Distributed Signaling Games*

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Abstract

The study of the algorithmic and computational complexity of designing efficient signaling schemes for mechanisms aiming to optimize social welfare or revenue is a recurring theme in recent computer science literature. In reality, however, information is typically not held by a central authority, but is distributed among multiple sources (third-party "mediators"), a fact that dramatically changes the strategic and combinatorial nature of the signaling problem.

In this paper we introduce *distributed signaling games*, while using display advertising as a canonical example for introducing this foundational framework. A distributed signaling game may be a pure coordination game (i.e., a distributed optimization task), or a non-cooperative game. In the context of pure coordination games, we show a wide gap between the computational complexity of the centralized and distributed signaling problems, proving that distributed coordination on revenue-optimal signaling is a much harder problem than its "centralized" counterpart.

In the context of non-cooperative games, the outcome generated by the mediators' signals may have different value to each. The reason for that is typically the desire of the auctioneer to align the incentives of the mediators with his own by a compensation relative to the marginal benefit from their signals. We design a mechanism for this problem via a novel application of Shapley's value, and show that it possesses a few interesting economical properties.

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1 Introduction

The topic of signaling has recently received much attention in the computer science literature on mechanism design [2, 4, 6, 5, 7, 12]. A recurring theme of this literature is that proper design of a signaling scheme is crucial for obtaining efficient outcomes, such as social welfare maximization or revenue maximization. In reality, however, sources of information are typically not held by a central authority, but are rather distributed among third party mediators/information providers, a fact which dramatically changes the setup to be studied, making it a game between information providers rather than a more classic mechanism

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design problem. Such a game is in the spirit of work on the theory of teams in economics [14], whose computational complexity remained largely unexplored. The goal of this paper is to initiate an algorithmic study of such games, which we term *distributed signaling games*, via what we view as a canonical example: Bayesian auctions; and more specifically, display advertising in the presence of third party external mediators (information providers).

Consider a web-site owner that auctions each user's visit to its site, a.k.a. impression. The impression types are assumed to arrive from a commonly known distribution. The bidders are advertisers who know that distribution, but only the web site owner knows the impression type instantiation, consisting of identifiers such as age, origin, gender and salary of the web-site visitor. As is the practice in existing ad exchanges, we assume the auction is a second price auction. The web-site owner decides on the information (i.e., signal) about the instantiation to be provided to the bidders, which then bid their expected valuations for the impression given the information provided. The selection of the proper signaling by the web-site is a central mechanism design problem. Assume, for example, an impression associated with two attributes: whether the user is male or female on one side, and whether he is located in the US or out of the US on the other side. This gives 4 types of possible users. Assume for simplicity that the probability of arrival of each user type is 1/4, and that there are four advertisers each one of them has value of \$100 for a distinguished user type and \$0 for the other types, where these values are common-knowledge. One can verify that an auctioneer who reveals no information receives an expected payoff of \$25, an auctioneer who reveals all information gets no payoff, while partitioning the impression types into two pairs, revealing only the pair of the impression which was materialized (rather than the exact instantiation) will yield a payoff of \$50, which is much higher revenue.

While the above example illustrates some of the potential benefits of signaling and its natural fit to mechanism design, its major drawback is in the unrealistic manner in which information is manipulated: while some information about the auctioned item is typically published by the ad network [18] (such information is modeled here as a public prior), and despite the advertisers' effort to perform "behavioral targeting" by clever data analysis (e.g., utilizing the browsing history of a specific user to infer her interests), the quantity of available contextual information and market expertise is often way beyond the capabilities of both advertisers and auctioneers. This reality gave rise to "third-party" companies which develop technologies for collecting data and online statistics used to infer the contexts of auctioned impressions (see, e.g., [15] and references therein). Consequently, a new *distributed* ecosystem has emerged, in which many third-party companies operate within the market aiming at maximizing their own utility, while significantly increasing the effectiveness of display advertising, as suggested by the following article recently published by Facebook:

> *"Many businesses today work with third parties such as Acxiom, Datalogix, and Epsilon to help manage and understand their marketing efforts. For example, an auto dealer may want to customize an offer to people who are likely to be in the market for a new car. The dealer also might want to send offers, like discounts for service, to customers that have purchased a car from them. To do this, the auto dealer works with a third-party company to identify and reach those customers with the right offer."*
> (www.facebook.com, "Advertising and our Third-Party Partners", April 10, 2013.)

Hence, in reality sources of information are distributed. Typically, the information is distributed among several mediators or information providers/brokers, and is not held (or mostly not held) by a central authority. In the display advertising example, one information source may know the gender and another may know the location of the web-site visitor, while the web-site itself often lacks the capability to track such information. The information sources need to decide on the communicated information. In this case the information sources become players in a game. To make the situation clearer, assume (as above) that the value of each impression type for each bidder is public-knowledge (as is typically the case in repeated interactions through ad exchanges which share their logs with the participants), and the only unknown entity is the instantiation of the impression type; given the information learned from the information sources each bidder will bid his true expected valuation; hence, the results of this game are determined solely by the information providers. Notice that if, in the aforementioned example, the information provider who knows the gender reveals it while the other reveals nothing, then the auctioneer receives a revenue of \$50 as in the centralized case, while the cases in which both information providers reveal their information or none of them do so result in lower revenues. This shows the subtlety of the situation.

The above suggests that a major issue to tackle is the study of *distributed signaling games*, going beyond the realm of classical mechanism design. We use a model of the above display advertising setting, due to its centrality, as a tool to introduce this novel foundational topic. The distributed signaling games may be pure coordination games (a.k.a. distributed optimization), or non-cooperative games. In the context of pure coordination games each information source has the same utility from the output created by their joint signal. Namely, in the above example if the web-site owner pays each information source proportionally to the revenue obtained by the web-site owner then the aims of the information sources are identical. The main aim of the third parties/mediators is to choose their signals based on their privately observed information in a distributed manner in order to optimize their own payoffs. Notice that in a typical embodiment, which we adapt, due to both technical and legal considerations, the auctioneer does not synthesize reported signals into new ones nor the information providers are allowed to explicitly communicate among them about the signals, but can only broadcast information they individually gathered. The study of the computational complexity of this highly fundamental problem is the major technical challenge tackled in this paper. Interestingly, we show a wide gap between the computational complexity of the centralized and distributed setups, proving that coordinating on optimal signaling is a much harder problem than the one discussed in the context of centralized mechanism design. On the other hand we also show a natural restriction on the way information is distributed among information providers, which allows for an efficient constant approximation scheme.

In the context of non-cooperative games the outcome generated by the information sources' reports may result in a different value for each of them. The reason for that is typically the desire of the auctioneer to align the incentives of the mediators with his own by a compensation relative to the marginal benefit from their signals. In the above example one may compare the revenue obtained without the additional information sources, to what is obtained through their help, and compensate relatively to the Shapley values of their contributions, which is a standard (and rigorously justified) tool to fully divide a gain yielded by the cooperation of several parties. Here we apply such division to distributed signaling games, and show that it possesses some interesting properties: in particular the corresponding game has a pure strategy equilibrium, a property of the Shapley value which is shown for the first time for signaling settings (and is vastly different from previous studies of Shapley mechanisms in non-cooperative settings such as cost-sharing games [16]).

