09391 Abstracts Collection
Algorithms and Complexity for Continuous Problems
— Dagstuhl Seminar —

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Abstract. From 20.09.09 to 25.09.09, the Dagstuhl Seminar 09391 Algorithms and Complexity for Continuous Problems was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Computational complexity of continuous problems, partial information, high-dimensional problems, tractability analysis, quasi-Monte Carlo methods, operator equations, non-linear approximation, stochastic computation, ill posed-problems

09391 Summary - Algorithms and Complexity for Continuous Problems

This was already the 10th Dagstuhl Seminar on Algorithms and Complexity for Continuous Problems over a period of 18 years. It brings together researchers from different communities working on computational aspects of continuous problems, including computer scientists, numerical analysts, applied and pure mathematicians, and statisticians. Although the Seminar title has remained the same many of the topics and participants change with each Seminar. Each seminar in this series is of a very interdisciplinary nature.

Continuous problems arise in diverse areas of science and engineering. Examples include multivariate and path integration, approximation, optimization, operator equations, (stochastic) ordinary as well as (stochastic) partial differential equations. Typically, only partial and/or noisy information is available, and the aim is to solve the problem with a given error tolerance using the minimal amount of computational resources. For example, in multivariate numerical integration one wants to compute an ε-approximation to the integral with the minimal number of function evaluations.

Still growing need of efficiently solving more and more complicated computational problems makes this branch of science both important and challenging.

The current seminar attracted 58 participants from 11 different countries all over the world. About 30% of them were young researchers including PhD students. There were 53 presentations covering in particular the following topics:

- tractability of high dimensional problems
- computational stochastic processes
- numerical analysis of operator equations
- inverse and ill-posed problems
- applications in computer graphics and finance

The work of the attendants was supported by a variety of funding agencies. This includes the Deutsche Forschungsgemeinschaft, the National Science Foundation and the Defense Advanced Research Projects Agency (USA), and the Australian Research Council. Many of the attendants from Germany were supported within the DFG priority program SPP 1324 on "Extraction of Quantifiable Information from Complex Systems", which is strongly connected to the topics of the seminar.

As always, the excellent working conditions and friendly atmosphere provided by the Dagstuhl team have led to a rich exchange of ideas as well as a number of new collaborations.

Selected papers related to this seminar will be published in a special issue of the Journal of Complexity.

Approximating Extrema of Diffusions

Jan Baldeaux (Univ. of New South Wales, AU)

In this work, we consider the following one-dimensional stochastic differential equation

$$dS_t = \mu(S_t)dt + \sigma(S_t)dW_t\,,\ S_0 = s \in \mathbb{R},$$

where $\mu \in C^1$, $\sigma \in C^2$ and $(W_t)_{t\geq 0}$ is a one-dimensional Brownian motion. We are interested in computing

$$\mathbb{E}\left[g(S_T, \max_{0\leq t\leq T} S_t)\right]$$

and consider Monte Carlo methods to approximate this quantity. Under additional assumptions, an unbiased estimator for this quantity is obtained and subject to a different set of assumptions, a biased estimator is produced. Having introduced a cost model, we can discuss the complexity of the algorithm generating the biased estimator.

Keywords: Extreme-values, Diffusions, Monte Carlo methods, Option pricing

Joint work of: Baldeaux, Jan; Chen, Nan

Lower bound on complexity of optimization under r-fold integrated Wiener measure

Jim Calvin (NJIT - Newark, US)

We consider the problem of approximating the global minimum of a univariate function defined on the unit interval. Algorithms can adaptively choose points at which to evaluate the function and its derivatives, and the number of points can also be chosen adaptively.

The conditional r-fold integrated Wiener measure is obtained by integrating the Brownian motion paths r times and then translating the integrated paths by suitable polynomials so that prescribed boundary conditions are met at 0 and 1. We require that the derivative at 0 is negative and at 1 is positive, so that the global minimum occurs in the interior of the interval. Under this probability, the average number of function and derivative evaluations needed to obtain an ϵ approximation must be at least a constant times $\log(1/\epsilon)^{\frac{1}{2r+1}}$.

Keywords: Optimization, Wiener measure

A Carleman estimate and the balancing principle in the quasi-reversibility method for solving the Cauchy problem for Laplace equation

Hui Cao (RICAM - Linz, AT)

The quasi-reversibility method to solve the Cauchy problem for the Laplace's equation in a bounded domain Ω is considered. With the help of the Carleman estimations technique improved error and stability bounds in a subdomain $\Omega_\sigma \subset \Omega$ are obtained. This paves a way for the use of the balancing principle for *a posteriori* choice of the regularization parameter ε in the quasi-reversibility method. As an adaptive regularization parameter choice strategy, the balancing principle does not require *a priori* knowledge of either the solution smoothness or a constant K appearing in the stability bound estimation. Nevertheless, this principle allows an *a posteriori* parameter choice that up to controllable constant achieves the best accuracy guaranteed by the Carleman estimate. This is joint work with Michael Klibanov and Sergei Pereverzev.

Keywords: Quasi-reversibility method, Cauchy problem, Carleman estimate, balancing principle

Full Paper:
http://www.iop.org/EJ/abstract/0266-5611/25/3/035005/

See also: Inverse Problems 25 (2009), no. 3

Construction of lattice rules: from K-optimal lattices to extremal lattices

Ronald Cools (K.U. Leuven, BE)

The concept of K-optimal lattices was introduced by Cools and Lyness (2001) to set up searches for lattice rules for multivariate integration with a low number of points for a given trigonometric degree of precision. The original searches were restricted to 3 and 4 dimensions. Refinements were later made to make the searches possible in 5 and 6 dimensions. The idea behind K-optimal lattices was extended in Russia using extremal lattices.

