Neurons and Symbols: A Manifesto

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Abstract

We discuss the purpose of neural-symbolic integration including its principles, mechanisms and applications. We outline a cognitive computational model for neural-symbolic integration, position the model in the broader context of multi-agent systems, machine learning and automated reasoning, and list some of the challenges for the area of neural-symbolic computation to achieve the promise of effective integration of robust learning and expressive reasoning under uncertainty.

1 Overview

The study of human behaviour is an important part of computer science, artificial intelligence (AI), neural computation, cognitive science, philosophy, psychology and other areas. Among the most prominent tools in the modelling of behaviour are computational-logic systems (classical logic, non-monotonic logic, modal and temporal logic) and connectionist models of cognition (feedforward and recurrent networks, symmetric and deep networks, self-organising networks).

Recent studies in cognitive science, artificial intelligence and evolutionary psychology have produced a number of cognitive models of reasoning, learning and language that are underpinned by computation [19, 21, 23]. In addition, recent efforts in computer science have led to the development of cognitive computational systems integrating machine learning and automated reasoning [6, 8, 27]. Such systems have shown promise in a range of applications, including computational biology, fault diagnosis, training and assessment in simulators and software verification [6]. In neural computing, it is assumed that the mind is an emergent property of the brain, and that computational cognitive modelling can lead to valid theories of cognition and

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offer an understanding of certain cognitive processes [23]. It follows that connectionism should be able to offer an appropriate representational language for AI as well.

In particular, a connectionist computational theory of the mind should be able to replicate the parallelism and kinds of adaptive learning processes seen in neural networks, which are generally accepted as responsible for the necessary robustness and ultimate effectiveness of the system in dealing with commonsense knowledge. As a result, a purely symbolic approach would not be sufficient, as argued by Valiant in [29].

On the other hand, it is undeniable that logic is a fundamental tool in the modelling of behaviour. Logic has been viewed generally as the calculus of computer science. The importance of nonclassical logic cannot be ignored. Temporal logic, for instance, has had an impact in both academia and industry [20]. Modal logics have become a *lingua franca* for the specification and analysis of knowledge and communication in multi-agent and distributed systems [10]. Epistemic logics have found a large number of applications, notably in game theory and in models of knowledge and interaction in multi-agent systems. Nonmonotonic reasoning has dominated the research on practical reasoning in artificial intelligence, and intuitionistic logic is considered by many as providing an adequate logical foundation for several core areas of theoretical computer science, including type theory and functional programming [30]. Both artificial intelligence and computer science have made extensive use of decidable modal logics, including the analysis and model checking of distributed systems, program verification and specification, and hardware model checking. More recently, description logics, which are similar to Kripke models, have been instrumental in the study of the semantic web [1].

From the above, it becomes clear that, if neural networks are to represent rich models of human reasoning, nonclassical logic should be at the core of this enterprise. Hence, we seek to provide a coherent, unifying view for nonclassical logic and connectionism, contributing to the modelling and understanding of behaviour, and producing better computational tools. To this end, we study logic and network models together as part of an integrated model of computation [8].

Our methodology is to transfer principles and mechanisms between nonclassical computation and neural computation. In particular, we consider how principles of symbolic computation can be implemented by connectionist mechanisms. Connectionism provides the *hardware* upon which different levels of abstraction can be built according to the needs of the application. This methodology, looking at principles, mechanisms and applications, has proven a fruitful way of progressing the research in the area of neural-symbolic integration [8, 12].

In [8], this methodology has led to a connectionist system for nonclassical reasoning that seems to strikes an adequate balance between complexity and expressiveness. In this system – known as a neural-symbolic system – neural networks provide the machinery for parallel computation and robust learning, while logic provides the necessary explanation to the network models, facilitating the necessary interaction with the world and other systems. In this integrated model, no conflict arises between a continuous and a discrete component of the system. Instead, a tightly-coupled hybrid system exists that is continuous by nature (the neural network), but that has a clear discrete interpretation (its logic) at different levels of abstraction.

