Decentralized Asynchronous Crash-Resilient Runtime Verification*

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Abstract

Runtime Verification (RV) is a lightweight method for monitoring the formal specification of a system during its execution. It has recently been shown that a given state predicate can be monitored consistently by a set of crash-prone asynchronous *distributed* monitors, only if sufficiently many different verdicts can be emitted by each monitor. We revisit this impossibility result in the context of LTL semantics for RV. We show that employing the four-valued logic RV-LTL will result in inconsistent distributed monitoring for some formulas. Our first main contribution is a family of logics, called LTL_{2k+4}, that refines RV-LTL incorporating $2k+4$ truth values, for each $k \geq 0$. The truth values of LTL_{2k+4} can be effectively used by each monitor to reach a consistent global set of verdicts for each given formula, provided k is sufficiently large. Our second main contribution is an algorithm for monitor construction enabling fault-tolerant distributed monitoring based on the aggregation of the individual verdicts by each monitor.

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1 Introduction

Runtime Verification (RV) is a technique where a *monitor* process determines whether or not the current execution of a system under inspection complies with its formal specification. The state-of-the-art RV methods for distributed systems exhibit the following shortcomings. They (1) employ a central monitor, (2) employ several monitors but lack a systematic way to monitor formally specified properties of a system (e.g., [12, 10, 11]), or (3) assume a fault-free setting, where each individual monitor is resilient to failures [16, 7, 15, 17, 19, 5, 8]. Relaxing the latter assumption, that is, handling monitors subject to failures, poses significant challenges as individual monitors would become unable to agree on the same perspective of the execution, due to the impossibility of consensus [9]. Thus, it is unavoidable

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that individual monitors emit different *local* verdicts about the current execution, so that a consistent *global* verdict with respect to a correctness property can be constructed from these verdicts.

The necessity of using more than just the two truth values of Boolean logic is a known fact in the context of RV with a single monitor. For instance, RV-LTL [3] has four truth values $\mathbb{B}_4 = \{\top, \bot, \top_p, \bot_p\}$. These values identify cases where a finite execution (1) permanently satisfies, (2) permanently violates, (3) presumably satisfies, or (4) presumably violates an LTL formula. For example, consider a request/acknowledge property, where a request r_1 is eventually responded by acknowledgement a_1, and a_1 should not occur before r_1; i.e., LTL formula $\varphi = \mathbf{G}(\neg a_1 \wedge \neg r_1) \vee [(\neg a_1 \, \mathbf{U} \, r_1) \wedge \mathbf{F} a_1]$. In RV-LTL, a finite execution containing r_1 and ending in a_1 (i.e., the request has been acknowledged) yields the truth value 'permanently satisfied', whereas an execution containing only r_1 (i.e., the request has not yet been acknowledged) yields 'presumably violated'.

Although RV-LTL can monitor φ (see Figure 1 for its monitor automaton) in a centralized setting, we show $\mathbb{B}_4$ is not sufficient to *consistently* monitor a conjunction of two such formulas in a framework of several asynchronous unreliable monitors. Namely, the set of verdicts emitted by the monitors may not be sufficient to distinguish executions that satisfy the formula from those that violate it. Intuitively, this is because each monitor has only a partial view of the system under scrutiny, and after a finite number of rounds of communication among monitors, still too many different perspectives about the global system state remain. In fact, it was proved in [10] using algebraic topology techniques [13] that fault-tolerant distributed monitoring requires that the individual verdicts are taken from a set whose size depends on the formula being monitored.

Our results. In this paper, we propose a framework for distributed fault-tolerant RV. To this end, we make a novel connection between RV and consensus in a failure-prone distributed environment by proposing a *multi-valued temporal logic*. This new logic is a refinement of RV-LTL. More specifically, we propose a family of $(2k + 4)$-valued logics, denoted LTL_{2k+4}, for $k \geq 0$. In particular, LTL_{2k+4} coincides with RV-LTL when $k = 0$. The syntax of LTL_{2k+4} is identical to that of LTL. Its semantics is based on FLTL [14] and LTL_3 [4], two LTL-based finite trace semantics for RV. For each $k \geq 0$, the kth instance of the family has $2k + 4$ truth values, that intuitively represent a *degree of certainty* that the formula is satisfied. We characterize the formulas that when verified at run time with LTL_{2k+4}, no additional information is gained if they are verified with $\text{LTL}_{2k'+4}$, for a larger value k'. We present a monitor construction algorithm that generates a finite-state Moore machine for any given LTL formula and $k \geq 0$.

For example, for formula $\varphi = \varphi_1 \wedge \ldots \wedge \varphi_t$, where each φ_i is an independent request/acknowledgement formula, LTL_{2k+4} can be used to consistently monitor φ, whenever $k \geq t$. In particular, when $t = 2$, the set of truth values is $\mathbb{B}_8 = \{\top_0, \bot_0, \top_1, \bot_1, \top_2, \bot_2, \top, \bot\}$. Moreover, formula φ evaluates to: $\top_0$ (presumably true with the lowest degree of certainty) in a finite execution that does not contain neither r_1 nor a_1, then to $\bot_1$ in an extension where r_1 appears (presumably true with a higher degree of certainty), to $\top_1$ in an extension that includes both r_1 and a_1, to $\bot_2$ if r_2 appears, and finally to $\top$ (permanently true) in an execution that contains r_1, a_1, r_2, and a_2.

Our second contribution is an algorithm for fault-tolerant distributed RV, where the monitors are asynchronous *wait-free* processes that communicate with each other via a read/write shared-memory, and any of them can fail by crashing. (For simplicity we use this abstract model, which is well-understood [2, 13], and is known to be equivalent, with respect to task computability, to a message-passing model where less than half the processes can crash.) Each monitor gets a partial view of the system's global state, communicates with the other monitors a fixed number of rounds, and then emits a verdict from $\mathbb{B}_{2k+4}$. We show how, given any LTL formula and a large enough k, the truth values of LTL_{2k+4} can be effectively used such that a set of verdicts collectively provided by the monitors can be mapped to the verdict computed by a centralized monitor that has full view of the system under inspection. It follows from the general lower bound result in [10] that our algorithm is optimal, meaning that for any $k \geq 0$, there exists an LTL formula that cannot be monitored consistently in LTL_{2k+4}, if k is not sufficiently large. Finally, we prove that the value of k is solely a function of the structure of the LTL formula.

