The Price of Local Power Control in Wireless Scheduling*

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Abstract

We consider the problem of scheduling wireless links in the physical model, where we seek an assignment of power levels and a partition of the given set of links into the minimum number of subsets satisfying the signal-to-interference-and-noise-ratio (SINR) constraints. Specifically, we are interested in the efficiency of local power assignment schemes, or *oblivious power schemes*, in approximating wireless scheduling. Oblivious power schemes are motivated by networking scenarios when power levels must be decided in advance, and not as part of the scheduling computation.

We present the first $O(\log\log\Delta)$-approximation algorithm, which is known to be best possible (in terms of Δ) for oblivious power schemes, where Δ is the longest to shortest link length ratio. We achieve this by representing interference by a conflict graph, which allows the application of graph-theoretic results for a variety of related problems, including the weighted capacity problem. We explore further the contours of approximability and find the choice of power assignment matters; that not all metric spaces are equal; and that the presence of weak links makes the problem harder. Combined, our results resolve the *price of local power* for wireless scheduling, or the value of allowing unfettered power control.

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1 Introduction

We treat the fundamental *scheduling* problem of partitioning a given set of transmission requests (links) in a wireless network into the fewest possible feasible subsets. Scheduling problems arise from the MAC layer of wireless networks, and solving them requires effective spatial reuse while dealing with wireless interference. We use the *SINR model* for modeling interference, where signal decays as it travels and a transmission is successful if its strength at the receiver exceeds the accumulated signal strength of interfering transmissions by a sufficient factor. Although the standard analytic assumption that signal decays polynomially with the distance traveled is far from realistic [36, 31], it has been shown that results obtained with that assumption can be translated to the setting of arbitrary measured signal decay [2, 13]. A number of studies dedicated to elucidating algorithmic properties of the SINR model has appeared in recent years (e.g., [32, 11, 1, 6, 29, 27, 21, 5, 25]).

Abstracting wireless interference by *conflict graphs* is much more common in wireless research. Arbitrary graphs are too general to be useful (in the worst case) for scheduling

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problems. Instead, the standard modus operandi is to assume geometric intersection graphs, where nodes are represented by disks, such as *unit disk graphs* or the *protocol model* [14]. Unfortunately, disk graphs provably lack fidelity to the reality of wireless signals, being simultaneously too conservative and too loose [34, 33]. Yet, the graph abstraction is cleaner and connects better to the literature, which leads us to search for alternative classes.

Problem Formulations. Given as input is a set Γ of n communication *links*; each link is a pair of a sender and receiver, which are nodes in a metric space. The senders can adjust their power as needed; in each of the problems we consider, finding the appropriate power assignment is a part of the problem. A subset $S \subseteq \Gamma$ of links is *feasible* if there exists a power assignment for which the transmission on each link satisfies the SINR formula (see Section 2) when the links in S transmit simultaneously. We treat the following two problems:

Scheduling: Partition Γ into fewest number of feasible sets.

WCapacity: Find the maximum weight feasible subset of Γ, when the links have positive weights. When WCapacity is restricted to unit weights, we get the related unweighted Capacity problem. The WCapacity problem is of fundamental importance to dynamic scheduling where requests arrive over time, as it is demonstrated in a celebrated paper [37].

Optimal solutions to our problems may require global power management: the power assigned to a link may depend on all the other links. However, it is desirable in various restricted settings to let the power chosen for a link depend only on the link itself, specifically on the link length. Such local power regimes are called *oblivious*. The use of oblivious power assignments may be forced in cases when having a separate power control phase is not affordable. Oblivious powers may also be preferable due to scalability concerns: when a new link is added, the power assignments of other links are left intact (as opposed to globally managing power).

The main goal of this paper is to solve the Scheduling and WCapacity problems using *only* oblivious power assignments, while still comparing the quality of the solution to the optimal solution that can use arbitrary power assignment. It was shown in [8] that every oblivious power assignment can be worst possible factor n from optimal. In terms of the parameter Δ, the bound becomes $\Omega(\log\log\Delta)$ [8, 15], i.e. the factor n may appear only when the network contains exponentially long links. Indeed, there is an algorithm using oblivious power that achieves $O(\log\log\Delta\log n)$-approximation (compared with best achievable with arbitrary power) [15]. For Capacity, this was improved to $O(\log\log\Delta)$-factor [18], which holds for every metric space and for a wide range of oblivious power assignments.

Finding constant-factor approximations for Scheduling is still an open problem. However, there are several approaches giving logarithmic approximation. The (unweighted) Capacity problem has an efficient constant-factor approximation, due to Kesselheim [27] that holds for general metrics [28], as well as for versions with various fixed power assignments [19]. This immediately yields $O(\log n)$-approximation for Scheduling. A different approach is to divide the links into groups of nearly equal length and schedule each group separately. Following this approach, numerous $O(\log\Delta)$-approximation results have been argued [12, 10, 15], where Δ is the ratio between the longest and shortest link length. In a recent work, we propose a novel conflict-graph based approach that yields $O(\log^*\Delta)$ approximation for Scheduling and WCapacity using *non-oblivious* power assignment [22].

Results. Our main result is $O(\log\log\Delta)$-approximation algorithms for Scheduling and WCapacity using oblivious power assignments. This is an exponential improvement over existing approximations using oblivious powers and matches the known lower bounds [8, 15].

This is also the first improvement over logarithmic approximations for fixed power scheduling, where we compare to the optimum schedule with respect to *given* power assignment. Even though $O(\log\log\Delta)$ is weaker than the approximation of $O(\log^*\Delta)$ obtained in [22], it still demonstrates the remarkable closeness of oblivious powers to the optimum power assignments. After all, $\log\log\Delta \le 5$ in all practical applications.

Unlike the state of affairs for the Capacity problem, our results are surprisingly sensitive to the metric and the exact power assignment. They hold for doubling metrics, but provably fail in general (or even tree) metrics, and they hold for a range of power assignments, while for others they provably fail (including the most common assignments – *uniform* and *linear*).

