Using Options with Set Exercise Prices to Reduce Bidder Exposure in Sequential Auctions*
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Abstract. This report studies the benefits of using priced options for solving the exposure problem that bidders with valuation synergies face in sequential auctions. We consider a model in which complementary-valued items are auctioned sequentially by different sellers, who have the choice of either selling their good directly or through a priced option, after fixing its exercise price. We analyze this model from a decision-theoretic perspective and we show, for a setting where the competition is formed by local bidders, that using options can increase the expected profit for both buyers and sellers. Furthermore, we derive the equations that provide minimum and maximum bounds of the synergy buyer's bid in order for both sides to have an incentive to use the options mechanism. Next, we perform an experimental analysis of a market in which multiple synergy buyers are active simultaneously.

1 Introduction

The exposure problem appears whenever a bidder with complementary valuations (i.e. synergies) tries to acquire a bundle of goods sold through sequential auctions. Informally, the problem occurs whenever an agent may buy a single good at a price higher than what it is worth to her, in the hope of obtaining extra value through synergy with another good, which is sold in a later auction. However, if she then fails to buy this other good at a profitable price, she is exposed to the risk of a potential loss. In the analysis presented in this paper, we call such a global bidder a *synergy buyer*.

The exposure problem is well known in auction theory and multi-agent systems research. The usual way to tackle this problem in the mechanism design

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community is to replace sequential allocation with a one-shot mechanism, such as a combinatorial auction [7]. However, this approach has the disadvantage of typically requiring a central point of authority, which handles all the sales. Moreover, many allocation problems occurring in practice are inherently decentralized and sequential. Possible examples range from items sold on Ebay by different sellers, loads appearing over time in distributed transportation logistics, dynamic resource allocation in hospitals, etc.

Note that this is a very complex problem, and this paper provides a first decision-theoretic analysis of how priced options can be used to address this problem, as well as a first mathematical model to compute option and exercise prices. However, we do stress options are not a "silver bullet" that completely removes the exposure problem, rather, they are a mechanism that, under some assumptions, removes part of the risk exposure and is preferable to both sides (buyers and sellers), by comparison to a direct sale. In fact, auctions for direct sale of the good (as will become apparent in Section 1.3) becomes, in our option model, a particular sub-case.

1.1 Options: basic definition

An option can be seen as a contract between the buyer and the seller of a good, subject to the following rules:

- The writer or seller of the option has the *obligation* to sell the good for the *exercise price*, but not the right.
- The holder or buyer of the option has the *right* to buy the good for the *exercise price*, but not the obligation.

Since the buyer gains the right to choose in the future whether or not she wants to buy the good, an option comes with an *option price*, which she has to pay regardless of whether she chooses to exercise the option or not.

Options can thus help a synergy buyer reduce the exposure problem she faces. She still has to pay the option price, but if she fails to complete her desired bundle, then she does not have to pay the exercise price as well and thus she limits her loss. So part of the uncertainty of not winning subsequent auctions is transferred to the seller, who may now miss out on the exercise price if the buyer fails to acquire the desired bundle. At the same time, the seller can also benefit indirectly, from the additional participation in the market by additional synergy buyers, who would have otherwise stayed out, because of the exposure to a potential loss.

1.2 Related work

In existing multi-agent literature, to our knowledge, there has been only limited work to study the use of options.

The first work to introduce an explicit option-based mechanism for sequential-auction allocation of goods to the MAS community is Juda & Parkes [3, 4]. They create a market design in which global bidders are awarded free (i.e. zero-priced) options, in order to cover their exposure problem and, for this setting, they propose truth-telling as a dominant strategy. In their case, the exposure problem is entirely solved for the synergy buyers, because they do not even have a possible loss consisting of the option price. However, this approach also introduces some limitations. First, there may be cases when the market entry effects are not sufficient to motivate the sellers of items to use options. Because the options are assumed to be offered freely (zero-priced), there may be cases in which sellers do not have a sufficient incentive to offer free options, because of the risk of remaining with their items unsold. The sellers could, however, demand a premium (in the form of the option price) to cover their risk. Thus, in such cases, only positively-priced options can provide sufficient incentive for for both sides to use the mechanism.

Also, the mechanism described in [3, 4] assumes synergy buyers bid their entire valuation (monetary utility) for their desired bundle on each good of that bundle. This design works with a single synergy buyer - but fails when several such buyers are active in the market simultaneously.

Priced options have a long history of research in finance (see [2] for an overview). However, the underlying assumption for all financial option pricing models is their dependence on an underlying asset, which has a current, public value that moves independently of the actions of individual agents (e.g. this motion is assumed to be Brownian for Black-Scholes models). This type of assumption does not hold for the online, sequential auctions setting we consider. In our case, each individual synergy buyer has its own private value for the goods/bundles on offer, and bids accordingly.

Another relevant work that studies the use of options in online auctions is that of Gopal et al [1]. Gopal et al. discuss the benefits of using options to increase the expected revenue of a seller of multiple copies of the same good. They do not consider the use of options to solve the exposure problem of buyers with complementary valuations over a bundle of goods (i.e. the synergy buyers in our model). Furthermore, in [1], it is the seller that fixes both the option price and the exercise price when writing the option, which requires rather restrictive assumptions on the behaviour of the bidders.

Finally, there is a connection between options and leveled commitment mechanisms, first proposed by Sandholm & Lesser [8]. In leveled commitment, both parties have the possibility to decommit (i.e. unilaterally break a contract), against paying a pre-agreed decommitment penalty. However, as [8] show, setting the level of the decommitment penalty can be hard, due to the complex game-theoretic reasoning required. There are situations in which both parties would find it beneficial to decommit but neither does, hoping the other party would do so first, to avoid paying the decommitment penalty. This differs from option contracts, where the right to exercise the option is paid by one party in advance. In our model, this right is sold through an auction, thus the option price is established through an open market.

1.3 Outline and contribution of our approach

The goal of this paper is to study the use of priced options to solve the exposure problem and to identify the settings in which using priced options benefits both the synergy buyer and the seller.

An option consists out of two prices, so an adjustment needs to be made to the standard auction with bids of a single price. The essence of options, in our model, is that buyers obtain the right to buy the good for a certain exercise price in the future. The value of such an option may be different for different market participants at different times. Throughout this study, in order to make the analysis tractable, we have a fixed exercise price and a flexible option price. The seller determines the exercise price of an option for the good she has for sale and then sells this option through a first price auction. Buyers bid for the right to buy this option, i.e. they bid on the option price.

Note that, in this model, direct auctioning of the items appears as a particular sub-case of the proposed mechanism, assuming free disposal on the part of the buyers. If the seller fixes the future exercise price for the option at zero, then a buyer basically bids for the right to get the item for free. Since such an option is always exercised (assuming free disposal), this is basically equivalent to auctioning the item itself.

Based on the above description, we provide both an analytical and an experimental investigation of the setting. Our analysis of the problem can be characterized as decision-theoretic, meaning both buyer and seller reason with respect to expected future price. In summary, our contribution to the literature can be characterized as being twofold:

First, we consider a setting in which n complementary-valued goods (or options for them) are auctioned sequentially, assuming there is only one synergy buyer or global bidder (the rest of the competition is formed by local bidders desiring only one good). For this setting, we show analytically (under some assumptions), that using priced options can increase the expected profit for both the synergy buyer and the seller, compared to the case when the goods are auctioned directly. Furthermore, we derive the equations that provide minimum and maximum bounds between which the bids of the synergy buyer are expected to fall, in order for both sides to have an incentive to use options.

In the second part of the paper, we consider market settings in which multiple synergy buyers (global bidders) are active simultaneously, and study it through experimental simulations. In such settings, we show that, while some synergy buyers lose because of the extra competition, other synergy buyers may actually benefit, because sellers are forced to fix exercise prices for options at levels which encourages participation of all buyers.

The structure for the rest of this paper is as follows. Sect. 2 lays the foundation for further analysis by deriving the expected profits of synergy buyers and sellers for both the direct sale, respectively for a sale with options. Sect. 3 provides the analytical results and proofs of the paper, for a market of sequential auctions with one synergy buyer. Sections 4 and 5 summarize the results from our experimental investigations, while Sect. 6 concludes with a discussion.

2 Expected profit for a sequence of n auctions and 1 synergy buyer

Section 3 will analytically prove, that options can be profitable to both synergy buyer and seller. In order to do that, this section derives the expected profit functions (which depend on the bids of the synergy buyer) for the synergy buyer and the seller. Throughout this study it is assumed that both sellers and buyers are risk neutral and that they want to maximize their expected utility, respectively - in this case - their expected profit.

