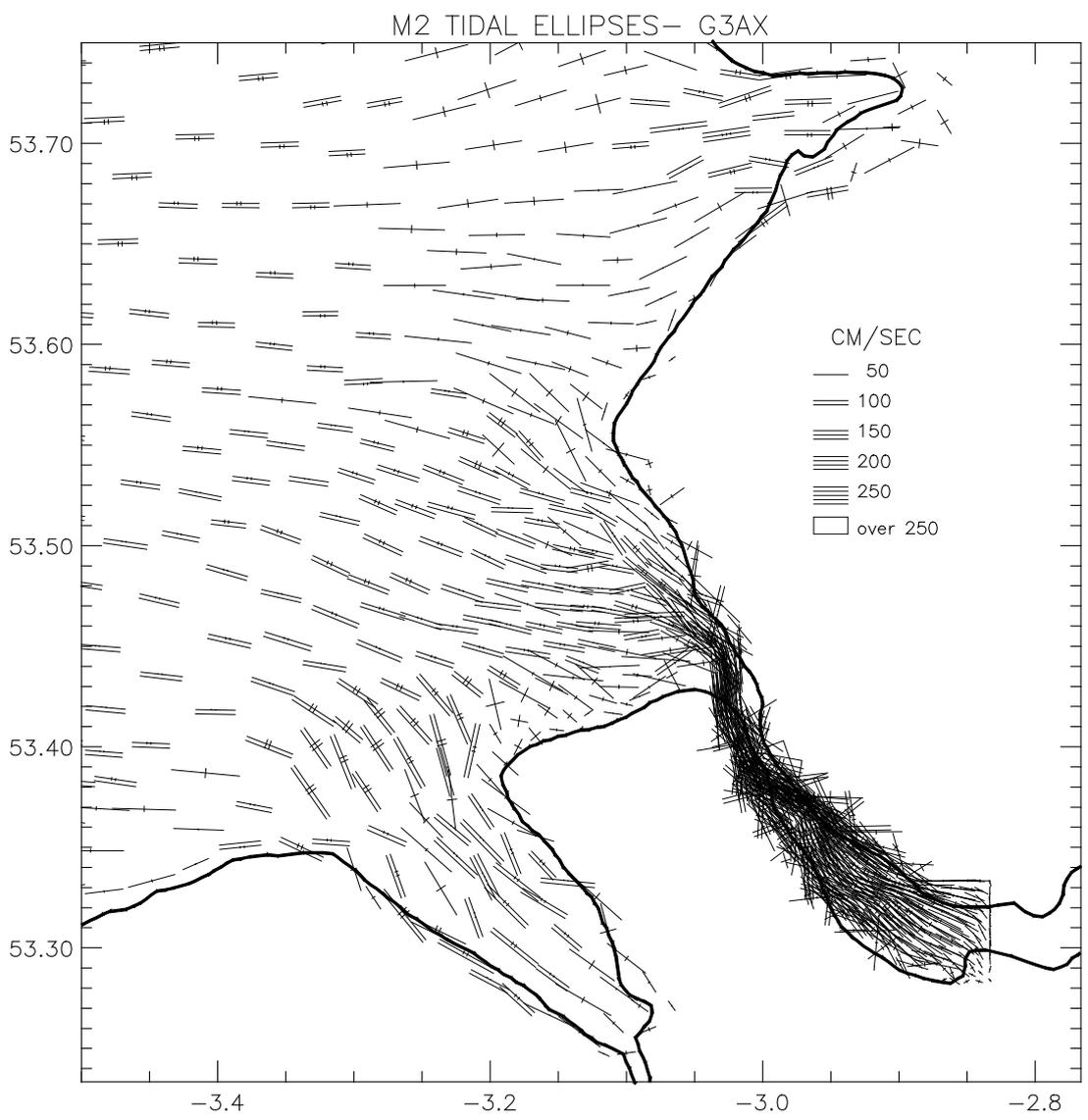


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7
8
9
10
11
12
13

1    FIG5c(i)



M6 TIDAL ELLIPSES— G3AX

2

3

4