In this talk these approaches to lattice rule construction will be described and a survey of the obtained results will be presented.

Optimal Approximation of Elliptic Problems by Linear and Nonlinear Mappings: Errors in L_2 and Other Norms

Stephan Dahlke (Universität Marburg, DE)

We shall be concerned with optimal approximations of the solution of an operator equation $Au = f$ by linear and different types of nonlinear mappings.

We assume that that A is a boundedly invertible operator from H_0^s to H^{-s}. In the first part of the talk, we recall some results concerning optimal approximations with respect to the H^s-norm. Then, in the second part, we study error bounds with respect to weaker norms, i.e., we assume that H_0^s is continuously embedded into a space X and we measure the error in the norm of X. We show that in general the order of convergence is improved if one takes these weaker norms. In particular, the case of the Poisson equation in a Lipschitz domain is discussed in detail with special emphasis on best n-term wavelet approximation.

Keywords: Elliptic operator equations, worst case error, nonlinear widths, manifold widths, wavelets, best n-term approximation

Full Paper:
http://www.dfg-spp1324.de/download/preprints/preprint016.pdf

On the Randomized Solution of Initial Value Problems

Thomas Daun (TU Kaiserslautern, DE)

In this talk we discuss the randomized solution of initial value problems for systems of ordinary differential equations

$$y'(x) = f(x,y),\ x \in [a,b],\ y(a) = y_0\ \in \mathbb{R}^d.$$

In a recent paper S. Heinrich and B. Milla gave an order optimal randomized algorithm for solving this problem for γ-smooth input data (i.e. $\gamma = r + \rho$: the r-th derivatives of f satisfy a ρ-Hölder condition).
This algorithm uses function values and values of derivatives of the right-hand side.

We present an order optimal randomized algorithm for the class of γ-smooth functions that uses only values of f.

For this purpose we show how to obtain an order optimal randomized algorithm from an order optimal deterministic one.

Consistency for Markov Chain Monte Carlo using completely uniformly distributed sequences

Josef Dick (Univ. of New South Wales, AU)

The random numbers driving Markov chain Monte Carlo (MCMC) simulation are usually modeled as independent $U(0,1)$ random variables.

Tribble reports substantial improvements when those random numbers are replaced by carefully balanced inputs from completely uniformly distributed sequences.

The previous theoretical justification for using anything other than IID $U(0,1)$ points showed consistency for estimated means, but only applies for discrete stationary distributions.

We extend those results to some MCMC algorithms for continuous stationary distributions.

The main motivation is the search for quasi-Monte Carlo versions of MCMC. As a side benefit, the results also establish consistency for the usual method of using pseudo-random numbers in place of random ones.

Keywords: Markov Chain Monte Carlo, quasi-Monte Carlo, completely uniformly distributed

L_p-Approximation of stochastic integrals driven by the Brownian motion and Besov spaces

Stefan Geiss (University of Jyväskylä, FI)

Firstly, given a functional $f(W_1) \in L_p$ with $2 \leq p < \infty$, where $W = (W_t)_{t\in[0,1]}$ is a standard Brownian motion, we consider a Riemann approximation

$$f(W_1) = Ef(W_1) + \int_0^1 L_t dW_t \sim Ef(W_1) + \sum_{k=1}^{n} L_{\frac{k-1}{n}} \left(W_{\frac{k}{n}} - W_{\frac{k-1}{n}} \right)$$

and define the corresponding error by

$$C_1(f,n) := \int_0^1 L_t dW_t - \sum_{k=1}^{n} L_{\frac{k-1}{n}} \left(W_{\frac{k}{n}} - W_{\frac{k-1}{n}} \right).$$

We characterize $\|C_1(f,n)\|_{L_p} \leq cn^{-\frac{\theta}{2}}$ for $0 < \theta < 1$ by the condition that f belongs to the weighted Besov space $B^{\theta}_{p,\infty}(R,\gamma)$ obtained by real interpolation between $L_p(R,\gamma)$ and $D_{1,p}(R,\gamma)$ with parameters (θ,∞). This result generalizes the case $p = 2$ from [1,2] to the case $2 < p < \infty$.

For us the case $2 < p < \infty$ is very different from the case $p = 2$ as one main idea of [1,2] was to reduce the probabilistic problem to a deterministic one and to work on this deterministic problem.

In the case $2 < p < \infty$ there is (so far) no such reduction to a deterministic problem. Instead we have use interpolation between BMO and L_2 and [3] for one direction of our main result and a completely different route for the opposite direction.

The second part of the talk concerns a relation between BMO-estimates for the error process in discretizations of stochastic integrals and the Hölder continuity of the terminal condition. The idea is to use special time-nets adapted to the Hölder continuity to get the optimal BMO-estimates. The inverse implication is also considered. Having the BMO-estimates one can use John-Nirenberg type theorems to obtain tail-estimates for the error process that improve the known ones.

This generalizes the result about Lipschitz functions proved in [3]. The result about Lipschitz functions is used in the first part of the talk which gives the link between the two parts.

References

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Joint work of: Geiss, Stefan; Toivola, Anni

Discrepancy Bounds for Mixed Sequences

Michael Gnewuch (Christian-Albrechts-Universität zu Kiel, DE)

A mixed sequence is a sequence in the s-dimensional unit cube which one obtains by concatenating a d-dimensional low-discrepancy sequence with an $s-d$-dimensional random sequence.

We discuss some probabilistic bounds on the star discrepancy of mixed sequences.