2.1 Profit with n unique goods without options

This section describes the expected profit of the synergy buyer and the sellers as a function of the synergy buyer's bids for a market with n unique, complementary goods, which are sold without options.

Let G be the set of n goods for sale in a temporal sequence of auctions and $v_{syn}(G_{sub})$ be the valuation the synergy buyer has for $G_{sub} \subseteqq G$. Then assume that $v_{syn}(G) > 0$ and $\forall G_{sub} \subsetneqq G,\ v_{syn}(G_{sub}) = 0$. In other words, the synergy buyer only desires a bundle of all the goods considered in the model.

The goods $G_1..G_n \in G$ are sold individually through sequential, first-price, sealed-bid auctions. Here we choose the auctions to be first price, as they are more tractable to study using game-theoretic analysis. Furthermore, in a sequential setting with valuation complementarities of the agents, second-price auctions do not have the nice dominant strategies properties, described by Vickrey. Furthermore, in many settings where such a model could be used in practice, such as request-for-quotes (RFQ) auctions in logistics or supply chains, first-price auctioning is often used.

The time these auctions take place in is $t = 1 \ldots n$, such that at time t good $G_t \in G$ is auctioned. The above assumptions mean that if the synergy buyer has failed to obtain G_t, then she cannot achieve a bundle, for which she has a positive valuation. So if G_{t+1} is auctioned with a positive reserve price, then obtaining G_{t+1} will only cost the synergy buyer money. Therefore, if the synergy buyer fails to obtain G_t, then it is rational for her to not place bids in subsequent auctions.

The bids of the synergy buyer are $\boldsymbol{B} = (b_1, \ldots, b_n)$, where b_t is the bid the synergy buyer will place for good G_t, conditional on having won the previous auctions. Because of the first-price auction format, b_t is also the price the synergy buyer has to pay if she has won the auction.

Throughout this analysis, we assume the competition the synergy buyer faces for each good G_t (sold at time t) is formed by local bidders that only require the good G_t. We further assume that these local bidders are myopic, i.e. the bids placed by the synergy buyer have no effect on their bidding behaviour. Therefore, from the perspective of the synergy buyer, the competition can be modeled as a distribution over the expected closing prices at each time point t, more precisely as a distribution over a value bm_t, which is the maximal bid placed by the competition not counting b_t.

Denote by $F_t(b_t)$ the probability that the synergy buyer wins good G_t with bid b_t - where $F_t(b_t)$ depends on whether b_t can outbid the maximal bid bm_t placed by the competition, excluding b_t. For each good G_t, there exists a strictly positive reserve price of $b_{t,res}$, which is the seller's own valuation for that good. Then bm_t is the highest bid of the local bidders (who only want G_t), if that bid is higher than $b_{t,res}$. Otherwise bm_t equals $b_{t,res}$. To deal with ties, we assume the synergy buyer only wins G_t if $b_t > bm_t$ and not if the bids are equal. Then $F_t(b_t)$ can be defined as follows:

$$F_t(b_t) = Prob(b_t > bm_t) \tag{1}$$

The synergy buyer only has a strictly positive valuation for the bundle of goods G, which includes all the goods G_t, sold at times $t = 1..n$. Therefore, in a market without options, the a-priori expected profit π_{syn}^{dir} of the synergy buyer is:

$$E(\pi_{syn}^{dir}) = \left[v_{syn}(G) \prod_{i=1}^{n} F_i(b_i) \right] + \left[\sum_{j=1}^{n} (-b_j) \prod_{k=1}^{j} F_k(b_k) \right] \tag{2}$$

The synergy buyer wants to maximize her expected profit. So her optimal bids $\boldsymbol{B}^* = (b_1^*, \ldots, b_n^*)$ maximize equation 2:

$$\boldsymbol{B}^* = argmax_{\boldsymbol{B}^*}\ E(\pi_{syn}^{dir}) \tag{3}$$

Next the profit of the sellers are examined. It is assumed that all sellers have their own valuation for the good that they sell and that they set their reserve price of $b_{t,res}$ equal to this private valuation. So when the good is sold for b_t, the seller of G_t has a profit π_t^{dir} of $b_t - b_{t,res}$. As previously shown, the synergy buyer only participates when she has won the previous auctions; otherwise bm_t is the maximal placed bid. The expected profit of the seller of the good G_t sold at time t is:

$$\begin{aligned} E(\pi_t^{dir}) = (E(bm_t) - b_{t,res})(1 - \prod_{i=1}^{t-1} F_i(b_i)) + \Big(F_t(b_t)(b_t - b_{t,res}) \\ + (1 - F_t(b_t))(E(bm_t | bm_t \geq b_t) - b_{t,res}) \Big) \prod_{i=1}^{t-1} F_i(b_i) \end{aligned} \tag{4}$$

Intuitively explained, the equation defines the expected utility over 3 disjoint cases: one in which the optimal bids b_i of the synergy bidder were sufficient to win all auctions up to time t, in which case the expected profit of the seller is the highest expected bid of the local bidders $E(bm_t)$, minus its own reservation value $b_{t,res}$; the second case in which the synergy bidder wins all previous auctions, including the current one (i.e. the one at time t), in which case the expected profit is this bid minus reservation $b_t - b_{t_res}$, and the third in which the synergy buyer won all previous auctions but fails to win the current one, in which case still the highest bid by the local bidders is taken.

2.2 Profit with n unique goods with options

Section 2.1 derived the expected profit functions for the synergy buyer and the sellers in a market without options. The next step is to do the same for a market with options. This section has the same setting as the general model with n goods being sold, only now an option on G_t is auctioned at time t. Therefore, all the sellers in the market will sell options for their goods, instead of directly the goods themselves. After the n auctions have taken place, the buyers need to determine whether or not they will exercise their option. It is assumed that an option is only exercised if a buyer has obtained her entire, desired bundle. The local bidders are only interested in G_t, so they will always exercise an option on G_t should they have one. The synergy buyer is only interested in a bundle of all goods, so she will only exercise an option (and pay the corresponding exercise price) if she has options on all the goods required.

The option exists out of a fixed exercise price K_t and the synergy buyer's bids on the option price are $\boldsymbol{OP} = (op_1, \ldots, op_n)$. The maximal bid without the synergy buyer was bm_t, but now opm_t is the maximal placed option price.

Since the competition only wants one good, they do not benefit from having an option and they will always exercise any option they acquire. Therefore the competition's best policy is to keep bidding the same total price, which is the bid without options minus the exercise price. Thus the distribution of the competition is only shifted horizontally to the left, by the reduction of the exercise price: $opm_t = bm_t - K_t$. Thus, if the synergy buyer bids the same total price (option + exercise), then she has the same probability of winning the auction in both models. Let $F_t^o(op_t)$ be the probability that op_t wins the auction for the option on G_t. So if $op_t + K_t = b_t$, then $F_t^o(op_t) = F_t^o(b_t - K_t) = F_t(b_t)$.

The synergy buyer's expected profit with options then is:

$$E(\pi_{syn}^{op}) = \left[(v_{syn}(G) - \left[\sum_{h=1}^{n} K_h \right] \right] \prod_{i=1}^{n} F_i^o(op_i) + \left[\sum_{j=1}^{n} (-op_j) \prod_{k=1}^{j} F_k^o(op_k) \right] \tag{5}$$

So her optimal bids $\boldsymbol{OP}^* = (op_1^*, \ldots, op_n^*)$ maximize the profit equation 5:

$$\boldsymbol{OP}^* = argmax_{\boldsymbol{OP}^*} \; E(\pi_{syn}^{op})) \tag{6}$$

The main difference for the seller of G_t, is that if the synergy buyer wins, then she only earns $K_t - b_{t,res}$ when the option is exercised. She then gains the exercise price, but loses the value the good has to her, which is the reserve price. And the probability of exercise is the probability that the synergy buyer wins all the other auctions. Therefore, the total expected profit of the seller at time

t is:

$$E(\pi_t^{op}) = (E(opm_t) + K_t - b_{t,res})(1 - \prod_{i=1}^{t-1} F_i^o(op_i))$$
$$+ \Big(F_t^o(op_t)(op_t + \Big[(K_t - b_{t,res}) \prod_{h=t+1}^{n} F_h^o(op_h)\Big])$$
$$+ (1 - F_t^o(op_t))(E(opm_t|opm_t \geq op_t) + K_t - b_{t,res})\Big) \prod_{i=1}^{t-1} F_i^o(op_i) \tag{7}$$

Briefly explained, this equation has the same 3-case structure as Eq. 4 above. In two cases: when the synergy buyer loses an auction for one the earlier items in the sequence (before the items sold at time t), or when she wins all the earlier auctions, but not the auction at time t, the expected payoffs are equivalents to the direct auctioning case, although this time expressed slightly differently, based on both the exercise and option price. However in one case, when the synergy buyer acquires all the previous items and the current one (middle line in Eq. 7), the payoff is composed of two amounts. The option price op_t will be gained for sure, in this case. However, the difference between the exercise and reserve price $K_t - b_{t,res}$ (which signifies the item actually changes hands) is acquired only if the synergy bidder also wins all the subsequent auctions at times $h = t + 1..n$.

