This House Proves That Debating is Harder Than Soccer

Stefan Neumann[1] and Andreas Wiese[2]

1 University of Vienna, Faculty of Computer Science, Vienna, Austria
stefan.neumann@univie.ac.at
2 Max Planck Institute for Computer Science, Saarbrücken, Germany
awiese@mpi-inf.mpg.de

Abstract

During the last twenty years, a lot of research was conducted on the sport elimination problem: Given a sports league and its remaining matches, we have to decide whether a given team can still possibly win the competition, i.e., place first in the league at the end. Previously, the computational complexity of this problem was investigated only for games with two participating teams per game. In this paper we consider Debating Tournaments and Debating Leagues in the British Parliamentary format, where four teams are participating in each game. We prove that it is NP-hard to decide whether a given team can win a Debating League, even if at most two matches are remaining for each team. This contrasts settings like football where two teams play in each game since there this case is still polynomial time solvable. We prove our result even for a fictitious restricted setting with only three teams per game. On the other hand, for the common setting of Debating Tournaments we show that this problem is fixed parameter tractable if the parameter is the number of remaining rounds k. This also holds for the practically very important question of whether a team can still qualify for the knock-out phase of the tournament and the combined parameter $k+b$ where b denotes the threshold rank for qualifying. Finally, we show that the latter problem is polynomial time solvable for any constant k and arbitrary values b that are part of the input.

1998 ACM Subject Classification F.2.2 Nonnumerical Algorithms and Problems

Keywords and phrases complexity, elimination games, soccer, debating

Digital Object Identifier 10.4230/LIPIcs.FUN.2016.25

1 Introduction

Debating and soccer are deeply rooted in our society. Debating dates back to the times of the ancient greek when already in 460 BC the citizens of Athens were meeting in one of the first parliaments of the world for discussions and votings [4]. This gave rise to the fine art of rhetoric, the skill to speak in a public debate in a convincing manner, to give a solid argumentation for the provided claims, and to win the support of the audience for the own case. Since the ancient Greece the art of debating has developed, and great speeches became milestones of history such as the famous speech delivered by Martin Luther King on August 28, 1963 containing the dictum “I have a dream” [12]. Nowadays, all over the world there are debating societies at universities and outside academia that are devoted to debates and public speaking. This has a long tradition, for instance, the Cambridge Union Society was founded in 1815 and has been run continuously for more than 200 years now [3]. Important for this paper is that there are debating competitions: teams of debaters meet and argue for and against the case of a previously specified motion. The roles (pro and

8th International Conference on Fun with Algorithms (FUN 2016).
Editors: Erik D. Demaine and Fabrizio Grandoni; Article No. 25; pp. 25:1–25:14
Leibniz International Proceedings in Informatics
LIPICS Schloss Dagstuhl – Leibniz-Zentrum für Informatik, Dagstuhl Publishing, Germany

contra) are assigned randomly and thus the debaters do not necessarily argue for the side that they personally support.

Like debating, soccer is an integral part of the contemporary societies in many countries. It is played by 250 million players in more than 200 countries which makes it the world's most popular sport [7]. Even more people are passionate for watching the matches and supporting their favorite teams. For instance, the final of the last world cup 2014 was watched by more than one billion people world wide [9].

It is clear that debating and soccer play a significant role in modern societies. However, one question has remained open: what is harder, debating or soccer? Empirically there are only very few indications. There are quotes by soccer players such as "We lost because we didn't win." (Ronaldo [20]), "I also told him that verbally." (Mario Basler [1]), "It doesn't matter if it is Milano or Madrid as long as it is Italy." (Andreas Möller [16]), or "I can see the carrot at the end of the tunnel." (Stuart Pearce [19]) which suggest that excelling rhetorically might be harder than playing soccer. On the other hand, the political careers of heads of states typically surpass their soccer careers by orders of magnitude. For instance, Gerhard Schröder, the former chancellor of Germany, played only in the Bezirksliga [22] which is nowadays the 7th level of the soccer league system in Germany. For the current German chancellor Angela Merkel we are not aware of any non-trivial soccer abilities. However, she is known to occasionally frequent the German national team's changing room after important matches [17].

From a scientific point of view it is difficult to compare debating and soccer since they have only few intersection points that allow a scientifically accurate comparison. One of the few is the following: consider a league in which soccer/debating teams play matches against each other according to a pre-defined schedule that indicates on which match days which respective teams play each other. Consider your favorite team t_1. The question is: are there outcomes for all remaining matches such that t_1 wins the championship?

In soccer, this question is polynomial time solvable if there are at most two remaining matches per team and NP-hard for at most three matches per team under the three-point rule [2, 13]. The latter is nowadays ubiquitous in soccer leagues and tournaments (such as in all FIFA world cups since 1994, in most national soccer leagues since 1995, and in some of them even much earlier [6]). It specifies that if a team wins a match it scores three points for the league ranking and the losing team scores zero points, if the match is a draw then both teams score one point.

For debating, we focus in this paper on the British parliamentary style format that enjoys great popularity world wide and is played for instance in the world universities debating championships [24]. In this format, four teams are playing in each game and the winning team scores three points, the second team scores two points, the third team scores one point, and the fourth team scores zero points. If in the final ranking multiple teams have the same number of points, then a tiebreaker is used. For simplicity, in this paper we assume that this tie-breaker is the total number of FUN papers written by members of the team and that the team t_1 has written the most FUN papers among all participating teams. Thus, t_1 wins the championship if there is no team with more points than t_1 and the corresponding problem is called DebatingLeague.

1.1 Our contribution

In this paper we prove that DebatingLeague is NP-hard, even if there are only two remaining matches to play for each team. This shows that debating is computationally harder than soccer in two ways: first, if there are only two remaining matches to play for each team then in soccer we can decide in polynomial time whether a given team can still win [2]. Secondly, for an arbitrary number of remaining matches soccer is easy under the two-point rule [2], i.e., the winning team scores two points, rather than three. The two-point rule has the important feature that for each match there is a given number of points (two) that are completely distributed among the participating teams. This is also the case in debating: in each match there are six points available and they are all distributed. While with this feature soccer is easy, DebatingLeague is NP-hard despite of this which underlines the complexity of the latter problem. To the best of our knowledge, this is the first time that the elimination problem has been studied for games with more than two teams per match. In fact, we prove that our hardness result even holds in a fictitious setting in which only three teams participate in a game and they score two, one, and zero points, respectively.