1.1 Model

Our model is a generalization of the one defined in [11]. There is a ground set $I = [n]$ of potential items (contexts) to be sold and a set $B = [k]$ of bidders. The value of item j for bidder i is given as v_{ij}. Following the above discussion (and the previous line of work, e.g., [8, 11]), we assume the valuation matrix $V = \{v_{i,j}\}$ is publicly known. An auctioneer is selling a single random item j_R, distributed according to some publicly known prior distribution μ over I, using a second price auction (a more detailed description of the auction follows). There is an additional set $M = [m]$ of "third-party" mediators. Following standard practice in game theoretic information models [1, 9, 7], we assume each mediator $t \in M$ is equipped with a *partition* (signal-set) $\mathcal{P}_t \in \Omega(I)$[1]. Intuitively, $\mathcal{P}_t$ captures the extra information t has about the item which is about to be sold – he knows the set $S \in \mathcal{P}_t$ to which the item j_R belongs, but has no further knowledge about which item of S it is (except for the a priori distribution μ) – in other words, the distribution t has in mind is $\mu|_S$. For example, if the signal-set partition $\mathcal{P}_t$ partitions the items of I into pairs, then mediator t knows to which pair $\{j_1, j_2\} \in \mathcal{P}_t$ the item j_R belongs, but he has no information whether it is j_1 or j_2, and therefore, from her point of view, $\Pr[j_R = j_1] = \mu(j_1)/\mu(\{j_1, j_2\})$.

Mediators can signal some (or all) of the information they own to the network. Formally, this is represented by allowing each mediator t to report *any* super-partition $\mathcal{P}'_t$, which is obtained by merging partitions in her signal-set partition $\mathcal{P}_t$ (in other words $\mathcal{P}_t$ must be a refinement of $\mathcal{P}'_t$). In other words, a mediator may report any partition $\mathcal{P}'_t$ for which there exists a set $\mathcal{Q}'_t \in \Omega(\mathcal{P}_t)$ such that $\mathcal{P}'_t = \{\cup_{S \in A} S \mid A \in \mathcal{Q}'_t\}$. In particular, a mediator can always report $\{I\}$, in which case we say that he remains silent since he does not contribute any information. The signals $\mathcal{P}'_1, \mathcal{P}'_2, \ldots, \mathcal{P}'_m$ reported by the mediators are *broadcasted*[2] to the bidders, inducing a combined partition $\mathcal{P} \triangleq \times_{t=1}^m \mathcal{P}'_t = \{\cap_{i \in M} A_i \mid A_i \in \mathcal{P}'_i\}$, which we call the *joint partition* (or *joint signal*). $\mathcal{P}$ splits the auction into separate "restricted" auctions. For each bundle $S \in \mathcal{P}$, the item j_R belongs to S with probability $\mu(S) = \sum_{j \in S} \mu(j)$, in which case S is signaled to the bidders and a second-price auction is performed over $\mu|_S$. Notice that if the signaled bundle is $S \subseteq I$, then the (expected) value of bidder i for $j_R \sim \mu|_S$ is $v_{i,S} = \frac{1}{\mu(S)} \sum_{j \in S} (\mu(j) \cdot v_{ij})$, and the truthfulness of the second price auction implies that this will also be bidder i's bid for the restricted auction. The winner of the auction is the bidder with the maximum bid $\max_{i \in B} v_{i,S}$, and he is charged the second highest valuation for that bundle $\max^{(2)}_{i \in B} v_{i,S}$. Therefore, the auctioneer's revenue with respect to $\mathcal{P}$ is the expectation (over $S \in_R \mathcal{P}$) of the price paid by the winning bidder: $R(\mathcal{P}) = \sum_{S \in \mathcal{P}} [\mu(S) \cdot \max^{(2)}_{i \in B}(v_{i,S})]$.

The joint partition $\mathcal{P}$ signaled by the mediators can dramatically affect the revenue of the auctioneer. Consider, for example, the case where V is the 4×4 identity matrix, μ is the uniform distribution, and M consists of two mediators associated with the partitions $\mathcal{P}_1 = \{\{1, 2\}\ ,\ \{3, 4\}\}$ and $\mathcal{P}_2 = \{\{1, 3\}\ ,\ \{2, 4\}\}$. If both mediators remain silent, the revenue is $R(\{I\}) = 1/4$ (as this is the average value of all 4 bidders for a random item). However, observe that $\mathcal{P}_1 \times \mathcal{P}_2 = \{\{1\}, \{2\}, \{3\}, \{4\}\}$, and the second highest value in every column of V is 0, thus, if both report their partitions, the revenue drops to $R(\mathcal{P}_1 \times \mathcal{P}_2) = 0$. Finally, if mediator 1 reports $\mathcal{P}_1$, while meditor 2 keeps silent, the revenue increases from 1/4 to $R(\mathcal{P}_1) = 1/2$, as the value of each pair of items is 1/2 for two different bidders (thus, the second highest price for each pair is 1/2). This example can be easily generalized to show

[1] For a set S, $\Omega(S) \triangleq \{\mathcal{A} \subseteq 2^S \mid \bigcup_{A \in \mathcal{A}} A = S, \forall_{A,B \in \mathcal{A}} A \cap B = \varnothing\}$ is the collection of all partitions of S.

[2] By saying that a mediator reports $\mathcal{P}'_t$, we mean that he reports the bundle $S \in \mathcal{P}'_t$ for which $j_R \in S$. The reader may wonder why our model is a broadcast model, and does not allow the mediators to report their information to the auctioneer through private channels, in which case the ad network will be able to manipulate and publish whichever information that best serves its interest. The primary reason for the broadcast assumption is that online advertising is a highly dynamic marketplace in which mediators often "come and go", so implementing "private contracts" is infeasible. The second reason is that real-time bidding environments cannot afford the latency incurred by such a two-phase procedure in which the auctioneer first collects the information, and then selectively publishes it. The auction process is usually treated as a "black box", and modifying it harms the modularity of the system.

that in general the intervention of mediators can increase the revenue by a factor of $n/2$!

Indeed, the purpose of this paper is to understand how mediators' (distributed) signals affect the revenue of the auctioneer. We explore the following two aspects of this question:

1. (Computational) Suppose the auctioneer has control over the signals reported by the mediators. We study the computational complexity of the following problem. *Given a $k \times n$ matrix V of valuations and mediators' partitions $\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_m$, what is the revenue maximizing joint partition $\mathcal{P} = \mathcal{P}'_1 \times \ldots \times \mathcal{P}'_m$?* We call this problem the *Distributed Signaling Problem*, and denote it by $\mathbf{DSP}(n, k, m)$.
   We note that the problem studied in [11] is a special case of **DSP**, in which there is a *single* mediator ($m = 1$) with *perfect knowledge* about the item sold and can report any desirable signal (partition).[3]
2. (Strategic) What if the auctioneer cannot control the signals reported by the mediators (as the reality of the problem usually entails)? *Can the auctioneer introduce compensations that will incentivize mediators to report signals leading to increased revenue in the auction, when each mediator is acting selfishly?*
   This is a mechanism design problem: Here the auctioneer's goal is to design a payment rule (i.e., a mechanism) for allocating (part of) his profit from the auction among the mediators, based on their reported signals and the auction's outcome, so that global efficiency (i.e., maximum revenue) emerges from their signals.

