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# 10481 Executive Summary Computational Counting — Dagstuhl Seminar —

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**Abstract.** From November 28 to December 3 2010, the Dagstuhl seminar 10481 "Computational Counting" was held in Schloss Dagstuhl – Leibniz Center for Informatics. 36 researchers from all over the world, with interests and expertise in different aspects of computational counting, actively participated in the meeting.

**Key words.** computational complexity, counting problems, holographic algorithms, statistical physics, constraint satisfaction

## 1 Introduction

Computational complexity is typically concerned with decision problems, but this is a historical accident, arising from the origins of theoretical computer science within logic. Computing applications, on the other hand, typically involve the computation of numerical quantities. These applications broadly fall into two types: optimisation problems and counting problems. We are interested in the latter, broadly interpreted: computing sums, weighted sums, and integrals including, for example, the expectation of a random variable or the probability of an event. The seminar covered all aspects of computational counting, including applications, algorithmic techniques and complexity. Computational counting offers a coherent set of problems and techniques which is different in flavour from other algorithmic branches of computer science and is less well-studied than its optimisation counterpart.

Specific topics covered by the meeting include: techniques for exact counting, especially moderately exponential algorithms for intractable problems, techniques for approximate counting including Markov Chain Monte Carlo (MCMC), holographic algorithms and reductions, computational complexity of counting, algebraic complexity of counting, applications to statistical physics, and applications to constraint satisfaction.

Dagstuhl Seminar Proceedings 10481
Computational Counting
http://drops.dagstuhl.de/opus/volltexte/2011/2944

The questions addressed include: What algorithmic techniques are effective for exact counting and approximate counting? Do these techniques remain effective in the presence of weights (including negative and complex weights)? What inherent limitations arise from computational complexity? Are there inherent limitations for specific techniques such as MCMC? Our nominated application areas prompted many of those questions and hopefully will benefit from the answers.

Although each of these topics is important in its own right, the real goal of this seminar was to bring them together to allow cross-fertilisation. Here is an example. A key issue for MCMC is the rate at which a Markov chain converges to equilibrium, which determines the length of simulation needed to get a good estimate. An important insight has been that this mixing rate is connected to the phenomenon of phase transitions in statistical physics. But it also seems likely that phase transitions are connected with computational intractability more generally, i.e., resistance to all efficient approximation algorithms, not just those based on MCMC. A further example is provided by the way algebra pervades several of our topics - holographic algorithms, complexity of counting, and constraint satisfaction - and yet the connections between these are only now being explored. For example, algebraic methods permit semi-automatic generation of reductions between counting problems, and open up the speculative possibility of resolving the P versus NP question positively through "accidental algorithms".

We are interested in the complexity of counting in different models of computation. Counting in models of arithmetic circuits is intimately connected with the permanent versus determinant problem. The latter has recently triggered the study of several specific counting problems such as the computation of Littlewood-Richardson coefficients. Another direction of research that is relevant to the meeting is the classification of counting problems in computational algebraic geometry (counting irreducible factors, connected components, etc).

Two key applications areas, statistical physics and constraint satisfaction, have a central role. The problem of computing and approximating weighted sums already arises frequently in statistical physics, where such sums are referred to as partition functions. Constraint Satisfaction is a wide class of problems which arose in the context of AI - many computer science problems can be cast in this framework. Weights are not traditionally considered in CSP, but with this addition, many applications can be viewed in terms of counting CSPs.

## 2 Participation and Programme

The seminar brought together 36 researchers from Canada, China, Europe, India, Israel, and the United States with interests and expertise in different aspects of computational counting. Among them there was a good mix of senior participants, postdoctoral researchers and PhD students. Altogether, there were 36 talks over the week including three overview presentations and 7 ultra short five minute introductions by those participants that did not wish to give a full talk.

The first overview was presented by Jin-Yi Cai (Madison, Wisconsin) on holographic algorithms and the complexity of counting problems. He explained Valiant's notion of holographic reductions by matchgates with its twofold applications. One the one hand for exhibiting new efficient counting algorithms in a surprising way. On the other hand, holographic reductions provide new tools for showing the hardness of problems. The last part of Jin-Yi's survey focused on the various dichotomy results that have been recently obtained by him and his coauthors.

The second overview was devoted to the complexity of arithmetic circuits and delivered by Pascal Koiran (ENS Lyon and Toronto). It was a nice mix of old facts and new insights. Pascal gave a description of the hierarchy of algebraic complexity classes arising by studying arithmetic circuit models under various restrictions (formula, skew, weakly skew), explained the different characterizations and parallelization results. He then led over to newer results on depth four arithmetic circuits and presented his fascinating real tau-conjecture, which would imply that VP is different from VNP.

The third survey was presented by Martin Dyer (Leeds) on the complexity of the counting version of constraint satisfaction problems. The main focus of the talk was on Bulatov's dichotomy theorem, which says that each #CSP problem is either solvable in polynomial time or it is #P-complete, with no intermediate cases. After explaining the history of this result, Martin went on to explain the ideas of his simpler proof (found with David Richerby) that also yields more information, among other things the decidability of the dichotomy.

Other topics covered by the talks included phase transitions, graph polynomials, subexponential time algorithms, approximate (weighted) counting, and exact counting algorithms. The topic of Markov Chain Monte Carlo simulations was less represented than originally conceived, which is probably due to the fact that there was recently little progress in this subject due to its maturity.

One of the main aims of the seminar was to bring together researchers from different, but related fields, covering all aspects of computational counting with the goal of fostering the exchange of knowledge and to stimulate new research. This goal was fully achieved according to our opinion and the participants' feedback. It was an intense week with relatively tight schedule. Still, there were stimulating discussions in the afternoon breaks and in the evenings, some of them even leading to improvements of results that had been presented in the talks. New contacts and maybe even friendships were made.

Due to the snow and heavy cold we abandoned the traditional Wednesday hike. Instead we organized a trip to the UNESCO world heritage "Völklinger Hütte" on which most participants joined and had a good and relaxing time.

The organizers and participants thank the staff and the management of Schloss Dagstuhl for their assistance and support in the arrangement of a very successful and productive meeting.