# Convex Configurations on Nana-kin-san Puzzle*

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## Abstract

We investigate a silhouette puzzle that is recently developed based on the golden ratio. Traditional silhouette puzzles are based on a simple tile. For example, the tangram is based on isosceles right triangles; that is, each of seven pieces is formed by gluing some identical isosceles right triangles. Using the property, we can analyze it by hand, that is, without computer. On the other hand, if each piece has no special property, it is quite hard even using computer since we have to handle real numbers without numerical errors during computation. The new silhouette puzzle is between them; each of seven pieces is not based on integer length and right angles, but based on golden ratio, which admits us to represent these seven pieces in some nontrivial way. Based on the property, we develop an algorithm to handle the puzzle, and our algorithm succeeded to enumerate all convex shapes that can be made by the puzzle pieces. It is known that the tangram and another classic silhouette puzzle known as Sei-shonagon chie no ita can form 13 and 16 convex shapes, respectively. The new puzzle, Nana-kin-san puzzle, admits to form 62 different convex shapes.

**1998 ACM Subject Classification** G.2.1 Counting problems

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## 1 Introduction

A *silhouette puzzle* is a game where, given a certain set of polygons, one must decide whether all of them can be placed in the plane in such a way that their union is a target figure or letter[1]. Rotation and reflection are allowed but scaling and overlapping are not. Formally, a set of polygons $S$ can *form* a polygon $P$ if there is an isomorphism up to rotation and reflection between a partition of $P$ and the polygons of $S$ (i.e. a bijection $f(\cdot)$ from a partition of $P$ to $S$ such that $x$ and $f(x)$ are congruent for all $x$).

The *tangram* is a set of polygons consisting of a square of material cut by straight incisions into different-sized pieces. See the left diagram in Figure 1. Of anonymous origin, their first known reference in literature is from 1813 in China [7]. The tangram has grown to be extremely popular throughout the world; now, over 2000 dissection and related puzzles exist for it ([7, 3]). Less famous is a quite similar Japanese puzzle called *Sei-shonagon Chie no Ita*

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[1] This is also called a *dissection puzzle* in some context. However, dissection puzzle rather asks how to cut a given polygon into a few pieces so that they can be rearranged to the target polygon. The most famous one is called the haberdasher's problem created by Henry Dudeney [1], which asks to transform a square into a regular triangle by cutting into four pieces.


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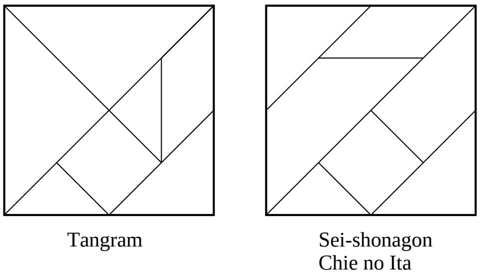


**Figure 1** (Left) Tangram and (Right) Sei-shonagon Chie no Ita.

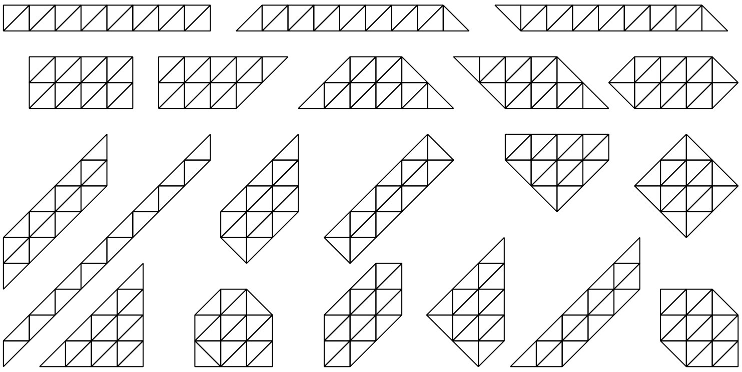

**Figure 2** All 20 potential convex polygons by tangram type puzzles.

(the right diagram in Figure 1). Sei-shonagon was a courtier and famous novelist in Japan, but there is no evidence that the puzzle existed a millennium ago when she was living. *Chie no ita* means *wisdom plates*, which refers to this type of physical puzzle. This puzzle is said to be named after Sei-shonagon's wisdom. Historically, it first appeared in literature in 1742, bit older than the tangram [7].

Wang and Hsiung considered the number of possible convex (filled) polygons formed by the tangram [9]. They first noted that, given sixteen identical isosceles right triangles, one can create the tangram pieces by gluing some edges together. (In technical word in puzzle society, each piece is a *polyabolo*.) So, clearly, the set of convex polygons one can create from the tangram is a subset of those that sixteen identical isosceles right triangles can form. Embedded in the proof of their main theorem, Wang and Hsiung [9] demonstrate that sixteen identical isosceles right triangles can form exactly 20 convex polygons. These 20 polygons are illustrated in Figure 2. The tangram can realize thirteen of those 20. Since the same idea works for the Sei-shonagon Chie no Ita, it is quite natural to ask how many of these twenty convex polygons the Sei-shonagon Chie no Ita pieces can form. In [2], Fox-Epstein, Katsumata and Uehara showed that (1) Sei-shonagon Chie no Ita achieves sixteen convex polygons out of twenty, (2) there are four sets of seven convex polygons in this manner that can form nineteen convex polygons out of twenty (Figure 3), and we cannot improve any more in this context.