Section 1.2 summarizes our findings regarding the two above problems.

1.2 Our Results

Ghosh et al. [11] showed that computing the revenue-maximizing signal in their "perfect-knowledge" setup is NP-hard, but present an efficient algorithm for computing a 2-approximation of the optimal signal (partition). We show that when information is distributed, the problem becomes much harder. More specifically, we present a gap-preserving reduction from the *Maximum Independent Set* problem to **DSP**.

▶ **Theorem 1.1** (Hardness of approximating **DSP**). *If there exists an $O(m^{1/2-\varepsilon})$ approximation (for some constant $\varepsilon > 0$) for instances of $\mathbf{DSP}(2m, m+1, m)$, then there exists a $O(N^{1-2\varepsilon})$ approximation for Maximum Independent Set (MIS_N), where N is the number of nodes in the underlying graph of the MIS instance.*

Since the Maximum Independent Set problem is NP-hard to approximate to within a factor of $n^{1-\rho}$ for any fixed $\rho > 0$ [13], Theorem 1.1 indicates that approximating the revenue-maximizing signal, even within a multiplicative factor of $O((\min\{n, k, m\})^{1/2-\varepsilon})$, is NP-hard. In other words, one cannot expect a reasonable approximation ratio for $\mathbf{DSP}(n, k, m)$ when the three parameters of the problem are all "large". The next theorem shows that a "small" value for either one of the parameters n or k indeed implies a better approximation ratio.

▶ **Theorem 1.2** (Approximation algorithm for small n or k). *For $k \geq 2$, there is a polynomial time $\min\{n, k-1\}$-approximation algorithm for $\mathbf{DSP}(n, k, m)$.*[4]

We leave open the problem of determining whether one can get an improved approximation ratio when the parameter m is "small". For $m = 1$, the result of [11] implies immediately

[3] In other words, $\mathcal{P}_1$ is the partition of I into singletons.

[4] For $k = 1$, any algorithm is optimal since the use of a second price auction implies that the revenue of any strategy profile is 0 when there is only one bidder.

a 2-approximation algorithm. However, even for the case of $m = 2$ we are unable to find an algorithm having a non-trivial approximation ratio. We mitigate the above results by proving that for a natural (and realistic) class of mediators called *local experts* (defined in Section 3), there exists a polynomial time 5-approximation algorithm for **DSP**.

▶ **Theorem 1.3** (A 5-approximation algorithm for Local Expert mediators)**.** *If mediators are local experts, there exists a polynomial time* 5*-approximation algorithm for* **DSP***.*

In the strategic setup, we design a fair (symmetric) payment rule $\mathcal{S} : (\mathcal{P}'_1, \mathcal{P}'_2, ..., \mathcal{P}'_m) \to \mathbb{R}^m_+$ for incentivizing mediators to report useful information they own, and refrain from reporting information with negative impact on the revenue. This mechanism is inspired by the *Shapley Value* – it distributes part of the auctioneer's surplus among the mediators according to their expected relative marginal contribution to the revenue, when ordered randomly.[5] We first show that this mechanism always admits a *pure* Nash equilibrium, a property we discovered to hold for arbitrary games where the value of the game is distributed among players according to Shapley's value function.

▶ **Theorem 1.4.** *Let $\mathcal{G}_m$ be a non-cooperative m-player game in which the payoff of each player is set according to $\mathcal{S}$. Then $\mathcal{G}_m$ admits a pure Nash equilibrium. Moreover, best response dynamics are guaranteed to converge to such an equilibrium.*

We then turn to analyze the revenue guarantees of our mechanism $\mathcal{S}$. Our first theorem shows that using the mechanism $\mathcal{S}$ never decreases the revenue of the auctioneer compared to the initial state (i.e., when all mediators are silent).

▶ **Theorem 1.5.** *For every Nash equilibrium $(\mathcal{P}'_1, \mathcal{P}'_2, \dots, \mathcal{P}'_m)$ of $\mathcal{S}$, $R(\times_{t\in M}\mathcal{P}'_t) \geq R(\{I\})$.*

The next two theorems provide tight bounds on the *price of anarchy* and *price of stability* of $\mathcal{S}$.[6] Unlike in the computational setup, even restricting the mediators to be local experts does not enable us to get improved results here.

▶ **Theorem 1.6.** *For $k \geq 2$, the price of anarchy of $\mathcal{S}$ under any instance* $\mathbf{DSP}(n, k, m)$ *is no more than* $\min\{k - 1, n\}$.

▶ **Theorem 1.7.** *For every $n \geq 1$, there is a* $\mathbf{DSP}(3n + 1, n + 2, 2)$ *instance for which the price of stability of $\mathcal{S}$ is at least n. Moreover, all the mediators in this instance are local experts.*

Interestingly, an adaptation of Shapley's uniqueness theorem [17] to our non-cooperative setting asserts that the price of anarchy of our mechanism is inevitable if one insists on a few natural requirements – essentially anonymity and efficiency[7] of the payment rule – and assuming the auctioneer alone can introduce payments. We discuss this further in the full version of this paper [10].

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[5] Shapley's value was originally introduced in the context of cooperative games, where there is a well defined notion of a coalition's value. In order to apply this notation to a non-cooperative game, we assume the game has some underlying global function ($v(\cdot)$) assigning a value to every strategy profile of the players, and the Shapley value of each player is defined with respect to $v(\cdot)$. In this setting, a "central planner" (the auctioneer in our case) is the one making the utility transfer to the "coalised" players. For the formal axiomatic definition of a value function and Shapley's value function, see [17].

[6] The price of anarchy (stability) is the ratio between the revenue of the optimum and the worst (best) Nash equilibrium.

[7] I.e., the sum of payments is equal to the total surplus of the auctioneer.

1.3 Additional Related Work

The formal study of internet auctions with contexts was introduced by [8] where the authors studied the impact of contexts in the related Sponsored Search model, and showed that *bundling contexts* may have a significant impact on the revenue of the auctioneer. The subsequent work of Ghosh et al. [11] considered the computational algorithmic problem of computing the revenue maximizing partition of items into bundles, under a second price auction in the full information setting. Recently, Emek et al. [7] and Bro Miltersen and Sheffet [2] studied signaling (which generalizes bundling) in the context of display advertising. They explore the computational complexity of computing a signaling scheme that maximizes the auctioneer's revenue in a Bayesian setting. On the other hand, Guo and Deligkas [12] studied a special case of bundling where only "natural" bundles are allowed. Unlike our distributed setup, all the above models are *centralized*, in the sense that the auctioneer has full control over the bundling process (which in our terms corresponds to having a single mediator with a perfect knowledge about the item sold).