Related Work. While there has been significant progress in sequential monitoring in the past decade, there has been less work devoted to distributed monitoring. Lattice-theoretic centralized and decentralized online predicate detection in distributed systems has been studied in [7, 15]. This line of work does not address monitoring properties with temporal requirements. This shortcoming is partially addressed in [17], but for offline monitoring. In [19], the authors design a method for monitoring safety properties in distributed systems using the past-time linear temporal logic (PLTL). In such a work, however, the valuation of some predicates and properties may be overlooked. This is because monitors gain knowledge about the state of the system by piggybacking on the existing communication among processes. That is, if processes rarely communicate, then monitors exchange little information and, hence, some violations of properties may remain undetected. Runtime verification of LTL for synchronous distributed systems where processes share a single global clock has been studied in [5, 8]. In [6], the authors introduce parallel algorithms for runtime verification of sequential programs. As already mentioned, our work is inspired by the research line of [10, 12, 11], the first one to study the effects of monitor failures in distributed RV. Distributed applications that can be runtime monitored with three *opinions* were studied in [12], and the number of opinions needed to runtime monitor set agreement was analyzed in [11]. More generally, [10] proves a tight lower bound on the number of opinions needed to monitor a property based on its alternation number. The goal of this paper is to give a formal semantics to the opinions studied in [10, 12, 11], and derive a framework in the actual formal context of runtime verification.

2 Background: Linear Temporal Logics for RV

Let AP be a set of *atomic propositions* and $\Sigma = 2^{AP}$ be the set of all possible *states*. A *trace* is a sequence $s_0 s_1 \cdots$, where $s_i \in \Sigma$ for every $i \geq 0$. We denote by Σ^* (resp., Σ^ω) the set of all finite (resp., infinite) traces. Throughout the paper, we denote infinite traces by the letter σ, and finite traces by the letter α. We denote the empty trace by ϵ. For a finite trace $\alpha = s_0 s_1 \cdots s_n$, $|\alpha|$ denotes its *length*, i.e., its number of states $n+1$. Finally, by α^i, we mean trace $s_i s_{i+1} \cdots s_n$ of α. We assume that the syntax and semantics of standard LTL is common knowledge.

Example. We use the following *request/acknowledgement* LTL formula throughout the paper to explain the concepts:

$$\varphi_{ra} = \mathbf{G}(\neg a \wedge \neg r) \ \vee \ [(\neg a \, \mathbf{U} \, r) \ \wedge \ \mathbf{F} a]$$

That is (1) if a request is emitted (i.e., $r = \mathit{true}$), then it should eventually be acknowledged (i.e., $a = \mathit{true}$), and (2) an acknowledgement happens only in response to a request.

Finite LTL (FLTL). In the context of runtime verification, the semantics of LTL is not fully appropriate as it is defined over infinite traces. Finite LTL (FLTL, see [14]) allows us to reason about finite traces for verifying properties at run time. The syntax of FLTL is identical to that of LTL and the semantics is based on the truth values $\mathbb{B}_2 = \{\top, \bot\}$. The semantics of FLTL for atomic propositions and Boolean operators are identical to those of LTL. We now recall the semantics of FLTL for the temporal operators. Let φ, φ_1, and φ_2 be LTL formulas, $\alpha = s_0 s_1 \cdots s_n$ be a non-empty finite trace, and $\models_F$ denote satisfaction in FLTL. We have

$$[\alpha \models_F \mathbf{X}\,\varphi] = \begin{cases} [\alpha^1 \models_F \varphi] & \text{if } \alpha^1 \neq \epsilon \\ \bot & \text{otherwise} \end{cases}$$

and

$$[\alpha \models_F \varphi_1 \,\mathbf{U}\, \varphi_2] = \begin{cases} \top & \text{if } \exists k \in [0,n] : ([\alpha^k \models_F \varphi_2] = \top) \,\wedge\, (\forall \ell \in [0,k), [\alpha^\ell \models_F \varphi_1] = \top) \\ \bot & \text{otherwise} \end{cases}$$

To illustrate the difference between LTL and FLTL, let $\varphi = \mathbf{F}p$ and $\alpha = s_0 s_1 \cdots s_n$. If $p \in s_i$ for some $i \in [0, n]$, then we have $[\alpha \models_F \varphi] = \top$. Otherwise, $[\alpha \models_F \varphi] = \bot$, and this holds even if the program under inspection extends α in the future to a state where p becomes true.

Multi-valued LTLs. As illustrated above, for a finite trace α, FLTL ignores the possible future extensions of α, when evaluating a formula. 3-valued LTL (LTL_3, see [4]) evaluates LTL formulas for finite traces with an eye on possible future extensions. In LTL_3, the set of truth values is $\mathbb{B}_3 = \{\top, \bot, ?\}$, where '$\top$' (resp., '$\bot$') denotes that the formula is permanently satisfied (resp., violated), no matter how the current execution extends, and '?' denotes an unknown verdict; i.e., there exist an extension that can falsify the formula, and another extension that can truthify the formula.

Now, let $\alpha \in \Sigma^*$ be a non-empty finite trace. The truth value of an LTL_3 formula φ with respect to α, denoted by $[\alpha \models_3 \varphi]$, is defined as follows:

$$[\alpha \models_3 \varphi] = \begin{cases} \top & \text{if} \quad \forall \sigma \in \Sigma^\omega : \alpha\sigma \models \varphi \\ \bot & \text{if} \quad \forall \sigma \in \Sigma^\omega : \alpha\sigma \not\models \varphi \\ ? & \text{otherwise.} \end{cases}$$

RV-LTL [3], which we will denote in this paper LTL_4, refines the truth value ? into $\bot_p$ and $\top_p$. That is, $\mathbb{B}_4 = \{\top, \top_p, \bot_p, \bot\}$. More specifically, evaluation of a formula in LTL_4 agrees with LTL_3 if the verdict is $\bot$ or $\top$. Otherwise, (i.e., when the verdict in LTL_3 is ?), LTL_4 utilizes FLTL to compute a more refined truth value.

Now, let $\alpha \in \Sigma^*$ be a finite trace. The truth value of an LTL_4 formula φ with respect to α, denoted by $[\alpha \models_4 \varphi]$, is defined as follows:

$$[\alpha \models_4 \varphi] = \begin{cases} \top & \text{if} \quad [\alpha \models_3 \varphi] = \top \\ \bot & \text{if} \quad [\alpha \models_3 \varphi] = \bot \\ \top_p & \text{if} \quad [\alpha \models_3 \varphi] = ? \,\wedge\, [\alpha \models_F \varphi] = \top \\ \bot_p & \text{if} \quad [\alpha \models_3 \varphi] = ? \,\wedge\, [\alpha \models_F \varphi] = \bot \end{cases}$$

The LTL_4 *monitor* of a formula φ is the unique deterministic finite state machine $\mathcal{M}_4^\varphi = (\Sigma, Q, q_0, \delta, \lambda)$, where Q is a set of states, q_0 is the initial state, $\delta : Q \times \Sigma \to Q$ is the

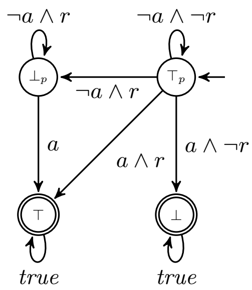

Figure 1 LTL$_4$ monitor of φ_{ra}.

transition function, and $\lambda : Q \to \mathbb{B}_4$, is a function such that:

$$\lambda(\delta(q_0, \alpha)) = [\alpha \models_4 \varphi]$$

for every finite trace $\alpha \in \Sigma^*$. In [4], the authors introduce an algorithm that takes as input an LTL formula and constructs as output an LTL$_4$ monitor. For example, Figure 1 shows the LTL$_4$ monitor for the request/acknowledgement formula $\varphi_{ra} = \mathbf{G}(\neg a \wedge \neg r) \vee [(\neg a\, \mathbf{U}\, r) \wedge \mathbf{F} a]$.