For the tangram and the Sei-shonagon Chie no Ita, the key idea is that each piece consists of congruent right isosceles triangles. On the other hand, even if the pieces are in quite simple forms, the silhouette puzzle can be intractable when it has many pieces. In [4], they prove that this problem is NP-complete for general $n$ pieces even if each piece is a rectangle of size $1 \times x_i$ for some integer $x_i$ except only one polygon with 6 vertices. We remark that,

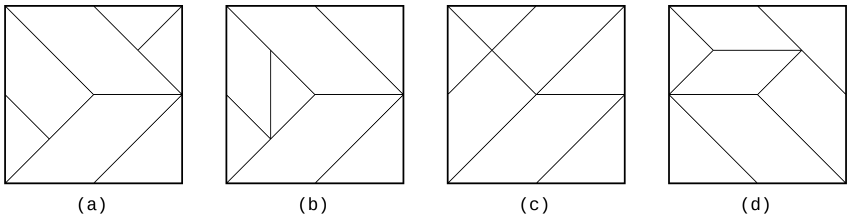


**Figure 3** Four patterns that can form nineteen convex polygons.

the goal in their paper is to form "line symmetric shape," that is, the target shape itself is not explicitly given. However, even if the goal rectangle is explicitly given, the proof in [4] still works and we obtain NP-completeness in our framework with the same set of pieces. In fact, in our framework of silhouette puzzle, we can further improve their result to the set of only rectangles by splitting the last polygon with 6 vertices into three rectangles. In this case, all pieces and the goal shape are just rectangles.

Another interesting problem is ETS polygons discussed in [5] and [6]: Given an integer $n$, we determine whether a convex $n$-gon can be obtained by gluing equilateral triangles and squares in an edge-to-edge manner. In [5], this problem is treated as a dissection problem, and it is shown that $n$ can only be an integer from 3 to 12. In [6], all possible sets of exterior angles for $n$-gons are listed, and for each of the sets, an edge-to-edge glued shape is given as an example.

As seen the fact that there are over 2,000 dissection for the tangram [7, 3], when we consider arbitrary polygons, even a set of seven pieces seems to be intractable since there are essentially infinitely many polygons that can form from the seven pieces of the tangram. (We remark that the tangram itself contains seven convex pieces.) Therefore, it is reasonable to consider as the framework of a silhouette puzzle with the following two assumptions; (1) each piece is convex and (2) target polygon is also convex. Even under this assumption, we still have many variants of problems; in fact, even with only two pieces, the following theorem is mentioned by Uematsu [8]:

▶ **Theorem 1** ([8]). *For any given positive integer $n > 2$, we have a set of two convex polygons that can form $2n$ different convex polygons.*

**Proof.** (Sketch) We here show for general case $n > 4$ since $n = 3$ and $n = 4$ are special cases. Let $P_n$ be a regular $n$-gon. Then we first construct the first piece $P'_n$ by bending $P_n$ little a bit to satisfy the following conditions; each edge has length 1, and every angle is distinct. The second piece $T$ is a shallow triangle; its base edge is of length 1, and two other edges are of length $1/2 + \epsilon_1$ and $1/2 - \epsilon_2$ with $0 < \epsilon_1 - \epsilon_2$. By setting $\epsilon_1$ and $\epsilon_2$ sufficiently small, attaching $T$ at the base edge to $P'_n$, we have $n$ different convex polygons, and by flipping $T$, we also have the other $n$ different convex polygons. ◀

We note that $P$ and its mirror image $P^R$ are regarded as the same shape. We also generalize this idea and obtain exponential lower bound for the number of pieces:

▶ **Theorem 2.** *For any given positive integer $n > 4$, we have a set of $n + 1$ convex polygons of $4n$ vertices in total that can form $2^n \cdot n!$ different convex polygons.*

**Proof.** Let $P'_n$ be the $n$-gon used in the proof of Theorem 1, which is obtained by bending a regular $n$-gon little a bit without changing the length of each edge. We construct a set $S$ of $n$ shallow triangles constructed in a similar manner in the proof of Theorem 1. We make all

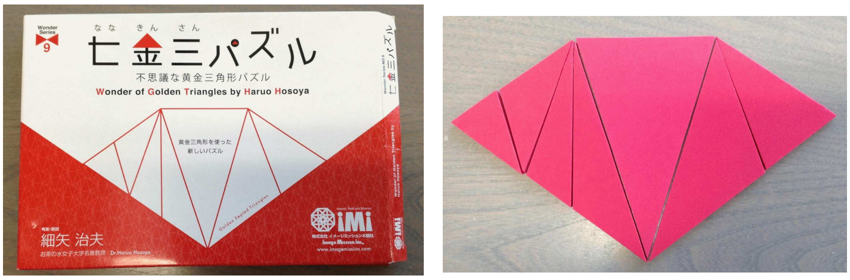


**Figure 4** Package (left) and seven pieces (right) of Nana-kin-san puzzle.

triangles in $S$ distinct from each other. In this time, we can attach these $n$ triangles at each of $n$ edges of $P'$, and each different way produces a distinct convex shape. We count them up. Once we fix an edge of $P'$ as a special edge, the ordering of the triangles makes $n!$ ways. In each ordering, each triangle has two ways to attach it to $P'$ by flipping. Therefore, in total, we have $2^n \cdot n!$ different convex polygons from this set of $n+1$ pieces. ◀

From the viewpoint of the design of algorithms, such detailed real numbers in Theorems 1 and 2 are not welcome since we have to take care of numerical errors in computation. On the other hand, the tangram and the Sei-shonagon Chie no Ita are rather too simple to use computer since they are based on the unit tile of identical isosceles right triangles. (In fact, most results in [2] are obtained without using computer.)

Recently, one of the authors produces a new silhouette puzzle named "Nana-kin-san puzzle[2]." This puzzle is designed based on the golden ratio triangles (Figure 4). As the same as the tangram and Sei-shonagon Chie no Ita, it consists of seven pieces. However, each piece is a triangle based on the golden ratio, and hence these edges are not of integer lengths.

From the property of the golden ratio, this puzzle has beautiful features from the viewpoint of both of lengths and angles. That is, as shown in Figure 5, most edge lengths can be described quite simple form using golden ratio $\varphi = \frac{\sqrt{5}+1}{2} = 1.68\ldots$ that satisfies $\varphi^2 - \varphi - 1 = 0$. Moreover, each of all angles of these triangles can be represented by a multiple of 18°. These facts are useful from the viewpoint of design of algorithms. Namely, each angle equal to $18k°$ for some $k$ can be represented by just positive integer $k$, and each edge can be represented by a simple equation $a_0 + a_1\varphi + a_2\alpha$ for some natural numbers $a_0, a_1, a_2$ in $\{0, 1, 2\}$. That is, all computations can be done over integer operations, which mean that we do not take care of numerical errors in computation.