Keywords: Star Discrepancy, Mixed Sequence, Hybrid Method, Monte Carlo, Quasi-Monte Carlo, Probabilistic Bounds

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2009/2297

Weighted L_2 B Discrepancy and Approximation of Integrals over Reproducing Kernel Hilbert Spaces

Michael Gnewuch (Christian-Albrechts-Universität zu Kiel, DE)

We extend the notion of L_2 B discrepancy provided in [E. Novak, H. Woźniakowski, L_2 discrepancy and multivariate integration, in: Analytic number theory. Essays in honour of Klaus Roth. W. W. L. Chen, W. T. Gowers, H. Halberstam, W. M. Schmidt, and R. C. Vaughan (Eds.), Cambridge University Press, Cambridge, 2009, 359 – 388] to the weighted L_2 $\mathcal{B}$ discrepancy.

This newly defined notion allows to consider weights, but also volume measures different from the Lebesgue measure and classes of test sets different from measurable subsets of some Euclidean space.

We relate the weighted L_2 $\mathcal{B}$ discrepancy to numerical integration defined over weighted reproducing kernel Hilbert spaces and settle in this way an open problem posed by Novak and Woźniakowski.

Keywords: Discrepancy, Numerical Integration, Quasi-Monte Carlo, Reproducing Kernel Hilbert Space

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2009/2296

On the complexity of a two-point boundary value problem in different settings

Maciej Gocwin (AGH Univ. of Science & Technology-Krakow, PL)

We study the complexity of a two-point boundary value problem. We concentrate on the linear problem of order k with separated boundary conditions. Right-hand side functions are assumed to be r times differentiable with all partial derivatives bounded by a constant.

We consider three models of computation: deterministic with standard and linear information, randomized and quantum. In each setting we construct an algorithm for solving the problem, which allows us to establish upper complexity bounds.

In the deterministic setting, we show that the use of linear information gives us a speed-up of at least one order of magnitude compared to standard information. For randomized algorithms, we show that the speed-up over standard deterministic algorithms is by 1/2 in the exponent. For quantum algorithms, we can achieve a speed-up by one order of magnitude.

We also provide lower complexity bounds. They match upper bounds in the deterministic setting with standard information, and almost match upper bounds in the randomized and quantum settings. In the deterministic setting with linear information, a gap still remains between upper and lower complexity bounds.

Keywords: Boundary value problems, Complexity, Worst-case setting, Randomized computing, Quantum computing

Joint work of: Gocwin, Maciej; Szczêsny, Marek

Best m-term approximation and tensor products of Sobolev and Besov spaces

Markus Hansen (Universität Jena, DE)

We consider the problem of best m-term approximation of functions with respect to suitable wavelet-bases.

After recalling a basic result by Pietsch on approximation in l_p, we discuss its consequences for approximation of functions from the tensor product of Sobolev and Besov spaces.

We obtain nonperiodic counterparts of results by Temlyakov and Dinh Dung.

Dimension reduction for monomolecular reactions

Markus Hegland (Australian National University - Canberra, AU)

Metabolic networks are described by monomolecular reactions.

The solution of the chemical master equation is a multinomial distribution for such networks. This probability distribution does not have full dimension and we show how the dimension changes over time for the case of a chain reaction.

A Multilevel Monte Carlo Algorithm for Levy Driven SDEs

Felix Heidenreich (TU Darmstadt, DE)

We introduce and analyze multilevel Monte Carlo schemes for the evaluation of the expectation $\mathbb{E}[f(Y)]$, where $Y = (Y_t)_{t\in[0,1]}$ is a solution of a stochastic differential equation driven by a square integrable Lévy process X. Upper bounds are provided for the worst case error over the class of all measurable functionals $f : D[0,1] \to \mathbb{R}$, that are Lipschitz continuous with respect to supremum norm with Lipschitz constant one, where $D[0,1]$ denotes the Skorohod space of càdlàg functions. Moreover, the worst case errors are compared with the computational cost of the corresponding algorithms. If the driving Lévy process has Blumenthal-Getoor index β, we obtain as dominant term in the upper estimate $n^{-(\frac{1}{\beta\vee 1}-\frac{1}{2})}$, when the computational cost n tends to infinity. In my talk, I focus on the coupled simulation of strong approximations of X. Such simulations are crucial for the implementation of the multilevel Euler scheme, which is considered.

Pointwise Approximation of a Stochastic Heat Equation with Additive Space-Time White Noise

Daniel Henkel (TU Darmstadt, DE)

We consider a stochastic heat equation on the spatial domain $(0, 1)$ with additive space-time white noise, and we study approximation of the mild solution at a fixed time point with respect to the average L_2-distance. In this talk we consider algorithms, which use a total of N evaluations of the one-dimensional components of the driving Wiener process W and we present upper and lower error bounds in terms of N. In particular we compare uniform with non-uniform time discretizations.

Complexity of Numerical Integration over Spherical Caps

Kerstin Hesse (University of Sussex - Brighton, GB)

The construction of positive weight rules for numerical integration over spherical caps

$$\mathcal{C}(\mathbf{z};\gamma) := \left\{\mathbf{x} \in \mathbb{S}^d \,:\, \mathbf{x}\cdot\mathbf{z} \geq \cos\gamma\right\}$$

on the unit sphere $\mathbb{S}^d := \{\mathbf{x} \in \mathbb{R}^{d+1} \,:\, \|\mathbf{x}\| = 1\}$ with a high polynomial degree of exactness has only very recently attracted interest.

Let $H^s(\mathbb{S}^d)$, where $s > d$, be the Sobolev space of functions on the sphere $\mathbb{S}^d$ whose distributional derivatives up to order s exist and are square-integrable.