While DebatingLeague is NP-hard if only two matchdays are remaining, we can show something different for the system that is typically played in debating *tournaments*. There, the matches of the teams are defined in a similar way as in Swiss-system tournaments [8] (which are for instance common in chess): after each round the teams are ordered according to the number of points they scored so far. Then the teams ranked 1st-4th play one match, the teams ranked 5th-8th play the second match, and so on. Since the pairings in each round depend on the initial ranking and the outcomes of the previous rounds, the above hardness result for DebatingLeague does not apply. In practice debating tournaments have a first phase organized as above and a second phase that is played as a knock-out tournament. There is a threshold b specifying that the first b teams of the final ranking after the first phase qualify for the knock-out phase, denoted as *breaking*. A key question that a team typically asks itself during a tournament is whether it can still break. Formally, we denote by DebatingTournament the problem of deciding whether t_1 can finish on place b or better with k rounds left in the tournament.

We show that DebatingTournament can be solved in time $O(f(k+b) \cdot n)$, i.e., the problem is fixed parameter tractable for the combined parameter $k+b$. In particular, this implies that for any constant k it is polynomial time solvable to decide whether t_1 can win the tournament, while for $k = 2$ DebatingLeague is NP-hard. For our algorithm we first prove that if initially the team t_1 is "too far behind", i.e., has a too large initial rank depending on k and b, then it cannot break anymore. For the remaining case we provide an algorithm with a running time of $O(f(k+b) \cdot n)$ for a suitable function f. Additionally, we show that for constant k the problem is polynomial time solvable (for an arbitrary value of b that is part of the input). Thus, even for arbitrary b the case that $k = 2$ is in P, in contrast to DebatingLeague.

1.2 Other related work

In 1966, Schwartz [21] proved that using flow networks it can be decided in polynomial time whether a baseball team can still win a baseball league. In baseball the winner of a game wins a single point and the looser gets zero points, there is no tie. McCormick [15] generalized this result by giving a polynomial time algorithm which allowed to fix a number of losses for the team that is supposed to win the league. Wayne [23] characterised all teams of a baseball league which can still win the league by giving a threshold value for the number of points and the number of matches a team must have to be able to win the league. He further gave a polynomial time algorithm to compute this threshold. This result was later improved by Gusfield and Martel [11] who gave thresholds for a bigger set of possible outcomes of the matches. For baseball leagues they gave a faster algorithm to determine the threshold and further allowed leagues with multiple divisions and wild-cards [11].

A major difference between baseball and soccer leagues is which outcomes are possible in a single game. For soccer leagues with the three-point-rule it was proven by [13] and [2] independently that it is NP-hard to determine whether a team can win the league. Pálvölgyi [18] proved that when we are given the table of a soccer league and a list of games that were played so far without their outcomes, it is NP-hard to decide whether this table is valid, i.e., whether the distribution of points to the teams can be achieved by real outcomes of games. In [5], the authors construct a hypergraph representing the teams and their remaining matches. Depending on certain properties of this graph they prove multiple hardness results for the question whether a certain team can still win the competition.

In [14], Kern and Paulusma consider games with two teams, but allow a game to have many different outcomes. They prove that it can be decided in polynomial time whether a team can still win the competition if and only if in each match exactly m points can be distributed arbitrarily to both teams (for any positive integer m).

2 Debating League

In this section we prove that DebatingLeague is NP-hard, even if each team has at most two remaining matches to play. First, let us define the problem formally. Let $T = \{t_1, \dots, t_n\}$ be the set of teams participating in the debating league. We denote the set of remaining matches by $M \subset T^4$, i.e., we have $(t_i, t_j, t_k, t_l) \in M$ iff the teams t_i, t_j, t_k and t_l still have to play against each other in a match. We assume that each possible match occurs at most once; further, throughout the whole section the game schedule of remaining matches is fixed. The winner of each match scores 3 points, the second placed team scores 2 points, the third placed team scores 1 point and the loosing team does not get any point. We are given a *score vector* $s \in \mathbb{R}^n$ with an entry s_i for each team t_i that indicates how many points team t_i already obtained before playing the remaining matches. Notice that the tuple (T, M, s) encodes all information we need about the competition. In the DebatingLeague problem we want to find out whether team t_1 can still win the competition.

▶ **Definition 1.** In the DebatingLeague problem we are given a tuple (T, M, s) and we want to answer the question whether there are outcomes for all matches M, such that at the end there is no team that has more points than team t_1.

We will prove that this problem is already NP-hard when each team has at most two remaining matches. We prove this first for a variant of DebatingLeague where we have only 3 teams per match and each team has at most two matches left to play. In a game the winner gets 2 points, the second placed team gets 1 point and the looser gets 0 points. We still want to decide whether team t_1 can win the competition. We denote this problem ThreeTeamDebating. It can also be characterised by a tuple (T, M, s) similarly to above.

▶ **Theorem 2.** *The ThreeTeamDebating problem is NP-hard even when each team has at most two remaining matches to play.*

Before we start giving the proof of Theorem 2, we introduce a way to visualize instances of ThreeTeamDebating as graphs. Suppose we are given an instance (T, M, s) of ThreeTeam-Debating in which each team plays at most two matches. We visualize its matches via a *game graph* $G = (V, E)$ in the following way: For each game $g \in M$, we introduce a *game vertex* $v_g \in V$. For each team t_i that participates in two matches g, g', i.e. if $t_i \in g$ and $t_i \in g'$, we introduce an edge e_i connecting v_g and $v_{g'}$. Such an edge will be called a *team edge*. Each edge will receive a weight w_i which encodes how many points team t_i can still get without obtaining more points than team t_1. If a team has only one game remaining, we do not introduce an edge for it. Notice that later team t_1 will not be part of the game graph as we can assume w.l.o.g. that it wins all of its remaining games and has no games left.

We prove Theorem 2 via a reduction from 3-Bounded-3-SAT [10] to ThreeTeamDebating. Let φ be a 3-Bounded-3-SAT formula with variables $x_1, \ldots, x_n$ and clauses $C_1, \ldots, C_m$. We can assume that each variable occurs in two or three different clauses and that it occurs at least once positively and at least once negatively. We can further assume that each clause has two or three literals.