This is an important difference, and it would seem from these equations that the seller has no interest to use options, since in one important case, part of the amount she is about to receive depends on the outcome of future auctions. The key, however, rests in the observation that the synergy buyer should be willing to bid more in total (i.e. $K_t + op_t$) than in the direct auctioning case. This will be analyzed in the next Section.

3 When options can benefit both synergy buyer and seller

Section 2 resulted in the a-priori, expected profit for the synergy buyer and the sellers as a function of the synergy buyer's bids for a market with and without options. This section uses these functions to determine the difference in profit between the two markets, which is $\pi_{\delta t}$ and $\pi_{\delta syn}$ for the seller of good G_t and the synergy buyer respectively, where:

Definition 1.

$$\pi_{\delta t} = \pi_t^{op} - \pi_t^{dir},$$
$$\pi_{\delta syn} = \pi_{syn}^{op} - \pi_{syn}^{dir}$$

So if $\pi_{\delta t}$ and $\pi_{\delta syn}$ are positive, then both agents are better off with options.

3.1 When agents are better off with options

Let $\boldsymbol{B}^*$ denote the synergy buyer's optimal bidding policy in a market where goods are sold directly (without options). We assume for the rest of Sect. 3 that for $1 \leq t \leq n$, $F_t(b_t^*) > 0$ and $F_t(b_t^*) < 1$. So she may complete her bundle, but may also end up paying for a worthless subset of goods. Thus she faces an exposure problem. For the market with options, we define a benchmark strategy $\boldsymbol{OP}'$ for the synergy buyer, so that the two markets can easily be compared.

Definition 2. *The benchmark of the synergy buyer's bids with options* $\boldsymbol{OP}' = (op_1', \ldots, op_n')$ *is that for* $1 \leq t \leq n$*:*

$$op_t' = b_t^* - K_t$$

In other words, the benchmark strategy implies that the synergy buyer will bid the same total amount for the good, as if she used her optimal bidding policy in a direct sale market. Clearly this does not have to be her profit-maximizing bid in a market where priced options are used. In fact, it is almost always the case that the synergy buyer will bid a different value in a market in with priced options. This deviance from the benchmark is denoted by λ_t:

Definition 3. *Let* λ_t *denote the deviation in the bid of the synergy buyer on the item* G_t *sold at time* t*, in a model with options, with respect to her profit-maximizing bid* b_t^* *in a model without options. So her bid on an option for* G_t *will be* $op_t' + \lambda_t$*.*

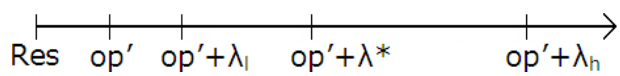

Fig. 1. A possible situation in which options are desirable.

These definitions enable us to rigorously define the bounds within which the use of options (with a given exercise price) are desirable for both the synergy buyer and the seller, for each good in the auction sequence (except the last one, for which there is no uncertainty, so the use of options is indifferent). Fig. 1 gives the visual description of a generic setting in which options are beneficial for both sides. It shows the possible bids a synergy buyer can place for an option. First, valid bids have to be bigger than the reserve price Res, for each good in the sequence. The point op' is where the synergy buyer keeps bidding the same total price as in a market without options, c.f. Def. 2.

The deviations, in an option model, from the benchmark bid op' is measured by three levels, all denoted with λ: λ_l is the minimal risk premium the seller requires to benefit from using options, λ_h is the maximal extra amount the synergy buyer is willing to pay for an option and $op^* = op' + \lambda^*$ is the synergy buyer's profit-maximizing bid in an option market. So, if it is rational for the synergy buyer to bid an additional quantity between λ_l and λ_h (as shown in Fig. 1), then both she and the seller are better off with options.

In the rest of Sect. 3, we derive the analytical expressions which can be used to determine the values for λ_l, λ_h and λ^* and compare them. Before this, however, we describe an important assumption behind the proofs in the remainder of this Section.

Assumption on the proof structure Performing an exhaustive theoretical analysis of the minimum, maximum and optimal bidding levels of the λ-s for all auctions in a sequence would not be tractable, as they all influence each other. Therefore, we simplify our proof structure by focusing only on one of the λ parameters: the one corresponding to the first good. This is possible since, as explained in the introduction, each seller sells one good and is only interested in maximizing the expected profit from that sale. The decision of using options contract or a direct sale is a decision taken bilaterally by each seller and the synergy buyer, thus has to benefit both of them. The reason why we focus on the first good in the sequence is that, for this good, the buyer's probability of not completing her desired bundle, hence her exposure problem, is the greatest. Our proof structure could be generalized as a recursive procedure: if one shows that options are beneficial to use for the first item in a sequence, given a remaining [non-empty] sequence of auctions, this can be generalized to all remaining subsequences, (except perhaps, for the very last item, for which the analysis is trivial, as options cannot bring a benefit by comparison to direct sale).

In order to analytically examine the benefits of deviating from the benchmark strategy op'_1 in the first auction, the proofs in this paper use the additional assumption that the synergy buyer will use the benchmark strategy from Def. 2 for all remaining goods in the sequence. This is a reasonable assumption for this model (as defined above), as sellers of items in subsequent auctions can only benefit from (or are indifferent to) the fact that items sold earlier in the sequence were sold through options, rather than directly. To explain, if there are no complementarities between the earlier items and the good they are currently selling, then sellers are indifferent to the use of options in earlier sales. If there are such complementarities however, subsequent sellers also benefit, because the synergy buyer has a higher chance of acquiring the first good, she also has a higher probability of participating in subsequent auctions. Therefore, subsequent sellers can only benefit if earlier sellers use options to sell their items, and may likely benefit further from using options themselves[3].

[3] As a caveat, note, however, that if there are substitutabilities with the earlier items, then subsequent sellers may suffer from the use of options earlier in the sequence. This is because the increased probability of acquiring an item earlier in the sequence reduces their chances of attracting bids for a substitute item, sold later. We note, however, that in the analysis in this paper, we explicitly do not consider substituabilities, such as to keep the model tractable. But even if we did, since the decision to use or not to use options to sell an item rests with each seller, there is little that

When synergy buyer is better off with options This part of Section 3.1 examines for which bids the synergy buyer is better off with options. This is done by determining the maximal amount she is willing to pay for options.

Lemma 1. *Let $\boldsymbol{B}^* =< b_t >$ for $1 \leq t \leq n$ be the vector of optimal bids of the synergy buyer in the model without options, and $op'_t + \lambda_t$ be the bids in a model with options. Then the expected gain (i.e. difference in expected profit) from using options $E(\pi_{\delta syn})$ can be written as:*

$$
\begin{aligned}
E(\pi_{\delta syn}) = & \Big[v_{syn}(G)(\prod_{i=1}^{n} F_i(b_i^* + \lambda_i) - \prod_{i=1}^{n} F_i(b_i^*))\Big] \\
+ & \Big[\sum_{j=1}^{n} K_j(\prod_{k=1}^{j} F_k(b_k^* + \lambda_k) - \prod_{i=1}^{n} F_i(b_i^* + \lambda_i))\Big] \\
+ & \sum_{j=1}^{n}(-\lambda_j) \prod_{k=1}^{j} F_k(b_k^* + \lambda_k) \\
+ & \Big[\sum_{j=1}^{n}(-b_j^*)(\prod_{k=1}^{j} F_k(b_k^* + \lambda_k) - \prod_{k=1}^{j} F_k(b_k^*))\Big]
\end{aligned}
$$

Proof. We compute the different in profit between a model with options and a model without options, using expected profit equations (5) and (2), as defined in the previous section. In a model without options, the optimal bids of the synergy buyer at each time step t are given by b_t^*. In a model with options, we express the bidding policy as a deviation with respect to the benchmark strategy with options, i.e. $op'_t + \lambda_t$. This gives the difference:

$$
\begin{aligned}
E(\pi_{\delta syn}) = & \Big[(v_{syn}(G) - \Big[\sum_{h=1}^{n} K_h\Big]) \prod_{i=1}^{n} F_i^o(op'_i + \lambda_i)\Big] \\
+ & \Big[\sum_{j=1}^{n}(-(op'_j + \lambda_j) \prod_{k=1}^{j} F_k^o(op'_k + \lambda_k)\Big] \\
- & \Big[v_{syn}(G) \prod_{i=1}^{n} F_i(b_i)\Big] - \Big[\sum_{j=1}^{n}(-b_j^*) \prod_{k=1}^{j} F_k(b_k^*)\Big]
\end{aligned}
$$