In more practical terms, a rational agent is said to perform concept acquisition (generally unsupervised and statistical) and concept manipulation (generally supervised and symbolic) as part of a permanent cycle of perception and action. This process has to be permeated by a strong attention focus [24]. The question of how to reconcile the statistical nature of learning with the logical nature of reasoning, in an attempt to build such robust computational models integrating concept acquisition and manipulation, has been identified as a key research challenge and fundamental problem in computer science [28]. We see neural-symbolic integration as a way of addressing this challenge through the mechanisms of knowledge translation and knowledge extraction between symbolic logic and networks.

There are also important applications of neural-symbolic systems. The merging of theory (known as background knowledge in machine learning) and data learning (i.e. learning from examples) in neural networks has been shown more effective than purely symbolic or purely connectionist systems, especially in the case of real-world, noisy, unstructured datasets [25]. Specific application areas include: business process modelling, service-oriented computing (trust management and fraud prevention in e-commerce), synchronisation and coordination in large multi-agent systems, for instance, the web or an economic market, multimodal processing and integration, e.g. video classification based on annotated videos and sensor data.

In multimodal processing, for example, there are several forms of reasoning: a scene classification can be achieved by the well-trained network; the network gives an immediate answer following a number of assumptions.

A change in the scene, however, may require more specific temporal, non-monotonic reasoning and learning from data (based, for example, on the amount of change in the scene). Some assumptions may need to be revised, information from an image annotation may provide a different context, abduction and similarity reasoning by intersecting network ensembles may be needed, probability distributions may have to be reasoned about, and so on. The integrated system will need to respond quickly, revise its answers in the presence of new information, and control the inevitable accumulation of errors derived from real-world data (robustness). This provides an excellent opportunity for the application of neural-symbolic systems.

2 Neural-Symbolic Systems

The goals of neural-symbolic computation are to provide a coherent, unifying view for logic and connectionism, contribute to the modelling and understanding of behaviour, and produce better computational tools for integrated machine learning and reasoning. To this end, logic and network models are studied together as integrated models of computation. Typically, translation algorithms from a symbolic to a connectionist representation and vice-versa are employed to provide either (*i*) a neural implementation of a logic, (*ii*) a logical characterisation of a neural system, or (*iii*) a hybrid learning system that brings together features from connectionism and symbolic artificial intelligence.

A general framework for neural-symbolic systems uses modular deep networks[1] organised in multiple layers of abstraction [8]. Each layer represents the knowledge evolution of multiple agents over time. Each agent is represented by a network encoding commonsense (nonmonotonic) knowledge and preferences. The networks in different layers can be connected upwards to represent relational knowledge and downwards to represent specialisations, following a network-fibring methodology [7].

The fibring methodology offers a principled way of combining networks. The main idea of network fibring is simple. Fibred networks may be com-

[1] Growing attention has been given recently to symmetric deep networks where it is hoped that high level abstract representations will emerge from low level unprocessed datasets [13]. Most modern neural-symbolic systems use feedforward and recurrent networks, but seminal work in the area used symmetric networks [18] and recent work [9] is starting to address real applications of symmetric neural-symbolic networks.

posed of interconnected neurons, as usual, but also of other networks, forming a recursive structure. A fibring function defines how this network architecture behaves; it defines how the networks should relate to each other. Typically, the fibring function will allow the activation of neurons in one network (A) to influence the change of weights in another network (B). Intuitively, this may be seen as training network B at the same time that network A is running. Albeit being a combination of simple and standard neural networks, fibred networks can approximate any polynomial function in an unbounded domain, thus being more expressive than standard feedforward networks. As an example, network A could have been trained with a robot's visual system, while network B could, for example, have been trained with its planning system; fibring can perform the composition of the two systems.

Fibring is a good example of how principles from symbolic computation, in this case, *recursion*, can be used by connectionism to advance the research in this area. Below, we discuss in more detail the principles, mechanisms and applications that drive the research in neural-symbolic integration.