We construct an instance (T, M, s) of ThreeTeamDebating. First, we describe gadgets out of which our construction is composed and prove some of their properties. Afterwards, we describe how to combine the gadgets to the final instance. In the sequel, we will prove some properties about our construction. We will use the term "We can assume that ..." for the claim that team t_1 can still win the championship if and only if it can still win the championship for outcomes of the matches where the respective following statement is true. In our construction, t_1 has no remaining game to play. We distinguish the other teams into *two-game teams* and *one-game teams*, where the former type has two remaining games to play and the latter type has one remaining game to play. For each team, we will define how many points it can still score without getting more points than t_1. We will not exactly specify how many points each team has initially since it matters only how many points it can still get without overtaking t_1.

For each variable x in φ we introduce a *ring gadget*. Assume that x occurs in the three clauses C_i, C_j, C_k. The ring gadget for a variable x consists of the six games given by the set $G_x := \{g^1_{x,C_i}, g^2_{x,C_i}, g^1_{x,C_j}, g^2_{x,C_j}, g^1_{x,C_k}, g^2_{x,C_k}\}$ and six teams two-game teams as specified by $T_x := \{t^1_{x,C_i}, t^2_{x,C_i}, t^1_{x,C_j}, t^2_{x,C_j}, t^1_{x,C_k}, t^2_{x,C_k}\}$. If x appears in only two clauses C_i, C_j we use the same setup for a fictitious clause C_k.

The games of the teams in T_x are visualized in Figure 1. Ignoring teams which are not from the set T_x and which we will introduce later, the game g^1_{x,C_i} is played by the teams t^2_{x,C_k}, t^1_{x,C_i}, the game g^2_{x,C_i} is played by the teams t^1_{x,C_i}, t^2_{x,C_i}, the game g^1_{x,C_j} is played by the teams t^2_{x,C_i}, t^1_{x,C_j}, the game g^2_{x,C_j} is played by the teams t^1_{x,C_j}, t^2_{x,C_j}, the game g^1_{x,C_k} is played by the teams t^2_{x,C_j}, t^1_{x,C_k}, and the game g^2_{x,C_k} is played by the teams t^1_{x,C_k}, t^2_{x,C_k}. Thus, when visualizing the games in G_x and the teams in T_x they form a cycle. Each team g^1_{x,C_ℓ} with $\ell \in \{i, j, k\}$ is allowed to get 2 points and each team g^2_{x,C_ℓ} with $\ell \in \{i, j, k\}$ can get 3 points. The other teams participating in the games G_x (to be defined later) will only be able to score exactly 1 point and hence they will not be able to win a game. Hence, we can assume that in each game $g \in G_x$ one team in T_x that plays in g must score 2 points. Furthermore, each team in T_x can win at most one game and since there are six games in G_x and six teams in T_x, each team in T_x must win exactly one game.

One way to visualize the outcome of the circle games is to orient each edge in the game graph. The team edge of a team $t \in T_x$ points towards the unique game in which t scores 2 points. In this viewpoint, the following lemma implies that we can assume that all edges of the cycle are either oriented clockwise or counter-clockwise.

▶ **Proposition 3.** *We can assume that either the ring gadget is* oriented clockwise*, i.e. game g^z_{x,C_ℓ} is won by team t^z_{x,C_ℓ} for $\ell \in \{i, j, k\}$ and $z \in \{1, 2\}$, or the ring gadget is* oriented counter-clockwise*, i.e. game g^2_{x,C_ℓ} for $\ell \in \{i, j, k\}$ is won by team t^1_{x,C_ℓ} and the games g^1_{x,C_i},g^1_{x,C_j},g^1_{x,C_k} have winners t^2_{x,C_k},t^2_{x,C_i},t^2_{x,C_j}, respectively.*

Later, the two possible orientations of the ring gadget for variable x will correspond to setting the variable x to true or to false. Next, we introduce a *clause game* g_C for each clause C in φ. Let C be a clause with variables x, y, z. We introduce three two-game teams

$t^4_{x,C}, t^4_{y,C}, t^4_{z,C}$ that play g_C and each of them will play in another game that we will define later. Each of them can still score 2 points. Intuitively, the team among them that scores 2 points in g_C will correspond to the variable that satisfies the clause C in a satisfying assignment. Note that for the names of the teams we do not distinguish whether a variable x occurs positively or negatively in C.

We describe now how we connect the clause games with the ring gadgets, see Figure 1. Let x be a variable that occurs in a clause C. For this occurrence, we introduced the team $t^4_{x,C}$ above. We now introduce a game $g^3_{x,C}$, a two-game team $t^3_{x,C}$, and a one-game team $t^3_{x,C,d}$. The team $t^3_{x,C}$ can still get 1 point and the team $t^3_{x,C,d}$ can still get 2 points. We define that $g^3_{x,C}$ is the second game of $t^4_{x,C}$, the only game of $t^3_{x,C,d}$, and one of the two games that $t^3_{x,C}$ plays. The intuition behind this construction is that if $t^3_{x,C}$ gets 0 points in its second game (that we have not specified yet) then the team $t^4_{x,C}$ can score up to 2 points in game g_C (without getting more points in total than t_1). On the other hand, if $t^3_{x,C}$ gets 1 point in its other game, then $t^4_{x,C}$ can score only up to 1 point in g_C and in particular, it cannot score 2 points in g_C anymore. Later, the first case will correspond to the case that x satisfies C whereas the second case will correspond to the case that x does not satisfy C.

▶ **Proposition 4.** *Let x be a variable appearing in a clause C. We can assume that*
- *if $t^3_{x,C}$ scores 1 point in a game different than $g^3_{x,C}$ that it plays, then $t^4_{x,C}$ scores at most 1 point in game g_C, and*
- *if $t^3_{x,C}$ scores 0 points in a game different than $g^3_{x,C}$ that it plays, then $t^4_{x,C}$ can score up to 2 points in the game g_C.*

We specify the second game for the team $t^3_{x,C}$ (i.e., the game different than $g^3_{x,C}$ that it plays). If x appears positively in clause C then this second game is defined to be $g^2_{x,C}$, otherwise, this second game is defined to be $g^1_{x,C}$. If the variable x appears in three clauses C_i, C_j, C_k then three games from the G_x are still missing one team, exactly one per clause. For these games we add the one-game teams $T^d_x := \{t_{x,d_i}, t_{x,d_j}, t_{x,d_k}\}$, each of them playing the game with the corresponding clause in the subscript and each of them allowed to score 1 point. If x appears in only two clauses then we similarly add two one-game teams such that each of them is allowed to score 1 point and by this we ensure that each game in G_x has three teams. This completes the definition of the instance.

