New Models and Methods for Arc Routing

Stefan Irnich[1]

RWTH Aachen University,
Chair for Operations Research and Supply Chain Management
Templergraben 64, 52074 Aachen, Germany
stefan.irnich@rwth-aachen.de

Abstract. The talk presents two non-standard extensions for single-vehicle arc-routing problems a.k.a. postman problems: First, street segments that require a service on both sides of the street can be covered either by two separate services or by a single zigzag service. Instead of deciding the type of service beforehand, we propose to take into account the zigzagging option when designing a tour. We present MIP models for the extension of Undirected Chinese and Rural Postman Problem (UCPP, URPP). We show that these models can be solved reasonable well using a cutting-plane or branch-and-cut algorithm.
Second, capacitated postman problems occur as subproblems in column-generation and Lagrangian-relaxation approaches of the capacitated arc-routing problem. In order to model these and similar subproblems or submodels, we present the Profitable Capacitated Rural Postman Problem (PCRPP): In the PCRPP, edges that are serviced give a profit, but deadheading through edges generates costs. Both service and deadheading consume time. The task is to find a tour that maximizes the difference of profits and costs, while the overall duration of the tour must not exceed a given bound. The solution approach for this problem is again based on branch-and-cut.

Keywords. Postman problems, branch-and-cut

1 Introduction

Single-vehicle arc-routing problems a.k.a. postman problems are among the oldest and best studied discrete optimization problems (see Dror, 2000; Eiselt et al., 1995a,b). The talk presents two non-standard extensions of classical postman problems, i.e. the *Chinese and Rural Undirected Postman Problems with Zigzagging Option (UCPPZ, URPPZ)* and the Profitable Capacitated Rural Postman Problem (PCRPP).

2 Undirected Postman Problems with Zigzagging Option

In postal delivery and garbage collection applications, street segments that require a service on both sides of the street can be covered either by two separate

Dagstuhl Seminar Proceedings 09261
Models and Algorithms for Optimization in Logistics
http://drops.dagstuhl.de/opus/volltexte/2009/2163

services or by a single zigzag service. Instead of deciding the type of service beforehand, we propose to take into account the zigzagging option when designing a tour. This problem was first formally stated by Irnich (2005), where a transformation of the windy rural postman problem with zigzagging option was transformed into a asymmetric traveling salesman problem. Later, Irnich (2008) showed that the undirected Chinese postman problem with zigzagging option can be solved as a T-join problem (Schrijver, 2003, p. 485f) and, therefore, belongs to the complexity class $\mathcal{P}$. Moreover, a MIP-formulation was presented and the corresponding cutting-plane approach was able to solve large-scale instances. The rural postman version of the problem was solved by branch-and-cut.

3 Profitable Capacitated Rural Postman Problem

Up to now, exact methods for the Capacitated Arc Routing Problem (CARP) either rely on branch-and-cut (Belenguer and Benavent, 2003) or on the transformation into the corresponding node-routing problem (see, e.g., Baldacci and Maniezzo, 2006), the well-known Vehicle-Routing Problem (VRP). An alternative exact approach that has not been analyzed before (as far we know) is the solution of the CARP by column generation (Desaulniers et al., 2005) or Lagrangian-relaxation (Lemaréchal, 2001), where the subproblem is treated as a postman problem (and not transformed). The Profitable Capacitated Rural Postman Problem (PCRPP) is this subproblem and is formally defined in the following.

We consider the PCRPP over an undirected graph $G = (V, E)$. The startpoint and endpoint of a postman tour is the depot $d \in V$. Edges $e \in E$ that are serviced generate a profit p_e, but traversing an edge costs c_e. Traversing an edge without servicing it is called deadheading. Both service and deadheading consume time. Let q_e be the time for service and r_e be the time for deadheading through edge e. The task is to find a postman tour that maximizes the difference of profits and costs, while the overall duration of the tour must not exceed a given bound Q. Note that one can earn the profit p_e only once (when first traversing an edge). Furthermore, providing no service to an edge but deadheading through it can make sense: due to the maximum tour duration Q it might be better to not serve an edge, but to deadhead through that edge in order to reach other (profitable) edges.