3 Distributed Runtime Monitoring and Insufficiency of LTL$_4$

In this section, we present a general computation model for asynchronous distributed wait-free monitoring. Throughout the rest of the paper, the system under inspection produces a finite trace $\alpha = s_0 s_1 \cdots s_k$, and is inspected with respect to an LTL formula φ by a set $\mathcal{M} = \{M_1, M_2, \ldots, M_n\}$ of asynchronous distributed wait-free monitors.

Algorithm sketch: For every $j \in [0, k-1]$, between each s_j and s_{j+1}, each monitor, in a wait-free manner:

1. reads the value of propositions in s_j, which may result in a *partial* observation of s_j;
2. repeatedly communicates its partial observation with other monitors through a single-writer/multi-reader shared memory;
3. updates its knowledge resulting from the aforementioned communication, and
4. evaluates φ and emits a verdict from $\mathbb{B}_4$.

Since each monitor observes and maintains only a partial view of s_j, and since the monitors run asynchronously, different read/write interleavings are possible, where each interleaving may lead to a different collective set of verdicts emitted by the monitors in $\mathcal{M}$ for s_j. In Subsection 3.1, we formally introduce our notion of *wait-free distributed monitoring*.

To ensure *consistent* distributed monitoring, one has to be able to map a collective set of verdicts of monitors (for any execution interleaving) to one and only one verdict of a centralized monitor that has the full view s_j. A necessary condition for this mapping is that, for every two finite traces $\alpha, \alpha' \in \Sigma^*$, if $[\alpha \models_F \varphi] \neq [\alpha' \models_F \varphi]$, then the monitors in $\mathcal{M}$ should compute different collective sets of verdicts for α and α', no matter what their initial partial observation and subsequent read/write interleavings are. We call this condition *global consistency*, described in detail in Subsection 3.2.

3.1 Wait-Free Distributed Monitoring

We consider a set $\mathcal{M} = \{M_1, M_2, \ldots, M_n\}$ of *monitors*, each observing a system under inspection. We assume that each monitor in $\mathcal{M}$ has only a *partial view* of the system under inspection.

▶ **Definition 1.** A *partial state* is a mapping $\mathcal{S}$ from the set AP of atomic propositions to the set $\{true, false, \natural\}$, where $\natural$ denotes an *unknown* value.

When a state s is reached in a finite trace, each monitor $M_i \in \mathcal{M}$, for $1 \leq i \leq n$, takes a *sample* from s, which results in obtaining a partial state. More formally:

▶ **Definition 2.** A *sample* of a state $s \in \Sigma$ by monitor M_i is a partial state $\mathcal{S}_i^s$ such that, for all $ap \in AP$, we have: $(\mathcal{S}_i^s(ap) = true \rightarrow ap \in s) \wedge (\mathcal{S}_i^s(ap) = false \rightarrow ap \notin s)$.

Definition 2 entails that, in a sample, if the value of an atomic proposition is not unknown, then the sampled value is consistent with state s. Thus, two monitors M_i and M_j cannot take inconsistent samples. That is, for any state s and samples $\mathcal{S}_i^s$, $\mathcal{S}_j^s$, and for every $ap \in AP$, we have: $(\mathcal{S}_i^s(ap) \neq \mathcal{S}_j^s(ap)) \rightarrow (\mathcal{S}_i^s(ap) = \natural \vee \mathcal{S}_j^s(ap) = \natural)$.

We say that a set of monitors *cover* a state if the collection of partial views of these monitors covers the value of the all atomic propositions. Formally:

▶ **Definition 3.** A set $\mathcal{M} = \{M_1, M_2, \ldots, M_n\}$ satisfies *state coverage* for a state s if and only if for every $ap \in AP$, there exists $M_i \in \mathcal{M}$ such that $\mathcal{S}_i^s(ap) \neq \natural$.

Each monitor M_i in $\mathcal{M}$ is a process, and the monitors run in the standard *asynchronous wait-free read/write shared memory* model [2]. Each monitor (1) runs at its own speed, that may vary along with time and (2) may fail by *crashing* (i.e., halt and never recover). We assume that up to $n-1$ monitors can crash, and thus a monitor never "waits" for another monitor (since this may cause a livelock). Every monitor that does not fail is required to output; i.e., to emit a verdict. Hence, a distributed algorithm in this settings consists for each monitor in a bounded sequence of read/write accesses to the shared memory at the end of which a verdict is emitted. If the number of possible inputs is bounded, the lengths of such sequences are globally bounded. We thus assume without loss of generality that each monitor accesses the shared memory a *fixed* number of times before emitting a verdict [13].

More specifically, for every state s_j in $\alpha = s_0 s_1 \cdots s_k$, each monitor M_i maintains a so-called *local snapshot* $LS_i[j]$ consisting of n *registers*, one per monitor in $\mathcal{M}$ (i.e., the local snapshot is organized as an array of registers). We denote by $LS_i^l[j]$ the local register of monitor M_i associated with monitor M_l for state s_j. Each register has $|AP|$ elements, one for each atomic proposition in AP. The monitors in $\mathcal{M}$ communicate by means of *shared memory*. The structure of the shared memory SM is similar to monitor local snapshots: for each state s_j, $SM[j]$ consists of n atomic registers, one per monitor, and each register has $|AP|$ elements one for each atomic proposition (i.e, single-writer/multiple-reader (SWMR) registers). Thus, for state s_j, each monitor M_i can read the entire content of $SM[j]$, but can only write into register $SM_i[j]$[1].

The distributed monitoring algorithm. Each monitor $M_i \in \mathcal{M}$, $i \in [1, n]$, runs Algorithm 1 that we shall now describe in detail. For any given new state s_j, Monitor M_i first initializes all registers of its local snapshot to $\natural$ (cf. Line 1). Then, M_i takes a sample from state s_j (cf. Line 2). Recall from Def. 2 that the value of an atomic proposition in a sample is either true, false, or $\natural$. The set of values in the sample is copied in local register $LS_i^i[j]$.

[1] We assume that each monitor is aware of the change of state of the system under inspection. Thus, for a state s_j, a monitor M_i reads and writes in the associated local and shared memory locations, i.e., $LS_i[j]$ and $SM[j]$.