We can now replace op'_t with the definition of the benchmark strategy (i.e. same total bid amount, as in the case without options), using the properties: $op'_t = b_t^* - K_t$ and $F_t^o(op'_t + \lambda_t) = F_t(b_t^* + \lambda_t)$. This gives:

future sellers can do to control the profit-maximizing decisions of sellers earlier in the auction sequence.

$$
\begin{aligned}
E(\pi_{\delta syn}) = & \Big[(v_{syn}(G) - \Big[\sum_{h=1}^{n} K_h\Big]) \prod_{i=1}^{n} F_i(b_i^* + \lambda_i)\Big] \\
& + \Big[\sum_{j=1}^{n}(-b_j^* + K_j - \lambda_j) \prod_{k=1}^{j} F_k(b_k^* + \lambda_k)\Big] \\
& - \Big[v_{syn}(G) \prod_{i=1}^{n} F_i(b_i)\Big] - \Big[\sum_{j=1}^{n}(-b_j^*) \prod_{k=1}^{j} F_k(b_k^*)\Big]
\end{aligned}
$$

This formula is now re-grouped, separating the terms $v_{syn}G$, $\sum_{j=1}^{n} K_j$, $\sum_{j=1}^{n}(-\lambda_j)$ and $\sum_{j=1}^{n}(-b_j^*)$, each with its corresponding probabilities to complete the proof the proof:

$$
\begin{aligned}
E(\pi_{\delta syn}) = & \Big[v_{syn}(G)(\prod_{i=1}^{n} F_i(b_i^* + \lambda_i) - \prod_{i=1}^{n} F_i(b_i^*))\Big] \\
& + \Big[\sum_{j=1}^{n} K_j(\prod_{k=1}^{j} F_k(b_k^* + \lambda_k) - \prod_{i=1}^{n} F_i(b_i^* + \lambda_i))\Big] \\
& + \sum_{j=1}^{n}(-\lambda_j) \prod_{k=1}^{j} F_k(b_k^* + \lambda_k) \\
& + \Big[\sum_{j=1}^{n}(-b_j^*)(\prod_{k=1}^{j} F_k(b_k^* + \lambda_k) - \prod_{k=1}^{j} F_k(b_k^*))\Big]
\end{aligned}
$$

To explain intuitively Lemma 1, the difference in expected profits between the two models is formed of 4 parts (corresponding to the 4 lines). First, in an options model, the synergy bidder has a higher probability of getting the desired bundle, since she bids more in total (line 1). Furthermore, in an options model, the bidder does not have to pay exercise prices unless she acquires all n items in the desired bundle (line 2), but she does have to pay a set of additional amounts λ (line 3). Finally, because the

In the following, we turn our attention to providing equations that allow us to deduce the λ parameters that give the synergy buyer an incentive to use options. As previously explained in Sect. 3.1 above, we simplify the proof structure by only focusing on the most important option for the synergy buyer: the one on the first good (when bidding for this good, the probability of not completing her entire bundle is the greatest). This is done under the assumption that for the goods in the sequence, we assume the benchmark strategy is used (i.e. $\lambda_t = 0$ for $t > 1$). For the rest of the items in the sequence, the same proof technique can be applied recursively.

Theorem 1. *Let λ_1 be the deviation in the bidding strategy, compared to the benchmark strategy op'_1, as defined in Def. 2. If $\lambda_t = 0$ for $1 < t \leq n$, then*

$E(\pi_{\delta syn}) >= 0$ if $0 \leq \lambda_1 < \lambda_h$. The value of λ_h (corresponding to $E(\pi_{\delta syn}) = 0$) can be solved as the numerical solution to the following equation:

$$F_1(b_1^* + \lambda_h)\lambda_h = F_1(b_1^* + \lambda_h)\Big[\sum_{j=1}^{n} K_j(\prod_{k=2}^{j} F_k(b_k^*) - \prod_{i=2}^{n} F_i(b_i^*))\Big]$$
$$+ (F_1(b_1^* + \lambda_h) - F_1(b_1^*))\Big[v_{syn}(G)\prod_{i=2}^{n} F_i(b_i^*) - \sum_{j=1}^{n}(b_j^*)\prod_{k=2}^{j} F_k(b_k^*)\Big]$$

Proof. The proof is based on the difference in profit function derived in Lemma 1, using the assumption that $\lambda_t = 0$ for $1 < t \leq n$. As the expectation function of the synergy bidder is descending in the value of λ, we determine when $E(\pi_{\delta syn}) = 0$.

$$\Big[v_{syn}(G)(F_1(b_1^* + \lambda_h) - F_1(b_1^*))\prod_{i=2}^{n} F_i(b_i^*)\Big]$$
$$+ \Big[\sum_{j=1}^{n} K_j(F_1(b_1^* + \lambda_h)\prod_{k=2}^{j} F_k(b_k^*)) - (F_1(b_1^* + \lambda_h)\prod_{i=2}^{n} F_i(b_i^*))\Big]$$
$$+ (-\lambda_h)F_1(b_1^* + \lambda_h)$$
$$+ \Big[\sum_{j=1}^{n}(-b_j^*)(F_1(b_1^* + \lambda_h) - F_1(b_1^*))\prod_{k=2}^{j} F_k(b_k^*)\Big] = 0$$

Isolating the values of λ_h yields the formula in Th. 1.

$$F_1(b_1^* + \lambda_h)\lambda_h = (F_1(b_1^* + \lambda_h) - F_1(b_1^*))\Big[v_{syn}(G)\prod_{i=2}^{n} F_i(b_i^*)\Big]$$
$$+ F_1(b_1^* + \lambda_h)\Big[\sum_{j=1}^{n} K_j(\prod_{k=2}^{j} F_k(b_k^*) - \prod_{i=2}^{n} F_i(b_i^*))\Big]$$
$$+ (F_1(b_1^* + \lambda_h) - F_1(b_1^*))\Big[\sum_{j=1}^{n}(-b_j^*)\prod_{k=2}^{j} F_k(b_k^*)\Big]$$

$$F_1(b_1^* + \lambda_h)\lambda_h = F_1(b_1^* + \lambda_h)\Big[\sum_{j=1}^{n} K_j(\prod_{k=2}^{j} F_k(b_k^*) - \prod_{i=2}^{n} F_i(b_i^*))\Big]$$
$$+ (F_1(b_1^* + \lambda_h) - F_1(b_1^*))\Big[v_{syn}(G)\prod_{i=2}^{n} F_i(b_i^*) - \sum_{j=1}^{n}(b_j^*)\prod_{k=2}^{j} F_k(b_k^*)\Big]$$

When the first seller is better off with options We now determine the minimum or lower bound λ_l (the level of λ that, according to Def. 3, keeps the seller of G_1 indifferent about options). In order to compare this bid with the λ_h from the previous section, it is again assumed that $\lambda_t = 0$ for $1 < t \leq n$.

Theorem 2. *If without options the synergy buyer bids $\boldsymbol{B}^*$ and with options $op'_1 + \lambda_1$ for G_1 and op'_t for $1 < t \leq n$, then $E(\pi_{\delta 1})$ for the seller of G_1 is:*

$$
\begin{aligned}
E(\pi_{\delta 1}) &= F_1(b_1^*)(\lambda_1 + (b_{1,res} - K_1)\Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big]) \\
&+ (F_1(b_1^* + \lambda_1) - F_1(b_1^*))(b_1^* + \lambda_1 - E(bm_1 | b_1^* + \lambda_1 \geq bm_1 > b_1^*) \\
&+ (b_{1,res} - K_1)\Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big])
\end{aligned}
$$

By definition, λ_1 is the lower bound for λ_l that guarantees that the expected profit of the seller $E(\pi_{\delta 1}) > 0$. The value of λ_l can be obtained as the solution to the equation $E(\pi_{\delta 1}) = 0$, which using the equation above gives:

$$
\begin{aligned}
F_1(b_1^* + \lambda_l)(-\lambda_l) &= F_1(b_1^* + \lambda_l)((b_{1,res} - K_1)\Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big]) \\
&+ (F_1(b_1^* + \lambda_l) - F_1(b_1^*))(b_1^* - E(bm_1 | b_1^* + \lambda_l \geq bm_1 > b_1^*))
\end{aligned}
$$