Principles. From the beginning of connectionism [17]: arguably the first neural-symbolic system for Boolean logic, most neural-symbolic systems have focused on representing, computing and learning languages other than classical propositional logic [3, 4, 6, 8, 14, 21]. Much effort has been devoted to representing fragments of classical *first-order logic*. In [8], a new approach to knowledge representation and reasoning has been proposed, establishing connectionist *nonclassical logic*, including connectionist modal, intuitionistic, temporal, nonmonotonic, epistemic and relational logic. More recently, it has been shown that argumentation frameworks, abductive reasoning and normative multi-agent systems can also be represented by the same network framework. This is encouraging to the extent that a variety of reasoning can be realised by the same, simple network structure that is specialised in different ways.

A key characteristic of neural-symbolic systems is *modularity*. Neural-symbolic networks can be built through the careful engineering of network ensembles. Modularity is of course important for comprehensibility and maintenance. Each network in the ensemble can be responsible for a specific task or logic, with the overall model being potentially very expressive despite its relatively simple components.

Like deep networks, fibred networks are generally organised in a modular hierarchy [7]. The lowest-level network takes raw data as input and produces a model of the dataset. The next-level network would take the first network's output as its input and produce some higher-level representation of the information in the data. Other levels would increase the level of abstraction of the model until some high-level representation can be learned.

The idea is that such networks might be trained independently. They can also combine unsupervised and supervised learning at different levels of the hierarchy. This *parallel* model of computation can be very powerful. It offers the extra *expressiveness* required by complex applications at low computational cost, i.e. the cost of computing the fibring functions.

It is worth noting that the nonclassical approach to neuro-symbolism finds in propositional modal logic the most adequate language for the purposes of integration. Propositional modal logic seems capable of striking the right balance between expressiveness and complexity; it is decidable, strictly more expressive than propositional logic and, in fact, equivalent to the two-variable fragment of first-order logic. In contrast, the first-order approach to neuro-symbolism has to deal with the persisting problem of *variable* manipulation and binding. Nevertheless, this is an important problem, and new insight from the area of lifted message passing may be useful in tackling it [16].

Mechanisms. We subscribe to the view that representation precedes learning. Neural-symbolic networks can represent a range of expressive logics and implement certain important principles of symbolic computation. However, neural-symbolic computation is not just about representation. The mechanisms of propagation of activation and other message passing methods, gradient-descent and other learning algorithms, reasoning about uncertainty, massive parallelism, fault tolerance, etc. are a crucial part of neural-symbolic integration. Put simply, neural-symbolic networks are efficient computational models, not representational tools. It is the mechanisms in place, in the form of efficient algorithms, that enable the computational feasibility of neural-symbolic systems.

In the same way, fibred networks are computational models, not just graphical models or mathematical abstractions like graphs or networks generally. The neural-symbolic networks can be mapped directly onto hardware. An implementation in a VLSI chip should be straightforward and cost effective. The main architectural constraint, which here is brain-inspired, is that neural-symbolic systems should replicate and specialise simple neuronal structures to which a single algorithm can be applied efficiently at different levels of abstraction, with the resulting system being capable of exhibiting emergent behaviour.

Emergence prompts the need for mechanisms of knowledge extraction.

The cycle of neural-symbolic integration includes (i) translation of symbolic (background) knowledge into the network, (ii) learning of additional knowledge from examples (and generalisation), (iii) executing the network, and (iv) symbolic knowledge extraction. Extraction provides explanation, and facilitates maintenance and incremental learning.

We refer to the overall connectionist computational model as *fibred network ensembles*. In this model, a network ensemble A (representing, for example, a temporal theory) can be combined with another network ensemble B (representing, for example, an agent's epistemic state). Using the same mechanisms of fibring, metalevel knowledge in one network can be integrated with object-level knowledge in another network. For example, one may reason (in the metalevel) about the actions that are needed to be taken (in the object-level) to solve inconsistencies in a database. Relational knowledge can also be represented in the same way. Relations between concepts encoded in distinct (object-level) networks can be represented and learned through a metalevel network. For example, if two networks denote concepts $P(X,Y)$ and $Q(Z)$ containing variables X, Y and Z, respectively, a metalevel network can be used to map a representation of P and Q onto a new concept, say $R(X,Y,Z)$, such that, for example, the relation $P(X,Y) \wedge Q(Z) \rightarrow R(X,Y,Z)$ is valid [8].