Then the complexity in $H^s(\mathbb{S}^d)$ of numerical integration over a spherical cap with m nodes is of the order $m^{-s/d}$.

The proof of this complexity result consists of two parts:

On the one hand, we prove lower bounds of the order $m^{-s/d}$ for the worst-case error in $H^s(\mathbb{S}^d)$ of any rule with m nodes. This lower bound is obtained by constructing a 'bad function' for which the error is of the order $m^{-s/d}$. This 'bad function' has local support on a collection of m non-overlapping spherical caps of the same size that contain no nodes of the numerical integration rule in the interior.

On the other hand, we construct positive weight tensor product rules that are exact for all spherical polynomials of degree $\leq n$ and use $m \sim n^d$ nodes. For this class of rules, we derive an upper bound of the order $n^{-s} \sim m^{-s/d}$ for the worst-case error in $H^s(\mathbb{S}^d)$. The proof of this upper bound is quite technical and involves special functions and exploits the polynomial exactness of the rules.

Since the upper bound and the lower bound have the same order, the complexity of numerical integration over a spherical cap in $H^s(\mathbb{S}^d)$ is established.

The construction of the tensor product rules for numerical integration over spherical caps is joint work with Robert Womersley. In this joint work, we also prove several properties of positive weight rules with polynomial exactness for numerical integration over spherical caps that are used in the the proofs of the complexity result.

Keywords: Complexity, numerical integration, Sobolev spaces, sphere, spherical cap, worst-case error

Evaluating Expectations of Functionals of Brownian Motions: a Multilevel Idea

Fred J. Hickernell (Illinois Inst. of Technology, US)

Prices of path dependent options may be modeled as expectations of functions of an infinite sequence of real variables. This talk presents recent work on bounding the error of such expectations using quasi-Monte Carlo algorithms. The expectation is approximated by an average of n samples, and the functional of an infinite number of variables is approximated by a function of only d variables. A multilevel algorithm employing a sum of sample averages, each with different truncated dimensions, d_l, and different sample sizes, n_l, yields faster convergence than a single level algorithm. This talk presents results in the worst-case error setting.

Keywords: Brownian motions, multilevel, option pricing, worst-case error

Joint work of: Hickernell, Fred J.; Müller-Gronbach, Thomas; Niu, Ben; Ritter, Klaus

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2009/2298

Optimal Importance Sampling for the Approximation of Integrals

Aicke Hinrichs (Universität Jena, DE)

We consider optimal importance sampling for approximating integrals

$$I(f) = \int_D f(x)\rho(x)\,dx$$

of functions f in a Hilbert space $H \subset L_1(\rho)$ where ρ is a given probability density on $D \subset \mathbb{R}^d$.

We show that whenever the embedding of H into $L_1(\rho)$ is a bounded operator then there exists another density ω such that the worst case error of importance sampling with density function ω is of order $n^{-1/2}$.

This applies in particular to reproducing kernel Hilbert spaces with a non-negative kernel.

As a result, for multivariate problems generated from nonnegative kernels we prove strong polynomial tractability of the integration problem in the randomized setting.

The density function ω is obtained from the application of change of density results used in the geometry of Banach spaces in connection with a theorem of Grothendieck about 2-summing operators.

Keywords: Importance Sampling, Monte-Carlo algorithm, tractability, multivariate integration

Parallel Quasi-Monte Carlo Methods

Alexander Keller (mental images - Berlin, DE)

A general concept for parallelizing quasi-Monte Carlo methods is introduced.

By considering the distribution of computing jobs across a multiprocessor as an additional problem dimension, the straightforward application of quasi-Monte Carlo methods implies parallelization.

The approach in fact partitions a single low-discrepancy sequence into multiple low-discrepancy sequences. This allows for adaptive parallel processing without synchronization, i.e. communication is required only once for the final reduction of the partial results.

The resulting algorithms are deterministic, independent of the number of processors, and include previous approaches as special cases.

Keywords: Parallel programming, Quasi-Monte Carlo methods

Covering numbers in Gaussian RKHS

Thomas Kuehn (Universität Leipzig, DE)

The topic of the talk is motivated by a problem in mathematical learning theory. In the pioneering paper by Cucker and Smale (*Bull. AMS* 2001) it was shown that functional analytic concepts could be very fruitful in statistical learning theory. Modern machine learning methods, like support vector machines, often use Gaussian kernels and their corresponding reproducing kernel Hilbert spaces (RKHS). Then, in estimating the probabilistic error and the number of required samples, bounds for the covering numbers of the unit ball of the RKHS are needed.

Gaussian kernels are functions of the form

$$K(x,y) = \exp(-\sigma^2 |x-y|^2), \quad x, y \in X,$$

where X is a subset of $I\!R^d$ and $\sigma > 0$ a parameter. Since these kernels are continuous and positive definite, they generate a RKHS which consists of continuous functions only.

Given $\varepsilon > 0$, the covering number $N(\varepsilon, A; M)$ of a subset A in a metric space M is defined as the minimal number of balls in M of radius ε that are necessary to cover A. The function $H(\varepsilon, A; M) := \log N(\varepsilon, A; M)$, which is often sufficient in applications, is Kolmogorov's famous $\varepsilon-$entropy.

In the context of learning theory, Ding-Xuan Zhou (*J. Complexity* 2002) proved that for $X = [0,1]^d$ the covering numbers $N(\varepsilon)$ of the unit ball of a Gaussian RKHS on X, considered as a subset of $C(X)$, satisfy the upper estimate

$$\log N(\varepsilon) = \mathcal{O}\left(\left(\log \tfrac{1}{\varepsilon}\right)^{d+1}\right) \quad \text{as } \varepsilon \to 0$$

and conjectured that the correct bound is $\left(\log \frac{1}{\varepsilon}\right)^{d/2+1}$.