Proof. The difference in profit is equation (7) minus equation (4):

$$
\begin{aligned}
&E(\pi_1^{op}) - E(\pi_1^{dir}) = \\
&\Big(F_1^o(op_1)\big[op_1 + (K_1 - b_{1,res}) \prod_{h=2}^{n} F_h^o(op_h)\big] \\
&+ (1 - F_1^o(op_1))(E(opm_1 | opm_1 \geq op_1) + K_1 - b_{1,res})\Big) \\
&- \Big(F_1(b_1^*)(b_1^* - b_{1,res}) + (1 - F_1(b_1^*)(E(bm_1 | bm_1 \geq b_1^*) - b_{1,res})\Big)
\end{aligned}
$$

Recall the the the price op_1 bid in an options model can be expressed in terms of the benchmark strategy op'_1 and the deviation λ_1.

$$
\begin{aligned}
E(\pi_{\delta 1}) &= F_1^o(op'_1 + \lambda_1)(op'_1 + \lambda_1 + \Big[(K_1 - b_{1,res}) \prod_{h=2}^{n} F_h^o(op'_h)\Big]) \\
&+ (1 - F_1^o(op'_1 + \lambda_1))(E(opm_1 | opm_1 \geq op'_1 + \lambda_1) + K_1 - b_{1,res}) \\
&- F_1(b_1^*)(b_1^* - b_{1,res}) - (1 - F_1(b_1^*))(E(bm_1 | bm_1 \geq b_1^*) - b_{1,res})
\end{aligned}
$$

Furthermore, we can make the substitution to replace op'_1 with its definition, as follows: $op_1 = op'_1 + \lambda_1 = b_1^* - K_1 + \lambda_1$ and $F_1^o(op_1) = F_1^o(op'_1 + \lambda_1) = F_1(b_1^* + \lambda_1)$:

$$
\begin{aligned}
E(\pi_{\delta 1}) &= F_1(b_1^* + \lambda_1)(b_1^* - K_1 + \lambda_1 + \Big[(K_1 - b_{1,res}) \prod_{h=2}^{n} F_{oh}(op'_h)\Big]) \\
&+ (F_1(b_1^* + \lambda_1) - F_1(b_1^*))(-E(bm_1 | b_1^* + \lambda_1 \geq bm_1 > b_1^*) + b_{1,res}) \\
&- F_1(b_1^*)(b_1^* - b_{1,res})
\end{aligned}
$$

Split $F_1(b_1^* + \lambda_1)$ into $F_1(b_1^*)$ and $F_1(b_1^* + \lambda_1) - F_1(b_1^*)$ and combine some K_1 and $b_{1,res}$.

$$E(\pi_{\delta 1}) = F_1(b_1^*)(-K_1 + b_{1,res} + \lambda_1 + \Big[(K_1 - b_{1,res}) \prod_{h=2}^{n} F_h^o(op_h')\Big])$$
$$+ (F_1(b_1^* + \lambda_1) - F_1(b_1^*))(b_1^* - K_1 + \lambda_1 + \Big[(K_1 - b_{1,res}) \prod_{h=2}^{n} F_h^o(op_h')\Big]$$
$$- E(bm_1 | b_1^* + \lambda_1 \geq bm_1 > b_1^*) + b_{1,res})$$

$$E(\pi_{\delta 1}) = F_1(b_1^*)(\lambda_1 + (b_{1,res} - K_1)\Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big])$$
$$+ (F_1(b_1^* + \lambda_1) - F_1(b_1^*))(b_1^* + \lambda_1 - E(bm_1 | b_1^* + \lambda_1 \geq bm_1 > b_1^*)$$
$$+ (b_{1,res} - K_1)\Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big])$$

Since, by definition, $E(\pi_{\delta 1}) = 0$ gives the value of λ_l, this value can be solved via the equation in Th. 2.

$$F_1(b_1^* + \lambda_l)(-\lambda_l) = F_1(b_1^* + \lambda_l)((b_{1,res} - K_1)\Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big])$$
$$+ (F_1(b_1^* + \lambda_l) - F_1(b_1^*))(b_1^* - E(bm_1 | b_1^* + \lambda_l \geq bm_1 > b_1^*))$$

Intuitively, the difference in profit has two parts: the cases where the synergy buyer wins the auction in both markets and the ones where she only wins with options. With the first, the synergy buyer pays more than she used to and with the second, the synergy buyer pays more than the local bidders, who used to win if $\lambda_1 < \lambda_l$. But both cases have the downside for the seller that the synergy buyer may now not exercise her option.

Both agents can be better off with options The previous parts of Section 3.1 give the equations for the cases when the individual agents are better off with options. These results will now be combined to determine when they are both better off. This is done by simply stating that the minimum bid the seller of G_1 requires should be below the maximal value the synergy buyer is willing to pay.

Theorem 3. *If synergy buyer bids λ_x extra for an option on G_1, where $\lambda_l < \lambda_x < \lambda_h$, then both the seller of G_1 and the synergy buyer have a higher expected profit in a market with only options compared to one without options.*

Proof. The proof of this follows from the previous theorems. Say that the synergy buyer bids $op_1' + \lambda_x$, where $\lambda_l < \lambda_x < \lambda_h$ and op_t' for the other goods. Then the synergy buyer bids more than $op' + \lambda_l$, so according to Theorem 2 the seller of G_1 has a higher expected profit with options. Also, the synergy buyer bids between $0 < \lambda_x < \lambda_h$ extra, so according to Theorem 1 she too has a higher expected profit with options with these bids.

3.2 Synergy buyer's profit-maximizing bid

In the previous Section, we focused our attention on deriving equations for the bounds λ_l and λ_h between which the additional bids of the synergy buyer have to fall in order for both parties to be incentivised to use options. While these bounds were defined in relation to the expected-profit maximizing bid b^* in a model *without* options, we have not said much about the optimal (i.e.e expected profit maximizing) bid op^* in a model with options. The reason for this is that deriving this is much more involved than the optimal policy in a model without options. In this Section, we look at the synergy buyer's profit-maximizing bids op^*, but with the added assumption that $F_1(b_1)$ follows a uniform distribution in the range of the possible bids. Actually, we do this by using the same framework introduced in Def. 3 and Fig. 1 above. That means, we compute the deviation λ^* between the optimal bid in a model with options and the optimal bid in a model without options, i.e. the difference $\lambda^* = (K_1 + op_1^*) - b_1^*$ (the reason to do this will become apparent in the proof, but, basically, by taking the difference, several terms drop out).

If the profit-maximizing bid $op_1^* > op_1' + \lambda_l$, then according to Theorem 2 the seller of G_1 is better off with options. Therefore, it is in the rational interest of the seller to set the exercise price for selling her good such that the expected optimal bid of her buyers, in a model with options, will provide sufficient incentive for the seller to also use options, and thus the following condition holds: $op_1^* > op_1' + \lambda_l$. Note that in order to use Theorem 2, the bids for the other goods are fixed at op_t'. First op_1^* and λ_l are derived.

Lemma 2. *If $F_1(b_1)$ follows a uniform distribution, then $op_1^* + K_1 - b_1^* = \lambda^*$, where:*

$$\lambda^* = 0.5(K_1(1 - \prod_{i=2}^{n} F_i(b_i^*)) + \sum_{j=2}^{n} K_j(\prod_{k=2}^{j} F_k(b_k^*) - \prod_{i=2}^{n} F_i(b_i^*)))$$

Proof. With a uniform distribution, the price distribution has the following shape:

$$f_1(b_1) = 1/(ub - ua) = \alpha, \tag{8}$$
$$F_1(b_1) = (b_1 - ua)/(ub - ua) = \alpha(b_1 - ua) \tag{9}$$

For F_{o1} the variables α_o, ua_o and ub_o are used, where $ua_o = ua - K_1$ and $ub_o = ub - K_1$, so that $F_1(b_1) = F_{o1}(op_1)$ when $b_1 - K_1 = op_1$.