Algorithm 1: Behavior of Monitor M_i, for $i \in [1, n]$

Data: LTL formula φ and state s_j
Result: a verdict from $\mathbb{B}_4$
1 initialize all elements of $LS_i[j]$ with $\natural$;
2 $LS_i^i[j] \leftarrow \mathcal{S}_i^{s_j}$; /* *take sample from state* s_j */
3 **for** some fixed number of rounds **do**
4 $\quad SM_i[j] \leftarrow \mathbf{p}(LS_i[j])$; /* *write (i.e., project) current knowledge in shared memory* */
5 $\quad LS_i[j] \leftarrow SM[j]$; /* *take a snapshot of the shared memory* */
6 emit $[\mathbf{x}(LS_i[0]) \ldots \mathbf{x}(LS_i[j]) \models_4 \varphi]$; /* *evaluate* φ *using extrapolation function* */

After sampling, each monitor M_i executes a sequence of write/snapshot actions (cf. Lines 4 and 5) for some a priori known number of times, that we detail next[2].

In Line 4, M_i computes its knowledge about each proposition ap, given its content of $LS_i[j]$, and atomically *writes* it into its associated register $SM_i[j]$ in the shared memory. Function $\mathbf{p} = (\mathbf{p}_{ap})_{ap \in AP}$ where $\mathbf{p}_{ap} : \{\mathit{true}, \mathit{false}, \natural\}^n \rightarrow \{\mathit{true}, \mathit{false}, \natural\}$ is the *projection function* defined by

$$\mathbf{p}_{ap}(v_1, \ldots, v_n) = \begin{cases} \mathit{true} & \text{if } \exists i \in [1, n] : v_i = \mathit{true} \\ \mathit{false} & \text{if } \exists i \in [1, n] : v_i = \mathit{false} \\ \natural & \text{otherwise} \end{cases}$$

Given a local snapshot LS_i, $\mathbf{p}(LS_i)$ denotes the partial state obtained by applying $\mathbf{p}_{ap}$ to n values of each atomic proposition ap in LS_i. Notice that, based on Definition 2, $\mathbf{p}$ cannot receive contradicting values for an atomic proposition.

In Line 5, M_i reads of all the registers in $SM[j]$, and copies them into $LS_i[j]$, in a single atomic step. Finally, after a certain number of iterations, the for-loop ends, and M_i evaluates φ and emits a verdict based on the content of its local snapshots $LS_i[0] \cdots LS_i[j]$ (cf. Line 6). To evaluate φ on $s_0 s_1 \cdots s_j$, monitor M_i needs to compute one and only one Boolean value for each atomic proposition. To this end, we assume that for each atomic proposition $ap \in AP$, all monitors are provided with the same *extrapolation function* $\mathbf{x}_{ap}$ allowing them to associate a Boolean value to each atomic proposition, even if its truth value is unknown at some monitors. Such an extrapolation function must satisfy the following consistency condition.

▶ **Definition 4.** Given $ap \in AP$, a function $\mathbf{x}_{ap} : \{\mathit{true}, \mathit{false}, \natural\}^n \rightarrow \{\mathit{true}, \mathit{false}\}$ is an *extrapolation function* if and only if $\mathbf{p}_{ap}(v_1, \ldots, v_n) \neq \natural \;\rightarrow\; \mathbf{x}_{ap}(v_1, \ldots, v_n) = \mathbf{p}_{ap}(v_1, \ldots, v_n)$.

Given a local snapshot array LS, $\mathbf{x}(LS)$ denotes the state obtained by applying $\mathbf{x}_{ap}$ to n values of each atomic proposition ap in LS. Also given a state s_j, by $[\![LS_i[j]]\!]$, we mean the local snapshot of monitor M_i obtained after termination of the for loop in Algorithm 1.

Example. Let $\mathcal{M} = \{M_1, M_2\}$ and consider the formula for two requests and acknowledgements:

$$\varphi_{ra_2} = \Big(\mathbf{G}(\neg a_1 \wedge \neg r_1) \vee [(\neg a_1 \,\mathbf{U}\, r_1) \wedge \mathbf{F} a_1]\Big) \wedge \Big(\mathbf{G}(\neg a_2 \wedge \neg r_2) \vee [(\neg a_2 \,\mathbf{U}\, r_2) \wedge \mathbf{F} a_2]\Big)$$

[2] Algorithm 1 uses snapshot operations for the sake of simplifying the presentation. We emphasize that atomic snapshots can be implemented using atomic read/write operations in a wait-free manner [1].

Figure 2 shows different execution interleavings of monitors M_1 and M_2 when running Algorithm 1 from states $s_0 = \{r_1, a_1\}$ and $s'_0 = \{r_1, a_1, r_2\}$. Based on the order of monitor write-snapshot actions: M_1, M_2 (resp., M_2, M_1) denotes the case where monitor M_1 (resp., M_2) executes a write-snapshot before monitor M_2 (resp., M_1) does, and $M_1||M_2$ denotes the case where monitors M_1 and M_2 execute their write-snapshot actions concurrently. In case of s_0, after executing Line 2 of Algorithm 1, monitor M_1's sample, i.e., the local snapshot $LS_1^1[0]$, consists of $\mathcal{S}_1^{s_0}(r_1) = \mathit{true}$, $\mathcal{S}_1^{s_0}(a_1) = \natural$, and $\mathcal{S}_1^{s_0}(r_2) = \mathcal{S}_1^{s_0}(a_2) = \mathit{false}$. Moreover, initially, M_1 has no knowledge of M_2's sample. Monitor M_2's sample from s_0, i.e., the local snapshot $LS_2^2[0]$, consists of $\mathcal{S}_2^{s_0}(r_1) = \mathcal{S}_2^{s_0}(a_1) = \mathit{true}$, $\mathcal{S}_2^{s_0}(r_2) = \natural$, and $\mathcal{S}_2^{s_0}(a_2) = \mathit{false}$ while it initially has no knowledge of M_1's sample. Likewise, for state s'_0, Figure 2 shows different local snapshots by M_1 and M_2. Given two values v_1 and v_2, we define (an arbitrary) extrapolation function as follows:

$$\mathbf{x}_{ap}(v_1, v_2) = \begin{cases} \mathit{true} & \text{if } (v_1 = \mathit{true}) \vee (v_2 = \mathit{true}) \\ \mathit{false} & \text{otherwise} \end{cases}$$

where $ap \in \{a_1, r_1, a_2, r_2\}$. Finally, starting from s_0, if (1) the for loop of Algorithm 1 terminates after 1 communication round, and (2) the interleaving is M_1, M_2, then $\mathbf{x}([\![LS_2[0]]\!]) = \{r_1, a_1\}$, and evaluation of φ_{ra_2} by M_2 in LTL$_4$ results in $[\mathbf{x}([\![LS_2[0]]\!]) \models_4 \varphi_{ra_2}] = \top_p$.