5

6

1    FIG5c(ii)



M6 TIDAL ELLIPSES— G3AX
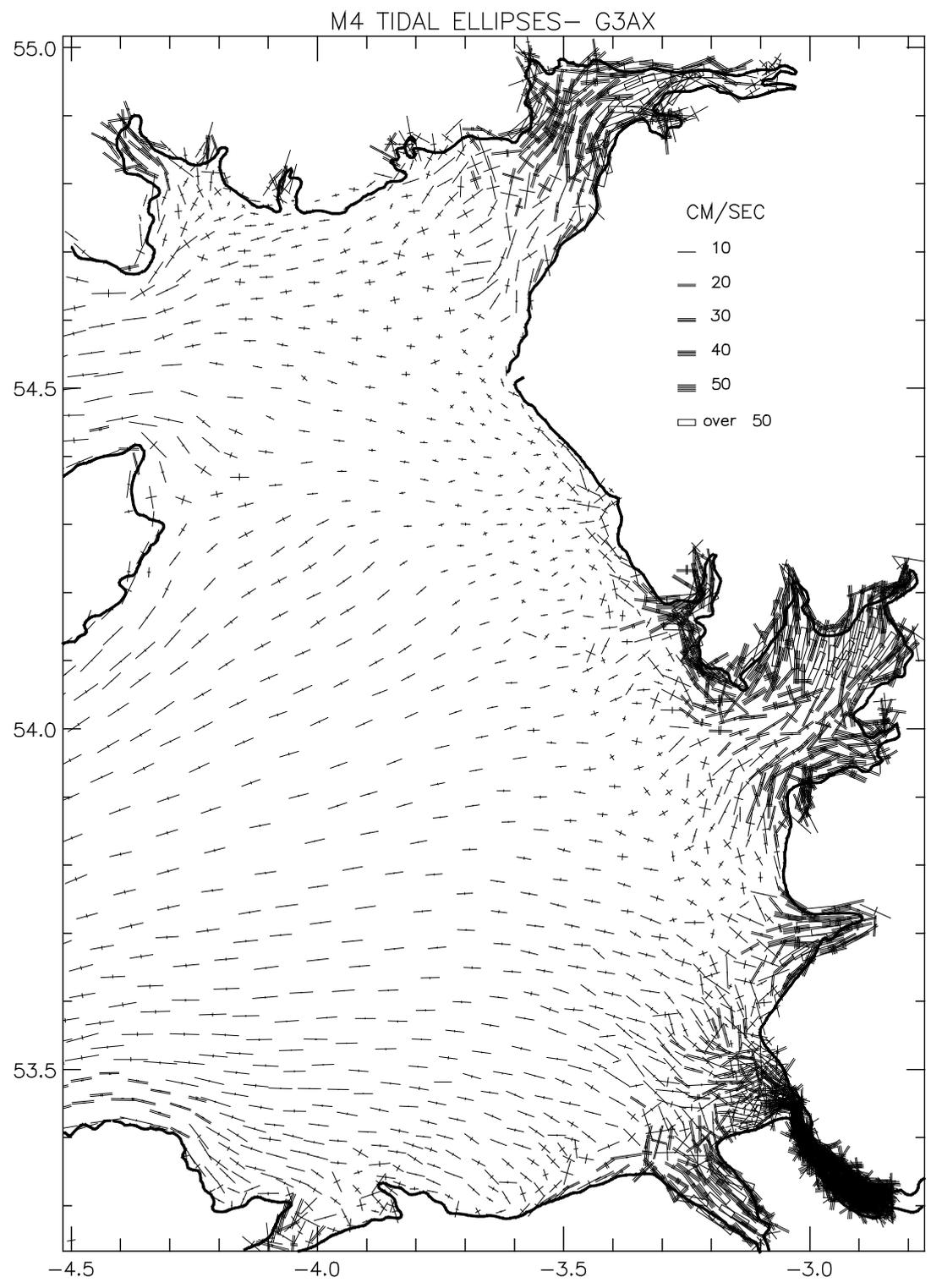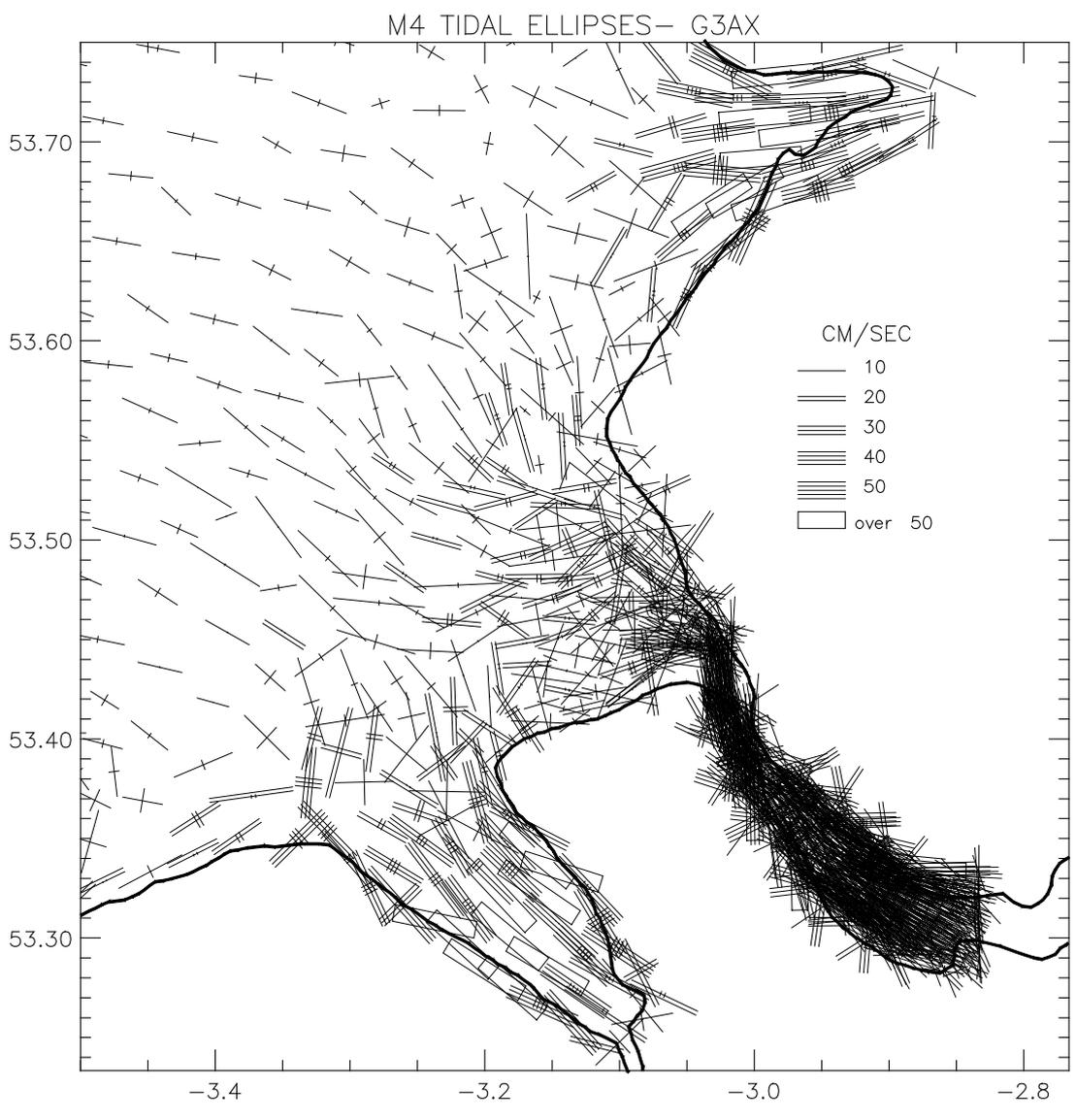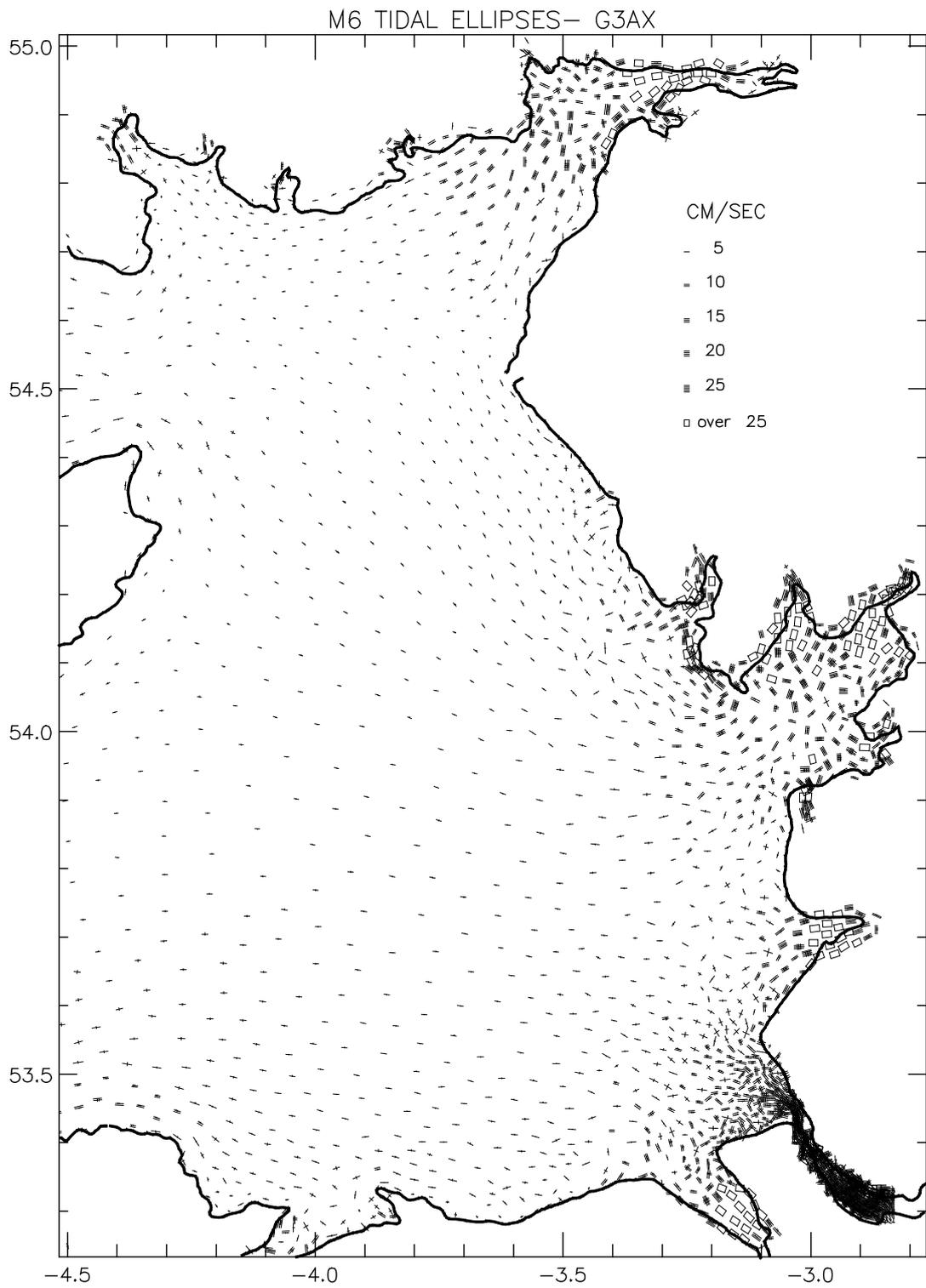