Figure 1 illustrates the fibred-network ensembles. The model, in its most general form, allows a number of network ensembles to be combined at different levels of abstraction through fibring. In the figure, each level is represented by a network ensemble in a horizontal plane, while network fibring takes place vertically among networks at different ensembles. Specialisation occurs downwards when a neuron is fibred onto a network. Relational knowledge is represented upwards when multiple networks are combined onto a metalevel network. Knowledge evolution through time occurs at each level. Alternative outcomes, nonmonotonic and epistemic reasoning for multiple, interacting agents also occur at each level. Modular learning takes place inside each network, but is also applicable across multiple networks in the ensemble. The same brain-inspired structure is replicated throughout the model so that a single algorithm is applied at each level and across levels.

Applications. Ultimately, it will be through useful or important applications that the promise of neural-symbolic integration will be realised. Real applications are a crucial ingredient and have been a permanent feature of the research on neural-symbolic computation. Theoretically, we are interested in finding the limits of representation and better machine learning methods

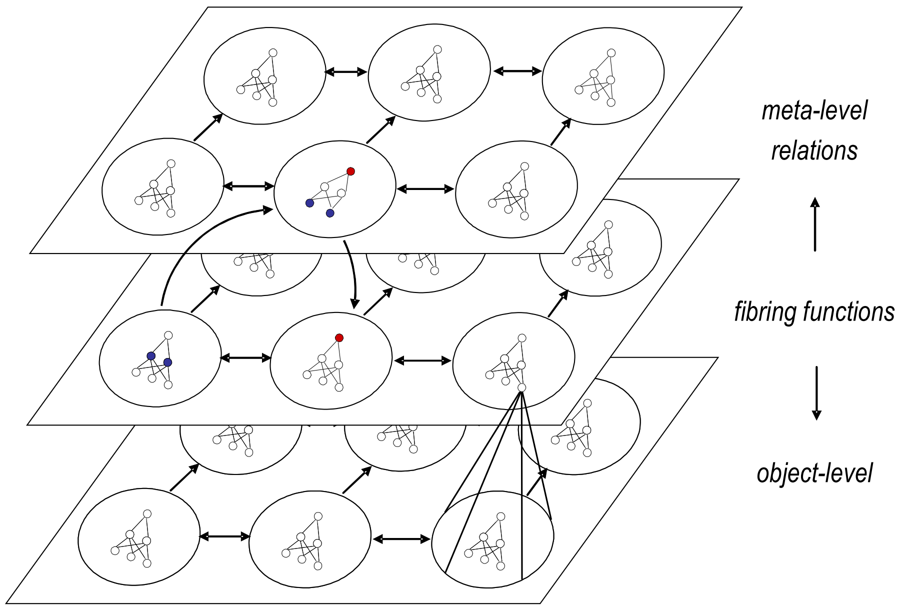

Figure 1: Fibred network ensembles

which may open up new applications for consideration. In practice, already at the current state of the research, real applications are possible in areas like bioinformatics, semantic web, fault diagnosis, robotics, software systems verification, business process modelling, fraud prevention, multimodal processing, noisy text analytics, training and assessment in simulators. In some of these areas, computing can offer real benefits to society and have a real impact in the economy.

All these different application areas have something in common: a computational system is required that is capable of learning from experience and reason about what has been learned [3, 28]. For this learning-reasoning process to be successful, the system must be robust (in the way advocated by Valiant so that the accumulation of errors resulting from the intrinsic uncertainty associated with the problem domain can be controlled [28]). One neural-symbolic system that is already providing a contribution to problems in bioinformatics and fault diagnosis is the Connectionist Inductive Learning and Logic Programming (CILP) System [6].