A straightforward model for the PCRPP uses decision variables $x = (x_e)_{e \in E} \in \{0,1\}^{|E|}$ to indicate that edges are serviced with (not necessarily positive) contribution margin $\phi = (\phi_e) \in \mathbb{R}^{|E|}$ (profit minus traversal cost; $\phi_e = p_e - c_e \in \mathbb{R}$). Decision variables $y = (y_e)_{e \in E} \in \mathbb{Z}_+^{|E|}$ indicate the deadheading through edges with costs $c = (c_e)_{e \in E} \in \mathbb{R}_+^{|E|}$. It is easy to see that in an optimal solution the variables y_e can only take the values 0, 1, and 2.

$$
\begin{aligned}
z_{PCRPP} = \max\, & \phi^\top x - c^\top y && && \text{(1a)}\\
\text{s.t.}\quad & x(\delta(i)) + y(\delta(i)) = 2w_i && \text{for all } i \in V && \text{(1b)}\\
& x(\delta(S)) + y(\delta(S)) \geq 2x_e && \text{for all } e \in E, S \subseteq V \setminus \{d\} : e \subseteq S && \text{(1c)}\\
& q^\top x + r^\top y \leq Q && && \text{(1d)}\\
& x \in \{0,1\}^{|E|} && && \text{(1e)}\\
& y \in \{0,1,2\}^{|E|} && \text{for all } e \in E && \text{(1f)}\\
& w \in \mathbb{Z}_+^{|V|} && && \text{(1g)}
\end{aligned}
$$

The objective (1a) is the maximization of the contribution margin, i.e., profit generated from services minus costs for traversals. Equalities (1b) use additional integer variables w_i for each node $i \in V$ in order to ensure that in the tour every node has an even degree. Connectivity of the tour with the depot results from the connectivity constraints (1c). Constraint (1d) ensures that the tour duration does not exceed the upper bound Q.

The PCRPP is an extension of the so-called Prize-collecting Rural Postman Problem (PRPP) introduced and solved with branch-and-cut in the work of Araoz et al. (2007). However, there are several important differences between the PCRPP and PRPP: First, Araoz et al. (2007) show that variables for dead-heading can be restricted to take only values 0 and 1 in the PRPP. This is not true for the PCRPP. Second, Araoz et al. (2007) consider connected components built by the edges $e \in E$ with nonnegative contribution margin when traversed twice, i.e., with $\phi_e - c_e > 0$. They prove that these connected components can be treated similar as the components of required edges in in the undirected rural postman problem (URPP) (cf. Corberán and Sanchis, 1994). In this way Araoz et al. (2007) are able to come up with a pure binary programm with reduced variable set and additional constraints to strengthen the formulation.

We follow another idea, already used by (Ghiani and Laporte, 2000) for the URPP: variables $y_e \in \{0,1,2\}$ can be replaced by two binary variables $y_e, z_e \in \{0,1\}$. The novelty of the following formulation for the PCRPP is that with these binary variables y_e and z_e the node degree constraints can be fully replaced by cocircuit-inequalities:

$$
\begin{aligned}
z_{PCRPP} = \max\, & \phi^\top x - c^\top (y+z) && \text{(2a)}\\
\text{s.t.}\quad & x(\delta(S)\setminus F) + y(\delta(S)\setminus G) + z(\delta(S)\setminus G) - x(F) - y(G) - z(H) \\
& \geq 1 - |F| - |G| - |H| \\
& \qquad \text{for all } S \subseteq V; F, G, H \subseteq \delta(S) \text{ with } |F|+|G|+|H| \text{ odd} && \text{(2b)}\\
& x(\delta(S)) + y(\delta(S)) + z(\delta(S)) \geq 2x_e \\
& \qquad \text{for all } e \in E, S \subseteq V \setminus \{d\} : e \subseteq S && \text{(2c)}\\
& q^\top x + r^\top (y+z) \leq Q && \text{(2d)}\\
& y \geq z && \text{(2e)}\\
& x \in \{0,1\}^{|E|} && \text{(2f)}\\
& y, z \in \{0,1\}^{|E|} && \text{(2g)}
\end{aligned}
$$