The aim of this talk is to prove that the exact asymptotic behaviour is

$$\log N(\varepsilon) = \mathcal{O}\left(\frac{\left(\log \frac{1}{\varepsilon}\right)^{d+1}}{\left(\log\log \frac{1}{\varepsilon}\right)^{d}}\right) \quad \text{as } \varepsilon \to 0\,.$$

This shows that Zhou's upper bound was almost sharp, up to a double logarithmic factor, but his conjecture was far too optimistic. While Zhou derived his result from smoothness properties of the Fourier transform of the kernel, our proof is based on the explicit description of Gaussian RKHS, which was established recently by Steinwart, Hush and Scovel (*IEEE Trans. Information Theory* 2006).

Approximation Complexity of Additive Random Fields

Mikhail A. Lifshits (St. Petersburg University, RU)

Let $X(t,\omega), (t,\omega) \in [0,1]^d \times \Omega$ be an additive random field. We investigate the complexity of finite rank approximation

$$X(t,\omega) \approx \sum_{k=1}^{n} \xi_k(\omega)\varphi_k(t).$$

The results obtained in asymptotic setting $d \to \infty$, as suggested H.Woźniakowski, provide quantitative version of dimension curse phenomenon: we show that the number of terms in the series needed to obtain a given relative approximation error depends exponentially on d and find the explosion coefficients.

Keywords: Approximation complexity, dimension curse, Gaussian processes, linear approximation error, random fields, tractability

Joint work of: Lifshits, Mikhail A.; Zani, Marguerite

See also: M.A.Lifshits, M.Zani. Approximation complexity of additive random fields , J. of Complexity, 2008, 24, 362–379.

The Research Progress of BNU Group on Relative Widths

Yongping Liu (Beijing Normal University, CN)

In this talk, I will state some works with the joined authors Xiao Weiwei* and Yang Wei**.

What we consider are the problems on the relative n-widths of two kinds of periodic convolution classes, $\widetilde{\mathcal{K}}_p(K)$ and $\widetilde{\mathcal{B}}_p(G)$, whose convolution kernels K and G are NCVD-kernel and B-kernel. The asymptotic estimations of $K_n(\widetilde{\mathcal{K}}_p(K), \widetilde{\mathcal{K}}_p(K))_q$ and $K_n(\widetilde{\mathcal{B}}_p(G), \widetilde{\mathcal{B}}_p(G))_q$ are obtained for $p = 1$ and ∞, $1 \leq q \leq \infty$. We also defined a new concept of the average relative widths and obtain the exact results of the average relative widths of the classes of some smooth functions in $L_2(R^d)$. In addition, some open problems are proposed on the relative widths and the average relative widths.

Keywords: Relative n-width, periodic convolution class

Multi-parameter regularization and its numerical realization

Shuai Lu (RICAM - Linz, AT)

In this talk we propose and analyse a choice of parameters in the multi-parameter regularization. A modified discrepancy principle is presented within the multi-parameter regularization framework. An order optimal error bound is obtained under standard smoothness assumptions. We also propose a numerical realization of the multi-parameter discrepancy principle based on the model function approximation. Numerical experiments on a series of test problems support theoretical results.

Joint research with Sergei V Pereverzev (RICAM).

Inverse Problems in different settings

Peter Mathe (Weierstraß Institut - Berlin, DE)

One major problems in regularization nof inverse problems is the determination of the regularization parameter. Among the rules for determining such parameter appropriately there are such which have a sound mathematical foundation and others which are based on heuristics. The latter ones are successfully used in practice, however, a justification is lacking.

We shall indicate, that by altering/relaxing the error criteria, one can give justification to some of the known heuristic parameter choices.

On the expressive power of Conjunctive-Normal-Form polynomials

Klaus Meer (BTU Cottbus, DE)

We study the expressive power of certain families of polynomials over a field K. The latter are defined via Boolean formulas in conjunctive normal form.

We attach in a natural way a graph to such a formula and study the resulting polynomial families when the tree-width of the graph is assumed to bounded.

This is joint work with I. Briquel and P. Koiran from ENS Lyon.

Keywords: Permanent polynomials, expressiveness, bounded tree-width

Marcinkiewicz–Zygmund inequalities and quadrature on manifolds

Hrushikesh N. Mhaskar (California State Univ. - Los Angeles, US)

Let $\mathbb{X}$ be a compact, Riemannian manifold, μ be the volume measure on $\mathbb{X}$, $\{\phi_j\}$ (respectively, $\{\ell_j^2\}$) be the system of eigenfunctions (respectively, eigenvalues) of the positive Laplace–Beltrami operator on $\mathbb{X}$.

Let $\Pi_L := \text{span}\{\phi_j : \ell_j \leq L\}$, $\mathcal{C}$ be a finite subset of $\mathbb{X}$. We discuss inequalities of the form

$$\sum_{y\in\mathcal{C}} W_y |P(y)|^p \sim \int_{\mathbb{X}} |P(y)|^p d\mu(y)$$

for suitable weights W_y, and their applications. Among these are the existence of quadrature formulas exact for Π_L, and which yield the best approximation order for integrating functions in a suitably defined Sobolev potential space on $\mathbb{X}$. This last part is analogous to our paper with Hesse and Sloan, J. Complexity, **23** (2007), 528–552. The present work is joint with F. Filbir.

