Formulations, Bounds and Heuristic Methods for a Two-Echelon Adaptive Location-Distribution Problem

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Abstract. We consider a two-echelon location-distribution problem arising from an actual application in fast delivery service. This problem belongs to the class of adaptive logistics problems, as the locations of the facilities (typically, parking spaces) are revised on a daily basis according to demand variations. We present and compare two formulations for this problem: an arc-based model and a path-based model. Since these formulations cannot be solved in reasonable time for large-scale instances, we introduce a heuristic method based on a variable neighborhood search approach.

Keywords. two-echelon location problem, formulations, relaxations, variable neighborhood search

1 Problem Description

We consider a multi-echelon location-distribution problem arising from an actual application in fast delivery service [1]: a mail-order company offers several products (typically, packages containing various types of goods, such as clothes, electronic devices, appliances,...) that must be delivered on time to the customers requesting them. To satisfy these requests, the firm operates a multi-echelon distribution system: starting their trips from a small set of hubs (their locations are assumed known and fixed, following a preliminary strategic analysis), a fleet of medium-size trucks delivers the products to depots, where they are transferred on small-size trucks, and then shipped to satellites, where the products

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are sorted and delivered to the customers. The company exploits existing facilities for the depots and the satellites, but has to pay to use them. The problem is to ensure that customers' requests are satisfied on time at minimum cost, taking into account the transportation costs and the location costs for using the depots and the satellites.

Typically, satellites are neither owned nor rented by the company. They can be warehouses owned by independent carriers or sites such as car parks where items are transferred from one vehicle to another. More traditional distribution systems, which do not use such satellites, can be very costly to run when the demand varies significantly from one period to another. Moreover, when few depots are present, services such as 24 hour-delivery can be cost ineffective or simply cannot be assured when a wide distribution area is considered. The multi-echelon system we consider allows to satisfy time constraints in a cost-effective way. It is also an *adaptive* system, in the sense that satellites can be opened or closed easily according to demand variations.

We model the problem by defining a network for which the only possible connections are those that ensure on time delivery of the products to the customers. In addition, we assume that for each satellite and each product, the set of customers and the routes used to satisfy their requests have been determined in a preprocessing phase. Hence, the model does not include any routing aspect. Each customer in our model represents a set of customers to which the same product is delivered using a single vehicle. The transportation cost between a satellite and a customer thus corresponds to the cost of the best route determined during this preprocessing phase.

Transportation costs between hubs and depots, and between depots and satellites, vary with the distance travelled, but more importantly, with the number of vehicles used on each arc, each type of vehicle (medium- or small-size truck) having an associated volumetric capacity. A fixed cost is incurred when using any depot, while the satellite location cost increases with the number of batches of products handled at the satellite (a batch corresponds to a fixed number of product units). This cost structure is similar to what can be found in telecommunications network design applications, where multiple facilities, each with an associated capacity, can be installed on the arcs or the nodes. In our problem, vehicles (at the arcs) and product batches (at the satellites) play the role of facilities. Note that this cost structure is more complex than what can be found in most location-distribution problems discussed in the literature, which typically exhibit fixed costs at the nodes and transportation costs that are linear in the number of product units.

Because the locations of the hubs are assumed to be fixed, there are no fixed costs associated to the hubs and we can always assign to each depot its closest hub without losing optimality. This simplification is also performed in the preprocessing phase. Note that we still need to determine how many product units, on how many medium-size vehicles, need to be transported between any depot and its closest hub, but we are now allowed to associate the corresponding decision variables to the depots, instead of the arcs between hubs and depots.

The resulting problem can therefore be considered as a *two-echelon* (from depots to satellites, and from satellites to customers) location-distribution problem.

2 Summary of the Contributions

We present and compare two mixed-integer programming (MIP) formulations for this problem: an arc-based model and a path-based model. We show that the linear programming (LP) relaxation of the path-based model provides a better bound than the LP relaxation of the arc-based model. We also compare the so-called *binary relaxations* of the models, which are obtained by relaxing the integrality constraints for all variables, except for the 0-1 design variables that determine which nodes and which arcs should be used to satisfy customers' requests. We show that the binary relaxations of the two models always provide the same bound, but that the path-based binary relaxation appears preferable from a computational point of view, since it can be reformulated as an equivalent *simple plant location problem* (SPLP), for which several efficient algorithms exist. We also show that the LP relaxation of this SPLP reformulation provides a better bound than the LP relaxation of the path-based model.

Since these formulations cannot be solved in reasonable time for large-scale instances, we develop a heuristic method based on a variant of variable neighborhood search (VNS), called *multi-layered VNS* (MLVNS). MLVNS is based on dividing the neighborhood structures into l_{max} layers, each layer l having associated neighborhood structures. The layers are ordered from 1 to l_{max} and then scanned in that order. For each layer l, a VNS is invoked, with an additional step, which consists in calling MLVNS itself up to layer $l-1$. This approach is useful when the neighborhood structures can be naturally divided in at least two classes: one for which the evaluation of each move is relatively straightforward, and another for which the evaluation of each move is to be performed by a search algorithm to be driven by the more simple neighborhood structures. We present computational results on on an actual application and on instances derived from this large-scale real network.

References

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