Our main result is obtained by using the conflict graph framework of [22], which essentially reduces the notoriously hard SINR optimization to graph problems, for which a large body of theory can be brought to bear.

Further Related Work. Gupta and Kumar [14] proposed the geometric version of SINR and initiated average-case analysis of capacity known as scaling laws. Moscibroda and Wattenhofer [32] initiated worst-case analysis in the SINR model. Early work on the Scheduling problem includes [4, 7, 3]. NP-completeness has been shown for Scheduling with different forms of power control: none [12], limited [26], and unbounded [30]. Distributed algorithms attaining $O(\log n)$-approximation are also known [29, 16]. Scheduling and WCapacity have also been considered for *fixed oblivious power assignments* [23, 8, 15, 20, 9]. The only known constant-factor approximation algorithms for these problems are obtained in the case of the *linear power scheme* [20, 39].

2 Model and Definitions

Communication Links. Consider a set Γ of n *links*, numbered from 1 to n. Each link i represents a unit-demand communication request from a sender s_i to a receiver r_i - point-size wireless transmitter/receivers located in a metric space with distance function d. We denote $d_{ij} = d(s_i, r_j)$ the distance from the sender of link i to the receiver of link j, $l_i = d(s_i, r_i)$ the *length* of link i and $d(i,j) = d(j,i)$ the minimum distance between a node of link i and a node of link j. We let $\Delta(\Gamma)$ denote the ratio between the longest and shortest link lengths in Γ, and drop Γ when clear from context. A set of links S is *equilength* if $\Delta(S) \le 2$.

Power Schemes. A *power assignment* for Γ is a function $P : \Gamma \to \mathbb{R}_+$. For each link i, $P(i)$ defines the power level used by the sender node s_i. We will be particularly interested in *power schemes* P_τ of the form $P_\tau(i) = C \cdot l_i^{\tau\alpha}$, where C is constant for the given network instance. These are called *oblivious power assignments* because the power level of each link depends only on a local information - the link length. Examples of such power schemes are *uniform* power scheme (P_0), *linear* power scheme (P_1) and *mean* power scheme ($P_{1/2}$) [8].

SINR Feasibility. In the *physical model (or SINR model)* of communication [35], a transmission of a link i is successful if and only if

$$\mathcal{S}_i \ge \beta \cdot \left(\sum_{j \in S \setminus \{i\}} \mathcal{I}_{ji} + N \right), \tag{1}$$

where $\mathcal{S}_i$ denotes the received signal power of link i, $\mathcal{I}_{ji}$ denotes the interference power on link i caused by link j, $N \ge 0$ is a constant denoting the ambient noise, $\beta > 0$ is the minimum

SINR (Signal to Interference and Noise Ratio) required for a message to be successfully received and S is the set of links transmitting concurrently with link i. If P is the power assignment used, then $\mathcal{S}_i = \frac{P(i)}{l_i^\alpha}$ and $\mathcal{I}_{ji} = \frac{P(j)}{d_{ji}^\alpha}$, where $\alpha \in (2,6)$ is the path-loss exponent.

A set L of links is called *P-feasible* if the condition (1) holds for each link $i \in L$ when using power assignment P. We say L is *feasible* if there exists a power assignment P for which L is P-feasible. Similarly, a collection of sets is P-feasible/feasible if each set in the collection is.

Capacity and Scheduling Problems. Scheduling denotes the problem of partitioning a given set Γ into the minimum number of feasible subsets (or *slots*). WCapacity denotes the problem where we are also given a weight function $\omega : \Gamma \to \mathbb{R}_+$ on the links and we seek a maximum weight feasible subset S of Γ.

Affectance. Following [23], we define the *affectance* $a_P(i,j)$ of link i by link j under power assignment P by

$$a_P(j,i) = c_i \frac{\mathcal{I}_{ji}}{\mathcal{S}_i} = c_i \frac{P(j) l_i^\alpha}{P(i) d_{ji}^\alpha},$$

where $c_i = 1/(1 - \beta N l_i^\alpha / P(i))$ is a factor depending on the properties of link i[1]. We let $a_P(j,j) = 0$ and extend a_P additively over sets: $a_P(S,i) = \sum_{j\in S} a_P(j,i)$ and $a_P(i,S) = \sum_{j\in S} a_P(i,j)$. It is readily verified that a set of links S is feasible if and only if $a_P(S,i) \le 1/\beta$ for all $i \in S$. We call a set of links *p-P-feasible* for a parameter $p > 0$ if $a_P(S,i) \le 1/p$.

The following is an important tool showing that modifying the threshold value β by a constant factor affects the solutions only by a constant factor.

▶ **Theorem 1** ([17]). *Any p-P-feasible set can be partitioned into $\lceil 2p'/p \rceil$ subsets, each of which is p'-P-feasible.*

We make the standard assumption that for all links i in the instance, received signal power is a little higher than necessary to overcome the noise term N alone in the absence of any other transmission: $P(i) \ge c\beta N l_i^\alpha$ for some constant $c > 1$. This can be achieved by scaling the power levels of links. This assumption helps to avoid the terms c_i in the affectance formula. Indeed, it implies that $c_i \le c/(c-1)$ for all i. Then given e.g. a Scheduling instance Γ, we can solve it with $c_i = 1$ for all i and $\beta' = (c-1)\beta/c$, getting a feasible solution for the original problem. Moreover, by Thm. 1, the number of slots obtained will be at most a constant factor away from the optimum of the original problem. Thus, we assume henceforth that $c_i = 1$ for all links i, i.e. $a_P(i,j) = \frac{P(j) l_i^\alpha}{P(i) d_{ji}^\alpha}$. We have in particular that $a_{P_\tau}(i,j) = l_i^{(1-\tau)\alpha} \cdot l_j^{\tau\alpha} / d_{ij}^\alpha$.