First, we determine, for this type of distribution, the equation for the optimal bid b_1^* in a model without options. To do this, we start from the expected profit equation (2):

$$E(\pi_{syn}^{dir}) = F_1(b_1)\big[v_{syn}(G)\prod_{i=2}^{n} F_i(b_i)\big] + F_1(b_1)(-b_1) + F_1(b_1)\big[\sum_{j=2}^{n}(-b_j)\prod_{k=2}^{j} F_k(b_k)\big]$$

$$E(\pi_{syn}^{dir}) = F_1(b_1)\Big[- b_1 + \big[v_{syn}(G)\prod_{i=2}^{n} F_i(b_i)\big] + \big[\sum_{j=2}^{n}(-b_j)\prod_{k=2}^{j} F_k(b_k)\big]\Big]$$

So the partial derivative is:

$$\frac{\partial E(\pi_{syn}^{dir})}{\partial b_1} = f_1(b_1)\Big[- b_1 + \big[v_{syn}(G)\prod_{i=2}^{n} F_i(b_i)\big] + \big[\sum_{j=2}^{n}(-b_j)\prod_{k=2}^{j} F_k(b_k)\big]\Big] + F_1(b_1)(-1) = 0$$

Filling in the equations for f_1 and F_1 leads to:

$$\big[v_{syn}(G)\prod_{i=2}^{n} F_i(b_i)\big] + \big[\sum_{j=2}^{n}(-b_j)\prod_{k=2}^{j} F_k(b_k)\big] + ua = 2b_1^*$$

Next, we calculate, through a similar procedure, the optimal bid op_1^* in a model *with* options:

$$E(\pi_{syn}^{op}) = \Big[(v_{syn}(G) - \Big[\sum_{h=1}^{n} K_h\Big])\prod_{i=1}^{n} F_i^o(op_i)\Big] + \Big[\sum_{j=1}^{n}(-op_j)\prod_{k=1}^{j} F_k^o(op_k)\Big]$$

First, we isolate op_1 in the above equation:

$$E(\pi_{syn}^{op}) = F_1^o(op_1)\Big[(v_{syn}(G) - \big[\sum_{h=1}^{n} K_h\big])\prod_{i=2}^{n} F_i^o(op_i)\Big] + F_1^o(op_1)(-op_1) + \Big[\sum_{j=2}^{n}(-op_j)\prod_{k=2}^{j} F_k^o(op_k)\Big]$$

$$E(\pi_{syn}^{op}) = F_{o1}(op_1)\Big[- op_1 + \big[(v_{syn}(G) - \big[\sum_{h=1}^{n} K_h\big])\prod_{i=2}^{n} F_{oi}(op_i)\big] + \big[\sum_{j=2}^{n}(-op_j)\prod_{k=2}^{j} F_{ok}(op_k)\big]\Big]$$

We take the partial derivative wrt. op_1:

$$\frac{\partial E(\pi^{op}_{syn})}{\partial op_1} = f^o_1(op_1)\Big[- op_1 + \big[(v_{syn}(G) - \big[\sum_{h=1}^{n} K_h\big]) \prod_{i=2}^{n} F^o_i(op_i)\big]$$
$$+ \big[\sum_{j=2}^{n}(-op_j) \prod_{k=2}^{j} F^o_k(op_k)\big]\Big] + F^o_1(op_1)(-1) = 0$$

In order to determine the optimal value op^*_1, we add the condition $\frac{\partial E(\pi^{op}_{syn})}{\partial op_1} = 0$:

$$\alpha_o\Big[- op^*_1 + \big[(v_{syn}(G) - \big[\sum_{h=1}^{n} K_h\big]) \prod_{i=2}^{n} F^o_i(op_i)\big]$$
$$+ \big[\sum_{j=2}^{n}(-op_j) \prod_{k=2}^{j} F^o_k(op_k)\big]\Big] + \alpha_o(op^*_1 - ua_o)(-1) = 0$$

Which finally yields the following equation for determining op^*_1:

$$\big[(v_{syn}(G) - \sum_{h=1}^{n} K_h) \prod_{i=2}^{n} F^o_i(op_i)\big] + \big[\sum_{j=2}^{n}(-op_j) \prod_{k=2}^{j} F^o_k(op_k)\big] + ua_o = 2op^*_1$$

We now focus our attention at computing the difference λ^* between the optima decision-theoretic bid in a model with options vs. a model without options. By definition, we have that: $\lambda^* = (K_1 + op^*_1) - b^*_1$, so $2\lambda^* = 2op^*_1 + 2K_1 - 2b^*_1$. When taking this difference, $ua_o = ua - K_1$ and op'_t are replaced according to $op'_t = b^*_t - K_t$ and $F_{ot}(op'_t) = F_1(b^*_1)$. Then all variables cancel each other out, except for the K_t:

$$2(b^*_1 + \lambda^* - K_1) = \Big[\big[(v_{syn}(G) - \big[\sum_{h=1}^{n} K_h\big]) \prod_{i=2}^{n} F_i(b^*_i)\big]$$
$$+ \big[\sum_{j=2}^{n}(-b^*_j + K_j) \prod_{k=2}^{j} F_k(b^*_k)\big]\Big] + ua - K_1$$

$$2\lambda^* = \Big[\big[(v_{syn}(G) - \big[\sum_{h=1}^{n} K_h\big]) \prod_{i=2}^{n} F_i(b^*_i)\big]$$
$$+ \big[\sum_{j=2}^{n}(-b^*_j + K_j) \prod_{k=2}^{j} F_k(b^*_k)\big]\Big] + ua + K_1 - 2b^*_1$$

$$\lambda^* = 0.5(\Big[[(v_{syn}(G) - [\sum_{h=1}^{n} K_h]) \prod_{i=2}^{n} F_i(b_i^*)]$$

$$+ [\sum_{j=2}^{n} (-b_j^* + K_j) \prod_{k=2}^{j} F_k(b_k^*)]\Big] + ua + K_1$$

$$- (\Big[[v_{syn}(G) \prod_{i=2}^{n} F_i(b_i)] + [\sum_{j=2}^{n} (-b_j) \prod_{k=2}^{j} F_k(b_k)]\Big] + ua))$$

$$\lambda^* = 0.5((-\sum_{h=1}^{n} K_h) \prod_{i=2}^{n} F_i(b_i^*) + \sum_{j=2}^{n} K_j \prod_{k=2}^{j} F_k(b_k^*) + K_1)$$

Which leads to the equation in Lemma 2:

$$\lambda^* = 0.5(K_1 - K_1 \prod_{i=2}^{n} F_i(b_i^*) - \sum_{h=2}^{n} K_h \prod_{i=2}^{n} F_i(b_i^*) + \sum_{j=2}^{n} K_j \prod_{k=2}^{j} F_k(b_k^*))$$

$$\lambda^* = 0.5(K_1(1 - \prod_{i=2}^{n} F_i(b_i^*)) + \sum_{j=2}^{n} K_j(\prod_{k=2}^{j} F_k(b_k^*) - \prod_{i=2}^{n} F_i(b_i^*)))$$

The main intuition behind this formula is that, in an options model, the synergy buyer saves the exercise price when she fails to complete her bundle. Therefore, it is her profit-optimizing strategy, in a model with options, to increase her bid with a part of the potential savings on the exercise prices of subsequent auctions.

Lemma 3. *If $F_1(b_1)$ follows a uniform distribution, then the lower bound is:*

$$\lambda_l = \frac{-(b_1^* - ua + \Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big](b_{1,res} - K_1)) + \sqrt{(b_1^* - ua + \Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big](b_{1,res} - K_1))^2}}{-2(b_1^* - ua)\Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big](b_{1,res} - K_1)}$$

Proof. Take the λ_l equation from Theorem 2. With a uniform distribution, $F_1(b_1) = \alpha(b_1^* - ua)$ and $E(bm_1|b_1^* + \lambda_l \geq bm_1 > b_1^*) = b_1^* + 0.5\lambda_l$. So the equation becomes:

$$\alpha(b_1^* + \lambda_l - ua)(-\lambda_l) = \alpha(b_1^* + \lambda_l - ua)((b_{1,res} - K_1)\Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big])$$

$$+ \alpha\lambda_l(b_1^* - b_1^* - 0.5\lambda_l)$$

Dividing both sides by α and reducing b_1^* in the last parenthesis gives:

$$(b_1^* + \lambda_l - ua)(-\lambda_l) = (b_1^* + \lambda_l - ua)((b_{1,res} - K_1)\Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big]) + \lambda_l(-0.5\lambda_l)$$

After re-arranging the terms and moving the left -hand side to the right, this yields:

$$(b_1^* + \lambda_l - ua)(\lambda_l + (b_{1,res} - K_1)\Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big]) - 0.5\lambda_l^2 = 0$$

The above equation can be brought to standard, 2nd order polynomial form in the unknown λ_l:

$$\begin{aligned} 0 = 0.5\lambda_l^2 + \lambda_l(b_1^* - ua + (b_{1,res} - K_1)\Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big]) \\ + (b_1^* - ua)((b_{1,res} - K_1)\Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big]) \end{aligned}$$

This polynomial equation can then be solved via the quadratic formula:

$$\begin{aligned} \lambda_l = -(b_1^* - ua + \Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big](b_{1,res} - K_1)) \\ \pm \sqrt{(b_1^* - ua + \Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big](b_{1,res} - K_1))^2 - 2(b_1^* - ua)\Big[1 - \prod_{h=2}^{n} F_h(b_h^*)\Big](b_{1,res} - K_1)} \end{aligned}$$

The seller should set K_1 at a value for which $\lambda_l < \lambda^*$ is true. Actually, we found that deriving a closed-form solution for this condition from the above equations is not possible analytically. However, the framework developed above is sufficient to enable the seller to solve the condition numerically using a standard solver and, thus, choose the optimal level for the exercise price K.