▶ **Lemma 5.** *If φ is satisfiable then there is an outcome of the defined instance of* ThreeTeam-Debating *such that no team gets more points than t_1.*

Proof. Suppose we are given a satisfying assignment to the variables in φ. From this satisfying assignment we will construct outcomes of the games, such that team t_1 wins the championship. Intuitively, we will want to assign all points as depicted in Figure 1. We will now describe this formally.

Let x be a variable. If x is true, then we orient the ring gadget of x counter-clockwise according to the left image in Figure 1; formally, the winners of the games G_x are assigned as defined in Proposition 3. If x is false, then the ring gadget of x is oriented clockwise according to the right image in Figure 1. All one-game teams T^d_x will place second in their games and thus obtain a single point each.

Consider a clause C with variables x, y, z. In the satisfying assignment one of them must satisfy C. Assume w.l.o.g. that x satisfies C. Then we let team $t^4_{x,C}$ score 2 points in the game g_C and we let an arbitrary team among $t^4_{y,C}, t^4_{z,C}$ score 1 point and the other one 0 points. For the game $g^3_{x,C}$ we let the one-game team score 2 points and team $t^3_{x,C}$ score 1 point, team $t^4_{x,C}$ obtains no additional point from this game. Team $t^3_{x,C}$ further scores 0

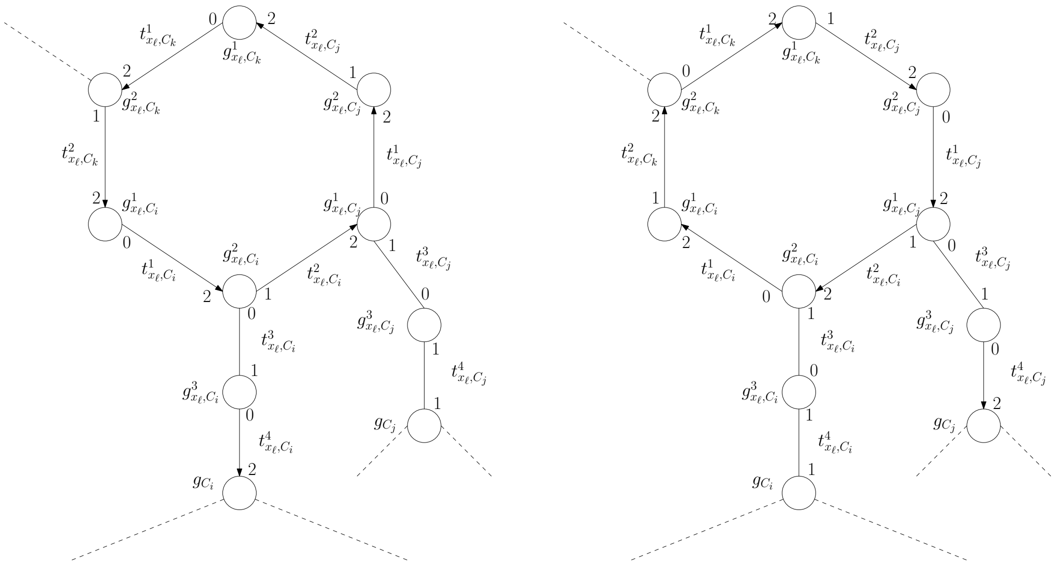

Figure 1 An excerpt of the game graph for variable x_ℓ which occurs in clauses C_i and C_k positively and in C_j negatively. In the left image the outcomes of the games for $x_\ell = true$ are visualised, in the right image we have $x_\ell =$ *false*. The edges are directed towards the game that was won by the corresponding team; the numbers close to the game vertices show how many points the associated two-game teams win in this game.

points in its remaining game (game $g^1_{x,C}$ or $g^2_{x,C}$, depending on whether x appears negatively or positively in C) and the remaining point of this game goes to the team which can obtain 3 points in total. In the game $g^3_{y,C}$ we let the team $t^4_{y,C}$ score 1 point and team $t^3_{y,C}$ scores 1 point from its game in G_y. For the teams and games for variable z we use the same distribution of points as for y. We define the outcomes of the games in the same way for each clause C. See Figure 1 for a sketch of the outcomes described above.

All games distribute all of their points: In the above assignment, for each clause game g_C we have distributed all points by construction. For all x, C, the games $g^3_{x,C}$ have a one-game team as a winner and by construction the second place goes to either $t^3_{x,C}$ or $t^4_{x,C}$. It is left to to argue about the games from the G_x. For each $g^z_{x,C} \in G_x$ with $z \in \{1, 2\}$ we must have a winner since we assigned the winners as defined in Proposition 3. If $g^z_{x,C}$ has a one-game team participating then this team can place second in our construction and hence all points are distributed. If $g^z_{x,C}$ has only two-game teams, then we constructed our variable assignment such that $g^z_{x,C}$ gives its last point to $t^3_{x,C}$, if x was not used to satisfy C. If x was used to satisfy C, then 1 point goes to its participating team which can still get 3 points (by the orientation for the ring gadget we picked, this team cannot have won $g_{x,C}$). Finally, by construction there is no team that scores more points than we had specified, i.e., there is no team that scores more points than t_1. ◀

▶ **Lemma 6.** *If there is an outcome of the games in the defined instance of ThreeTeamDebating such that no team gets more points than t_1 then the formula φ is satisfiable.*

Proof. Suppose there is an outcome of the games such that no team gets more points than t_1. We construct an assignment to the variables in φ that satisfies the formula. Let x be a variable. Consider the ring gadget for x. Due to Proposition 3 for the scores of the teams in G_x there are two possibilities. We set x to be true if its ring gadget (as presented in Figure 1) is oriented counter-clockwise in the sense of Proposition 3, otherwise, we set x to false.

We prove that this variable assignment satisfies φ. Consider a clause C with three variables x, y, z. Assume w.l.o.g. that $t^4_{x,C}$ scores 2 points in game g_C. We claim that then x satisfies C. Proposition 4 implies that since $t^4_{x,C}$ scores 2 points in game g_C, team $t^3_{x,C}$ cannot get any points in game $g^3_{x,C}$. Hence, the other game g of $t^3_{x,C}$ must be won by a team which can get 2 points and have a second placed team which can achieve 3 points.