Remark. In practice, there is an upper limit P_{max} on the available power level of links and for some links, even setting $P(i) = P_{max}$ can be insufficient for having $P(i) \ge c\beta N l_i^\alpha$. Such links are called *weak links*. Our assumption thus amounts to excluding weak links. Weak links are further discussed in Sec. 5.

[1] If the denominator of c_i is 0, i.e. $P(i) = \beta N l_i^\alpha$, then link i must always be scheduled separately from all other links. We assume that there are no such links.

Fading Metrics. The *doubling dimension* of a metric space is the infimum of all numbers $\delta > 0$ such that every ball of radius $r > 0$ has at most $C\epsilon^{-\delta}$ points of mutual distance at least ϵr where $C \geq 1$ is an absolute constant, $\delta > 0$ and $0 < \epsilon \leq 1$. Metrics with finite doubling dimension are called *doubling*. For instance, the m-dimensional Euclidean space is doubling with doubling dimension m [24]. We will assume for the rest of the paper that the links are located in a metric space with doubling dimension $m < \alpha$. Such metrics are called *fading metrics*.

3 Conflict Graphs and Oblivious Power Scheduling

Conflict graphs are graphs defined over the set of links. Let us call a conflict graph $A(L)$ an *upper bound graph* for a set L, if there is a power scheme P_τ such that each independent set in $A(L)$ is P_τ-feasible. Similarly, we call a graph $B(L)$ a *lower bound graph* for L if there is a power scheme $P_{\tau'}$ such that each $P_{\tau'}$-feasible set induces an independent set in $B(L)$. Note that the chromatic numbers of $A(L)$ and $B(L)$ give upper and lower bounds for Scheduling with oblivious power schemes. Moreover, if the vertex coloring problem for $A(L)$ can be efficiently approximated, then the upper bound is constructive. Now, our aim is to construct upper and lower bound graphs with $\tau = \tau'$ such that the gap between their chromatic numbers is bounded. The less the gap, the better colorings of $A(L)$ approximate Scheduling with oblivious power P_τ.

The outline of this section is as follows. First, we present a family of conflict graphs introduced in [22] and point out a sub-family $\mathcal{G}_\gamma$ that are lower bound graphs. Next, we present a family of upper bound graphs $\mathcal{G}_\gamma^\delta$ and show that the gap between the chromatic numbers is $O(\log\log\Delta)$. This section is concluded with the main theorem which, based on the method outlined above, presents $O(\log\log\Delta)$-approximation algorithms for Scheduling and WCapacity with oblivious power schemes.

Conflict Graphs, Lower Bound Graphs. Let $f : \mathbb{R}_+ \to \mathbb{R}_+$ be a positive monotonically non-decreasing function. Two links i, j are *f-independent* if

$$\frac{d(i,j)}{l_{min}} > f\left(\frac{l_{max}}{l_{min}}\right),$$

where $l_{min} = \min\{l_i, l_j\}, l_{max} = \max\{l_i, l_j\}$, and otherwise they are *f-conflicting*. A set of links is f-independent if they are pairwise f-independent.

Given a set L of links, $\mathcal{G}_f(L)$ denotes the graph with vertex set L where two vertices $i, j \in L$ are adjacent if and only if they are f-adjacent.

We will be particularly interested in conflict graphs $\mathcal{G}_f$ with $f(x) \equiv \gamma$ and $f(x) = \gamma \cdot x^\delta$ for constants $\gamma > 0$ and $\delta \in (0,1)$. We will use the notation $\mathcal{G}_\gamma$ in the former case and the notation $\mathcal{G}_\gamma^\delta$ in the latter case. We will refer to independence (conflict) in $\mathcal{G}_\gamma$ as γ-independence (γ-conflict, resp.) and to independence (conflict) in $\mathcal{G}_\gamma^\delta$ as (γ, δ)-independence ((γ, δ)-conflict, resp.). Note that $\mathcal{G}_\gamma$ is equivalent to $\mathcal{G}_\gamma^0$.

It will be useful to note that two links i, j are γ-independent iff $d(i,j) > \gamma l_j$ and are (γ, δ)-independent iff $d(i,j) > \gamma l_i^\delta l_j^{1-\delta}$, where l_i is the longer link, i.e. $l_j \leq l_i$.

We will need the following properties of conflict graphs $\mathcal{G}_\gamma^\delta$ that are obtained by applying the results of [22] to our conflict graphs (i.e. $\mathcal{G}_f$ with $f(x) = \gamma x^\delta$). These properties hold for any set L of links in a metric space of fixed doubling dimension. We let $\chi(G)$ denote the chromatic number of a graph G.

▶ **Theorem 2.** *Let $\delta \in (0,1)$ be constant. It holds that*
1. *for any constant $\gamma > 0$, $\chi(\mathcal{G}_\gamma^\delta(L)) = \chi(\mathcal{G}_\gamma(L)) \cdot O(\log\log\Delta)$,*
2. *if $\beta > 1$, there is a constant $\gamma > 0$ s.t. $\mathcal{G}_\gamma(L)$ is a lower bound graph,*
3. *vertex coloring and maximum weighted independent set problems are constant factor approximable in graphs $\mathcal{G}_\gamma^\delta(L)$.*

The first property ([22, Thm. 1]) bounds the gap between graphs $\mathcal{G}_\gamma^\delta(L)$ and $\mathcal{G}_\gamma(L)$. The second property ([22, Thm. 4]) shows that for an appropriate constant γ, the graph $\mathcal{G}_\gamma$ is a lower bound graph, i.e. each feasible set is independent in $\mathcal{G}_\gamma$ (this includes also P_τ-feasible sets for *any* τ). The last property ([22, Prop. 1]) shows that the graphs $\mathcal{G}_\gamma^\delta$ are algorithmically accessible.

Upper Bound Graphs. Here we show that for appropriate values of δ and γ, graphs $\mathcal{G}_\gamma^\delta(L)$ are upper bound graphs, i.e. each independent set in $\mathcal{G}_\gamma^\delta(L)$ is feasible with the appropriate oblivious power assignment. This complements the conflict graph framework described in the beginning of the section.