3.2 Global Consistency

For any state s_j, when a set of monitors execute Algorithm 1, different interleavings, and hence different sets of verdicts, are possible. Global consistency is the property enabling to map the set of verdicts of the distributed monitors to *the* verdict of a centralized monitor that has the full view of states.

▶ **Definition 5.** A *monitor trace* in LTL$_4$ for α is a sequence $m = m_0 m_1 \cdots m_k$, where, for every $j \in [0, k]$, $m_j \subseteq \mathbb{B}_4$, and each element of each m_j is the verdict of some monitor $M_i \in \mathcal{M}$ by evaluating $[\mathbf{x}([\![LS_i[0]]\!])\mathbf{x}([\![LS_i[1]]\!]) \cdots \mathbf{x}([\![LS_i[j]]\!]) \models_4 \varphi]$. For example, Figure 3, shows a concrete finite trace α and its corresponding monitor trace.

▶ **Definition 6.** Let φ be an LTL formula, α be a finite trace in Σ^*, and m be any of its monitor traces. We say that m satisfies *global consistency* in LTL$_4$ iff there exists a function $\mu : 2^{\mathbb{B}_4} \to \{\top, \bot\}$ such that $\mu(m_{|\alpha|-1}) = [\alpha \models_F \varphi]$.

We now show that LTL$_4$ is unable to consistently monitor all LTL formulas. To see this, observe that in Figure 2, the shaded collective verdicts m_0 and m'_0 are both equal to $\{\bot_p, \top_p\}$, but $[s_0 \models_4 \varphi] \neq [s'_0 \models_4 \varphi]$. This clearly does not meet global consistency (see the proof of Lemma 7 for details).

▶ **Lemma 7.** *Not all* LTL *formulas can be consistently monitored by a 1-round distributed monitor with traces in* LTL$_4$*, even if monitors satisfy state coverage, and even if no monitors crash during the execution of the monitor.*

Lemma 7 holds for an arbitrary number of communication rounds as well. Indeed, additional rounds of communication will not result into reaching global consistency. This impossibility result is a direct consequence of the main lower bound in [10], which can be rephrased as follows.

▶ **Theorem 8.** *Not all* LTL *formulas can be consistently monitored by a distributed monitor with traces in* LTL$_4$*, even if monitors satisfy state coverage, even if no monitors crash during the execution of the monitor, and even if the monitors perform an arbitrarily large number of communication rounds.*

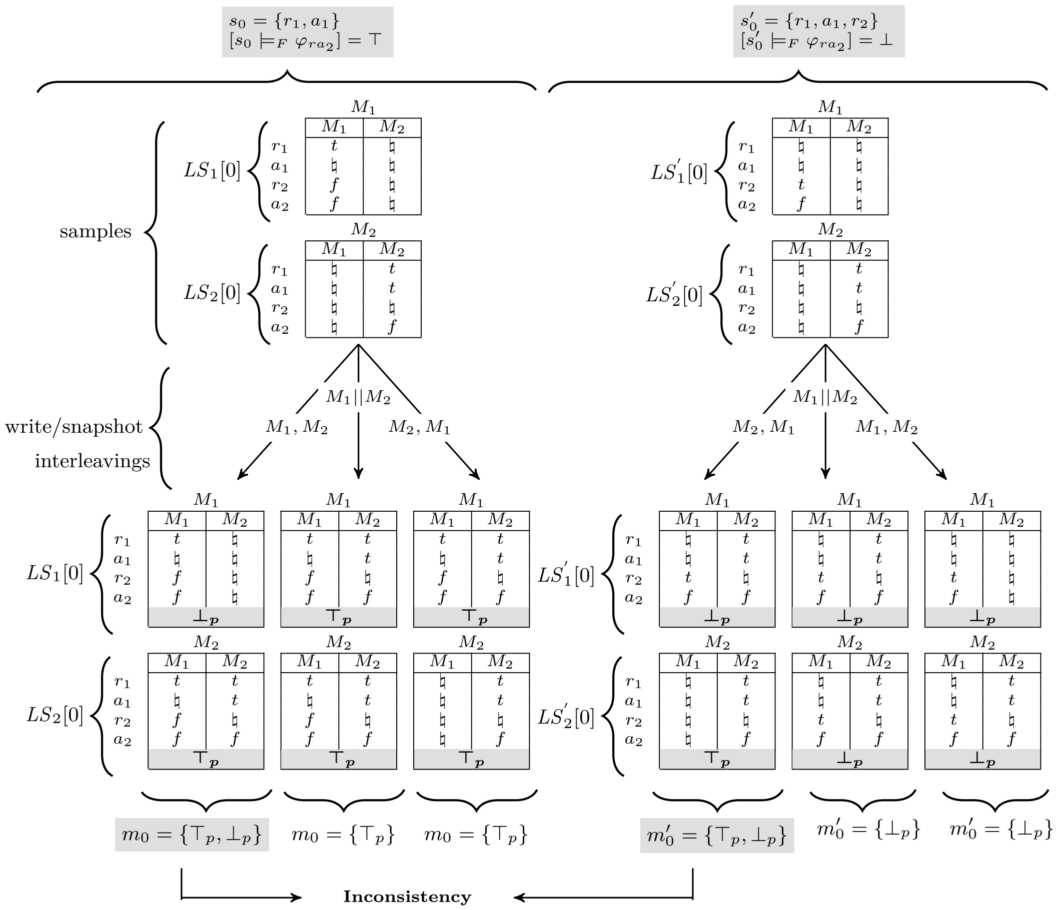

Figure 2 Example: Monitors M_1 and M_2 monitoring formula φ_{ra_2} from two different states s_0 and s'_0.

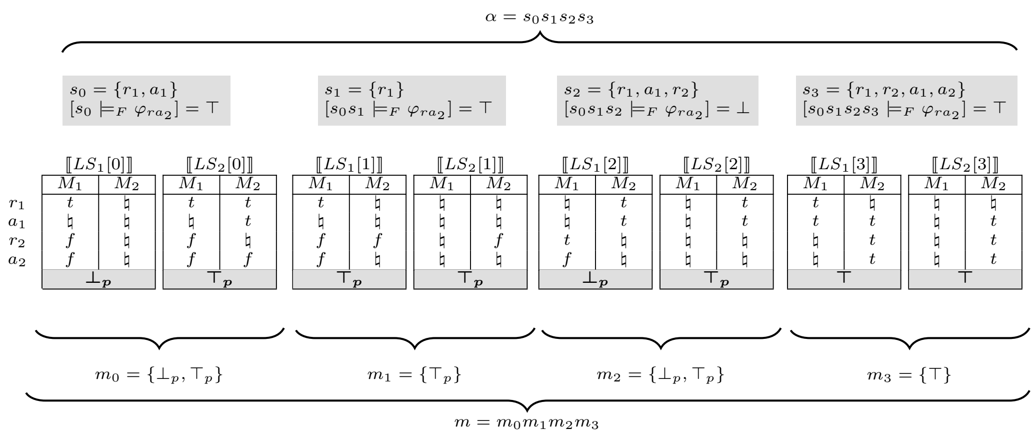

Figure 3 A monitor trace.