A different model with knowledgeable third parties was recently considered by Cavallo et al. [3]. However, the focus of this model is completely different then ours. More specifically, third parties in this model use their information to estimate the clicks-per-impression ratio, and then use this estimate to bridge between advertisers who would like to pay-by-click and ad networks which use a pay-by-impression payment scheme.

2 Preliminaries

Throughout the paper we use capital letters for sets and calligraphic letters for set families. For example, the partition $\mathcal{P}_t$ representing the knowledge of mediator t is a set of sets, and therefore, should indeed be calligraphic according to this notation. A mechanism $\mathcal{M}$ is a tuple of payment functions $(\Pi_1, \Pi_2, \ldots, \Pi_m)$ determining the compensation of every mediator given a strategy profile (i.e., $\Pi_t : \Omega(\mathcal{P}_1) \times \Omega(\mathcal{P}_2) \times \ldots \times \Omega(\mathcal{P}_m) \longrightarrow \mathbb{R}^+$). Every mechanism $\mathcal{M}$ induces the following game between mediators.

▶ **Definition 2.1** (**DSP** game)**.** Given a mechanism $\mathcal{M} = (\Pi_1, \Pi_2, \ldots, \Pi_m)$ and an instance $\mathbf{DSP}(n, k, m)$, the $\mathbf{DSP}_{\mathcal{M}}(n, k, m)$ game is defined as follows. Every mediator $t \in M$ is a player whose strategy space consists of all partitions $\mathcal{P}'_t$ for which $\mathcal{P}_t$ is a refinement. Given a strategy profile $\mathcal{P}'_1, \mathcal{P}'_2, \ldots, \mathcal{P}'_m$, the payoff of mediator t is $\Pi_t(\mathcal{P}'_1, \mathcal{P}'_2, \ldots, \mathcal{P}'_m)$.

Given a **DSP** instance and a set $S \subseteq I$, we use the shorthand $v(S) := \max^{(2)}_{i \in B}(v_{i,S})$ to denote the second highest bid in the restricted auction $\mu|_S$. Using this notation, the expected revenue $R(\mathcal{P})$ of the auctioneer under the (joint) partition $\mathcal{P}$ of the mediators can be restated as $R(\mathcal{P}) = \sum_{S \in \mathcal{P}} \mu(S) \cdot v(S)$.

For a $\mathbf{DSP}_{\mathcal{M}}$ game, let $\mathcal{E}(\mathcal{M})$ denote the set of Nash equilibria of this game and let $\mathcal{P}^*$ be a maximum revenue strategy profile. The *Price of Anarchy* and *Price of Stability* of $\mathbf{DSP}_{\mathcal{M}}$ are defined as:

$$\mathsf{PoA} := \max_{\mathcal{P} \in \mathcal{E}(\mathcal{M})} \frac{R(\mathcal{P}^*)}{R(\mathcal{P})} , \qquad \text{and} \qquad \mathsf{PoS} := \min_{\mathcal{P} \in \mathcal{E}(\mathcal{M})} \frac{R(\mathcal{P}^*)}{R(\mathcal{P})} ,$$

respectively. Notice that our definition of the price of anarchy and price of stability differs from the standard one by using revenue instead of social welfare.

Paper Organization. The proofs of our results for the computational and strategic setups are given in Sections 3 and 4, respectively. Unfortunately, due to space constraints, many proof are omitted from these sections, and are deferred to the full version of this paper [10]. Section 5 summarizes our contributions and discusses possible avenues for future research.

3 The Computational Complexity of Distributed Signaling (DSP)

This section explores **DSP** from a pure combinatorial optimization viewpoint. In other words, we assume the auctioneer can control the signals produced by each mediator. The objective of the auctioneer is then to choose a distributed strategy profile $\mathcal{P}'_1, \mathcal{P}'_2, \ldots, \mathcal{P}'_m$ whose combination $\times_t \mathcal{P}'_t$ yields maximum revenue in the resulting auction. In light of Theorem 1.1, an efficient algorithm with a reasonable approximation guarantee for general **DSP** is unlikely to exist when the three parameters of the problem are all "large". Subsection 3.1 gives a trivial algorithm which has a good approximation guarantee when either n or k is small. A more interesting result is given in Subsection 3.2, which proves a 5-approximation algorithm for **DSP** under the assumption that the mediators are *local experts*. Due to space constraints, the proof of our negative result (i.e., Theorem 1.1) is omitted from this extended abstract.

3.1 A Simple $\min\{n, k-1\}$-Approximation Algorithm for DSP

In this section we prove the following theorem:

▶ **Theorem 1.2.** *For $k \geq 2$, there is a polynomial time $\min\{n, k-1\}$-approximation algorithm for $\mathbf{DSP}(n, k, m)$.*

Proof. We show that the algorithm that simply returns the partition $\{I\}$, the joint partition corresponding to the case where all mediators are silent, has the promised approximation guarantee. For that purpose we analyze the revenue of $\{I\}$ in two different ways:

- Let $\mathcal{P}' = (\mathcal{P}'_1, \mathcal{P}'_2, \ldots, \mathcal{P}'_m)$ be an arbitrary strategy profile of the instance in question. The revenue of $\mathcal{P}'$ is:

$$R(\times_{t=1}^m \mathcal{P}'_t) = \sum_{S \in \times_{t=1}^m \mathcal{P}'_t} \mu(S) \cdot v(S) \leq |\times_{t=1}^m \mathcal{P}'_t| \cdot \max_{S \in \times_{t=1}^m \mathcal{P}'_t} \mu(S) \cdot v(S)$$
$$\leq n \cdot \max_{S \in \times_{t=1}^m \mathcal{P}'_t} \mu(S) \cdot v(S) \leq n \cdot R(\{I\}) \enspace ,$$

  where the last inequality holds since, for every set S, $R(\{I\}) = v(I) \geq v(S) \cdot \mu(S)$. This shows that the approximation ratio of the trivial strategy profile $\{I\}$ provides an n-approximation to the optimal revenue.

- Let $\mathcal{P}' = (\mathcal{P}'_1, \mathcal{P}'_2, \ldots, \mathcal{P}'_m)$ be an arbitrary strategy profile of the instance in question. The revenue of $\mathcal{P}'$ is:

$$R(\times_{t=1}^m \mathcal{P}'_t) = \sum_{S \in \times_{t=1}^m \mathcal{P}'_t} \mu(S) \cdot v(S) = \sum_{S \in \times_{t=1}^m \mathcal{P}'_t} \mu(S) \cdot \left(\max{}^{(2)}_{i \in B} \frac{\sum_{j \in S} \mu(j) \cdot v_{i,j}}{\mu(S)} \right)$$
$$= \sum_{S \in \times_{t=1}^m \mathcal{P}'_t} \left(\max{}^{(2)}_{i \in B} \sum_{j \in S} \mu(j) \cdot v_{i,j} \right) \enspace .$$