In order to bound the affectance of a given link i by an independent set S of links, we first split S into length classes (i.e. equilength subsets) and bound the affectance of i by each length class separately (lemmas 6 and 7). Then we combine the obtained bounds in a series that converges under the assumption that the links are in a fading metric (Cor. 8). The affectance of link i by each length class is bounded by using the common "concentric annuli" argument where the rough idea is to partition the metric space into concentric annuli centered at an endpoint of i, bound the number of links in each annulus using link independence and the doubling property of the space, use these bounds to bound the affectance by links from different annuli and combine them into a converging series. This scheme has also been used in [22]. The main difficulty here is that we have to deal with the affectance of a given link by both longer and shorter links, as opposed to [22] where we had to consider only shorter links. The parameter δ of $\mathcal{G}_\gamma^\delta$ has to be chosen very carefully in order to guarantee bounded affectance by both longer and shorter links.

We will obtain a slightly stronger result than feasibility. Our results hold in terms of the function $f_\tau(i,j)$ with a parameter $\tau \in [0,1]$, where

$$f_\tau(i,j) = \frac{l_i^{\tau\alpha} \cdot l_j^{(1-\tau)\alpha}}{d(i,j)^\alpha}.$$

Note that for any pair of links i,j, $a_{P_\tau}(i,j) \le f_\tau(i,j)$. The function $f_\tau(i,j)$ is extended additively to sets of links, similar to the function $a_P(i,j)$.

In the following core lemma we show that the affectance of a fixed link i by an independent equilength set S of links (i.e. $\Delta(S) \le 2$) can be bounded by the ratio of the length l_i and the minimum length in S if S and i are not too close to each other. The idea of the proof is the "concentric annuli" argument described above. We will use the following two facts.

▶ **Fact 3.** *Let $\alpha \ge 1$ and $r \ge 0$ be real numbers. Then $\frac{1}{r^\alpha} - \frac{1}{(r+1)^\alpha} \le \frac{\alpha}{(r+1)^{\alpha+1}}$.*

▶ **Fact 4.** *Let $g(x) = \dfrac{1}{(q+x)^\gamma}$, where $\gamma > 1$ and $q > 0$. Then $\sum_{r=0}^\infty g(r) \in O\left(\dfrac{1}{q^{\gamma-1}} + \dfrac{1}{q^\gamma}\right)$.*

▶ **Lemma 5.** *Let $\delta, \tau \in (0,1)$ and $\gamma \ge 1$, let S be an equilength set of 1-independent links, and let i be a link s.t. i,j are (γ,δ)-independent for all $j \in S$ and either $l_i \ge l_j$ for all $j \in S$ or $l_i \le l_j$ for all $j \in S$. Then,*

$$f_\tau(S,i) \in O\left(\gamma^{m-\alpha}\left(\frac{l_i}{\ell}\right)^{(1-\tau)\alpha-\delta'(\alpha-m)} \cdot \min\left\{1, \frac{l_i}{\ell}\right\}^{-\delta'}\right),$$

where ℓ denotes the shortest link length in S and $\delta' = \delta$ if $l_i \ge \ell$ and $\delta' = 1-\delta$ otherwise.

Proof. First, let us split S into two subsets S' and S'' such that S' contains the links of S that are closer to r_i than to s_i, i.e. $S' = \{j \in S : \min\{d(s_j,r_i), d(r_j,r_i)\} \le \min\{d(s_j,s_i), d(r_j,s_i)\}\}$ and $S'' = S \setminus S'$. Let us consider the set S' first.

For each link $j \in S'$, let p_j denote the endpoint of j that is closest to node r_i. Denote $q = \gamma l_i^{\delta'} \ell^{-\delta'}$. Consider the subsets $S_1, S_2, \dots$ of S', where $S_r = \{j \in S' : d(j,i) = d(p_j, r_i) \le q\ell + (r-1)\ell\}$. Note that S_1 is empty: $d(j,i) > \gamma l_i^{\delta'} \ell^{1-\delta'} = q\ell$ for all $j \in S'$ because i, j are (γ,δ)-independent, and so $S' = \cup_{r=2}^{\infty} S_r$. Let us fix an $r > 1$. Consider any two links $j,k \in S_r$ s.t. $l_j \ge l_k$. We have that $d(p_j,p_k) \ge d(j,k) > \ell$ (1-independence) and that $d(p_j,r_i) \le \gamma q\ell + (r-1)\ell$ for each $j \in S_r$ (by the definition of S_r), so using the doubling property of the metric space, we get the following bound:

$$|S_r| = |\{p_j\}_{j\in S_r}| \le C \cdot \left(\frac{q\ell + (r-1)\ell}{\ell}\right)^m = C\,(q+r-1)^m . \tag{2}$$

Note also that $l_j \le 2\ell$ and $d(i,j) \ge q\ell + (r-2)\ell$ for any link $j \in S_r \setminus S_{r-1}$ with $r > 1$; hence,

$$f_\tau(j,i) = \frac{l_j^{\tau\alpha} l_i^{(1-\tau)\alpha}}{d(i,j)^\alpha} \le \ell^{(\tau-1)\alpha} l_i^{(1-\tau)\alpha}\left(\frac{2\ell}{q\ell + (r-2)\ell}\right)^\alpha = \frac{2^\alpha \ell^{(\tau-1)\alpha} l_i^{(1-\tau)\alpha}}{(q+r-2)^\alpha}. \tag{3}$$