If x appears positively in C, then by construction we have $g = g^2_{x,C}$. But then with the previous observation and Proposition 3, the ring gadget is oriented counter-clockwise and thus we have set x to true. Thus, x must satisfy C. On the other hand, if x appears negatively in C, then we have $g = g^1_{x,C}$. This implies that the ring gadget is oriented clockwise and thus we have set x to false and hence x satisfies C. ◀

Finally, we observe that in the above construction each team has at most two remaining matches. This completes the proof of Theorem 2. Now we can show that DebatingLeague is NP-hard.

▶ **Theorem 7.** *The DebatingLeague problem is NP-hard even if each team has at most two remaining matches to play.*

Proof. Let (T', M', s') be an instance of ThreeTeamDebating. We modify it to an instance of DebatingLeague. We begin by letting team t_1 win all of its remaining matches in the ThreeTeamDebating instance and updating the score vector accordingly for all teams. In each of the games won by team t_1, we replace t_1 by a dummy team that plays exactly one match and can still score two points. Now we update the instance to have four teams per match: For each game g, we add a dummy team that plays only in g and that can still score three points. Let (T, M, s) denote the resulting instance of DebatingLeague. Observe that in this instance t_1 is not participating in any game.

If $(T', M', s') \in$ ThreeTeamDebating, then we can copy the outcomes of all games to (T, M, s) and then assign 3 points to each dummy team. This gives a solution for DebatingLeague.

On the other hand, consider an outcome of (T, M, s) where t_1 wins the championship. Then the newly added dummy teams do not necessarily have to win their respective games. However, we can resolve this in the following way: For each game won by a non-dummy team, we change the outcome of the match such that the newly added dummy team and the winning non-dummy team change positions. Hence, a newly added dummy team obtains 3 points and the other teams just get fewer points than before. Thus, all newly added dummy teams win their respective games. We do a similar manipulation to make sure that the dummy teams that replaced t_1 score exactly 2 points and we replace them by t_1. This implies that the outcomes of the matches disregarding the dummy teams give a solution for (T', M', s'). ◀

We would like to point out that the above construction can easily be adapted to show that it is also NP-hard to decide whether t_1 can finish among the b best teams for any constant b (and thus in particular if b is part of the input). This can be achieved by simply adding $b - 1$ dummy teams that do not participate in any game and initially have more points than t_1.

3 Debating Tournaments

In this section we will consider the DebatingTournament problem: We are given a set of teams $T = \{t_1, \dots, t_n\}$, where n is a multiple of 4, and a vector $s \in \mathbb{R}^n$, where entry s_i specifies how many points team t_i has scored so far. We further get a parameter k which indicates how many rounds (i.e., match days) are left to play. Contrary to the league setting from the previous section, the fixtures are not determined beforehand. At each match day the teams with ranks $4r+1, 4r+2, 4r+3$, and $4r+4$ for each $r \in \mathbb{N}_0$ play a game. The points for winning the games are distributed as in the DebatingLeague setting. Additionally, we are given a parameter b. We want to decide whether there are outcomes for all remaining matches such that at the end there are at most $b-1$ teams with more points than t_1. Since we assume that in case of ties t_1 is always preferred, this means that t_1 finishes among the b best teams. This is an interesting question since in debating tournaments it is common to have several rounds in the above format, after which only the best b teams are promoted to the playoffs in which a knock-out elimination mode is played. Teams who manage to finish among the b best teams are said to *break*. Note that for $b = 1$ this problem is identical to the question whether team t_1 can still place first. We prove that the problem is fixed parameter tractable (FPT) if both k and b are taken as parameters by giving an algorithm with a running time of $O(f(k+b) \cdot n)$.

Recall the assumption that in tie-breaking t_1 is always preferred. For the other teams, we assume w.l.o.g. that we have a fixed total order for the teams that specifies how to break ties if two teams have exactly the same number of points. The next lemma states a necessary condition for when t_1 can still break: t_1 has to be among the best $4^k b$ teams in the initial ranking s. For our algorithm, we use this lemma to output "no" if t_1 is not among the first $4^k b$ teams in s.

▶ **Lemma 8.** *Let t be a team that is among the best $4^\ell b$ teams when there are $\ell \in \{0, ..., k\}$ rounds left to be played. Then it has to be among the best $4^{\ell+1} b$ teams when there are $\ell + 1$ rounds left to be played. If a team is among the best b teams at the end of the tournament then it must be among the best $4^k b$ teams when there are k rounds left to be played.*

Proof. We start with the first claim. Assume for contradiction that team t is at a position larger than $4^{\ell+1} b$ when there are $\ell + 1$ rounds left to be played and it is among the best $4^\ell b$ teams when there are ℓ rounds left to be played. Observe that in the round when $\ell+1$ games are left, the $4^{\ell+1} b$ best placed teams will play $4^\ell b$ matches. Each of these games must have a winner and among the participating teams $4^\ell b$ teams must win their respective match (i.e., score 3 points) and thus will have more points than t when ℓ rounds are left, even if t wins its match. Hence, with ℓ rounds left to play, team t must have a position worse than $4^\ell b$.

The second claim can be shown by induction using the first claim as the inductive step: If a team t is among the best b teams when $\ell = 0$ rounds are left to be played then it must be among the best $4b$ teams before the last round, among the best $4^2 b$ teams before the last two rounds, ..., and among the best $4^k b$ teams when there are k rounds left. ◀

Now we describe a recursive FPT algorithm with parameters b and k, that solves a given instance of DebatingTournament. We define two sets $S_{>t_1} := \{t_i | s_i > s_1\}$ and $S_{\leq t_1} := \{t_i | s_i \leq s_1\}$. Both sets can be constructed in time $O(n)$. If $|S_{>t_1}| > 4^k b$, then the algorithm stops as team t_1 cannot break anymore by Lemma 8. Otherwise, the algorithm finds the best $4^k b$ teams by taking team t_1, all teams from $S_{>t_1}$ and filling the remaining $4^k b - |S_{>t_1}| - 1$ slots with teams from $S_{\leq t_1}$ in descending order of points. This step can be implemented in time $O(4^k b \cdot n)$: we iterate over all elements of $S_{\leq t_1}$ and keep track of the best team that was not yet added. When the iteration finished, we add the best team we found and mark it as added. We have one iteration over $O(n)$ elements for each free slot of of the $O(4^k b)$ teams, and thus we need a running time of $O(4^k b \cdot n)$. Denote by $T^{(k)}$ the obtained set of teams.