In a sense, the Nana-kin-san puzzle is a puzzle between the one based on a unit tile like polyominoes, polyabolos, and so on, and the general puzzle shown in Theorems 1 and 2. At a glance, since all pieces are "similar" triangles, this puzzle might seem to be simpler than the other silhouette puzzles like the tangram and the Sei-shonagon Chie no Ita that consist of more variant shapes. However, this is not the case. In this paper, we propose a simple algorithm that generates all convex shapes that can be formed by the Nana-kin-san puzzle. Comparing to the similar puzzles based on polyabolos (the tangram can form 13, and the Sei-shonagon Chie no Ita can form 16, and its theoretical upper bound is 19 in that framework), it is surprisingly many. The Nana-kin-san puzzle can form 62 different convex shapes.

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[2] In Japanese, "nana," "kin," and "san" mean "seven," "golden," and "three," respectively. That is, this name indicates seven golden-ratio triangles.

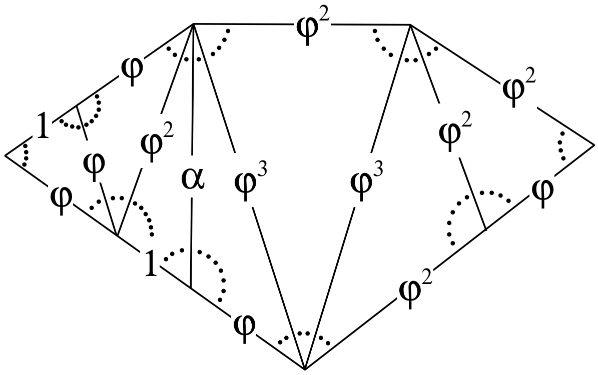

**Figure 5** Lengths and angles of Nana-kin-san puzzle.

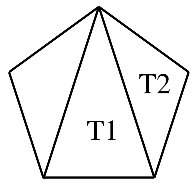


**Figure 6** Golden triangle (T1) and golden gnomon (T2).

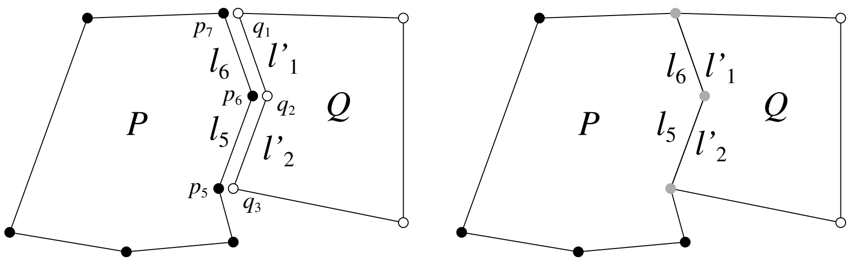


**Figure 7** Edge-to-edge gluing of $P$ and $Q$ at $(p_5, p_6, p_7)$ and $(q_1, q_2, q_3)$.

## 2 Nana-kin-san puzzle and its property

The details of the Nana-kin-san puzzle is described in Figure 5. In the figure, each black dot indicates 18°. That is, each angle can be represented by $18k°$ for some positive integer $k$. In the figure, the unit length is described by 1, and the other lengths are described by $\varphi$, $\varphi^2$, $\varphi^3$, and $\alpha$. Here, $\varphi$ means the golden ratio 1.618... that satisfies $1+\varphi=\varphi^2$, and $\alpha=\sqrt{5+2\sqrt{5}}$. We here remark that for the golden ratio, we have $\varphi^2 = 1+\varphi$ and $\varphi^3 = 1+2\varphi$. In general, for any positive integer $k>1$, $\varphi^k$ can be represented by a linear expression $a_0 + a_1\varphi$ for some positive integers $a_0$ and $a_1$ uniquely. We here remark that the Nana-kin-san puzzle is designed based on two triangles made from a regular pentagon; we can obtain two isosceles triangles from a regular pentagon (Figure 6). We call these triangles *golden triangle* (T1) and *golden gnomon* (T2), respectively.

For two polygons $P$ and $Q$, we introduce an *edge-to-edge gluing* of $P$ and $Q$ as follows. Let $P$ be a polygon with $n$ vertices $p_0, p_1, \ldots, p_{n-1}$ in counterclockwise order[3], and $Q$ a polygon with $m$ vertices $q_0, q_1, \ldots, q_{m-1}$. Let $\ell_i$ be the length of the edge $(p_i, p_{i+1})$, and $\ell'_j$ the length of the edge $(q_j, q_{j+1})$. Then we can edge-to-edge glue $P$ and $Q$ at some paths $(p_i, p_{i+1}, \ldots, p_{i+k})$ and $(q_j, q_{j+1}, \ldots, q_{j+k})$ if and only if for each $0 \le h \le k$, (1) $\ell_{i+h} = \ell'_{j+k-h-1}$, (2) at each pair of vertices $p_{i+h}$ and $q_{j+k-h}$, the summation of two angles at these two vertices makes 360°, (3) at pair of vertices $p_i$ and $q_{j+k}$, the summation of two angles at these two vertices makes less than 360°, and (4) at pair of vertices $p_{i+k}$ and $q_j$,

[3] In this paper, all indices are computed $\bmod n$, where $n$ is the number of vertices of the polygon.

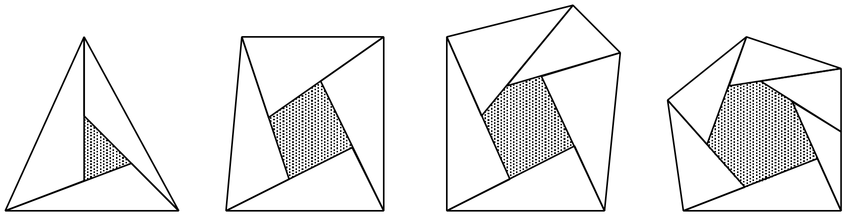


**Figure 8** Not pseudo-guillotine cut separable cases.

the summation of two angles at these two vertices makes less than 360°. See Figure 7 for a simple example of an edge-to-edge gluing of $P$ and $Q$ at $(p_5, p_6, p_7)$ and $(q_1, q_2, q_3)$.