Recall that $S_{r-1} \subseteq S_r$ for all $r > 1$, $S_1 = \emptyset$ and $S' = \cup_{r=2}^{\infty} S_r$. Using (3), we have:

$$\begin{aligned} f_\tau(S',i) &= \sum_{r\ge 2}\sum_{j\in S_r\setminus S_{r-1}} f_\tau(j,i) \\ &\le \sum_{r\ge 2}(|S_r| - |S_{r-1}|)\,\frac{2^\alpha \ell^{(\tau-1)\alpha} l_i^{(1-\tau)\alpha}}{(q+r-2)^\alpha} \\ &= 2^\alpha\left(\frac{l_i}{\ell}\right)^{(1-\tau)\alpha}\sum_{r\ge 2}|S_r|\left(\frac{1}{(q+r-2)^\alpha} - \frac{1}{(q+r-1)^\alpha}\right), \end{aligned} \tag{4}$$

where the last equality is just a rearrangement of the sum. The sum can be bounded as follows:

$$\begin{aligned} \sum_{r\ge 2}|S_r|\left(\frac{1}{(q+r-2)^\alpha} - \frac{1}{(q+r-1)^\alpha}\right) &\le \sum_{r\ge 2}|S_r|\frac{\alpha}{(q+r-1)^{\alpha+1}} \\ &\le \sum_{r\ge 2}\frac{C\alpha(q+r-1)^m}{(q+r-1)^{\alpha+1}} \\ &= O\left(\sum_{r\ge 2}\frac{1}{(q+r-1)^{\alpha-m+1}}\right) \\ &= O\left(\frac{1}{q^{\alpha-m}} + \frac{1}{q^{\alpha-m+1}}\right) \\ &= O\left(\frac{1}{(\gamma l_i^{\delta'}\ell^{-\delta'})^{\alpha-m}} + \frac{1}{(\gamma l_i^{\delta'}\ell^{-\delta'})^{\alpha-m+1}}\right) \\ &= O\left(\gamma^{m-\alpha}\left(\frac{\ell}{l_i}\right)^{\delta'(\alpha-m)}\left(1 + \left(\frac{\ell}{l_i}\right)^{\delta'}\right)\right), \end{aligned}$$

where the first line follows from Fact 3, the second one follows from (2) and the fourth one follows from Fact 4. Combined with (4), this completes the proof for the set S'.

The proof holds symmetrically for the set S''. Recall that S'' consists of the links of S which are closer to the sender s_i than to the receiver r_i. Now, we can re-define p_j to denote the endpoint of link j that is closer to s_i, for each $j \in S''$. The rest of the proof will be identical, by replacing r_i with s_i in the formulas. This is justified by the symmetry of (γ, δ)-independence. ◀

In the following two lemmas we bound the affectance of a fixed link i by a set L of independent links that is sufficiently separated from i. The two cases when L consists of links longer than i and shorter than i are treated separately because they impose different conditions on parameters δ and τ. The idea of the proof is to split L into length classes, bound the affectance by each length class using Lemma 5 then combine the obtained bounds in a geometric series.

▶ **Lemma 6.** *Let L be a 1-independent set of links and i be a link s.t. $l_i \geq l_j$ and i, j are (γ, δ)-independent for all $j \in L$. Then for each $\tau > 1 - \delta(1 - m/\alpha)$, $f_\tau(L, i) = O(\gamma^{m-\alpha})$.*

Proof. Let us split L into length classes $L_1, L_2, \ldots$ with $L_t = \{j \in L : 2^{t-1}\ell \leq l_j < 2^t\ell\}$ where ℓ is the shortest link length in L. Let ℓ_t be the shortest link length in L_t. Note that each L_t is an equilength 1-independent set of links that are (γ, δ)-independent from link i. Thus, the conditions of Lemma 5 hold for each L_t (with $\delta' = \delta$ since all links in L_t are shorter than link i):

$$f_\tau(L_t, i) = O\left(\gamma^{m-\alpha}\left(\frac{\ell_t}{l_i}\right)^{\delta(\alpha-m)-(1-\tau)\alpha}\right).$$

Recall that L_t are equilength sets and $\ell_t \geq 2^{t-1}\ell$. That allows us to combine the bounds above into a geometric series:

$$f_\tau(L, i) = \sum_1^\infty f_\tau(L_t, i) \leq \frac{C \cdot \gamma^{m-\alpha}}{l_i^{(1-\tau)\alpha-\delta(\alpha-m)}} \sum_{t=0}^{\lceil \log l_i/\ell \rceil} (2^t\ell)^{(1-\tau)\alpha-\delta(\alpha-m)},$$

where C is a constant. The upper limit of the last sum is obtained by the fact that link i is not shorter than the longest link in L. Recall that $\tau > 1 - \delta(1 - m/\alpha)$; hence, $\delta(\alpha - m) - (1 - \tau)\alpha > 0$. Thus, the last sum is the sum of a *growing* geometric progression and is $O(l_i^{(1-\tau)\alpha-\delta(\alpha-m)})$, implying the lemma. ◀

▶ **Lemma 7.** *Let L be a 1-independent set of links and i be a link s. t. $l_i \leq l_j$ and i, j are (γ, δ)-independent for all $j \in L$. Then for each $\tau < 1 - (1 - \delta)(\alpha - m + 1)/\alpha$, $f_\tau(L, i) = O(\gamma^{m-\alpha})$.*

Proof. Let us split L into length classes $L_1, L_2, \ldots$, where $L_t = \{j \in L : 2^{t-1}l_i \leq l_j < 2^t l_i\}$. Note that each L_t is a equilength 1-independent set of links that are (γ, δ)-independent from link i. Let ℓ_t denote the shortest link length in L_t. Recall that $\ell_t \geq 2^{t-1}l_i$. Thus, Lemma 5 implies (note that $\delta' = 1 - \delta$ in this case):

$$f_\tau(L_t, i) = O\left(\gamma^{m-\alpha}\left(\frac{l_i}{\ell_t}\right)^\eta\right) = O\left(\gamma^{m-\alpha}\left(\frac{1}{2^{t-1}}\right)^\eta\right),$$

where $\eta = (1 - \tau)\alpha - (1 - \delta)(\alpha - m + 1)$. Recall that $\tau < 1 - (1 - \delta)(\alpha - m + 1)/\alpha$, implying $\eta > 0$. Thus, we have:

$$f_\tau(L, i) = \sum_1^\infty f_\tau(L_t, i) \leq \gamma^{m-\alpha} \sum_{t=0}^{\lceil \log l_i/\ell \rceil} \frac{1}{2^{\eta t}} = O\left(\gamma^{m-\alpha}\right),$$

where C is a constant. ◀

By combining Lemmas 6 and 7 we find a family of upper bound graphs.