4 Simulation of a market with a single synergy buyer

This section presents an experimental examination of a market with one synergy buyer. It introduces the market entry effects in the synergy buyer's behaviour, as well as the threshold effects that may determine which exercise prices the seller chooses for her options. This experimental analysis is performed here for a market with one synergy bidder and several local bidders, while Sect. 6 considers a market with multiple synergy bidders.

The experimental setting is as follows: we consider a simulation where two goods A and B are auctioned n_A and n_B times respectively. The synergy buyer desires one copy of both goods and has zero valuation for the individual goods.

That is, each synergy (or global) bidder requires exactly one bundle of $\{A, B\}$[4] In the setting considered in this Section, local bidders only want one good and participate in one auction, thus their bids can be modeled as a distribution.

Furthermore, in order to simplify the simulation we assume there is a single seller who auctions all the goods. This is actually equivalent to studying whether *on average* sellers have an incentive to use options. To explain, on any single sequence of auctions taken in isolation, the sellers of different items may have highly diverging incentives to use options, based on their position in the auction queue. However, in a very large setting, where buyers enter the market randomly, it is difficult for any individual seller to strategise about her particular place in the sequence (and, furthermore, in most markets she may simply have no information to do this). Our goal is to study under which conditions, on average, sellers benefit from using options if there are synergy buyers in the market. Also, to somewhat reduce the number of test parameters, we further assume that the exercise price is the same for all goods of the same type. So the seller needs to determine which exercise price for A and which for B maximize her expected profit.

Note that, typically a seller has a resale value of for the goods that remain unsold, which is typically lower that the value at the start of the auction sequence. The reason for this may be that there is some time discounting associated with waiting for a sequence of auctions to resell her items, or even a listing cost, which is paid per auction (such as in the Ebay case). In this paper, we do not explicitly simulate resale, but we use a reservation value, which represents the expected resale value the seller expects to get, if she is forced to resell her items.

To summarize, simulations were run in Matlab and had the following parameters:

Name	Explanation
n	The number of auctions.
$mean$	The mean of price distribution.
std	The standard deviation of price distribution.
res	Reserve prices.
v_{syn}	Valuation synergy buyer for A and B combined.
k	Number of simulations for each auction run (i.e. how many times a sequence of auctions is repeated for one set of parameters).

A basic simulation run is as follows. First, all possible auction sequences are determined for the given number of auctions for A and B. The simulation is then run for all these sequences, both for a direct sale setting and for a setting where the items are sold through options with given exercise prices.

For each auction, in each simulation run, there is a set of local bidders, assumed myopic. The bids of these local bidders are therefore, assumed to follow a normal price distribution, with the parameters $n, mean, std$ and res consisting

[4] An intuitive way to think about this setting is as a sequential sale of individual shoes of exactly the same type, where A is the left shoe, and B is the right shoe, and each synergy buyer requires exactly one pair.

out of two values: one for good A and one for good B. For each simulation run, the synergy bidders(s) are asked to determine their profit-maximizing bid for that setting, as described in the next section. The optimization required for determining their optimal bid is done using the Matlab function "fminsearch" from the Optimization Toolbox.

Since there may be considerable variance in the bids of the local bidders (which are myopic) each possible auction sequence is run k times (typically, we had $k > 10000$). The average profit of the seller and the synergy buyer which are reported here, for both the case of with and without options, are averages over all these k simulations and also over all possible auction orders of items A and B in the sequence.

4.1 Synergy buyer's bid strategy

This section describes how the synergy buyer determines her bids in the simulation. In order to neutralize the effect that the exact order items are auctioned in plays on the bidding strategy, we add the assumption that the synergy buyer knows the number of remaining auctions, but not the order they will be held in. This remaining number of auctions of each type is common knowledge (i.e. the synergy bidders can always observe how many auctions of each type are left before they have to leave the market, and so does the seller).

The model described here is for a situation without options. But in order to apply it to a situation with options, one merely has to replace the variables: $b_t = op_t - K_t$ and $v_{syn}(A, B) := v_{syn}(A, B) - K_A - K_B$. As in the analytical section, we assume a bidder only wants a complete bundle of $\{A, B\}$. Therefore, $v_{syn}(A) = 0 = v_{syn}(B) = 0$.

Determining the synergy buyer's profit-maximizing bid b_t^* at state t basically involves solving the Markov Decision Process (MDP), where we select the optimal bid b_t^* at time t, subject to the optimal bid b_{t+1}^* being selected for the future time point $t + 1$ (which in this case, is an auction). We can, however, use the valuation function of the bidding agent to significantly reduce the state space of the MDP, as shown below. However, first we introduce some notation.

Let b^* be the immediate best response to the state, which depends on four variables: z_A, z_B, X and I_t. The variables z_A and z_B are the number of remaining auctions for A and B respectively (including the current auction), so $z_A \leq n_A,\ z_B \leq n_B$. The type of good, which is currently sold, is denoted by I_t. The set of goods the synergy buyer owns (i.e. the endowment) is described by X, which can either be $\emptyset, \{A\}$ or $\{B\}$. If X is $\{A, B\}$ then the synergy buyer is done. Let $Q(z_A, z_B, X, I_t, b_t)$ be the expected profit of the synergy buyer when bidding b_t. Note that, in these definitions, b_{t+1}^* and $V_{t+1}()$ denote the best available bid, respectively best expected value for the next state (as computed by recursion), while I_{t+1} is the type of the next item in the auction sequence. Therefore, using MDP notation, the profit-maximizing bid b_t^* is determined as follows:

$$b_t^* = argmax_{b_t}\ Q(z_A, z_B, X, I_t, b_t) \tag{10}$$

Where the expected profit is determined via:

$$
\begin{aligned}
&Q(z_A, z_B, X, I_t = A, b^*_{t+1}) = F_A(b_t)(-b_t\\
&+ V_{t+1}(z_A - 1, z_B, X \cup A, b^*_{t+1})) + (1 - F_A(b_t))V_{t+1}(z_A - 1, z_B, X, b^*_{t+1}) \quad (11)\\
&Q(z_A, z_B, X, I_t = B, b_t) = F_B(b_t)(-b_t\\
&+ V_{t+1}(z_A, z_B - 1, X \cup B, b^*_{t+1})) + (1 - F_B(b_t))V_{t+1}(z_A, z_B - 1, X, b^*_{t+1}) \quad (12)
\end{aligned}
$$

Where $V()$ is the value of a state, which simply means the maximum expected profit of that state:

$$V_t(z_A, z_B, X, b_t) = max_{b_t}\ Q(z_A, z_B, X, I_t, b_t) \tag{13}$$

Looking at the formula for $Q()$, it basically says that for the probability of winning the auction with her bid, the synergy buyer has to pay a price equal to her bid and the good is included in the endowment X of the next state. If she does not win the auction, then the value of the current state is equal to the value of the next state.

As we mentioned before, in computing its optimal bidding strategy used in the experimental Section, we assume the synergy buyer does not know whether the next auction will be for A or B, she only knows the total numbers of auctions for A and B remaining. We acknowledge this is a departure from the formulas in the theoretical analysis, where the exact order of the auctions was taken into account to compute the bidding strategies. There are two reasons to use this assumption here. The first is that it reduces considerable the state space that needs to be modeled when computed the optimization. But the second is that we also find this choice more realistic if this model is to be applied to real-life settings. For example, when bidding on a part-truck order in a logistic scenario, it is more realistic to assume that a carrier can approximate the number of future opportunities to buy a complementary load, but not the exact auction order in which future loads will be offered for auction.

If we assume the synergy buyer only knows the total numbers of auctions for A and B remaining (and not their exact order), then her bidding strategy is based on assuming each future auction has an equal probability to occur. Therefore, the probability of an auction for A occurring next is simply the number of remaining auctions A divided by the total number of remaining auctions. Thus, a weighted average can be used to determine the value of the next auction, while not knowing for which good it will be for.