Let assume that a polygon $P$ can be formed by two polygons $P_1$ and $P_2$ by an edge-to-edge gluing at two paths on $P_1$ and $P_2$. Then we call this shared path in $P$ *pseudo-guillotine cut* of $P$. Then we have the following useful property of the Nana-kin-san puzzle:

▶ **Theorem 3.** *In the Nana-kin-san puzzle, every convex polygon can be obtained by repeating edge-to-edge gluing.*

**Proof.** Let $P$ be any convex polygon formed by the Nana-kin-san puzzle. To derive a contradiction, we assume that $P$ cannot be split into two pieces by any pseudo-guillotine cut. As similar in the ordinary guillotine cut, if $P$ cannot be split into two pieces by any pseudo-guillotine cut, $P$ should be surrounded by a series of triangles as shown in Figure 8 (some triangles can be composed by two or more pieces which can be split by pseudo-guillotine cut).

In the figure, each gray area is a hole surrounded by three, four, five, and six triangles in (a), (b), (c) and (d), respectively. Here, each hole should be filled by a set of triangles. Hence, the cases (c) and (d) are already impossible since we have only seven triangles in the Nana-kin-san puzzle contains seven pieces. Therefore, only considerable cases are (a) and (b). It is not difficult to check that each case cannot be achieved by using the seven pieces of the Nana-kin-san puzzle. ◀

By Theorem 3, we can say that it is sufficient to check the repetition of edge-to-edge gluing of the Nana-kin-san puzzle to generate all convex shapes.

## 3 Algorithm

In this section, we describe the details of our algorithm for enumerating all convex polygons using seven pieces of the Nana-kin-san puzzle.

### 3.1 Data structure

We design a special data structure for this problem, which has applications to the other problems with similar properties. Let $P$ be a polygon with $n$ vertices $p_0, p_1, \ldots, p_{n-1}$ in counterclockwise order. Hereafter, we assume that $P$ is made from some pieces of the Nana-kin-san puzzle. Then $P$ is described by two linked lists $(\ell_0, \ell_1, \ldots, \ell_{n-1})$ and $(d_0, d_1, \ldots, d_{n-1})$, where $\ell_i$ is the length of the edge $e_i = (p_i, p_{i+1})$ and $d_i$ is the inner angle at the vertex $p_i$. We here note that each $d_i$ takes a value of $(18 \times k)°$ for some positive integer $k$. Therefore, each $d_i$ needs to store this integer $k$. Moreover, each $\ell_i$ can be represented by a 3-tuple $(\ell_{i,0}, \ell_{i,1}, \ell_{i,2})$ such that $\ell_i = \ell_{i,0} + \ell_{i,1} \times \varphi + \ell_{i,2} \times \alpha$. By the property of the golden ratio, we can confirm that any length $\ell_i$ of $P$ can be represented in this form for some positive integers $\ell_{i,0}, \ell_{i,1}$, and $\ell_{i,2}$, and these positive integers are uniquely determined.

For a polygon $P$, we denote by $P^R$ its mirror image. From the viewpoint of the representation, for a polygon $P$ represented by $p_0, p_1, \ldots, p_{n-1}$, its mirror image $P^R$ can be represented by $p_{n-1}, \ldots, p_1, p_0$. Based on this representation, we define a *canonical form* of $P$ as follows. First, we fix some vertex as $p_0$. Then we can obtain the corresponding string of integers $(d_0, d_1, \ldots, d_{n-1}, \ell_0, \ell_1, \ldots, \ell_{n-1})$ (precisely, each $d_i$ is represented by an integer $k_i$ with $d_i = (18 \times k_i)^\circ$, and each $\ell_i$ is represented by a sequence of three integers $\ell_{i,0}, \ell_{i,1}, \ell_{i,2}$). For each vertex of $P$, we can compute the corresponding string of integers. Among them, we employ the lexicographically first one as the canonical representation of this $P$. It is not difficult to see that any two polygons $P$ and $P'$, $P$ is congruent with $P'$ if and only if their canonical representations are the same string. We maintain each polygon $P$ by its canonical representation. (Note that $P$ and $P^R$ have the different canonical representation in general.)

## 3.2 Algorithm description

Now we describe the algorithm we use to check all convex polygons made from the seven pieces of the Nana-kin-san puzzle. Based on Theorem 3, if we have a convex polygon $P$ made from some pieces of the Nana-kin-san puzzle, $P$ always can be cut into two convex polygons by one pseudo-guillotine cut. Therefore, we can apply inductive construction of convex polygons made from these seven pieces. First, we initialize the set $S_0$ of polygons by the seven pieces $\{P_0, P_1, \ldots, P_6\}$ of the Nana-kin-san puzzle. In general, we keep the set of shapes $P$ such that each of them is made from some pieces of $S_0$. That is, $P$ consists of the shape described by its canonical representation, and the subset of $S$ that forms $P$. That is, in the first set $S_0$, each piece $P_i$ (with $0 \le i \le 6$) has its canonical representation and it is associated with the set $\{P_i\}$. In general step, we grow the set $S_0$ and add convex polygons that can be formed by some pieces of the Nana-kin-san puzzle. We denote this general set by $S$ which starts from $S_0$.

In general step, we pick up two polygons $P$ and $P'$ from $S$ such that they do not share any common piece in $S_0$. Then we glue $P$ to $P'$ in all possible ways, obtain new polygons $P''$, and add them into $S$ as follows (we also apply the same algorithm for $P^R$ and $P'$):

**Step 1:** For each $i$ and $j$, pick up $e_i$ from $P$ and $e_j$ from $P'$.

**Step 2:** If $\ell_i \ne \ell_j$, we do nothing for this pair. If $\ell_i = \ell_j$, we construct a new polygon by gluing $e_i$ to $e_j$ as follows. The new polygon is described by two linked lists $(\ell'_{j+1}, \ell'_{j+2}, \ldots, \ell'_{j-1}, \ell_{i+1}, \ell_{i+2}, \ldots, \ell_{i-1})$ and $(d_i+d'_{j+1}, d'_{j+2}, \ldots, d'_{j-1}, d_{i+1}+d'_j, d_{i+2}, \ldots, d_{i-1})$. Let the resulting polygon $P''$ be represented by $(\ell''_0, \ell''_1, \ldots)$ and $(d''_0, d''_1, \ldots)$.

