# Using Branch-and-Price to Find High-Quality Solutions Quickly

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When integer programming (IP) models are used in operational situations there is a need to consider the tradeoff between the conflicting goals of solution quality and solution time, since for many problems solving realistic-size instances to a tight tolerance is still beyond the capability of state-of-the-art solvers. However, by appropriately defining small instances, good primal solutions frequently can be found quickly.
This approach is taken, for example, within LP-based branch-and-bound algorithms using techniques such as local branching (Fischetti and Lodi (2003)) and RINS (Danna et al. (2005)). These techniques use information from the LP solution and incumbent solution to define a small IP, which is then optimized. These techniques can be applied to any integer program and are available in commercial solvers such as CPLEX.
Another approach to defining small instances is to use problem structure. Hewitt et al. (2009) present an IP-based local search algorithm for the Multicommodity Fixed Charge Network Flow Problem (MCFCNF) where small IPs are chosen according to the attributes of previous solutions, such as the arcs used by various commodities. Combining exact and heuristic search techniques by solving small IPs has received quite a lot of attention recently, see, for example, De Franceschi et al. (2006), Schmid et al. (2008), Archetti et al. (2008), and Savelsbergh and Song (2008).

In this paper we introduce a new approach to finding good solutions quickly that is capable of proving optimality as well. It is different from techniques such as local branching and RINS since it uses problem structure to define the small IPs to be solved. It is different from other IP-based local search methods since the IPs to be solved are determined by a column generation scheme. The embedding of this column generation scheme into a branch-and-price algorithm gives the dual bounds that provide the capability of proving optimality.
Our extended formulation, which requires column generation, is very different from typical column generation formulations that employ structurally different ob jects from the compact formulation, for example, paths rather than arcs. Our extended formulation keeps the original variables from the compact formulation and augments them with an exponential number of additional variables that are used to define problem restrictions to obtain small IPs. By preserving the original compact formulation, we are able to enrich it by preprocessing, cutting planes or any other techniques normally used in a branch-and-cut framework.
The computational results demonstrate that the approach achieves its goals. For the instances used in the computational study, the approach often produces in 15 minutes a proven near-optimal primal solution. More specifically, the primal solution found in 15 minutes is often better than the one CPLEX produces in 6 hours, and the dual bound is

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usually close to the one CPLEX produces in 6 hours.

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