In the next section, we revisit the notion of *alternation number* introduced in [10] in order to identify formulas that can be monitored by LTL$_4$, and to design a multi-valued logic to monitor LTL formulas that cannot be monitored in LTL$_4$.

4 Alternation Number

We now define the notion of *alternation number* [10] in the context of LTL. In the next section, we shall show that the alternation number essentially determines an upper bound on the number of truth values needed to ensure consistency in distributed monitoring.

Let $\alpha \in \Sigma^*$ be a finite trace, α' be the longest proper prefix of α, and φ be an LTL formula. We set the *alternation number* of φ with respect to α as follows:

$$AN(\varphi, \alpha) = \begin{cases} 0 & \text{if } |\alpha| = 1 \\ AN(\varphi, \alpha') + 1 & \text{if } (|\alpha| \geq 2) \wedge ([\alpha' \models_F \varphi] \neq [\alpha \models_F \varphi]) \\ AN(\varphi, \alpha') & \text{otherwise} \end{cases}$$

The alternation number with respect to infinite traces is defined as follows. Let $\sigma \in \Sigma^\omega$ be an infinite trace. If for any prefix α of σ, there exists a finite extension α', such that $AN(\varphi, \alpha) < AN(\varphi, \alpha')$, then we set $AN(\varphi, \sigma) = \infty$. Otherwise, we set $AN(\varphi, \sigma) = AN(\varphi, \alpha)$ where α is such that there does not exist a finite extension α' of α such that $AN(\varphi, \alpha) < AN(\varphi, \alpha')$. Finally, the alternation number of φ with respect to a (possibly infinite) set A of traces is

$$AN(\varphi, A) = \max \left\{ AN(\varphi, \alpha) \mid \alpha \in A \right\}$$

▶ **Definition 9.** The *alternation number* of an LTL formula φ is $AN(\varphi) = AN(\varphi, \Sigma^*)$.

Examples. We have $AN(\mathbf{G}\, p) = 1$ because, in any finite trace α, if the valuation of $\mathbf{G}\, p$ in FLTL changes from $\top$ to $\bot$, then, in no extension of α this value can change back to $\top$. We have $AN(\mathbf{G}(r \rightarrow \mathbf{F} a)) = \infty$, because any occurrence of $r \wedge \neg a$ evaluates the formula to $\bot$, and a subsequent occurrence of a evaluates the formula to $\top$ in FLTL. We have $AN(\varphi_{ra}) = AN(\mathbf{G}(\neg a \wedge \neg r) \ \vee\ [(\neg a\, \mathbf{U}\, r) \ \wedge\ \mathbf{F} a]) = 2$. Indeed, as long as $\neg r \wedge \neg a$ is true throughout a trace α, we have $[\alpha \models_F \varphi_{ra}] = \top$. When $r \wedge \neg a$ becomes true, the valuation of φ_{ra} changes to $\bot$. If a becomes true subsequently, then φ_{ra} evaluates to $\top$. By the same type of arguments, we show $AN(\varphi_{ra_2}) = 4$.

Interestingly, the alternation number of an LTL formula φ can be determined from the structure of its LTL_4 monitor automaton M_4^φ.

▶ **Theorem 10.** *Let φ be an LTL formula. The alternation number of φ, $AN(\varphi)$, is equal to the length of the longest alternating walk in its LTL_4 monitor M_4^φ.*

Example. Let $\varphi_{ra} = \mathbf{G}(\neg a \wedge \neg r) \ \vee\ [(\neg a\, \mathbf{U}\, r) \ \wedge\ \mathbf{F} a])$. We have $AN(\varphi_{ra}) = 2$, and one can check on Figure 1 that indeed the length of the longest alternating walk in $M_4^{\varphi_{ra}}$ is 2.

5 Multi-Valued LTL for Consistent Distributed Monitoring

In this section, we introduce a family of multi-valued logics (called LTL_{2k+4}), for every $k \geq 0$, and relate it to the notion of alternation number. For every $k \geq 0$, the syntax of LTL_{2k+4} is identical to that of LTL. We present the semantics, monitor synthesis, and proof of global consistency of LTL_{2k+4} in Subsections 5.1, 5.2, and 5.3, respectively.

5.1 Semantics of LTL_{2k+4}

Truth values. The semantics of LTL_{2k+4} refines LTL_4. LTL_{2k+4} employs the following set of $2k+4$ truth values:

$$\mathbb{B}_{2k+4} = \{\bot, \top, \bot_0, \ldots, \bot_k, \top_0, \ldots, \top_k\}.$$

Intuitively, for $i \in [0,k]$, truth value $\bot_i$ means *possibly false* with *degree of certainty* i, and truth value $\top_i$ means *possibly true* with degree of certainty i, while $\top$ and $\bot$ have the same meaning as their LTL_3 counterparts. Thus, LTL_{2k+4} coincides with LTL_4 for $k=0$. Consider a non-empty finite trace $\alpha = s_0 s_1 \cdots s_n$ in Σ^*. We denote the valuation of a formula φ with respect to α in LTL_{2k+4} by $[\alpha \models_{2k+4} \varphi]$. Since, for any $i \in [0,k]$, $\bot_i$ implies '?' in LTL_3, we require that $[\alpha \models_{2k+4} \varphi] = \bot_i \;\rightarrow\; [\alpha \models_3 \varphi] = ? \wedge [\alpha \models_F \varphi] = \bot$. The latter conjunct is to relate $\bot_i$ with the valuation of α in FLTL. Likewise, we require that, for any $i \in [0,k]$: $[\alpha \models_{2k+4} \varphi] = \top_i \;\rightarrow\; [\alpha \models_3 \varphi] = ? \wedge [\alpha \models_F \varphi] = \top$. We determine the degree of certainty of $[\alpha \models_{2k+4} \varphi]$ inductively according to the judgement rules below, where $\alpha' = s_0 s_1 \cdots s_{n-1}$.