Let us consider, as an example, the problem of payment card fraud. Card fraud is on the increase as a result of the expansion in computing services and globalisation. The proceeds of fraud are being used increasingly to fund organised crime and terrorism. Unchecked, the problem will continue to grow to a point where it may challenge the competitiveness of the digital economy model. While fraud is dynamic, fraud management systems have been mainly static and are failing to identify new trends in fraud patterns. Neural computing is a major technology being used to detect fraud from transaction data. However, fraud management systems will need to identify sudden changes in the patterns of behaviour of card users. The temporal dynamics of the data can offer valuable information about such new trends. Current systems are showing around 70% accuracy, while dynamic neuro-symbolic systems can achieve 75% on average, representing savings of more than £1.5 million per week in the UK alone.

The method used by the research on neural-symbolic integration to enable some of the above applications has been (i) to apply translation algorithms between logic and networks (making use of the associated equivalence proofs), (ii) to study the systems empirically through case studies (following the practical motivation from statistical machine learning and neural computation), and (iii) to focus on the needs of the application (noting that some potentially interesting applications require just even rudimentary logical representations, e.g. [2]).

3 Challenges

In multi-agent systems, an agent explores the environment, learns from it, reasons about its internal state given the input, acts and, in doing so, changes the environment, explores further, and so on as part of a permanent cycle [5]. To realise this vision, there are at least four options:

1. symbolic approach: here, reasoning and explanation are reasonably straightforward (albeit the complexity issues), learning is inadequate (lack of robustness, brittle representation); notable systems include: expert systems, inductive logic programming (ILP), classical and certain nonclassical logical systems such as logic programming with negation-by-failure, analytic tableaux, satisfiability solvers, etc.

2. connectionist approach: effective learning (parallelism, robustness, adaptive, continuous domains), but limited reasoning (no recursion, vari-

able unification, weak on explanation); effective on a range of applications, mainly classification and pattern recognition (despite the explanation drawback, difficult system maintenance and poor at incremental learning); notable systems include: feedforward networks and recurrent networks trained by gradient-descent, symmetric networks and deep belief networks, support vector machines, self-organising networks, etc.

3. embracing uncertainty: efficient learning with approximate inference (albeit complexity issues still); limited reasoning in some cases; notable systems include: Bayesian networks, independent choice logic, probabilistic ILP and other stochastic versions of the symbolic approach, Markov logic networks, probabilistic relational models, dynamic Bayesian networks, propositional modal logic, fuzzy logic, multi-valued logic, etc.

4. hybrid systems: combines discrete and continuous systems; seeks the integration of effective learning (in the sense of the connectionist approach) and reasoning under uncertainty; loosely-coupled systems include: hybrid symbolic and statistical systems, neuro-fuzzy systems, connectionist expert systems, etc. (problem: difficult interface); tightly-coupled systems include: neural-symbolic systems, connectionist modal logic, translations between the logical, connectionist and embracing-uncertainty options above (takes new ideas across areas).

As claimed in [3], if connectionism is an alternative paradigm to artificial intelligence, neural networks must be able to compute symbolic reasoning efficiently and effectively. Moreover, in hybrid learning systems usually the connectionist component is fault-tolerant, whilst the symbolic component may be "brittle and rigid". By integrating connectionist and symbolic systems, hybrid systems seek to tackle this problem and offer a principled way of computing and learning various knowledge.

Given the above landscape, it is possible to identify already some convergence: deep belief networks are based on an early symmetric neural-network model (Boltzmann machines) and related to Bayesian networks. Certain neuro-symbolic recurrent networks are very similar to dynamic Bayesian networks. Connectionist modal logic uses modal logic as a language for reasoning about uncertainty in a connectionist framework. Markov logic networks combine logical symbols and probability. The objectives seem to be converging; the mechanisms are varied though. The distinctive feature of neural-symbolic systems is that a connectionist machine provides the baseline upon which the system is built, in line with a computational theory of mind, differently from the Bayesian approach and the Markov-logic-networks approach, and similarly to the connectionist-modal-logic approach and the deep-belief-networks approach.

There are challenges and limits to integration though. There are, possibly, applications that will admit a loosely-coupled solution, but not a tightly-coupled one. We now turn to some of the challenges. We have seen that neural-symbolic system are composed of (i) translations from logic to network, (ii) machine learning and reasoning, (iii) translation from network to logic. A main challenge in (i) is finding the limits of representation, in (ii) finding representations that are amenable to integrated learning and reasoning, and in (iii) producing effective knowledge extraction from very large networks. Below is a list of research issues related to challenges (i) to (iii).