Keywords: Manifold learning, quadrature formulas, Marcinkiewicz-Zygmund inequalities, complexity

Optimal Pointwise Approximation of SDEs driven by Fractional Brownian Motion

Andreas Neuenkirch (TU Dortmund, DE)

Let $(B_t)_{t\geq 0} = (B_t^{(1)}, \ldots, B_t^{(d)})_{t\geq 0}$ be a d-dimensional fractional Brownian motion with Hurst parameter $H \in (1/4, 1)$. Moreover, let $a, \sigma^{(i)} : \mathbb{R}^m \to \mathbb{R}^m$, $i = 1, \ldots, d$, be smooth vectorfields and consider the stochastic differential equation

$$dX_t = a(X_t)\, dt + \sum_{i=1}^{d} \sigma^{(i)}(X_t)\, dB_t^{(i)}, \quad t \in [0,1], \qquad X_0 = x_0 \in \mathbb{R}^m.$$

In this talk, we will study the optimal approximation of such SDEs with respect to the root mean square error at $t = 1$ in two cases:

- The one-dimensional case ($m = d = 1$) for $H > 1/2$. Here we show that the optimal convergence rate that can be achieved by arbitrary schemes that are based on an equidistant discretization of the driving fractional Brownian motion with stepsize $1/n$ is of order $n^{-H-1/2}$.
- The fractional Lévy area ($m = d = 2$) for $H > 1/2$. The simple SDE

  $$dX_t^{(1)} = dB_t^{(1)}, \quad dX_t^{(2)} = X_t^{(1)}\, dB_t^{(2)}$$

  with $X_0^{(1)} = X_0^{(2)} = 0$ is the prototype example for an equation with non-commutative noise.
  Here we show that the optimal convergence rate for arbitrary schemes that use n linear functionals of the driving fractional Brownian motion is of order $n^{-2H+1/2}$.

Keywords: Fractional Brownian motion; stochastic differential equations; optimal approximation

Why Weights?

Erich Novak (Universität Jena, DE)

We consider L_∞-approximation for C^∞-functions on $[0,1]^d$ and prove that this problem suffers from the curse of dimension.

We also present tensor product Hilbert spaces, actually the space $W_2^{(1,1,\ldots,1)}([0,1]^d)$ with standard norms, where every non-trivial tensor product functional is intractable (for function values).

Both results are for non-weighted spaces and show once more that weighted norms are needed to obtain tractable problems.

Joint work with Henryk Woźniakowski.

Weighted QMC rules — Stop anywhere

Dirk Nuyens (K.U. Leuven, BE)

We study the worst case integration error of combinations of quadrature rules in a reproducing kernel Hilbert space. We show that the error, with respect to the total number N of function evaluations used, cannot decrease faster than $O(N^{-1})$ for an equal-weight rule. However, if the errors of the quadrature rules constituting a compound rule have an order of convergence $O(N^{-\alpha})$ for $\alpha > 1$ then, by introducing weights, this order of convergence can be shown to be recovered for the compound rule.

The theory applies to lattice sequences as well as digital sequences.

Keywords: Worst-case error, lattice sequences, digital sequences, higher order of convergence

Joint work of: Nuyens, Dirk; Hickernell, Fred J.; Kritzer, Peter; Kuo, Frances Y.

Tractability of linear tensor product problems in the average case setting

Anargyros Papageorgiou (Columbia University, US)

We consider linear tensor product problems in the average case setting and show necessary and sufficient conditions for weak tractability. In particular we characterize the eigenvalues for which we have weak tractability but not polynomial tractability.

Keywords: Tractability, multivariate problem

Multi-parameter regularization and its application in learning theory

Sergei V. Pereverzev (RICAM - Linz, AT)

We discuss Tikhonov-type regularization schemes with several penalty terms, and show how such schemes naturally appear in regularization of ill-posed problems with noisy problem instances. Multiparameter discrepancy principle is proposed as a systematic procedure for choosing multiple regularization parameters. We prove that under standard assumptions this procedure guarantees an accuracy of optimal order. Finally we show how proposed approach can be successfully implemented in learning from labeled and unlabeled examples.

Joint research with Shuai Lu (RICAM), in cooperation with Ulrich Tautenhahn, and Sivananthan Sampath.

On the Tractability of Linear Tensor Product Problems in the Worst case

Iasonas Petras (Columbia University, US)

It has been an open problem to derive a necessary and sufficient condition for a linear tensor product problem to be weakly tractable in the worst case. The complexity of linear tensor product problems in the worst case depends on the eigenvalues $\{\lambda_i\}_{i\in N}$ of a certain operator. It is known that if $\lambda_1 = 1$ and $\lambda_2 \in (0,1)$ then $\lambda_n = o((\ln n)^{-2})$ as $n \to \infty$, is a necessary condition for a problem to be weakly tractable. We show that this is a sufficient condition as well.

Keywords: Multivariate problem, Complexity, Tractability

Full Paper:
http://www.cs.columbia.edu/%7Eap/html/op26.pdf

Exponential Convergence and Tractability of Multivariate Integration for Korobov Spaces

Friedrich Pillichshammer (University of Linz, AT)

We study multivariate integration for a weighted Korobov space for which the Fourier coefficients of the functions decay exponentially fast.

This implies that the functions of this space are infinitely times differentiable.

Weights of the Korobov space monitor the influence of each variable and each group of variables.

We show that there are numerical integration rules which achieve an exponential convergence of the worst-case integration error.

We also investigate the dependence of the worst-case error on the number of variables s, and show various tractability results under certain conditions on weights of the Korobov space.