  For every bidder $i \in B$, let $\Sigma_i = \sum_{j \in I} \mu(j) \cdot v_{ij}$. It is easy to see that $v(I) = \max^{(2)}_{i \in B} \Sigma_i$ (in other words, the second highest Σ_i value is $v(I)$). Let $i^* \in B$ be the index maximizing Σ_{i^*} (breaking ties arbitrary). Consider a set $S \in \times_{t=1}^m \mathcal{P}'_t$. The elements of S contribute at least $\max^{(2)}_{i \in B} \sum_{j \in S} \mu(j) \cdot v_{i,j}$ to at least two of the values: $\Sigma_1, \ldots, \Sigma_n$. Thus, they contribute at least the same quantity to the sum $\sum_{i \in B \setminus \{i^*\}} \Sigma_i$. This means that at least one of the values $\{\Sigma_i\}_{i \in B \setminus \{i^*\}}$ must be at least:

$$\frac{\sum_{S \in \times_{t=1}^m \mathcal{P}'_t} \left(\max^{(2)}_{i \in B} \sum_{j \in S} \mu(j) \cdot v_{i,j}\right)}{k-1} = \frac{R(\times_{t=1}^m \mathcal{P}'_t)}{k-1} \ .$$

By definition Σ_{i^*} must also be at least that large, and therefore, $R(\{I\}) = v(I) \geq R(\times_{t=1}^m \mathcal{P}'_t)/(k-1)$. ◀

3.2 A 5-Approximation Algorithm for Local Expert Mediators

In this subsection we consider an interesting special case of **DSP** which is henceforth shown to admit a constant factor approximation.

▶ **Definition 3.1** (Local Expert mediators). A mediator t in a **DSP** instance is a *local expert* if there exists a set $I_t \subseteq I$ such that: $\mathcal{P}_t = \{\{j\} \mid j \in I_t\} \cup \{I \setminus I_t\}$.

Informally, a local expert mediator has perfect knowledge about a single set I_t – if the item belongs to I_t, he can tell exactly which item it is. In other words, a local expert mediator specializes in some kind of items to the extent that it knows everything about this kind of items, and nothing at all about other kinds of items. Our objective in the rest of the section is to prove Theorem 1.3, i.e., to describe a 5-approximation algorithm for instances of **DSP** consisting of only local expert mediators.

We begin the proof with an upper bound on the revenue of the optimal joint strategy, which we denote by $\mathcal{P}^*$. To describe this bound, we need some notation. We use $\hat{I}$ to denote the set of items that are within the experty field of some mediator (formally, $\hat{I} = \bigcup_{t \in M} I_t$). Additionally, for every item $j \in I$, h_j and s_j denote $\mu(j)$ times the largest value and second largest value, respectively, of j for any bidder (more formally, $h_j = \mu(j) \cdot \max_{i \in B} v_{i,j}$ and $s_j = \mu(j) \cdot \max^{(2)}_{i \in B} v_{i,j}$).

Next, we need to partition the items into multiple sets. The optimal joint partition $\mathcal{P}^*$ is obtained from partitions $\{\mathcal{P}^*_t\}_{t \in M}$, where $\mathcal{P}^*_t$ is a possible partition for mediator t. Each part of $\mathcal{P}^*$ is the intersection of $|M|$ parts, one from each partition in $\{\mathcal{P}^*_t\}_{t \in M}$. On the other hand, each part of $\mathcal{P}^*_t$ is a subset of I_t, except for maybe a single part. Hence, there exists at most a single part $I_0 \in \mathcal{P}^*$ such that $I_0 \not\subseteq I_t$ for any $t \in M$. For ease of notation, if there is no such part (which can happen when $\hat{I} = I$) we denote $I_0 = \varnothing$. To partition the items of $I \setminus I_0$, we associate each part $S \in \mathcal{P}^* \setminus \{I_0\}$ with an arbitrary mediator t such that $S \subseteq I_t$, and denote by A_t the set of items of all the parts associated with mediator t. Observe that the construction of A_t guarantees that $A_t \subseteq I_t$. Additionally, $\{I_0\} \cup \{A_t\}_{t \in M}$ is a disjoint partition of I.

A different partition of the items partitions them according to the bidder that values them the most. In other words, for every $1 \leq i \leq k$, H_i is the set of items for which bidder i has the largest value. If multiple bidders have the same largest value for an item, we assign it to the set H_i of an arbitrary one of these bidders. Notice that the construction of H_i guarantees that the sets $\{H_i\}_{i \in B}$ are disjoint.

Finally, for every set $S \subseteq I$, we use $\phi(S)$ to denote the sum of the $|B| - 1$ smaller values in $\{\sum_{j \in H_i \cap S} h_j\}_{i \in B}$, i.e., the sum of all the values except the largest one. In other words, we calculate for every bidder i the sum of its values for items in $H_i \cap S$, and then add up the $|B| - 1$ smaller sums. Using all the above notation we can now state our promised upper bound on $R(\mathcal{P}^*)$.

▶ **Lemma 3.2.** $R(\mathcal{P}^*) \leq \mu(I_0) \cdot v(I_0) + \sum_{j \in \hat{I}} s_j + \sum_{t \in M} \phi(A_t)$.

Proof. Fix an arbitrary mediator $t \in M$, and let i be the bidder whose term is not counted by $\phi(A_t)$. For every part $S \in \mathcal{P}^*$ associated with t, let i' be a bidder other than i that has one of the two largest bids for S. By definition:

$$\mu(S) \cdot v(S) = \max^{(2)}_{i'' \in B} \sum_{j \in S} \mu(j) \cdot v_{i'',j} \leq \sum_{j \in S} \mu(j) \cdot v_{i',j} \leq \sum_{j \in S \cap H_i} s_j + \sum_{j \in S \setminus H_i} h_j \ .$$

Summing over all parts associated with t, we get:

$$\sum_{\substack{S \in \mathcal{P}^* \\ S \subseteq A_t}} \mu(S) \cdot v(S) \leq \sum_{j \in A_t \cap H_i} s_j + \sum_{j \in A_t \setminus H_i} h_j \leq \sum_{j \in A_t} s_j + \phi(A_t) \ .$$

Summing over all mediators, we get:

$$R(\mathcal{P}^*) - \mu(I_0) \cdot v(I_0) \leq \sum_{t \in M} \left(\sum_{j \in A_t} s_j + \phi(A_t) \right) \leq \sum_{j \in \hat{I}} s_j + \sum_{t \in M} \phi(A_t) \ .$$ ◀

Our next step is to describe joint partitions that can be found efficiently and upper bound the different terms in the bound given by Lemma 3.2 (up to a constant factor). Finding such partitions for the first two terms is quite straightforward.