The teams in $T^{(k)}$ play $4^{k-1}b$ matches. We guess the outcomes of all these matches that still allow t_1 to be among the best b teams at the end. For each match there are 4! possible outcomes and thus there are $(4!)^{4^{k-1}b}$ possible game outcomes to enumerate. We update the scores of the teams accordingly. Denote by $T^{(k-1)}$ the first $4^{k-1}b$ teams in the resulting ranking. Lemma 8 implies that in any outcome of all matches of the n given teams all teams in $T^{(k-1)}$ must also be in $T^{(k)}$. This justifies that we enumerate only the matches for the teams in $T^{(k)}$, rather than the matches for all n given teams. Then we guess the outcome of the $4^{k-2}b$ matches for the teams in $T^{(k-1)}$ that allows t_1 to break eventually. We continue recursively for all remaining rounds. For each guess of the outcomes of a round, e.g., when there are only ℓ rounds remaining and we have $4^{\ell}b$ teams left to consider, we make one recursive call to our routine with $\ell - 1$ remaining rounds and $4^{\ell-1}b$ remaining teams.

To evaluate the complexity of the algorithm let us observe that for a single matchday there are at most $(4!)^{4^{k-1}b}$ possible outcomes, since each match has 4! possible outcomes and during a single round of the tournament there are at most $4^{k-1}b$ games to be played. The recursion depth is k which yields an overall running time of $\left((4!)^{4^{k-1}b}\right)^k = 2^{(2b)^{O(k)}}$ of our algorithm.

In total, we need time $O(4^k b \cdot n)$ for the first phase of the algorithm in which we determine the best $4^k b$ teams. For the simulation of all possible outcomes we need time $2^{(2b)^{O(k)}}$. Note that if we set $b = 1$, the algorithm decides in time $n \cdot 2^{2^{O(k)}}$ whether t_1 can place first in a tournament without playoffs. Thus, this problem is FPT for parameter k.

▶ **Theorem 9.** *If there are k remaining rounds to be played in a debating tournament, there is an algorithm that decides in time $n \cdot 2^{(2b)^{O(k)}}$ whether t_1 can place among the first b teams at the end of the tournament.*

3.1 Constant number of rounds

We present an algorithm that decides in time $n^{O(k^4)}$ whether a team can still break if there are k more rounds to play. In particular, this implies that for any constant k the problem is polynomial time solvable, in contrast to DebatingLeague.

As before, suppose we are given a ranking with n teams where for each team t_i we are given a value s_i that denotes how many points team i has scored so far. Again, assume that after the last round the first b teams in the ranking break (and thus participate in the play-offs). Also, we are given a value k that denotes the number of remaining rounds and we want to decide whether t_1 can still break. Consider a round such that including this round there are only $\ell \leq k$ more rounds to play. For each team t_i let s_i^{ℓ} denote its score at the beginning of the round. We distinguish three types of teams: teams t_i with $s_i^{\ell} > s_1^{\ell} + 3\ell$, teams t_i with $s_1^{\ell} - 3\ell \leq s_i^{\ell} \leq s_1^{\ell} + 3\ell$, and teams t_i with $s_i^{\ell} < s_1^{\ell} - 3\ell$. Denote those teams by T_T^{ℓ}, T_M^{ℓ}, and T_B^{ℓ}, respectively (for <u>t</u>op, <u>m</u>iddle, and <u>b</u>ottom). At the end of the tournament, the final score for each team t_i will be in $\{s_i^{\ell}, ..., s_i^{\ell} + 3\ell\}$. Thus, during the last k rounds team t_1 cannot overtake any of the teams in T_T^k and none of the teams in T_B^k can overtake t_1. Thus, intuitively, only the exact scores teams in T_M^k are relevant when deciding whether t_1 can still break. Our algorithm enumerates all possible remaining outcomes of the remaining matches but in doing so, it does not keep track of the scores of the teams in $T_T^k \cup T_B^k$. For the initial scores of the teams in T_M^k there are only $O(k)$ possibilities and during k rounds a team can score at most $O(k)$ points. Thus there are also only $O(k)$ possibilities for the scores of teams in T_M^k during the last k rounds. In order to describe the ranking for those teams, up to permutations it sufficies to keep track of the total number of teams with each of the $O(k)$ possible scores. This yields $n^{O(k)}$ many possibilities in total which allows us to solve the problem via a dynamic program.

Formally, we will pretend that all teams in T_T^k have exactly the same number of points initially and that the same is true for all teams in T_T^k. This is justified by the following lemma.

▶ **Lemma 10.** *Assume that there are only ℓ rounds left to play. Consider an initial ranking given by a number of points s_i^ℓ for each team t_i. Then t_1 can still break if and only if it can still break in any initial ranking given by a number of points $\bar{s}_i^\ell$ for each team t_i such that*

- $s_1^\ell = \bar{s}_1^\ell$,
- *there is a bijection $f : T_M^\ell \to \bar{T}_M^\ell := \{t_i | \bar{s}_i^\ell - 3\ell \leq \bar{s}_1^\ell \leq \bar{s}_i^\ell + 3\ell\}$ such that for each $t_i \in T_M^\ell$ we have that in s^ℓ and $\bar{s}^\ell$ the teams t_i and $f(t_i)$ have the same rank and the same scores and $f(t_1) = t_1$,*
- $|T_T^\ell| = |\bar{T}_T^\ell| := |\{t_i | \bar{s}_i^\ell > \bar{s}_1^\ell + 3\ell\}|$ *and* $|T_B^\ell| = |\bar{T}_B^\ell| := |\{t_i | \bar{s}_i^\ell < \bar{s}_1^\ell - 3\ell\}|$.

Proof. We prove the claim by induction. For $\ell = 0$ it is immediate since t_1 can still break if and only if $|T_T^0| < b$. Suppose now the claim is true for some value ℓ and we want to prove it for $\ell + 1$. It is immediate that t_1 can break in the initial ranking $s^{\ell+1}$ if it can break in any initial ranking $\bar{s}^{\ell+1}$ with the above properties since $s^{\ell+1}$ satisfies these properties.