**Step 3:** We here search the vertex $p_k$ of degree 360° and remove it. Let we have $(\ldots, d''_{k-1}, 20, d''_{k+1}, \ldots)$ (we remind that each $d_i$ represents $(18 \times d_i)^\circ$). Then, intuitively, the edges $e_{k-1}$ and $e_k$ are overlapping with sharing the point $p_k$. If $\ell_{k-1} \ne \ell_k$, we conclude that this gluing is fault since $P''$ is not convex. Otherwise, the list $(\ldots, d''_{k-1}, 20, d''_{k+1}, \ldots)$ is replaced by $(\ldots, d''_{k-1}+d''_{k+1}, \ldots)$ and $(\ldots, \ell''_{k-2}, \ell''_{k-1}, \ell''_k, \ell''_{k+1}, \ldots)$ is replaced by $(\ldots, \ell''_{k-2}, \ell''_{k+1}, \ldots)$.

**Step 4:** We here search the vertex $p_k$ of degree 180° and remove it. Let we have $(\ldots, d''_{k-1}, 10, d''_{k+1}, \ldots)$. Then, intuitively, the edges $e_{k-1}$ and $e_k$ are on the same line with sharing the point $p_k$. Then the list $(\ldots, d''_{k-1}, 10, d''_{k+1}, \ldots)$ is replaced by $(\ldots, d''_{k-1}, d''_{k+1}, \ldots)$ and $(\ldots, \ell''_{k-2}, \ell''_{k-1}, \ell''_k, \ell''_{k+1}, \ldots)$ is replaced by $(\ldots, \ell''_{k-2}, \ell''_{k-1} + \ell''_k, \ell''_{k+1}, \ldots)$.

**Step 5:** If $d''_k$ is greater than 20 for some $k$ (i.e., the inner angle at vertex $p_k$ is greater than 360), forget $P''$ and go to Step 1.

**Step 6:** Add the canonical form of $P''$ into $S$, and go to Step 1.

### 3.3 Correctness of the algorithm

We here show the correctness of the algorithm. If the algorithm outputs a convex polygon $P$ that uses seven pieces of the Nana-kin-san puzzle, it is easy to see that we can construct it. Therefore, it is sufficient to show that all possible convex polygons are enumerated by the algorithm. By Theorem 3, any convex polygon $P$ that uses all pieces can be divided into two polygons by pseudo-guillotine cut. Then, for each (not necessarily convex) polygon, using Theorem 3 repeatedly, we finally obtain the set $S_0$ of the seven pieces of the Nana-kin-san puzzle. Therefore, the algorithm is correct and we obtain all possible convex polygons made from the Nana-kin-san puzzle.

## 4 Results

In this section, we show the results of the algorithm. In our experiment, we use the following system: Intel Core i7-3770K (3.50GHz), 32 GB Memory. The computation time is 675 seconds, and the memory consumption is 15 MB. We obtain 563 possible convex polygons that can be formed by a subset of the seven pieces of the Nana-kin-san puzzle. Among them, the number of the convex polygons that can be formed by the all of the seven pieces of the Nana-kin-san puzzle is 62. They consist with 3 tetragon, 24 pentagons, 29 hexagons and 6 heptagons. Their shapes are shown in Figure 9. Since this is a silhouette puzzle, we just only show the all possible convex polygons without cutting lines. Their solutions are given in Appendix. The whole 563 convex polygons can be found at `http://www.al.ics.saitama-u.ac.jp/horiyama/research/puzzle/7kin3_puzzle/`.

We also applied the algorithm to the tangram and the Sei-shonagon Chie no Ita. We obtain 93 and 100 possible convex polygons that can be formed by a subset of the seven pieces of the tangram and the Sei-shonagon Chie no Ita, respectively. Among them, the number of the convex polygons that can be formed by the all of the seven pieces of the tangram and the Sei-shonagon Chie no Ita, respectively, is 13 and 16. The 62 out of 563 possible convex polygons of the Nana-kin-san puzzle suggests the rich potential of that puzzle. On the computation time, interestingly, they have a significant difference : 65 seconds for the tangram, and 40,920 seconds for the Sei-shonagon Chie no Ita.