▶ **Observation 3.3.** *The joint partitions where all mediators are silent $\{I\} = \times_{i \in B} \{I\}$ obeys: $R(\{I\}) \geq \mu(I_0) \cdot v(I_0)$.*

Proof.

$$R(\{I\}) = \max^{(2)}_{i \in B} \left(\sum_{j \in I} \mu(j) \cdot v_{i,j} \right) \geq \max^{(2)}_{i \in B} \left(\sum_{j \in I_0} \mu(j) \cdot v_{i,j} \right) = \mu(I_0) \cdot v(I_0) \ .$$ ◀

▶ **Observation 3.4.** *The joint partitions $\mathcal{P}_S = \times_{t \in M} \mathcal{P}_t$ where every mediator reports all his information obeys:*

$$R(\mathcal{P}_S) = R(\{\{j\}_{j \in \hat{I}}\} \cup \{I \setminus \hat{I}\}) \geq \sum_{j \in \hat{I}} \mu(j) \cdot \max^{(2)}_{i \in B} v_{i,j} = \sum_{j \in \hat{I}} s_j \ .$$

It remains to find a joint partition that upper bounds, up to a constant factor, the third term in the bound given by Lemma 3.2. If one knows the sets $\{A_t\}_{t \in M}$, then one can easily get such a partition using the method of Ghosh et al. [11]. In this method, one partitions every set A_t into the parts $\{A_t \cap H_i\}_{i=1}^t$ and sort these parts according to the value of $\sum_{j \in A_t \cap H_i} h_j$. Then, with probability $1/2$ every even part is united with the part that appears after it in the above order, and with probability $1/2$ it is united with the part that appears before it in this order. It is not difficult to verify that if the part of bidder i is not the first in the order, then with probability $1/2$ it is unified with the part that appears before it in the order, and then it contributes $\sum_{j \in A_t \cap H_i} h_j$ to the revenue. Hence, the expected contribution to the revenue of the parts produced from A_t is at least $1/2 \cdot \phi(A_t)$.

Algorithm 1 can find a partition that is competitive against $\sum_{t \in M} \phi(A_t)$ without knowing the sets $\{A_t\}_{t \in M}$. The algorithm uses the notation of a *cover*. We say that a set S_j is a cover of an element $j \in I_t \cap H_i$ if $S_j \subseteq I_t \cap H_{i'}$ for some $i \neq i'$.

Notice that the definition of cover guarantees that a part containing both j and S_j contributes to the revenue at least $\min\{h_j, \sum_{j' \in S_j h_{j'}}\}$. Using this observation, each iteration of Algorithm 1 can be viewed as trying to extract revenue from element j. Additionally, observe that the partition $\mathcal{P}$ produced by Algorithm 1 can be presented as a joint partition since every part in it, except for $I \setminus \hat{I}$, contains only items that belong to a single set I_t (for some mediator $t \in M$).

Algorithm 1: Local Experts - Auxiliary Algorithm

1 Let $I' \leftarrow \hat{I}$ and $\mathcal{P} \leftarrow \{I \setminus \hat{I}\}$.
2 **while** $I' \neq \varnothing$ **do**
3 Let j be the element maximizing h_j in I'.
4 Find a cover $S_j \subseteq I'$ of j obeying $h_j \leq \sum_{j' \in S_j} h_{j'} \leq 2h_j$, or maximizing $\sum_{j' \in S_j} h_{j'}$ if no cover of j makes this expression at least h_j.
5 Add the part $S_j \cup \{j\}$ to $\mathcal{P}$, and remove the elements of $S_j \cup \{j\}$ from I'.
6 **return** $\mathcal{P}$

▶ **Observation 3.5.** *Algorithm 1 can be implemented in polynomial time.*

Proof. One can find a cover S_j maximizing $\sum_{j' \in S_j} h_{j'}$ in line 4 of the algorithm by considering the set $I_t \cap H_{i'} \cap I'$ for every mediators t and bidder i' obeying $j \in I_t$ and $j \notin H_{i'}$. Moreover, if this cover is of size larger than $2h_j$, then by removing elements from this cover one by one the algorithm must find a cover S'_j obeying $h_j \leq \sum_{j' \in S'_j} h_{j'} \leq 2h_j$ because j is the element maximizing h_j in I'. ◀

The following lemma relates the revenue of the set produced by Algorithm 1 to the sum $\sum_{t \in M} \phi(A_t)$.

▶ **Lemma 3.6.** *No iteration of the loop of Algorithm 1 decreases the value of the expression* $R(\mathcal{P}) + {}^1\!/_3 \cdot \sum_{t \in M} \phi(A_t \cap I')$.[8]

Proof. Fix an arbitrary iteration. There are two cases to consider. First, assume $h_j \leq \sum_{j' \in S_j} h_{j'} \leq 2h_j$. In this case the increase in $R(\mathcal{P})$ during this iteration is:

$$\mu(S_j \cup \{j\}) \cdot v(S_j \cup \{j\}) \geq \min\left\{h_j, \sum_{j' \in S_j} h_{j'}\right\} = h_j \quad .$$

On the other hand, one can observe that, when removing an element j' from S, the value of $\phi(S)$ can decrease by at most $h_{j'}$. Hence, the decrease in $\sum_{t \in M} \phi(A_t \cap I')$ during this iteration can be upper bounded by: $h_j + \sum_{j' \in S_j} h_{j'} \leq 3h_j$.

Consider now the case $\sum_{j' \in S_j} h_{j'} < h_j$. In this case the increase in $R(\mathcal{P})$ during the iteration is:

$$\mu(S_j \cup \{j\}) \cdot v(S_j \cup \{j\}) \geq \min\left\{h_j, \sum_{j' \in S_j} h_{j'}\right\} = \sum_{j' \in S_j} h_{j'} \quad .$$

If j does not belong to A_t for any mediator t, then by the above argument we can bound the decrease in $\sum_{t \in M} \phi(A_t \cap I')$ by $\sum_{j' \in S_j} h_{j'}$. Hence, assume from now on that there exists a mediator t' and a bidder i such that $j \in A_{t'} \cap H_i$. Let $i' \neq i$ be a bidder maximizing $\sum_{j' \in H_{i'} \cap A_{t'} \cap I'} h_{j'}$. Clearly, the removal of a single element from I' can decrease $\phi(A_{t'} \cap I')$ by no more than $\sum_{j' \in H_{i'} \cap A_{t'} \cap I'} h_{j'}$. Hence, the decrease in $\sum_{t \in M} \phi(A_t \cap I')$ during the iteration of the algorithm can be upper bounded by:

$$\sum_{j' \in H_{i'} \cap A_{t'} \cap I'} h_{j'} + \sum_{j' \in S_j} h_{j'} \quad .$$

[8] Before the algorithm terminates $\mathcal{P}$ is a partial partition in the sense that some items do not belong to any part in it. However, the definition of $R(\mathcal{P})$ naturally extends to such partial partitions.

On the other hand, $H_{i'} \cap A_{t'} \cap I'$ is a possible cover for j, and thus, by the optimality of S_j:

$$\sum_{j' \in H_{i'} \cap A_{t'} \cap I'} h_{j'} \leq \sum_{j' \in S_j} h_{j'} \quad . \qquad \blacktriangleleft$$

▶ **Corollary 3.7.** *$R(\mathcal{P}_A) \geq 1/3 \cdot \sum_{t \in M} \phi(A_t)$, where $\mathcal{P}_A$ is the partition produced by Algorithm 1.*

Proof. After the initialization step of Algorithm 1 we have:

$$R(\mathcal{P}) + 1/3 \cdot \sum_{t \in M} \phi(A_t \cap I') \geq 1/3 \cdot \sum_{t \in M} \phi(A_t) \quad .$$

On the other hand, when the algorithm terminates:

$$R(\mathcal{P}) + 1/3 \cdot \sum_{t \in M} \phi(A_t \cap I') = R(\mathcal{P}_A)$$

because $I' = \varnothing$. The corollary now follows from Lemma 3.6. ◀

We are now ready to prove Theorem 1.3.