This means that the dependence on s is never exponential, and sometimes the dependence on s is polynomial or there is no dependence on s at all.

Keywords: Multivariate integration, tractability, lattice rules

On the optimal approximation of stochastic Itô integrals

Pawel Przybylowicz (AGH Univ. of Science & Technology-Krakow, PL)

We study optimal approximation of stochastic integrals in the Itô sense, of the form $I(f,B) = \int_0^T f(t,B_t)dB_t$, when linear information, consisting of certain integrals of trajectories of Brownian motion, is available. Upper bounds on the nth minimal error, where n is the fixed cardinality of information, are obtained by the Wagner–Platen algorithm and are $O(n^{-3/2})$ or $O(n^{-2})$, depending on considered class of integrands.

We also show that $\Omega(n^{-2})$ is a lower bound which holds even for very smooth integrands.

Moreover, we deal with numerical approximation of stochastic Itô integrals $I(f,B) = \int_0^T f(t)dB_t$ of singular functions $f : [0,T] \to \mathbb{R}^d$.

We first consider the regular case of integrands belonging to the Hölder class with parameters r and ϱ.

We show that in this case the classical Itô–Taylor algorithm has the optimal error $\Theta(n^{-(r+\varrho)})$.

In the singular case, we consider a class of piecewise regular functions that have continuous derivatives, except for a finite number of unknown singular points. We show that any nonadaptive algorithm cannot efficiently handle such a problem, even in the case of a single singularity. The error of such algorithm is no less than $n^{-\min\{1/2,r+\varrho\}}$. Therefore, we must turn to adaptive algorithms. We construct the adaptive Itô–Taylor algorithm that, in the case of at most one singularity, has the optimal error $O(n^{-(r+\varrho)})$.

The best speed of convergence, known for regular functions, is thus preserved. For multiple singularities, we show that any adaptive algorithm has the error $\Omega(n^{-\min\{1/2,r+\varrho\}})$, and this bound is sharp.

Keywords: Stochastic Itô integrals, singular problems, optimal algorithm, standard information, r–fold integrated Brownian motion

See also: B. Kacewicz, P. Przybylowicz, *Optimal adaptive solution of initial-value problems with unknown singularities*, J.Complexity 24 (2008), 455-476; P. Przybylowicz, *Linear information for the approximation of the Itô integrals*, Numerical Algorithms, Published online: 25 June 2009, to appear.

Multilevel Preconditioning for Adaptive Sparse Optimization

Thorsten Raasch (Universität Marburg, DE)

We are concerned with linear inverse problems under sparsity constraints. For the numerical reconstruction, iterative thresholding algorithms are of special interest. Under mild assumptions on the forward operator, they converge strongly and with linear error reduction per iteration. We discuss acceleration techniques that involve nonuniform stepsizes, adaptivity and in particular preconditioning strategies. Numerical examples are given to illustrate the theoretical findings.

Keywords: Inverse problems, sparsity, iterative methods, preconditioning, adaptivity

Joint work of: Dahlke, Stephan; Fornasier, Massimo; Raasch, Thorsten

Multi-level Algorithms for Integration on Sequence Spaces

Klaus Ritter (TU Darmstadt, DE)

We study randomized algorithms for numerical integration with respect to a product probability measure on the sequence space IR^IN.

We consider integrands from reproducing kernel Hilbert spaces, whose kernels are superpositions of weighted tensor products. We combine tractability results for finite-dimensional integration with the multi-level technique to construct new algorithms for infinite-dimensional integration.

These algorithms use variable subspace sampling, and we compare the power of variable and fixed subspace sampling by an analysis of minimal errors.

Joint work with Fred Hickernell, Thomas Müller-Gronbach, and Ben Niu.

Explicit error bounds for reversible Markov chain Monte Carlo methods

Daniel Rudolf (Universität Jena, DE)

Markov chain Monte Carlo methods for approximating the expectation play a crucial role in numerous applications. The problem is to compute the integral of f with respect to a measure π. A straight generation of the desired distribution is in general not possible. Thus it is reasonable to use Markov chain sampling with a burn-in n_0. An explicit error bound of

$$S_{n,n_0}(f) := \frac{1}{n}\sum_{i=1}^{n} f(X_{i+n_0})$$

with respect to the L_p-norm (for $p \geq 4$) of the function f will be presented. By the estimation the well known asymptotical limit of the error is attained, i.e. there is no gap between the estimate and the asymptotical behavior.

L_2 Discrepancy of Generalized Two-Dimensional Hammersley Point Sets

Wolfgang Ch. Schmid (Universität Salzburg, AT)

The L_2 discrepancy is a quantitative measure for the irregularity of distribution of a finite point set. We consider the L_2 discrepancy of so-called generalized Hammersley point sets which can be obtained from the classical Hammersley point sets by introducing some permutations on the base b digits. While for the classical Hammersley point set it is not possible to achieve the optimal order of L_2 discrepancy with respect to a general lower bound due to Roth this disadvantage can be overcome with the generalized version thereof. For special permutations we obtain an exact formula for the L_2 discrepancy from which we obtain two-dimensional finite point sets with the lowest value of L_2 discrepancy known so far.

Joint work with Henri Faure, Friedrich Pillichshammer, and Gottlieb Pirsic

Spline Interpolation on Sparse Grids

Winfried Sickel (Universität Jena, DE)

We apply the Smolyak algorithm to spline interpolation on $\mathbb{R}$.

Then the results are discussed on $\mathbb{R}^d$ and the hypercube.