▶ **Corollary 8.** *If $\delta \in (\delta_0, 1)$ and the constant $\gamma > 1$ is large enough, the graphs $\mathcal{G}_\gamma^\delta(L)$ are upper bound graphs for any set L, where $\delta_0 = \frac{\alpha-m+1}{2(\alpha-m)+1}$. Namely, there exists $\tau \in (0,1)$ s.t. any (γ, δ)-independent set is P_τ-feasible. Moreover, we can choose $\tau = \delta$ whenever $\delta > \alpha/(2\alpha - m)$ and $m > 1$.*

Proof. We need to show that Lemmas 6 and 7 hold simultaneously for the given δ and certain $\tau \in (0,1)$. Then we can adjust γ in order to make L feasible. The constraints of the mentioned lemmas on δ and τ are as follows:

$$\tau > 1 - \delta\frac{\alpha - m}{\alpha} \text{ and } \tau < 1 - (1-\delta)\frac{\alpha - m + 1}{\alpha}. \tag{5}$$

So it is enough to show that any $\delta \in (\delta_0, 1)$ is a solution for the following system of inequalities:

$$0 < 1 - \delta\frac{\alpha - m}{\alpha} < 1 - (1-\delta)\frac{\alpha - m + 1}{\alpha} < 1. \tag{6}$$

The first and third inequalities hold whenever $\delta < 1$ and $\alpha > m$. The second inequality is equivalent to $\delta > \delta_0$. The conditions for choosing $\tau = \delta$ follow by setting $\tau = \delta$ in (5). ◀

Putting the Pieces Together. All the components of the conflict graph framework are ready now: a lower bound graph $\mathcal{G}_\gamma$, efficiently colorable upper bound graphs $\mathcal{G}_\gamma^\delta$ and a bound on the gap between the chromatic numbers of those graphs (by Thm. 2). Hence, we can apply the technique described at the beginning of this section to prove the main result – a $O(\log\log\Delta)$-approximation algorithm for Scheduling and WCapacity, using oblivious power schemes.

▶ **Theorem 9.** *There are $O(\log\log\Delta)$-approximation algorithms for Scheduling and WCapacity using oblivious power schemes. The approximation is obtained by approximating vertex coloring or maximum weighted independent set problems in $\mathcal{G}_\gamma^\delta(L)$ with appropriate constants γ and δ.*

4 Limitations of Oblivious Power Schemes

Euclidean Metrics. We proved that it is possible to approximate Scheduling and WCapacity within a factor of $O(\log\log\Delta)$ using oblivious power schemes. As shown in the theorem below, this bound is essentially best possible when using oblivious power assignments. The following was shown in greater generality in [15].

▶ **Theorem 10.** *[15] For every power scheme P_τ, there is an infinite family of feasible sets S arranged in a straight line such that any schedule of S using P_τ requires $\Omega(\log\log\Delta)$ slots.*

Recall that we obtained our approximations only for oblivious power schemes P_τ with τ falling in a specific sub-interval of $(0,1)$. What happens with the other oblivious power schemes? Interestingly, as we show below, oblivious power schemes P_τ with τ outside the range stipulated by Lemmas 6 and 7 yield only $O(\log\Delta)$-approximation for Scheduling and WCapacity.

We consider a family of sets L of $(1,1)$-independent links that are located in the Euclidean plane (hence, $m = 2$). With separation $\delta = 1$, the range of oblivious power schemes P_τ making L (almost) feasible according to Lemmas 6 and 7 is: $2/\alpha < \tau < 1$. In the following theorem

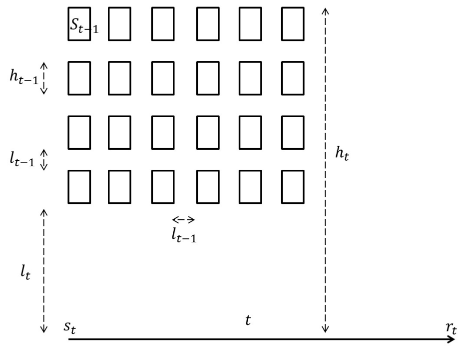

Figure 1 $(1,1)$-independent instance S_t. Each rectangle represents a sub-instance that is a translated copy of S_{t-1}.

we show that no scheduling algorithm can achieve better than $O(\log \Delta)$-approximation of Scheduling for the set L using a scheme P_τ with $\tau < \frac{2}{\alpha}$. An equivalent lower bound applies to WCapacity.

▶ **Theorem 11.** *Any algorithm for Scheduling that uses power assignment P_τ, $\tau < 2/\alpha$, is no better than $\Omega(\log n)$ $(\Omega(\log \Delta))$-approximate in terms of n (in terms of Δ, resp.). The same holds for WCapacity.*

Proof. We will prove that for infinitely many n, there is a set of n pairwise $(1,1)$-independent links in the plane that requires $\Omega(\log n)$ slots when using P_τ, $\tau < 2/\alpha$. In terms of Δ, the number of slots required is $\Omega(\log \Delta)$.

We assume that $\beta = 1$. We inductively construct a weighted set of links $S_t = S_t(q)$ in the plane, given a parameter q. We shall denote by $S_t^{(x,y)}$ a copy of the instance S_t translated by the vector (x, y).

The instance S_0 consists of the single link 0 of length $l_0 = 1$, with s_0 at the origin and r_0 at $(l_0, 0) = (1, 0)$. For $t \geq 1$, the instance S_t consists of the link t of length $3ql_{t-1}$ and of weight $\omega(t) = q^{2t}$ with s_t at the origin and r_t at $(l_t, 0)$, along with q^2 sub-instances $S_{t-1}^{(x_i,y_j)}$ with $i, j = 0, 1, \ldots q-1$, $x_i = 2il_{t-1}$ and $y_j = l_t + j(l_{t-1} + h_{t-1})$, where h_t is the height of S_{t-1}. This completes the construction. See Figure 1.