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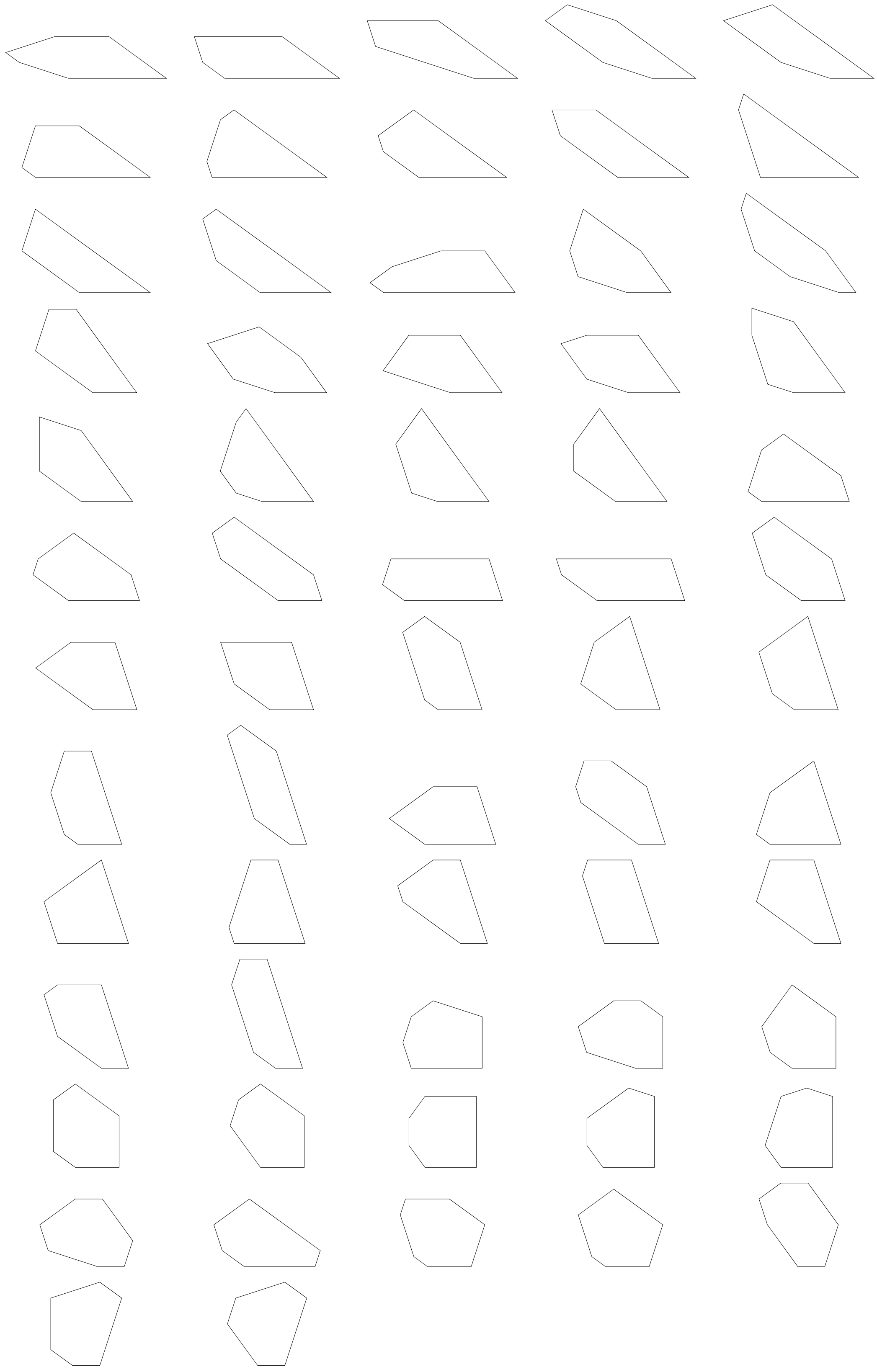


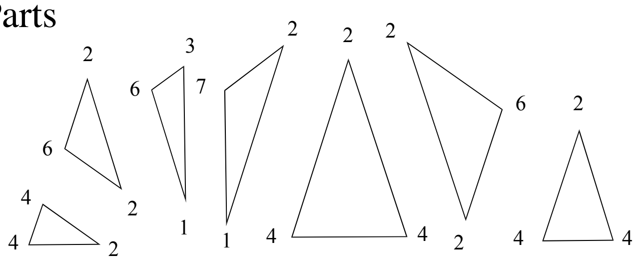
**Figure 9** 62 convex polygons formed by 7 pieces of the Nana-kin-san puzzle.

## Appendix

# Catalogue of Snug Golds (Golden Septet Triangles)

Haruo Hosoya (Jan. 2016)

Parts




Angles
1: 18°
2: 36°
3: 54°
4: 72°
5: 90°
6: 108°
7: 126°
8: 144°
9: 162°

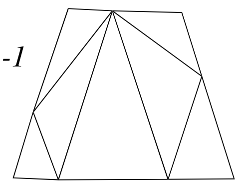
S: mirror symmetric

Credit

K: Kofu Satoh T: Takashi Horiyama

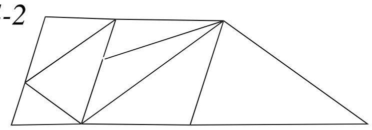
Tetragons (3)

*4S-1*



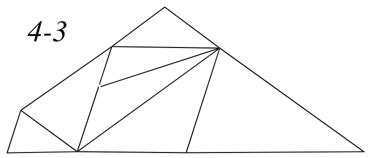
$4^2 6^2$ (4466)

*4-2*



2468 (2468)

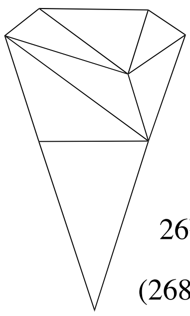
*4-3*



2468 (2486)

Pentagons (24)

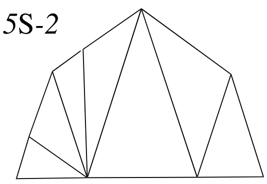
*5S-1*



$2 6^2 8^2$

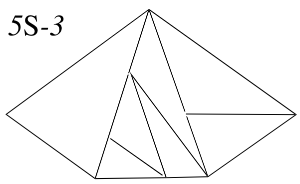
(26886)

*5S-2*



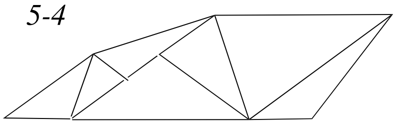
$4^2 6 8^2$ (44868)

*5S-3*



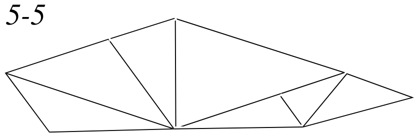
$4^2 6 8^2$ (46488)

*5-4*

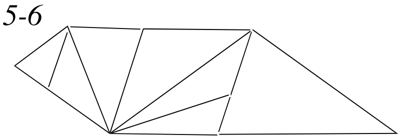


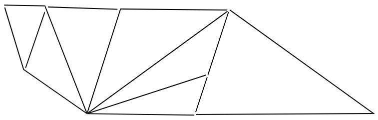
$2 3 7 9^2$ (27399)