Observe that the degree of certainty does not change if the FLTL valuation does not change in α' and α, or change from $\bot$ to $\top$. On the contrary, the degree of certainty does change if the FLTL valuation changes in α' and α from $\top$ to $\bot$, respectively.

$$[\alpha \models_{2k+4} \varphi] = \begin{cases} \bot & \text{if} \quad [\alpha \models_3 \varphi] = \bot \\ \top & \text{if} \quad [\alpha \models_3 \varphi] = \top \\ \bot_0 & \text{if} \quad |\alpha| = 1 \ \wedge\ [\alpha \models_3 \varphi] = ? \ \wedge\ [\alpha \models_F \varphi] = \bot \\ \top_0 & \text{if} \quad |\alpha| = 1 \ \wedge\ [\alpha \models_3 \varphi] = ? \ \wedge\ [\alpha \models_F \varphi] = \top \\ \top_i \quad \text{with } i \in [0,k] & \text{if} \quad |\alpha| \geq 2 \ \wedge\ [\alpha \models_3 \varphi] = ? \ \wedge\ [\alpha \models_F \varphi] = \top\ \wedge \\ & \qquad [\alpha' \models_{2k+4} \varphi] \in \{\top_i, \bot_i\} \\ \bot_i \quad \text{with } i \in [0,k) & \text{if} \quad |\alpha| \geq 2 \ \wedge\ [\alpha \models_3 \varphi] = ? \ \wedge\ [\alpha \models_F \varphi] = \bot\ \wedge \\ & \qquad [\alpha' \models_{2k+4} \varphi] \in \{\bot_i, \top_{i-1}\} \\ \bot_k & \text{if} \quad |\alpha| \geq 2 \ \wedge\ [\alpha \models_3 \varphi] = ? \ \wedge\ [\alpha \models_F \varphi] = \bot\ \wedge \\ & \qquad [\alpha' \models_{2k+4} \varphi] \in \{\bot_k, \top_k, \top_{k-1}\} \end{cases}$$

5.2 Monitorability and Monitor Synthesis for LTL_{2k+4}

Pnueli and Zaks [18] characterize an LTL formula φ as *monitorable* for a finite trace α, if α can be extended to one that can be evaluated with respect to φ at run time. That is, an LTL formula φ is *monitorable* in LTL_3 if and only if: $\forall \alpha \in \Sigma^* : \exists \alpha' \in \Sigma^* : [\alpha\alpha' \models_3 \varphi] \neq ?$. We stick to the same definition for LTL_{2k+4}.

▶ **Definition 11.** Let φ be an LTL formula. The LTL_{2k+4} *monitor* of φ is the unique deterministic finite state machine $\mathcal{M}^{\varphi}_{2k+4} = (\Sigma, Q, q_0, \delta, \lambda)$, where Q is a set of states, q_0 is the initial state, $\delta : Q \times \Sigma \rightarrow Q$ is the transition function, and $\lambda : Q \rightarrow \mathbb{B}_{2k+4}$, such that, for every non-empty finite trace $\alpha \in \Sigma^*$, we have $[\alpha \models_{2k+4} \varphi] = \lambda(\delta(q_0, \alpha))$.

Algorithm 2 constructs LTL_{2k+4} monitors. Intuitively, our algorithm creates $k+1$ copies of LTL_4 [3] monitors by invoking Function ConstructMonitor, and cascades them in such a way that incrementing the degree of certainty is implemented as prescribed by our definition of LTL_{2k+4}. Observe that for a given value $i \in [0,k]$, Function ConstructMonitor renames truth value $\top_p$ (respectively, $\bot_p$) in LTL_4 to $\top_i$ (respectively, $\bot_i$) (see Lines 14-18). Cascading

Algorithm 2: Monitor construction for LTL_{2k+4}

Input: Alphabet Σ, LTL formula φ, $k \geq 0$
Output: LTL_{2k+4} monitor $M^{\varphi}_{2k+4} = (\Sigma, Q, q_0, \delta, \lambda)$

1 $(Q, q_0, \delta, \lambda) \leftarrow$ ConstructMonitor(Σ, φ, 0);
2 **for** $i \leftarrow 1$ **to** k **do**
3     $(\bar{Q}, \bar{q}_0, \bar{\delta}, \bar{\lambda}) \leftarrow$ ConstructMonitor(Σ, φ, i);
4     $Q \leftarrow Q \cup \bar{Q}$; $\delta \leftarrow \delta \cup \bar{\delta}$; $\lambda \leftarrow \lambda \cup \bar{\lambda}$;
5   **forall the** $q \in Q$, $\bar{q} \in \bar{Q}$ **do**
6   **if** $(\lambda(q) = \top_{i-1} \wedge \lambda(\bar{q}) = \bot_i)$ **then**
7   **forall the** $q' \in Q$, $a \in \Sigma$ **do**
8   **if** $\lambda(q') = \bot_{i-1} \wedge \delta(q, a) = q'$ **then**
9                     $\delta = \delta - \{(q, a, q')\}$;
10                     $\delta = \delta \cup \{(q, a, \bar{q})\}$;

11 **return** $M^{\varphi}_{2k+4} = (\Sigma, Q, q_0, \delta, \lambda)$;

12 **Function** `ConstructMonitor(`*alphabet* Σ, LTL *formula* φ, $i \geq 0$`)`
13 **Let** $\mathcal{M}^{\varphi}_4 = (\Sigma, Q, q_0, \delta, \lambda)$;
14 **forall the** $q \in Q$ **do**
15   **if** $(\lambda(q) = \top_p)$ **then**
16         $\lambda(q) \leftarrow \top_i$;
17   **if** $(\lambda(q) = \bot_p)$ **then**
18         $\lambda(q) \leftarrow \bot_i$;

19 **return** $(Q, q_0, \delta, \lambda)$;

the monitors in Algorithm 2 is as follows. Initially, we generate an LTL_4 monitor for $k = 0$ (Line 1). Then, in each step $i \in [1, k]$ of the for-loop, we generate a new LTL_4 monitor (cf. Line 3). We ensure incrementing the degree of certainty by removing monitor transitions (q, a, q'), where q is annotated by $\top_{i-1}$ and q' is annotated by $\bot_{i-1}$, and adding transitions $(q, a, \bar{q})$, where $\bar{q}$ is annotated by $\bot_i$ (Lines 5-10).

▶ **Theorem 12.** *Let φ be an* LTL *formula, and let $\mathcal{M}^{\varphi}_{2k+4} = (\Sigma, Q, q_0, \delta, \lambda)$ be its LTL_{2k+4} monitor such as constructed by Algorithm 2. Then, for any non-empty finite trace $\alpha \in \Sigma^*$, we have $\lambda(\delta(q_0, \alpha)) = [\alpha \models_{2k+4} \varphi]$.*

5.3 Monitoring Algorithm and Global Consistency in LTL_{2k+4}

Monitoring Algorithm Let $\alpha = s_0 s_1 \cdots s_k$ be a finite trace in Σ^*. As discussed in Section 3, for any state s_j, where $j \in [0, k]$, each monitor runs Algorithm 1 and emits a verdict. In order to employ LTL_{2k+4} and ensure consistency, each monitor has to compute the highest possible degree of certainty by considering all possible monitor communication interleavings that result in state s_j. Formally, the set of all interleavings that reach a state $s \in \Sigma$ is the set of sequences of partial states defined as follows:

$$\mathcal{I}_s = \Big\{ \mathcal{S}_0 \mathcal{S}_1 \cdots \mathcal{S}_l \;\Big|\; (\forall ap \in AP : \mathcal{S}_0(ap) = \natural) \wedge (\mathcal{S}_l = s) \wedge \\ [\forall i \in [0, l) : \forall ap \in AP : (\mathcal{S}_i(ap) \neq \natural) \rightarrow (\forall m \in (i, l] : \mathcal{S}_i(ap) = \mathcal{S}_m(ap))] \Big\}$$