Now suppose that t_1 can break in the initial ranking $s^{\ell+1}$ and consider an initial ranking $\bar{s}^{\ell+1}$ with the above properties. Consider the outcome of the games in the current round for the ranking $s^{\ell+1}$ such that t_1 breaks after the last round. We construct an outcome of the games of the current round for the initial ranking $\bar{s}^{\ell+1}$. Consider a game $\bar{g}$ in which the teams $\{\bar{t}^{(1)}, \bar{t}^{(2)}, \bar{t}^{(3)}, \bar{t}^{(4)}\}$ participate. There is a corresponding game g, played by team $\{t^{(1)}, t^{(2)}, t^{(3)}, t^{(4)}\}$ according to the initial ranking $s^{\ell+1}$ such that for each $j \in \{1, 2, 3, 4\}$ we have that

- if $\bar{t}^{(j)} \in \bar{T}_M^\ell$ then $t^{(j)} = f^{-1}(\bar{t}^{(j)}) \in T_M^\ell$ and thus in $s^{\ell+1}$ and $\bar{s}^{\ell+1}$ the teams $\bar{t}^{(j)}$ and $t^{(j)}$ have exactly the same rank and the same score,
- if $\bar{t}^{(j)} \in \bar{T}_T^\ell$ then $t^{(j)} \in T_T^\ell$, and
- if $\bar{t}^{(j)} \in \bar{T}_B^\ell$ then $t^{(j)} \in T_B^\ell$.

Note that the first property implies that if $\bar{t}^{(j)} = t_1$ then $t^{(j)} = t_1$. For defining the outcome of $\bar{g}$ we simply the take of the outcome of game g from the known outcomes for all remaining matches that let t_1 break eventually. For each $j \in \{1, 2, 3, 4\}$ we assign the team $\bar{t}^{(j)}$ exactly the same score as team $t^{(j)}$ in those outcomes. We do this operation with all games $\bar{g}$. Denote by $\bar{s}^\ell$ the resulting ranking and by s^ℓ the ranking resulting if we apply those outcomes to $s^{\ell+1}$. Based on the induction hypothesis, we claim that t_1 can still break in $\bar{s}^\ell$. First, it is clear that $s_1^\ell = \bar{s}_1^\ell$ since $f(t_1) = t_1$. Consider a team t_i. If $t_i \in \bar{T}_T^{\ell-1}$ then $t_i \in \bar{T}_T^\ell$ and also if $t_i \in T_T^{\ell-1}$ then $t_i \in T_T^\ell$. Similarly, if $t_i \in \bar{T}_B^{\ell-1}$ then $t_i \in \bar{T}_B^\ell$ and also if $t_i \in T_B^{\ell-1}$ then $t_i \in T_B^\ell$. Finally, if $t_i \in \bar{T}_M^{\ell-1}$ then

- $t_i \in \bar{T}_M^\ell$ if and only if $f^{-1}(t_i) \in T_M^\ell$ and then t_i and $f^{-1}(t_i)$ have the same score in s^ℓ and $\bar{s}^\ell$
- $t_i \in \bar{T}_T^\ell$ if and only if $f^{-1}(t_i) \in T_T^\ell$, and
- $t_i \in \bar{T}_B^\ell$ if and only if $f^{-1}(t_i) \in T_B^\ell$.

Therefore, $|T_T^\ell| = |\bar{T}_T^\ell|$ and $|T_B^\ell| = |\bar{T}_B^\ell|$ and also there is a bijection $f : T_M^\ell \to \bar{T}_M^\ell$ with the properties required by the induction hypothesis. Thus, the induction hypothesis implies that t_1 can still break when starting with the initial ranking $\bar{s}^\ell$. ◀

We use Lemma 10 to justify that we can work with a new initial ranking s' instead of s. Note that the sets $T_B^k \dot\cup T_M^k \dot\cup T_T^k$ form a partition of the participating teams. For each team

$t_i \in T_B^k$ we define $s'_i := 0$. For each team $t_i \in T_M^k$ we define $s'_i := s_i$. For each team $t_i \in T_T^k$ we define $s'_i := s_1 + 3k + 1$. The next proposition follows immediately from Lemma 10.

▶ **Proposition 11.** *The team t_1 can break with the initial ranking s' if and only if it can break with the initial ranking s.*

In our algorithm, we use a dynamic program in order to enumerate all possible outcomes of the remaining k rounds when starting with the initial ranking s'. Key is that there are only $O(k)$ different scores that a team can have during these k rounds since there are only $O(k)$ different initial scores and each team can score at most $3k$ many points. We call two score vectors $\tilde{s}, \tilde{s}'$ *equivalent* if $\tilde{s}_1 = \tilde{s}'_1$ and if for each value x the number of teams with exactly x points is the same in $\tilde{s}$ and $\tilde{s}'$. The team t_1 can clearly break for an initial score vector $\tilde{s}$ if and only if it can still break in any equivalent initial score vector $\tilde{s}'$.

▶ **Lemma 12.** *When starting with the score vector s', there are only $n^{O(k)}$ equivalence classes for the score vectors arising during the last k rounds.*

Proof. For the number of points of t_1 there are only $O(k)$ possibilities. The other teams there can have at most $O(k)$ different scores. Thus, in order to describe an equivalence class it suffices to specify the points of t_1 and how many teams there are with each of the $O(k)$ possible different scores. This gives only $n^{O(k)}$ different possibilities in total. ◀

Our dynamic program works as follows: we have a DP-table entry (ℓ, C) for each $\ell \in \{0, ..., k\}$ and each equivalence class C of the possibly arising score vectors. We store either "yes" or "no" in this cell, corresponding to whether or not t_1 can still break if there are ℓ more rounds to play and we start with a score vector that is equivalent to C.

▶ **Lemma 13.** *Let $\ell \in \{0, ..., k\}$. Suppose we have computed the entry of the cell (ℓ, C') for each equivalence class C'. Then in time $n^{O(k^4)}$ we can compute the entry for a cell $(\ell + 1, C)$.*

Proof. Consider a score vector corresponding to C. We distinguish the different types of the games arising in the current round. We say that two games with teams $\{t^{(1)}, t^{(2)}, t^{(3)}, t^{(4)}\}$ and $\{\bar{t}^{(1)}, \bar{t}^{(2)}, \bar{t}^{(3)}, \bar{t}^{(4)}\}$, respectively, are of the same *type* if there exists a bijection $g : \{t^{(1)}, t^{(2)}, t^{(3)}, t^{(4)}\} \to \{\bar{t}^{(1)}, \bar{t}^{(2)}, \bar{t}^{(3)}, \bar{t}^{(4)}\}$ such that $t^{(j)}$ and $g(t^{(j)})$ have exactly the same score for each $j \in \{1, 2, 3, 4\}$. There are only $O(k^4)$ types of games at only 4! different outcomes for each game. Thus, in order to enumerate all possible outcomes of all games it suffice to guess how many games of each type have which of the 4! possible outcomes. Finally, there are 4! possible outcomes for the game that t_1 participates in. This gives $n^{O(k^4)}$ possibilities in total and for each possibility we obtain a cell (ℓ, C') for some equivalence class C'. ◀

Thus, in time $n^{O(k^4)}$ we can fill the entries of all DP-cells. There is one cell (k, C) such that C corresponds to the equivalence class that contains s'. The entry of this cell is "yes" if and only if t_1 can still break.