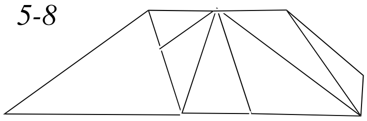
*5-5*

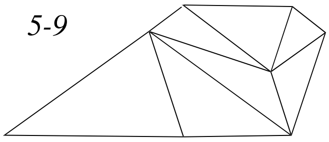


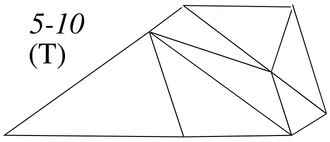
24789 (28479)

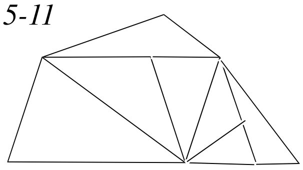
*5-6*

$248^3$ $(28488)_1$

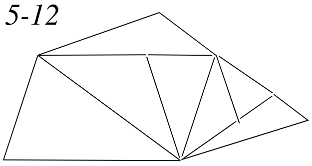
*5-7*

$248^3$ $(28488)_2$

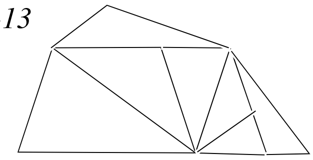
*5-8*

$26^28^2$ $(26688)_1$

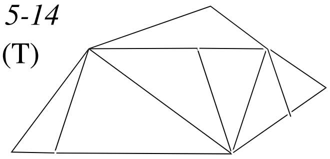
*5-9*

$26^28^2$ $(26688)_2$

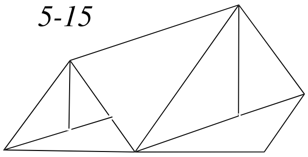
*5-10* (T)

$26^28^2$ $(28668)$

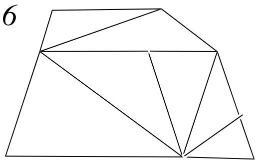
*5-11*

$347^29$ $(34779)$

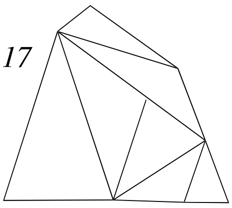
*5-12*

$347^29$ $(37749)$

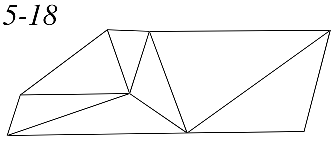
*5-13*

$3478^2$ $(34878)$

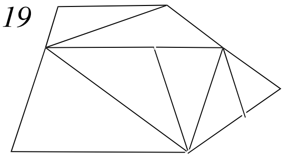
*5-14* (T)

$3478^2$ $(38478)$

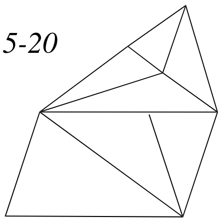
*5-15*

$36^278$ $(37668)$

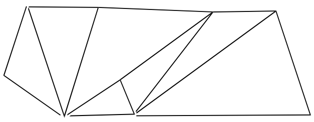
*5-16*

$4^268^2$ $(44688)$

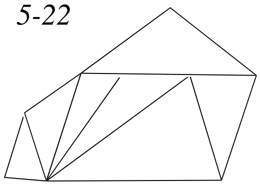
*5-17*

$4^268^2$ $(44868)$

*5-18*

$4^268^2$ $(46488)$

*5-19*

$4^268^2$ $(46848)_1$

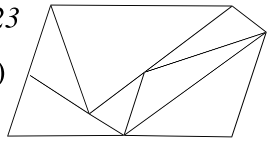
*5-20*

$4^268^2$ $(46848)_2$

*5-21* (T)

$46^38$ $(46668)_1$

*5-22*

$46^38$ $(46668)_2$

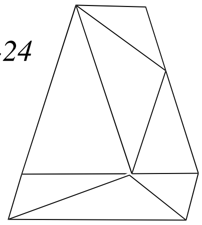
*5-23*

(T)



$46^38$ $(46686)_1$

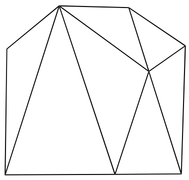
*5-24*



$46^38$ $(46686)_2$

## Hexagons (29)

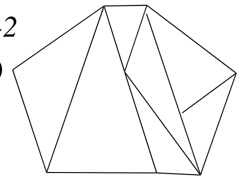
*6*S-*1*



$5^27^28^2$ (557887)

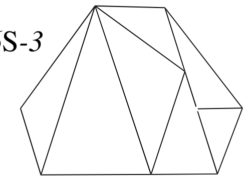
*6*S-*2*

(T)



$6^48^2$ (666688)

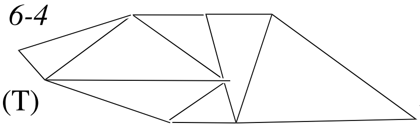
*6*S-*3*



$6^27^4$ (667777)

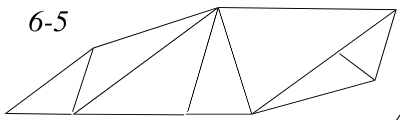
*6-4*

(T)



$2389^3$ (289399)

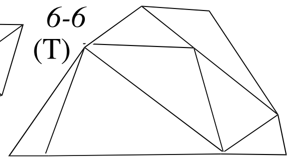
*6-5*



$2479^3$ (297499)

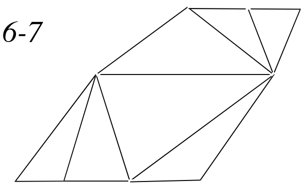
*6-6*

(T)



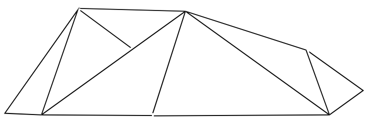
$34789^2$ (349789)

*6-7*



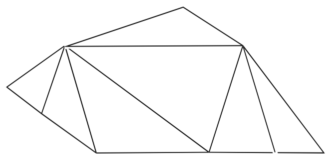
$34789^2$ (379489)