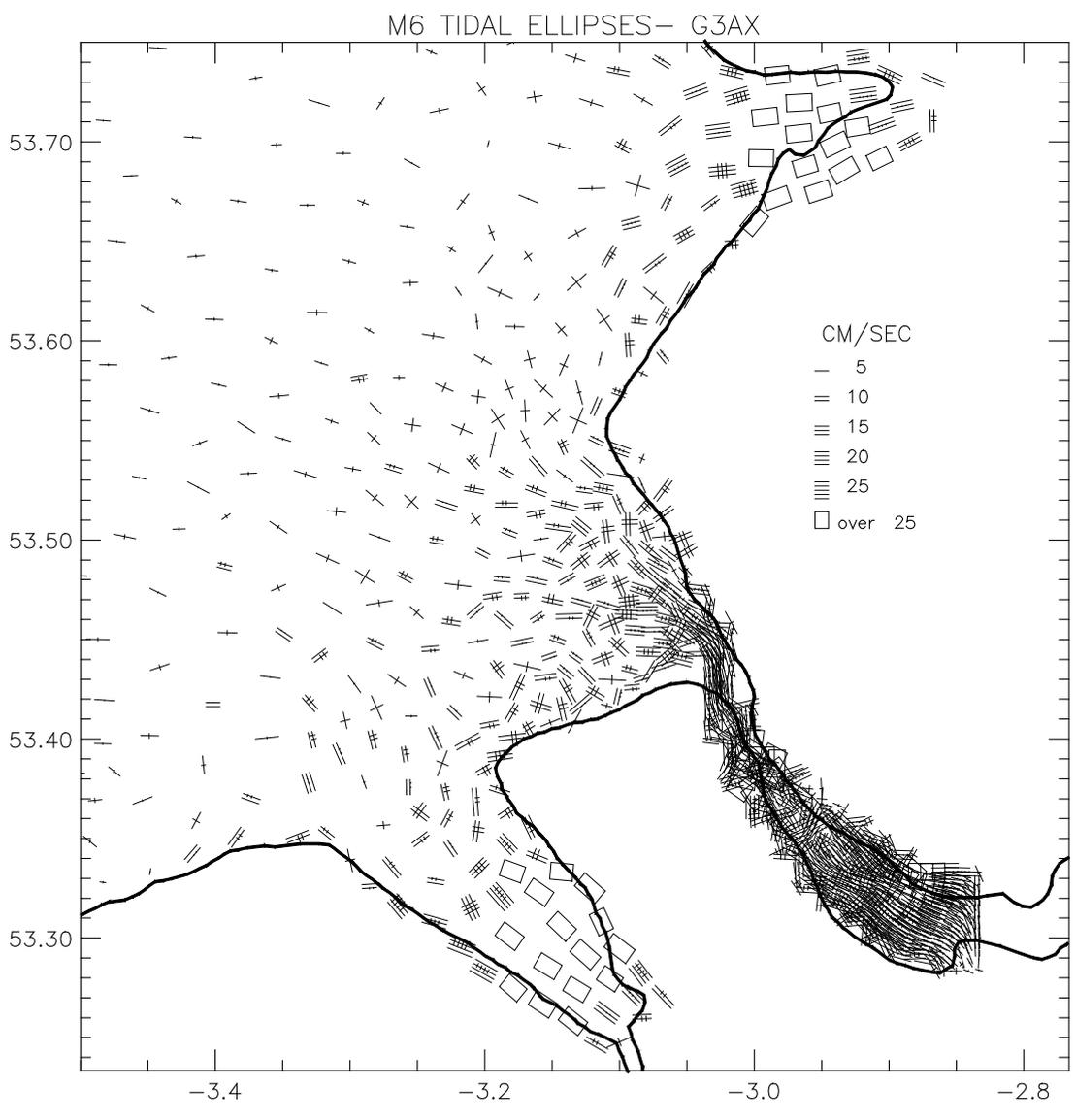
2
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4
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6
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8
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10
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12
13

1    FIG 6

## GRID G3A



2

3

4

1     FIG7a



M2 TIDAL ELLIPSES— G3A

2
3
4

1    FIG 7b

2

M4 TIDAL ELLIPSES— G3A



3
4
5

1     FIG 7c



M6 TIDAL ELLIPSES— G3A

2