Keywords: Interpolation on sparse grids, splines, Smolyak algorithm, tensor products of Sobolev and Besov spaces

Source Characterization of atmospheric releases using stochastic search and gradient optimization

Krzysztof Sikorski (University of Utah, US)

An inversion technique based on MC/QMC search and regularized gradient optimization was developed to solve the atmospheric source characterization problem. The Gaussian Plume Model was adopted as the forward operator and QMC/MC search was implemented in order to find good starting points for the gradient optimization. This approach was validated on the Copenhagen Tracer Experiments. The QMC approach with the utilization of clasical and scrambled Halton, Hammersley and Sobol points was shown to be 10-100 times more efficient than the Mersenne Twister Monte Carlo generator. Further experiments are needed for different data sets. Computational complexity analysis needs to be carried out.

Keywords: Atmospheric source problem, Gaussian Plume Model, Quasi Monte Carlo method, gradient optimization

Joint work of: Sikorski, K; B. Addepalli, E. Pardyjak, M. Zhdanov

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2009/2299

Smoothing effect of the ANOVA decomposition

Ian H. Sloan (Univ. of New South Wales, AU)

This talk describes joint work with Michael Griebel (Bonn) and Frances Kuo (University of New South Wales). The problem of option pricing has in the last decade and a half been a successful area of application, and at the same time an inspiration for theoretical development, for both quasi-Monte Carlo and sparse grid methods for high dimensional integration. But there is a problem: whereas the theories (for both quasi-Monte Carlo and sparse grid) assume that the integrand has all its mixed first derivatives square integrable, in the option pricing problem this is spectacularly not true for dimensions bigger than 1, because of the occurrence in the option pricing integrand of the 'max' function. In this work we prove, under a condition that is satisfied wholly or partly in option pricing problems, that the early terms of the ANOVA decomposition (which are widely considered to be the only terms of any significant size) do satisfy the smoothness requirement, even though the whole integrand does not. The principal theorem will be stated in simplified form and the key ides of the proof sketched. The results will be applied to the option pricing problem.

Keywords: ANOVA, decomposition of functions, quasi Monte Carlo, sparse grid, smoothing

Tight Randomized Complexity Bounds for Ordinary Differential Equations of Order k

Marek Szczesny (AGH Univ. of Science & Technology-Krakow, PL)

We study the complexity of initial-value problems for scalar equations of higher order. We consider randomized model of computation and class of r times differentiable right-hand side functions with bounded partial derivatives.

So far it has been shown that the complexity bounds depend on the regularity of the right-hand side function only and are independent of the order of equation. However, presented complexity bounds are not matching. The upper and lower complexity bounds differ by some arbitrarily small parameter. So, we settle the question about tight complexity bounds.

Sparse Recovery and Inverse Problems: Compression and Acceleration

Gerd Teschke (Hochschule Neubrandenburg, DE)

We consider (linear and nonlinear) inverse and ill-posed problems with sparsity constraints. In particular, we focus on iterative methods to approximate the solution of the operator equation. We study several approaches for reducing the computational complexity. One approach is numerical compression (adaptive procedures), another method relies on a projected steepest descent.

Keywords: Inverse and ill-posed problems, sparse recovery, adaptive approximation, l_1 projection, steepest descent

On isotropic high-dimensional integrals

Shu Tezuka (Kyushu University, JP)

In this talk, we discuss two types of high-dimensional integrals. One is the isotropic case considered by Keister as well as by Papageorgiou. The second one is the equally weighted case. We define the associated one dimensional discrepancy for both cases, and show some numerical results.

Keywords: High-diemensional integration, isotropic integrals, low-discrepancy sequences

On positive positive-definite functions and Bochner's Theorem

Jan Vybiral (Universität Jena, DE)

We recall an open problem on the error of quadrature formulas for the integration of functions from some finite dimensional spaces of trigonometric functions posed by Erich Novak in ten years ago and summarized recently in the book of Novak and Wozniakowski.

It is relatively easy to prove an error formula for the best quadrature rules with positive weights which shows intractability of the tensor product problem for such rules.

In contrast to that, the conjecture that also quadrature formulas with arbitrary weights can not decrease the error is still open.

We generalize Novak's conjecture to a statement about positive positive-definite functions and provide several equivalent reformulations, which show the connections to Bochner's Theorem and Toeplitz matrices.

Keywords: Positive positive-definite functions, Bochner's Theorem

Joint work of: Hinrichs, Aicke; Vybiral, Jan

Adiabatic Algorithm for Counting Problem

Chi Zhang (Columbia University, US)

We design adiabatic quantum algorithms for the counting problem, i.e., approximating the proportion, α, of the marked items in a given database. As the quantum system undergoes a designed cyclic adiabatic evolution, it acquires a Berry phase $2\pi\alpha$. By estimating the Berry phase, we can approximate α, and solve the problem. We discuss two different models for the adiabatic algorithm. The first one uses a single Hamiltonian, and is called "static" model. The other one uses $O(\log(\frac{1}{\epsilon}))$ Hamiltonians, and is called "dynamic" model. For an error bound ϵ, the algorithm can solve the problem with cost of order $(\frac{1}{\epsilon})^{3/2}$ in the static model, which is not as good as the optimal algorithm in the quantum circuit model, but better than the classical random algorithm. In the dynamic model, the algorithm can solve the problem in $O(\sqrt{\frac{1}{\epsilon}})$, which beats the quantum algorithm in the circuit model. Moreover, since the Berry phase is a purely geometric feature, the result may be robust to decoherence and resilient to certain noise.

Keywords: Adiabatic algorithm, quantum computing, counting problem, Geometric Phase, Phase Estimation

Joint work of: Zhang, Chi; Wei, Zhaohui; Papageorgiou, Anargyros