▶ **Theorem 1.3.** *If mediators are local experts, there exists a polynomial time 5-approximation algorithm for* **DSP**.

Proof. Consider an algorithm that outputs the best solution out of $\{I\}$, $\mathcal{P}_S$ and $\mathcal{P}_A$. The following inequality shows that at least one of these joint partitions has a revenue of $R(\mathcal{P}^*)/5$:

$$R(\{I\}) + R(\mathcal{P}_S) + 3R(\mathcal{P}_A) \geq \sum_{j \in \hat{I}} s_j + \mu(I_0) \cdot v(I_0) + \sum_{t \in M} \phi(A_t) \geq R(\mathcal{P}^*) \quad ,$$

where the first inequality holds by Observations 3.3 and 3.4 and Corollary 3.7; and the second inequality uses the upper bound on $R(\mathcal{P}^*)$ proved by Lemma 3.2. ◀

4 The Strategic Problem

This section explores the **DSP** problem from a strategic viewpoint, in which the auctioneer *cannot* control the signals produced by each mediator, and is, therefore, trying to solicit information from the mediators that would yield a maximal revenue in the auction. In other words, the objective of the auctioneer is to design a mechanism $\mathcal{M}$ whose equilibria (i.e., the signals $\mathcal{P}'_1, \mathcal{P}'_2, \ldots, \mathcal{P}'_m$ which are now chosen strategically by the mediators) induce maximum revenue. Due to space constraints we are only able to present in this extended abstract only a few of our contributions for the strategic settings. Namely, we introduce the *Shapley mechanism* $\mathcal{S}$ and prove some interesting properties of it (Theorems 1.4 and 1.5).

Our mechanism $\mathcal{S}$ aims to incentivize mediators to report useful information, with the hope that global efficiency emerges despite selfish behavior of each mediator. For the sake of generality, we describe $\mathcal{S}$ for a game generalizing **DSP**. Consider a game $\mathcal{G}_m$ of m players where each player t has a finite set A_t of possible strategies, one of which $\varnothing_t \in A_t$ is called the null strategy of t. The value of a strategy profile in the game $\mathcal{G}_m$ is determined by a value function $v : A_1 \times A_2 \times \ldots \times A_m \to \mathbb{R}$. A mechanism $M = (\Pi_1, \Pi_2, \ldots, \Pi_m)$ for $\mathcal{G}_m$ is a set of payments rules. In other words, if the players choose strategies $a_1 \in A_1, a_2 \in A_2, \ldots, a_m \in A_m$, then the payment to player t under mechanism M is $\Pi_t(v, a_1, a_2, \ldots, a_m)$. Notice that **DSP** fits the definition of $\mathcal{G}_m$ when $A_t = \Omega(\mathcal{P}_t)$ is the set of partitions that t can report for every mediator t, and $\varnothing_t$ is the silence strategy $\{I\}$. The appropriate value function v for **DSP** is the function $R(\times_{t=1}^{m}\mathcal{P}'_t)$, where $\mathcal{P}'_t \in A_t$ is the strategy of mediator t. In other words, the value function v of a **DSP** game is equal to the revenue of the auctioneer.

Given a strategy profile $a = (a_1, a_2, \ldots, a_m)$, and subset $J \in [m]$ of players, we write a_J to denote a strategy profiles where the players of J play their strategy in a, and the other players play their null strategies. We abuse notation and denote by $\varnothing$ the strategy profile $a_\varnothing$ where all players play their null strategies. Additionally, we write (a'_t, a_{-t}) to denote a strategy profile where player t plays a'_t and the rest of the players follow the strategy profile a. The mechanism $\mathcal{S}$ we propose distributes the increase in the value of the game (compared to $v(\varnothing)$) among the players according to their Shapley value: it pays each player his expected marginal contribution to the value according to a uniformly random ordering of the m player. Formally, the payoff for player t given a strategy profile a is

$$\Pi_t(a) = \frac{1}{m!} \cdot \sum_{\sigma \in S_m} \left[v\left(a_{\{\sigma^{-1}(j) | 1 \le j \le \sigma(t)\}}\right) - v\left(a_{\{\sigma^{-1}(j) | 1 \le j < \sigma(t)\}}\right) \right] \quad , \tag{1}$$

which can alternatively be written as

$$\Pi_t(a) = \sum_{J \subseteq [m] \setminus \{t\}} \gamma_J \left(v(a_{J \cup \{t\}}) - v(a_J) \right) \quad , \tag{2}$$

where $\gamma_J = \frac{|J|!(m-|J|-1)!}{m!}$ is the probability that the players of J appear before player t when the players are ordered according to a uniformly random permutation $\sigma \in_R S_m$. We use both definitions (1) and (2) interchangeably, as each one is more convenient in some cases than the other. We remark that the above payoffs can be implemented efficiently.[9]

Clearly, the mechanism $\mathcal{S}$ is anonymous (symmetric). The main feature of the Shapley mechanism is that it is efficient. In other words, the sum of the payoffs is exactly equal to the total increase in value (in the case of **DSP**, the surplus revenue of the auctioneer compared to the initial state).[10]

▶ **Proposition 4.1** (Efficiency property). *For every strategy profile $a = (a_1, a_2, \ldots, a_m)$, $v(a) - v(\varnothing) = \sum_{t=1}^{m} \Pi_t(a)$.*

Proof. Recall that the payoff of mediator t is:

$$\frac{1}{m!} \cdot \sum_{\sigma \in S_m} \left[v\left(a_{\{\sigma^{-1}(j) | 1 \le j \le \sigma(t)\}}\right) - v\left(a_{\{\sigma^{-1}(j) | 1 \le j < \sigma(t)\}}\right) \right] \quad .$$

Summing over all mediators, we get:

$$\sum_{t=1}^{m} \Pi_t(\mathcal{P}'_t, \mathcal{P}'_{-t}) = \sum_{t=1}^{m} \left\{ \frac{1}{m!} \cdot \sum_{\sigma \in S_m} \left[v\left(a_{\{\sigma^{-1}(j) | 1 \le j \le \sigma(t)\}}\right) - v\left(a_{\{\sigma^{-1}(j) | 1 \le j < \sigma(t)\}}\right) \right] \right\}$$

[9] Assuming value queries, we can calculate a payoff for every player by drawing a random permutation σ and paying $v\left(a_{\{\sigma^{-1}(j) | 1 \le j \le \sigma(t)\}}\right) - v\left(a_{\{\sigma^{-1}(j) | 1 \le j < \sigma(t)\}}\right)$ for each mediator t. Clearly this procedure produce the payoffs of our mechanism *in expectation*. Alternatively, the *expected* payoff of each player can be approximated using sampling.