▶ **Theorem 14.** *There is an algorithm with running time $n^{O(k^4)}$ that decides whether a given team t_1 can still break if there are at most k remaining rounds to play in a tournament, for an arbitrary breaking threshold b that is part of the input.*

Acknowledgments. The research leading to these results has received funding from the European Research Council under the European Union's Seventh Framework Programme (FP/2007-2013) / ERC Grant Agreement no. 340506. Additionally, this house would like to thank Nicolas McLardy for hosting Stefan in Berlin during the time when this paper was written.

References

1 Mario Basler. Das habe ich ihm dann auch verbal .... Fussballzitate.com. Accessed: 2016-04-22. URL: http://www.fussballzitate.com/zitate/das-habe-ich-ihm-dann-auch-verbal-gesagt.html.

2 Thorsten Bernholt, Alexander Gülich, Thomas Hofmeister, and Niels Schmitt. Football elimination is hard to decide under the 3-point-rule. In *Proceedings of the 24th International Symposium on Mathematical Foundations of Computer Science*, MFCS'99, pages 410–418, London, UK, UK, 1999. Springer-Verlag. doi:10.1007/3-540-48340-3_37.

3 Celebrating 200 years of free speech and the art of debating. The Cambridge Union Society. Accessed: 2015-12-18. URL: https://cus.org/.

4 Mark Cartwright. Athenian democracy. Ancient History Encyclopedia. Accessed: 2015-12-18. URL: http://www.ancient.eu/Athenian_Democracy.

5 Katarína Cechlárová, Eva Potpinková, and Ildikó Schlotter. Refining the complexity of the sports elimination problem. *Discrete Applied Mathematics*, 2015.

6 Nick Cholst. Why 'three points for a win' is a loss for football – a closer look into one of the most important rule changes in football history. Cafè Futebol. Accessed: 2015-12-18. URL: http://www.cafefutebol.net/2013/09/11/why-three-points-for-a-win-is-a-loss-for-football-a-closer-look-into-one-of-the-most-important-rules-in-football-history/.

7 Kelly Phillips Erb. Numerous FIFA Officials Arrested in Massive Corruption Scheme Tied to World Cup, Other Tournaments. Forbes. Accessed: 2015-12-18. URL: http://www.forbes.com/sites/kellyphillipserb/2015/05/27/numerous-fifa-officials-arrested-in-massive-corruption-scheme-tied-world-cup-other-tournaments/.

8 Handbook. FIDE. Accessed: 2015-12-18. URL: https://www.fide.com/fide/handbook.html?id=18&view=category.

9 2014 FIFA World Cup reached 3.2 billion viewers, one billion watched final. Fédération Internationale de Football Association. Accessed: 2015-12-18. URL: http://www.fifa.com/worldcup/news/y=2015/m=12/news=2014-fifa-world-cuptm-reached-3-2-billion-viewers-one-billion-watched--2745519.html.

10 Michael R. Garey and David S. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness*. W. H. Freeman & Co., New York, NY, USA, 1979.

11 Gusfield and Martel. The structure and complexity of sports elimination numbers. *Algorithmica*, 32(1):73–86, 2002. doi:10.1007/s00453-001-0074-y.

12 Marting Luther King Jr. Martin Luther King I Have a Dream Speech. American Rhetoric. Accessed: 2015-12-18. URL: http://www.americanrhetoric.com/speeches/mlkihaveadream.htm.

13 Walter Kern and Daniël Paulusma. The new FIFA rules are hard: complexity aspects of sports competitions. *Discrete Applied Mathematics*, 108(3):317–323, 2001. doi:10.1016/S0166-218X(00)00241-9.

14 Walter Kern and Daniël Paulusma. The computational complexity of the elimination problem in generalized sports competitions. *Discrete Optimization*, 1(2):205–214, 2004. doi:10.1016/j.disopt.2003.12.003.

15 S. Thomas McCormick. Fast algorithms for parametric scheduling come from extensions to parametric maximum flow. In *Proceedings of the Twenty-eighth Annual ACM Symposium on Theory of Computing*, STOC'96, pages 319–328, New York, NY, USA, 1996. ACM. doi:10.1145/237814.237978.

16 Andreas Möller. "Hauptsache Italien!" Legendäre Fußball-Sprüche. tz München. Accessed: 2016-04-22. URL: http://www.tz.de/sport/fussball/hauptsache-italien-legendaere-fussball-sprueche-fotostrecke-zr-3286394.html.

17 Philip Oltermann. Angela Merkel the mascot: German chancellor's relationship with the team. The Guardian. Accessed: 2015-12-18. URL: `http://www.theguardian.com/world/2014/jul/14/angela-merkel-chancellor-german-team-relationship`.

18 Dömötör Pálvölgyi. Deciding soccer scores and partial orientations of graphs. *Acta Univ. Sapientiae, Math*, 1(1):35–42, 2009.

19 Stuart Pearce. Stuart Pearce quotes. Think Exist. Accessed: 2016-04-22. URL: `http://thinkexist.com/quotation/i-can-see-the-carrot-at-the-end-of-the-tunnel/539152.html`.

20 Ronaldo. Ronaldo quotes. Think Exist. Accessed: 2016-04-22. URL: `http://thinkexist.com/quotation/we-lost-because-we-didn-t-win/539102.html`.

21 B. L. Schwartz. Possible winners in partially completed tournaments. *SIAM Review*, 8(3):302–308, 1966. `doi:10.1137/1008062`.

22 Jürgen Voges. Bundeskanzler: Als Gerhard Schröder noch den Rasen pflügte. Stern. Accessed: 2015-12-18. URL: `http://www.stern.de/politik/deutschland/bundeskanzler-als-gerhard-schroeder-noch-den-rasen-pfluegte-3067112.html`.

23 Kevin D. Wayne. A new property and a faster algorithm for baseball elimination. *SIAM Journal on Discrete Mathematics*, 14(2):223–229, 2001. `doi:10.1137/S0895480198348847`.

24 Rules. World Debating News. Accessed: 2015-12-18. URL: `http://worlddebating.blogspot.ie/p/rules.html`.