It is easily verified that links in S_t are $(1,1)$-independent; hence, S_t can be scheduled in constant number of slots using an oblivious power scheme, by Lemmas 6, 7 and Thm. 1. It remains to show that it requires $\Omega(\log n)$ slots when using P_τ, with $\tau < 2/\alpha$.

Note that the number n_t of links in S_t is $n_t = 1 + q^2 n_{t-1} = \sum_{i=0}^{t-1} q^{2i} = (q^{2t} - 1)/(q^2 - 1)$. Thus, $\log n_t = \theta(t \log q)$. Let us call t the *main link* of S_t. Let us fix an index $t > 0$. Let L_k denote the set of main links of the copies of S_k in S_t, where $k < t$. We call L_k *the k-th level of S_t*. All the links in L_k have equal length and weight q^{2k}. It is easy to check that $W_t = \omega(L_k) = q^{2t}$, and the total weight of all links is $tW_t = \theta(W_t \log n)$.

▶ **Lemma 12.** *Suppose $q \geq (2 \cdot 3^{\tau\alpha})^{1/(2-\tau\alpha)}$. Let T be a subset of links in $S_t(q)$ that is feasible under P_τ with $\tau < 2/\alpha$. Then, $\omega(T) \leq 2 \cdot 3^\alpha W_t$.*

Proof. First, an observation.

▶ **Claim 13.** *Let T_k be a subset of level k links in S_t s.t. $T_k \cup \{t\}$ is P_τ-feasible. Then, $\omega(T_k) \leq 3^\alpha 2^{-(t-k)} W_t$.*

Proof. Let us first estimate the distance $d(i,t)$ for each link $i \in S_t \setminus \{t\}$. Note that $l_t = (3q)^t$, $h_t = l_t + q(l_{t-1} + h_{t-1})$ and $h_0 = 0$ (because S_0 consists of one horizontal link), so we can see that $h_t \le 2l_t$. It follows that for each $i \in S_t \setminus \{t\}$, $d(i,t) \le 3l_t$, implying, for each $i \in T_k$,

$$a_{P_\tau}(i,t) = \frac{P_\tau(i) l_t^\alpha}{P_\tau(t) d_{i,t}^\alpha} = \left(\frac{l_i}{l_t}\right)^{\tau\alpha} \left(\frac{l_t}{d_{it}}\right)^{\alpha} \ge \frac{1}{3^\alpha}(3q)^{-(t-i)\tau\alpha}.$$

Since $a_{P_\tau}(L_k,t) \le 1$, T_k contains at most $3^\alpha (3q)^{(t-i)\tau\alpha}$ links, each of weight q^{2k}, for a total weight of

$$\omega(T_k) \le 3^\alpha (3q)^{(t-k)\tau\alpha} \cdot q^{2k} = 3^\alpha W_t \frac{(3q)^{(t-k)\tau\alpha}}{q^{2(t-k)}} = 3^\alpha W_t \left(\frac{(3q)^{\tau\alpha}}{q^2}\right)^{t-k}.$$

The bound on q ensures that $q^{2-\tau\alpha} \ge 2 \cdot 3^{\tau\alpha}$ or $q^2 \ge 2 \cdot (3q)^{\tau\alpha}$. Thus, $\omega(T_k) \le 3^\alpha W_t (1/2)^{t-k}$, as claimed. ◀

We now prove the lemma by induction on t. For $t = 0$, S_t consists of only one link of weight $1 = q^0 = W_0$. For the inductive step, we consider two cases. Suppose first that T contains the link t. Then, it follows from the claim that

$$\omega(T) \le \omega(t) + \sum_{k=0}^{t-1} \omega(T \cap L_k) \le W_t + 3^\alpha W_t \sum_{k=0}^{t-1} 2^{-(t-k)} < 2 \cdot 3^\alpha W_t.$$

If, on the other hand, T does not contain t, it follows from the inductive hypothesis that the total weight of links from T in each of the q^2 sub-instances $S_{t-1}^{x,y}$ is at most $2 \cdot 3^\alpha W_{t-1}$, for a grand total of $\omega(T) \le q^2 \cdot 2 \cdot 3^\alpha W_{t-1} = 2 \cdot 3^\alpha W_t$. ◀

Observe that the maximum length $\Delta(S_t) = \Delta_t$ of a link in S_t is the length $l_t = (3q)^t$ of link t, which implies that $\log \Delta_t = \theta(t \log q) = \theta(\log n)$. Thus, $\Omega(\log \Delta)$ is also a lower bound. ◀

As for the power schemes P_τ with $\tau \ge 1$, it is known that there is no algorithm using these power schemes that achieves better than $O(\log \Delta)$-approximation in terms of Δ. This is shown in [32] for $\tau = 1$ and easily follows from [38] for $\tau > 1$.

General Metrics. Recall that for Capacity, the $O(\log\log \Delta)$-approximation results hold in arbitrary metrics [18]. This begs the question whether this might also hold for Scheduling and WCapacity. A negative answer was given for Scheduling in [19, Thm. 5.1]: no bound of the form $f(\Delta)$, for any function f of Δ alone. Namely, a feasible instance of n *equal length links* (i.e. $\Delta = 1$) in a tree metric was given in [19], for which P_τ (which is necessarily uniform power (P_0) on equal length links) requires $\Omega(\log n)$ slots. Thus, there is a separation between possible bounds for Capacity and Scheduling. We simplify below this construction and show that it also gives the same lower bound for WCapacity.

▶ **Theorem 14.** *The use of oblivious power assignments cannot obtain approximations of Scheduling or WCapacity within $o(\log n)$ factor in arbitrary general metrics.*

Proof. We give a construction of a set of weighted *equal length* links that is feasible with a certain power assignment, but for which any subset that is feasible using oblivious power, contains at most $\Omega(\log n)$ fraction of the total weight. Since the links have equal lengths, the only possible oblivious assignment is the uniform one. This yields a $\Omega(\log n)$ lower bound on the price of oblivious power for the weighted capacity problem.