The advantage of this model compared to (2a)-(2g) is that the cocircuit-inequalities (2b) give a tight description of the underlying integer polyhedron. Constraints (2e) are added in order to reduce the inherent symmetry.

We propose solving this model (2a)-(2g) with branch-and-cut. The separation of violated cocircuit inequalities can be done with (fast) exact algorithm by Letchford et al. (2004). Violated connectivity constraints (2c) are easy to find (see Araoz et al., 2007) solving a sequence of max-flow/min-cut problems. Preliminary computational results of a branch-and-cut implementation are very promising.

4 Conclusions and Outlook

Two non-standard extensions to classical postman problems and their solution by cutting-plane approaches were presented. While model and methods for the postman problems with zigzagging option are already published, the new model and branch-and-cut algorithm for the PCRPP is ongoing research. Even without fine-tuning the branch-and-cut code, solution times are fast. We are convinced that the branch-and-cut algorithm for the PCRPP will be the key component when solving instances of the CARP using column generation or Lagrangian relaxation.

Bibliography

Araoz, J., Fernandez, E., and Meza, O. (2007). Solving the prize-collecting rural postman problem. Research Report DR-2007/12, EIO Departament, Technical University of Catalonia, Spain.

Baldacci, R. and Maniezzo, V. (2006). Exact methods based on node-routing formulations for undirected arc-routing problems. *Networks*, 47(1):52–60.

Belenguer, J. and Benavent, E. (2003). A cutting plane algorithm for the capacitated arc routing problem. *Computers & Operations Research*, 30:705–728.

Corberán, A. and Sanchis, J. (1994). A polyhedral approach to the rural postman problem. *European Journal of Operational Research*, 79:95–114.

Desaulniers, G., Desrosiers, J., and Solomon, M., editors (2005). *Column Generation*. Springer, New York, NY.

Dror, M., editor (2000). *Arc Routing: Theory, Solutions and Applications*. Kluwer, Boston.

Eiselt, H., Gendreau, M., and Laporte, G. (1995a). Arc routing problems, Part I: The chinese postman problem. *Operations Research*, 43(2):231–242.

Eiselt, H., Gendreau, M., and Laporte, G. (1995b). Arc routing problems, Part II: The rural postman problem. *Operations Research*, 43(3):399–414.

Ghiani, G. and Laporte, G. (2000). A branch-and-cut algorithm for the undirected rural postman problem. *Mathematical Programming*, 87(3):467–482.

Irnich, S. (2005). A note on postman problems with zigzag service. *INFOR*, 43(1):33–39.

Irnich, S. (2008). Undirected postman problems with zigzagging option: A cutting-plane approach. *Computers & Operations Research*, 35(12):3998–4009.

Lemaréchal, C. (2001). Lagrangian relaxation. In Jünger, M. and Naddef, D., editors, *Computational Combinatorial Optimization,*, volume 2241 of *Lecture Nodes in Computer Science*, pages 112–156. Springer, Berlin.

Letchford, A., Reinelt, G., and Theis, D. (2004). A faster exact separation algorithm for blossom inequalities. In Nemhauser, G. and Bienstock, D., editors, *Integer Programming and Combinatorial Optimization*, volume 3064, chapter 10. Springer, Berlin.

Schrijver, A. (2003). *Combinatorial Optimization. Polyhedra and Efficiency.* Number 24 in Algorithms and Combinatorics. Springer, Berlin Heidelberg New York.