*6-8*



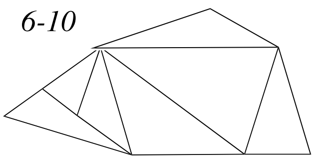
$34789^2$ (379948)

*6-9*



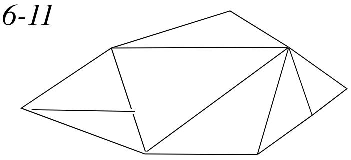
$34789^2$ (384979)

*6-10*



$34789^2$ (394879)

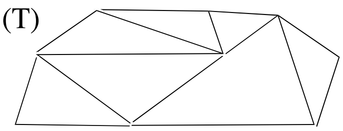
*6-11*



$34789^2$ (397489)

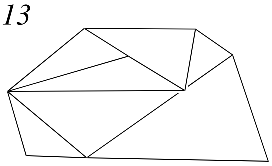
*6-12*

(T)



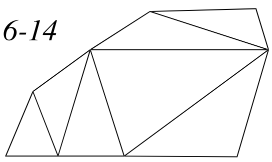
$46^28^3$ $(466888)_1$

*6-13*



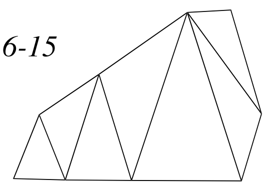
$46^28^3$ $(466888)_2$

*6-14*



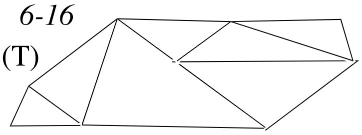
$46^28^3$ (468688)

*6-15*



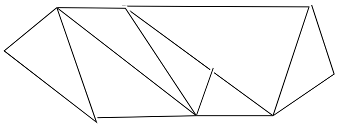
$46^28^3$ (468868)

*6-16* (T)



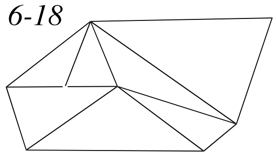
$46^28^3$ $(486688)_1$

*6-17* (T)



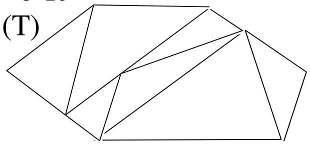
$46^28^3$ $(486688)_2$

*6-18*



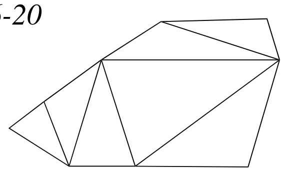
$46^28^3$ $(486688)_3$

*6-19* (T)



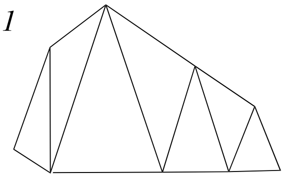
$46^28^3$ $(486688)_4$

*6-20*



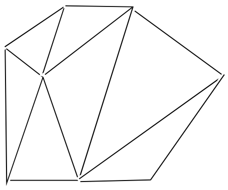
$46^28^3$ $(486868)_1$

*6-21*



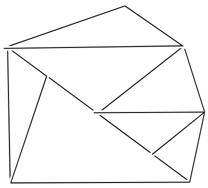
$46^28^3$ $(486868)_2$

*6-22* (K)



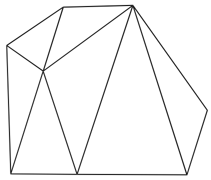
$5^27^28^2$ (575788)

*6-23*



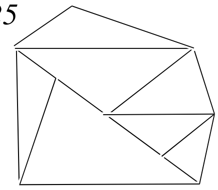
$56^278^2$ (567886)

*6-24*



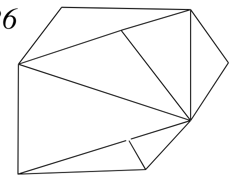
$567^38$ (567787)

*6-25*



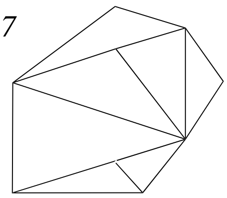
$567^38$ (568777)

*6-26*



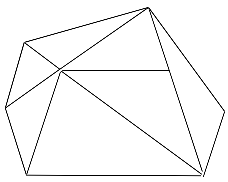
$567^38$ (576778)

*6-27*



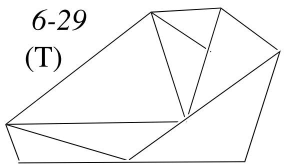
$567^38$ (576877)

*6-28*



$6^37^28$ (667678)

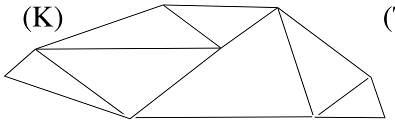
*6-29* (T)



$6^48^2$ (666688)

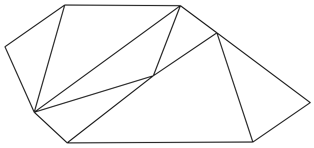
## Heptagons (6)

*7-1*
(K)



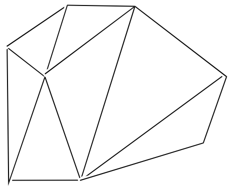
$348^{2}9^{3}$ (3948899)

*7-2*
(T)



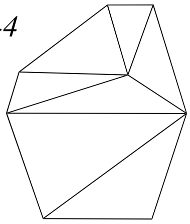
$468^{5}$ (4886888)

*7-3*



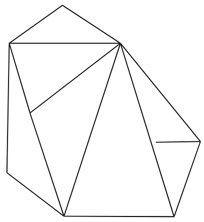
$567^{2}8^{2}9$ (5788679)

*7-4*



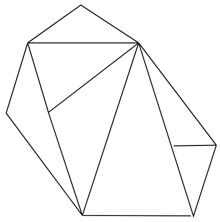
$6^{3}8^{4}$ (6686888)

*7-5*



$6^{2}7^{3}89$ (6778679)

*7-6*



$6^{2}7^{3}89$ (6778697)

In total: 62 convex polygons (Snug golds)

cf. Tangram: 13
Seisho-nagon: 16