A Robust PTAS for the Parallel Machine Covering Problem

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1 Introduction

Problem definition. We consider the classical scheduling problem of maximizing the minimum machine load on parallel machines. In this setting we are given a set of jobs J, a set of machines M to process the jobs and a processing time p_j for each job $j \in J$. For a given assignment of jobs to machines, the load of a machine i is defined as the sum of the processing of jobs assigned to i. Our objective is to maximize the minimum load of the machines. This problem is usually called the *machine covering problem*, and referred as $P||C_{\min}$ in the three-field notation (see Lawler et al. [1]).

Bounded migration. We study this problem in the framework of *online algorithms with bounded migration*, first introduced by Sanders et al. [2]. In this setting jobs arrive online one by one, and we need to maintained c-competitive solutions by only making slight changes to the current schedule. More precisely, if a job of size p_j arrives, the total processing time of jobs that are allowed to migrate (i.e., moved from one machine to another) is $\beta \cdot p_j$, for some constant $\beta \in I\!R_{\geq 0}$. The parameter β is called the *migration factor* of the algorithm. Note that if $\beta = 0$ we are not allowed to migrate any job, and thus this model generalizes the classical online setting. On the other hand if $\beta = \infty$, we obtain the usual offline model.

Robustness. The idea of bounded migration aims to answer the following natural questions concerning robustness and stability: can we maintain approximate solutions that are also stable? In other words, how much *recoverability* do we need to maintain solutions that are guaranteed to be close to optimal? We think that this two questions are an interesting motivation to study problems under this model.

2 Previous Results

Sanders et al. [2] first considered the idea of migration factor to analyze the problem of minimizing the makespan on parallel machines ($P||C_{\max}$). They show that a simple procedure gives a 3/2-competitive algorithm with migration factor

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4/3. Furthermore, they prove that this problem admits a *Robust PTAS*, i.e., for any given $\varepsilon > 0$, there exists an algorithm that maintains $(1+\varepsilon)$-competitive solutions by only using a constant migration factor $\beta(\varepsilon)$. They also consider the machine covering problem, giving a 2-competitive algorithm that uses a migration factor of 1.

In consecutive work Epstein and Levin [3] study the BIN-PACKING problem giving a Robust APTAS. Their approach uses similar techniques — although more involved — as in [2].

On the other hand the machine covering problem is a well studied problem from the classical approximation and online algorithms point of view. Csirik, Kellerer and Woeginger [4] show that the *longest processing time* rule (LPT) yields a $(3m-1)/(4m-2)$-approximation algorithm for this problem. Furthermore, Woeginger [5] developed a PTAS with running time $O(n \log m)$. On the online setting Azar and Epstein [6] give a lower bound of $\Omega(\sqrt{m})$ on the competitive ratio of any randomized algorithm, and a randomized $\widetilde{O}(\sqrt{m})$-competitive algorithm.

3 Our Contribution

As mentioned earlier we study the machine covering problem under the framework of bounded migration algorithms. We first give a negative result by showing a lower bound of 19/20 on the competitive factor of any online algorithm with constant migration factor. This clearly shows that there is no Robust PTAS for this problem. Nonetheless, if we slightly relax the notion of migration factor the problem admits a Robust PTAS. The relaxation consists in allowing to use the processing time of arriving jobs to migrate jobs at later iterations. More precisely, let L be a variable denoting the total processing time of jobs whose arrival caused no jobs to be migrated. We say that an algorithm uses a *cumulative migration factor* of β, if whenever a new job j arrives we migrate jobs whose total processing time is at most $\beta(L + p_j)$. It is worth remarking that if the arrival of a new job gives rise to the migration of jobs, we consider that all previous jobs have been used to migrate these jobs and thus redefine L as zero.

We show that is possible to maintain $(1+\varepsilon)$-competitive solutions by only using constant cumulative migration factor $\beta(\varepsilon)$. Similarly as in [2] and [3], the techniques we use consists in rounding the problem such that optimal schedules are characterized by solutions to an integer program in constant dimension. The guarantee on the migration factor is then given by a stability theorem from the theory of integer programming. Nonetheless, as oppose to the previously studied problems, not every optimal solution of the rounded instance can be used at each iteration by using constant cumulative migration factor. Our main contribution is to show how to overcome this problem by giving a specific structure of the optimal solutions that guarantees stability.

To be more precise, for a given schedule we define its *profile* as a vector ℓ, whose entry ℓ_i denotes the number of machines with load equals to $i \in IN$. Let ℓ and ℓ' be two profiles and i the first entry in which ℓ and ℓ' differ. We say that ℓ is lexicographically smaller than ℓ' if $\ell_i < \ell'_i$. It is not hard to see that a schedule whose profile is lexicographically minimal must be an optimal schedule. Furthermore, we show that in a rounded instance such a schedule can be found in polynomial time by solving a integer program in constant dimension. With this we can use the same sensitivity analysis as in [2] and [3] to obtain a Robust PTAS with bounded cumulative migration factor.

Finally, we can show that applying analogous ideas to the problem of minimizing the makespan, we can strengthen the results in [2] and also guarantee $(1+\varepsilon)$-competitive solutions with constant cumulative migration factor for $P||C_{\max}$ when jobs leave or arrive.

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