[10] One natural alternative for the Shapley mechanism is a VCG-based mechanism. The main disadvantage of this alternative mechanism is that it is not necessarily efficient. In fact, one can easily design instances where a VCG-based mechanism induces a total payoff which is significantly larger than the increase in the value.

$$= \frac{1}{m!} \cdot \sum_{\sigma \in S_m} \sum_{t=1}^{m} \left[v\left(a_{\{\sigma^{-1}(j) | 1 \leq j \leq \sigma(t)\}}\right) - v\left(a_{\{\sigma^{-1}(j) | 1 \leq j < \sigma(t)\}}\right) \right]$$

$$= \frac{1}{m!} \cdot \sum_{\sigma \in S_m} \left[v\left(a_{\{\sigma^{-1}(j) | 1 \leq j \leq m\}}\right) - v\left(a_\varnothing\right) \right] = v(a) - v(\varnothing) \ . \qquad ◀$$

Proposition 4.1 implies the following theorem. Notice that Theorem 1.5 is in fact a restriction of this theorem to the game $\mathbf{DSP}_{\mathcal{S}}$.

▶ **Theorem 4.2.** *For every Nash equilibrium a, $v(a) \geq v(\varnothing)$.*

Proof. A player always has the option of playing his null strategy, which results in a zero payoff for him. Thus, the payoff of a player in a Nash equilibrium can never be negative. Hence, by Proposition 4.1: $v(a) \geq v(\varnothing) + \sum_{i=1}^{m} \Pi_t(a) \geq v(\varnothing)$. ◀

Next, let us prove Theorem 1.4. For convenience, we restate it below.

▶ **Theorem 1.4.** *Let $\mathcal{G}_m$ be a non-cooperative m-player game in which the payoff of each player is set according to $\mathcal{S}$. Then $\mathcal{G}_m$ admits a pure Nash equilibrium. Moreover, best response dynamics are guaranteed to converge to such an equilibrium.*

Proof. We prove the theorem by showing that $\mathcal{G}_m$ is an exact potential game, which in turn implies all the conclusions of the theorem. Recall that an exact potential game is a game for which there exists a potential function $\Phi \colon A_1 \times A_2 \times \cdots \times A_t \to \mathbb{R}$ such that every strategy profile a and possible deviation $a'_t \in A_t$ of a player t obey:

$$\Pi_t(a'_t, a_{-t}) - \Pi_t(a) = \Phi(a'_t, a_{-t}) - \Phi(a) \ . \tag{3}$$

In our case the potential function is $\Phi(a) = \sum_{J \subseteq [m]} \beta_J \cdot v(a_J)$, where $\beta_J = \frac{(|J|-1)!(m-|J|)!}{m!}$. Let us prove that this function obeys (3). It is useful to denote by a' the strategy profile (a'_t, a_{-t}). By definition:

$$\Pi_t(a') - \Pi_t(a) = \sum_{J \subseteq [m] \setminus \{i\}} \gamma_J \left[v(a_{J \cup \{i\}}) - v(a_J) \right] - \sum_{J \subseteq [m] \setminus \{i\}} \gamma_J \left[v(a'_{J \cup \{i\}}) - v(a'_J) \right] \ . \tag{4}$$

For $J \subseteq [m] \setminus \{i\}$, we have $a_J = a'_J$. Plugging this observation into (4), and rearranging, we get:

$$\Pi_t(a') - \Pi_t(a) = \sum_{J \subseteq [m] \setminus \{i\}} \gamma_J \left[v(a_{J \cup \{i\}}) - v(a'_{J \cup \{i\}}) \right] \ . \tag{5}$$

For every J containing i we get: $\alpha_{J \setminus \{i\}} = \beta_J$. Using this observation and the previous observation that $a_J = a'_J$ for $J \subseteq [m] \setminus \{i\}$, (5) can be replaced by:

$$\Pi_t(a') - \Pi_t(a) = \sum_{J \subseteq [m]} \beta_J (v(a'_J) - v(a_J)) = \Phi(a') - \Phi(a) \ . \qquad ◀$$

Before concluding this section, a few remarks regarding the use of $\mathcal{S}$ to **DSP** are in order:

1. The reader may wonder why the auctioneer cannot impose on the mediators any desired outcome $\times_{t \in M} \mathcal{P}'_t$ by offering mediator t a negligible payment if he signals $\mathcal{P}'_t$, and no payment otherwise. However, implementing such a mechanism requires the auctioneer to know the information sets $\mathcal{P}_t$ of each mediator *in advance*. In contrast, our mechanism requires access only to the outputs of the mediators.

2. Proposition 4.1 implies that the auctioneer distributes the entire surplus among the mediators, which seems to defeat the purpose of the mechanism. However, in the target application she can scale the revenue by a factor $\alpha \in (0, 1]$ and only distribute the corresponding fraction of the surplus. As all of our results are invariant under scaling, this trick can be applied in a black box fashion. Thus, we assume, without loss of generality, $\alpha = 1$.
3. We assume mediators never report a signal which is inconsistent with the true identity of the sold element j_R. The main justification for this assumption is that the mediators' signals must be consistent with one another (as they refer to a single element j_R). Thus, given that sufficiently many mediators are honest, "cheaters" can be easily detected.
4. Note that for a particular ordering of the mediators $\sigma \in \mathcal{S}_m$ and a particular joint strategy profile, the marginal payoff of a mediator may be negative (if she is out of luck and contributes negatively to the revenue according to σ). However, we stress that the expected value (over σ) of each mediator is never negative in any equilibrium strategy (by Theorem 4.2). Since in realistic applications the process is assumed to be repeated over time, the probability that a mediator has overall negative payoff is negligible.

5 Discussion

In this paper we have considered computational and strategic aspects of auctions involving third party information mediators. Our main result for the computational point of view shows that it is NP-hard to get a reasonable approximation ratio when the three parameters of the problem are all "large". For the parameters n and k this is tight in the sense that there exists an algorithm whose approximation ratio is good when either one of these parameters is "small". However, we do not know whether a small value for the parameter m allows for a good approximation ratio. More specifically, even understanding the approximation ratio achievable in the case $m = 2$ is an interesting open problem. Observe that the case $m = 2$ already captures (asymptotically) the largest possible price of stability and price of anarchy in the strategic setup,[11] and thus, it is tempting to assume that this case also captures all the complexity of the computational setup.

Unfortunately, most of our results, for both the computational and strategic setups, are quite negative. The class of local experts we describe is a natural mediators class allowing us to bypass one of these negative result and get a constant approximation ratio algorithm for the computational setup. An intriguing potential avenue for future research is finding additional natural classes of mediators that allow for improved results, either under the computational or the strategic setup.

Another possible direction for future research is to study an extension of our distributed setup where bundling is replaced with randomized signaling (similarly to the works of [2] and [7] which introduced randomized signaling into the centralized model of [11]). In the centralized model it turned out that finding the optimal randomized signaling is easier then finding the optimal bundling [2, 7], which is counterintuitive since randomized signaling generalize bundling. Hence, one can hope that randomized signaling might also mitigate some of our inapproximability results.

[11] By Theorem 1.7 the price of stability can be as large as $O(\min\{k, n\})$ even for two mediators, and Theorem 1.6 shows that the price of anarchy cannot be larger than that for any number of mediators.

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