1. Reconciling first-order logic learning and first-order logic reasoning
2. Embracing semi-supervised and incremental learning
3. Evaluating large-scale gains of massive parallelism
4. Cognitive agent: implementing learn-reason-action cycle
5. Representations for learning: learning the ensemble structure
6. Rule extraction for networks with thousands of neurons
7. Applying fibring in practice: learning the fibring function
8. Proof theory and type theory for neural networks
9. Abductive neuro-symbolism: automating the scientific method
10. Attention focus: modelling emotions vs. utility functions

Finally, in terms of applications, we now discuss a so-called *killer app*: neural networks have been very effective at image processing and feature extraction [15]. Yet, large-scale symbolic systems have been the norm for text processing (as they rely on large ontologies, are inefficient to train from scratch and heavily dependent on data preprocessing). Even if networks could be trained from scratch to perform as well as, for example, wordnet [31], the networks would be very difficult to maintain and validate. Neural-symbolic systems capable of combining network models for image processing and legacy symbolic systems for text processing are needed for multimodal processing. Here the concept of fibring is brought to bear so that symbolic systems and network models can be integrated loosely at the functional level. In this process, inconsistencies may arise, and a key issue is how to resolve inconsistencies. Our approach is to see inconsistency as a trigger for learning, with new information in either part of the combined system serving to adjudicate the conflict. The immediate impact of this application can be considerable in areas like security, defense and the web.

4 Concluding Remarks

We have discussed the main characteristics and some challenges for neural-symbolic integration. In a nutshell:

neural-symbolic systems = connectionist machine + logical abstractions.

The need for rich, logic-based knowledge representation formalisms to be incorporated into learning systems has been argued since Valiant's seminal paper [26]. Connectionist modal logic (CML) is a case in point. It shows how the area of neural computation can contribute to the area of logic. CML offers parallel models of computation to modal logic that, at the same time, can be integrated with an efficient learning algorithm. Fibring, on the other hand, is an example of how logic can bring insight into neural computation. Fibring allows concepts from symbolic computation to help the development of neural models. Despite its origin in symbol processing, fibring does not necessarily conflicts with neural network's ambition of biological motivation, e.g. fibring functions can be understood as a model of *presynaptic weights*, which play an important role in biological neural networks.

Both the symbolic and connectionist paradigms have virtues and deficiencies. Research into the integration of the two has important implications that can benefit computing and cognitive science. The limits of effective integration can be pursued through the neural-symbolic method, following the needs of different applications. The results of principled integration must be shown advantageous in practice in comparison with purely symbolic or purely connectionist systems.

The question of how the human mind integrates reasoning and learning is only starting to be addressed [11, 22]. We argue that the prospects are better if we investigate the connectionist processes of the brain together with the logical processes of symbolic computation, and not as two isolated paradigms. The framework of *fibred network ensembles* seems expressive and tractable to address most current applications. Further development of the framework includes testing in controlled cognitive tasks.

The challenges for neural-symbolic integration today emerge from the goal of effective integration, expressive reasoning and robust learning. One cannot afford to lose learning performance while adding reasoning capability to neural models. This is because it is important to maintain the key idea that neural networks are composed of simple processing units (allowing some "clever" neuron to perform complex symbol processing would amount to cheating). Computationally, there are challenges associated with the more practical aspects of the application of neural-symbolic systems in areas such as engineering, robotics, semantic web, etc. These challenges include the effective computation of logical models, the efficient extraction of comprehensible knowledge and, ultimately, striking of the right balance between tractability and expressiveness.

In summary, by paying attention to the developments on either side of the division between the symbolic and the sub-symbolic paradigms, we are getting closer to a unifying theory, or at least promoting a faster and principled development of cognitive and computing sciences and artificial intelligence. This is the ultimate goal of neural-symbolic integration together with the associated provision of neural-symbolic systems with expressive reasoning and robust learning capabilities.

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