The set L of links consists of K subsets, $L_1, L_2, \ldots, L_K$ for $K > 0$. Each set L_k contains 4^{k-1} links, each of weight $1/|L_k|$, for a total weight of 1. L_k also has an associated number $t_k = (\gamma |L_k|)^{\frac{1}{\alpha}}$, for a constant parameter γ to be determined. The distance between a link in L_k and another link in $L_{k'}$ is simply $t_k + t_{k'}$. We assume that $\beta = 1$. This completes the construction. The total number of links $n = |L| = \sum_{k=1\ldots K} |L_k| = (4^K - 1)/3$, and the total weight is K.

It was shown in [19] that L is feasible using some power assignment. We give below a simplified proof. We first show that any feasible set using uniform power has weight $O(1)$, or $O(1/\log n)$-fraction of the whole.

▶ **Claim 15.** *Let $S \subseteq L$ be a subset of links of weight $\omega(S) \geq 1 + \gamma 2^\alpha$. Then, S is infeasible under uniform power.*

Proof. Let $\tilde{k}$ be the minimum value for which an element of $L_{\tilde{k}}$ exists in S. Consider an arbitrary link $l_j \in L_{\tilde{k}} \cap S$. Note that for $i \in L_k$ where $k > \tilde{k}$, $d_{ij} = t_k + t_{\tilde{k}} \leq 2t_k = 2(\gamma |L_k|)^{1/\alpha}$. The affectance $a(i,j) = a_{P_0}(i,j)$ under uniform power is then $a(i,j) = \frac{1}{d_{ij}^\alpha} \geq \frac{1}{\gamma 2^\alpha} \cdot \frac{1}{|L_k|}$. Now,

$$\sum_{i \in S} a(i,j) \geq \sum_{k > \tilde{k}} a(L_k \cap S, j) \geq \frac{1}{\gamma 2^\alpha} \sum_{k > \tilde{k}} \frac{|L_k \cap S|}{|L_k|} = \frac{\omega(S \setminus L_{\tilde{k}})}{\gamma 2^\alpha} \geq \frac{\omega(S) - 1}{\gamma 2^\alpha} > 1 \ . \qquad ◀$$

▶ **Claim 16.** *L is feasible, assuming $\gamma \geq 6$.*

Proof. We will use the power assignment P defined by $P(i) = \frac{1}{2^k}$, for $i \in L_k$. Consider $j \in L_{\tilde{k}}$ and $i \in L_k$, for some $\tilde{k}$, k. Then, $d_{ij}^\alpha > t_{\max(k,\tilde{k})} = \gamma 2^{2(\max(k,\tilde{k})-1)}$. Thus,

$$a_P(L_k, j) = |L_k| \frac{2^{\tilde{k}-k}}{d_{ij}^\alpha} \leq 2^{2(k-1)} \cdot \frac{2^{\tilde{k}-k}}{\gamma \cdot 2^{2(\max(k,\tilde{k})-1)}} = \frac{1}{\gamma} 2^{\min(k,\tilde{k}) - \max(k,\tilde{k}) + 1} \ .$$

It follows that

$$\begin{aligned} a_P(L, j) = \sum_{k > \tilde{k}} a_P(L_k, j) + \sum_{k \leq \tilde{k}} a_P(L_k, j) &< \frac{1}{\gamma} \left(\sum_{k > \tilde{k}} 2^{\tilde{k}-k+1} + \sum_{k \leq \tilde{k}} 2^{k-\tilde{k}+1} \right) \\ &< \frac{1}{\gamma} \left(\sum_{x=0}^{\infty} \frac{1}{2^x} + \sum_{x=0}^{\infty} \frac{2}{2^x} \right) = \frac{6}{\gamma} \ . \end{aligned}$$

Thus, for $\gamma \geq 6$, L is feasible. ◀

Thm. 14 now follows. ◀

5 Weak Links

Recall that in order to obtain our approximations, we assumed that for each link i, $P(i) \geq c\beta N l_i^\alpha$ for a constant $c > 1$. However, this is not always achievable when nodes have limited power. Suppose that each sender node has maximum power P_{max}. For concreteness, we assume that $c = 2$. A link i is called a *weak link* if $P_{max} \leq 2\beta N l_i^\alpha$. Note that a link is weak because it is too long for its maximum power, i.e. if $l_i \geq l_{max}/2^{1/\alpha}$, where $l_{max} = (P_{max}/\beta N)^{1/\alpha}$ is the maximum length a link can have to be able to overcome the noise when using maximum power. Scheduling weak links may be considered as a separate problem. Let τ-WScheduling denote the problem of scheduling weak links with power scheme P_τ. For a weak link i, let us call $e_i = c_i^{1/\alpha} l_i$ the *effective length* of link i and let $\Delta_e(S) = \max_{i,j \in S} e_i/e_j$. One approach to WScheduling is to split the set of weak links into effective length classes S' with $\Delta_e(S') \leq 2$. Note that in each class, c_i is almost same for all links. Then we can find constant factor approximate scheduling for each of the classes using known algorithms for non-weak scheduling. This leads to a $O(\log \Delta_e)$ approximation. Unfortunately, there is no known algorithm with approximation factor better than $O(\log \Delta_e)$ or $O(\log n)$. The following theorem shows that constant factor approximation of WScheduling is at least as hard as constant factor approximation of Scheduling with fixed uniform power scheme (denoted UScheduling). The proof is omitted due to space limitations.

▶ **Theorem 17.** *There is a polynomial-time reduction from UScheduling to τ-WScheduling for any $\tau \in [0, 1)$, transforming an arbitrary set L of links to a set W of weak links so that the scheduling number of L with P_0 is within a constant factor of the scheduling number of W using P_τ.*

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