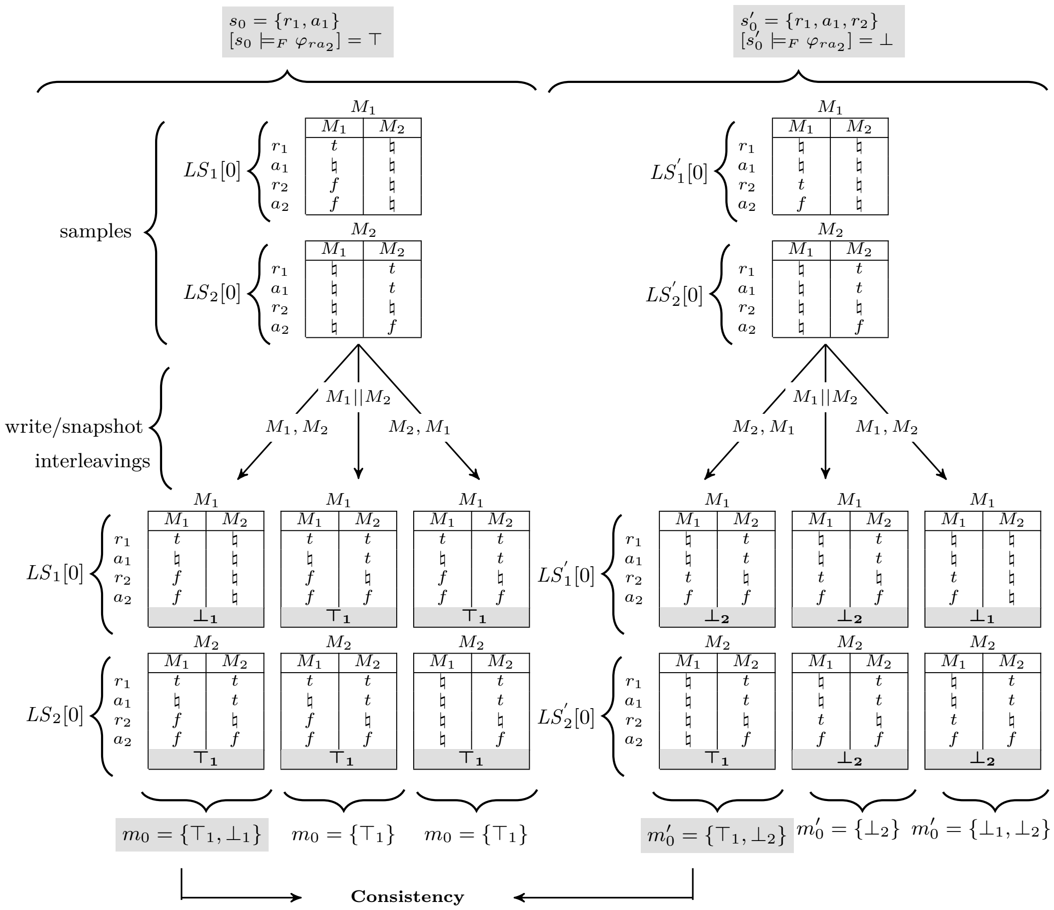

Figure 4 Global consistency of $\textsc{Ltl}_{2k+4}$ monitors M_1 and M_2 for formula φ_{ra_2}, where $k = 2$.

Now, for state s_j in α and formula φ, a monitor M_i computes $AN(\varphi, \mathcal{I}_{\mathbf{x}([\![LS_i[j]]\!])})$. This can be done by running each trace in $\mathcal{I}_{\mathbf{x}([\![LS_i[j]]\!])}$ on the $\textsc{Ltl}_{2k+4}$ monitor of φ. This is indeed the key idea to ensure global consistency.

▶ **Observation 13.** *For any state $s \in \Sigma$ and* LTL *formula φ, we have $AN(\varphi, \mathcal{I}_s) \leq AN(\varphi)$.*

Example. Figure 4 shows how monitors M_1 and M_2 evaluate formula φ_{ra_2} in $\textsc{Ltl}_{2k+4}$ with $k = 2$. Observe that the two sets of verdicts that were not distinguishable in Figure 2 (i.e., $m_0 = m'_0 = \{\bot_p, \top_p\}$) are now distinguishable (i.e., $m_0 = \{\bot_1, \top_1\}$, while $m'_0 = \{\top_1, \bot_2\}$), as we are now using 8 truth values instead of just 4. The ability of monitoring a formula in $\textsc{Ltl}_{2k+4}$ for a given $k \geq 0$ is strongly related to the alternation number of the formula.

Main Results. The following identifies an upper-bound on the number of truth values needed to monitor any LTL formula.

▶ **Theorem 14.** *An* LTL *formula φ can consistently be monitored by a wait-free distributed monitor in $\textsc{Ltl}_{2k+4}$, if*

$$k \geq \lceil \frac{1}{2}(\min(AN(\varphi), n) - 1) \rceil$$

where n is the number of monitors.

An immediate consequence of Theorem 14 is for computing μ (Definition 6) for $\textsc{Ltl}_{2k+4}$. For a set $m \in \mathbb{B}_{2k+4}$, one can compute $\mu(m)$ by identifying the supremum of m, for the total order $\bot_0 < \top_0 < \bot_1 < \top_1 < \ldots < \bot_k < \top_k$. It is straightforward to observe that such a μ results in global consistency for $\textsc{Ltl}_{2k+4}$. Also, notice that Theorem 14 is best possible. It matches the following generalization of Theorem 8. The proof is similar to the lower bound of [10].

▶ **Theorem 15.** *For each $k \geq 0$, there is an* Ltl *formula φ that cannot be consistently monitored by a wait-free distributed monitor in $\textsc{Ltl}_{2k+4}$, if*

$$k < \lceil \frac{1}{2}(\min(AN(\varphi), n) - 1) \rceil$$

where n is the number of monitors.

6 Conclusion and Future Work

In this paper, we proposed a family of multi-valued logics $\textsc{Ltl}_{2k+4}$, each one with $2k + 4$ truth values, for fault-tolerant distributed RV, refining existing finite Ltl semantics. We presented an idealized setting where a set of unreliable monitors emit consistent verdicts in $\textsc{Ltl}_{2k+4}$ about the correctness of the system under inspection, if k is sufficiently large.

We note that wait-free computing is a powerful and simple abstraction to model and reason about distributed algorithms. All results in this paper can theoretically be transformed to more practical refinements such as message passing frameworks. Of course, further research is needed to develop such transformations. From a more practical perspective, it would be interesting to relax the timing model enabling monitors to observe, communicate, and emit verdicts between any two global states; to study frameworks for message passing systems, and to address more severe, even Byzantine failures.

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