Apart from this general framework, we can prune the state space with the cases in which we know the synergy buyer's bid is zero:

$$b^*_t = argmax_{b_t}\ Q(0, z_B, X, B, b_t) = 0, \text{ with } A \notin X \tag{14}$$

$$b^*_t = argmax_{b_t}\ Q(z_A, 0, X, A, b_t) = 0, \text{ with } B \notin X \tag{15}$$

$$b^t* = argmax_{b_t}\ Q(z_A, z_B, X, I_t \in X, b_t) = 0 \tag{16}$$

With the first two cases, the synergy buyer can no longer obtain her desired bundle, because she does not own the complementary item and there is no chance left of acquiring it. The last equation is for the case when the synergy buyer already has a copy of the type of good (and, from her valuation function, she only wants exactly one copy of A and B). The corresponding values of these states are:

$$V(0, z_B, X, b_t^*) = 0, \text{ if } A \notin X(z_A = 0, asI_{t+1} = B) \tag{17}$$
$$V(z_A, 0, X, b_t^*) = 0, \text{ if } B \notin X(z_B = 0, asI_{t+1} = A) \tag{18}$$
$$V(z_A, z_B, \{A\}, b_t^*) = V(0, z_B, \{A\}, b_t^*) \tag{19}$$
$$V(z_A, z_B, \{B\}, b_t^*) = V(z_A, 0, \{B\}, b_t^*) \tag{20}$$

The first two equations correspond to the case when the buyer can no longer get the complementary-valued item, therefore the sequence of auctions of the same type has no value to her. In both these cases $b_t^* = 0$. The last two equations are important, since they help the most to reduce the state space. Basically, as already mentioned, we assume that a synergy bidder only wants exactly one bundle of $\{A, B\}$. If she already owns a good of one of the two types, she will no longer be interested in the remaining auctions for that type of good. Therefore, the valuation $V()$ of these states is equivalent to a state when no auctions are remaining for the type of good she already owns (as she would not take part in those anyway). All these techniques help reduce the recursive search.

To conclude, to determine the synergy buyer's bids in any situation, the values of b_t^* and $V()$ need to be calculated for the following states:

$$\begin{aligned}
\forall z_B > 0 &\quad Q(0, z_B, \{A\}, B, b_t) \\
\forall z_A > 0 &\quad Q(z_A, 0, \{B\}, A, b_t) \\
\forall z_A > 0, z_B > 0 &\quad Q(z_A, z_B, \emptyset, A, b_t) \\
\forall z_A > 0, z_B > 0 &\quad Q(z_A, z_B, \emptyset, B, b_t)
\end{aligned}$$

4.2 Experimental results: market entry effect for one synergy buyer

First, we study experimentally the incentives to use options for the sellers and buyers, in the case there is just one synergy bidder present in the market. In order to study different dimensions of such markets, we considered several combinations of parameter settings.

The first setting has $n_A = 2$ and $n_B = 2$. As mentioned above, the local bidders are considered myopic and only bid in one local auction. Therefore, their bids can be modeled as a distribution $\sim N(10, 4)$ for both goods. The goods A and B are, in this model, of equal rarity and attract an equal amount of independent competition during bidding. This choice is not random, as having a certain degree of symmetry in the experimental model allows us to reduce the number of parameter settings we need to consider. More specifically, we assume the same exercise prices are set for both goods of type A and B. This is a reasonable assumption, because A and B are of symmetric value and because bidders do not know in advance the exact order goods will be sold in.

Furthermore, for each good, the seller has a reservation value $res = 8$, which gives its estimate resell value in the case the synergy buyer acquires an option for the item, but fails to exercise it. Since, on average, myopic bidders bid have an expected mean of 10 for an item, 20% is a reasonably safe estimate of a resell value.

The value of a bundle of {A,B} for the synergy buyer is an important choice, especially in relation to the mean expectation μ of the bids placed by single-item bidders. We considered two settings: $v(A, B) = 24$ (thus 20% more, on average, than local competition) - with results shown in Fig. 2, and $v(A, B) = 21$ (which is only 5% more on average than local competition) - with results shown in Fig. 3.

Fig. 2. Percentage increase in profit for a model using options wrt. direct sale, for the case there is one synergy buyer is present in the market. In the setting, there are two items of type A sold and two items of type B. For all 4 items, the bids of the local bidders follow the distribution $N(10, 4)$, while the valuation of the synergy buyer is $v(A, B) = 24$ (thus 20% more, on average, than the local bidders). What is varied on the horizontal axis is the exercise price with which the items are sold (assuming they are set the same for all items, being of equal rarity). Note that the figure is superimposed: the left-hand side axis refers exclusively to the seller, while the right-hand side axis refers exclusively to the synergy bidder. From this picture, one can already see the important effect: synergy buyer prefers, on average, higher exercise prices, while seller prefers lower ones.

Fig. 3. Percentage increase in profit for a model using options wrt. direct sale, for the case there is one synergy buyer is present in the market. The settings are exactly the same as those is in Fig. 2 above: 2 auctions for A and 2 for B, with local, myopic bidders following $N(10, 4)$. However, now the valuation of the synergy buyer is $v(A, B) = 21$ (thus only 5% more, on average, than the local bidders). One can see, however, that there is an important difference by comparison to Fig. 2: the threshold effect in the profit increase for the seller when the exercise price $K \geq 2.5$. Intuitively, the reason this effect occurs is the market-entry effect on the part of the synergy buyer, who would otherwise stay out for this lower valuation

Looking at these two figures, some important effect can be observed. First, we mention that the seller has an immediately higher expected profit with options compared to direct sale. This is because an option is sometimes not exercised and then the seller gets to keep the good (for which she has a positive valuation), while the synergy buyer still pays the option price.

There are two main effects to be observed from Fig. 2 and 3:

- First, the synergy buyer in such a market always prefers *higher* exercise prices (an effect clearly seen in both Figs. 2 and 3). This may be counter-intuitive at first, but is a rational expectation. If the option for an item is sold with a higher exercise price, then the synergy buyer can bid more aggressively on the option price to get the item, since she is "covered" for the loss represented by the exercise price. The myopic bidders extract no advantage from being offered the good as an options vs. a direct sale, because, if they acquire the option, they would always exercise it regardless. Therefore, they will simply lower their bid for the option with the amount represented by the exercise price.

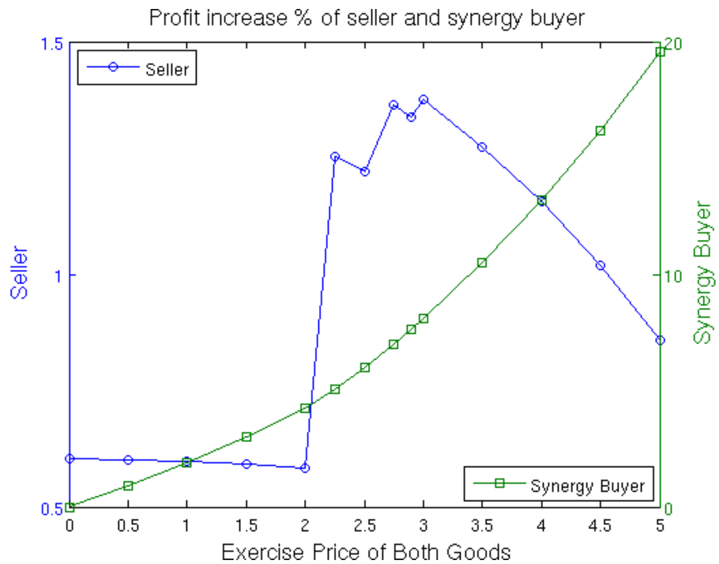

Fig. 4. Percentage increase in profit for the case of one synergy buyer, for longer auction sequences. The settings in terms of valuations are exactly the same as those is in Fig. 3 above: the synergy buyer has a value $v(A, B) = 21$, while single-item bidders bid according to $N(10, 4)$. One change is that now there are 4 auctions available for each type, i.e. 4 auctions for an item of type A and 4 for B. Notice that now there are multiple thresholds, since there are multiple points when the market entry effect of the synergy buyers appears. However, on average, the percentage increases in expected profits for the synergy buyers are lower, when compared to the direct auctions case. The reason for this is that, with multiple future buying opportunities, the exposure problems that synergy bidder faces decreases.

– Second, the expected profit of the seller seems to decrease between intervals if she has to sell the option with a higher exercise price. The main reason for this is that there is some chance that she or she would remain with her item unsold (because the option is not exercised), and thus only extract her reservation value for that item. There is, however, an important difference between the cases shown in Fig. 2 and 3, which is the participation thresholds (that appear as "peaks" in the picture), where the expected profit of the seller seems to "jump" at a new level. These can be explained by the synergy buyer joining the market, as the expected profit becomes non-negative. The threshold nature is determined by the discrete nature of the auction sequence, as is explained below.

Such a participation threshold is illustrated in Fig. 3 is the increase in the seller's expected profit when the exercise price is set above a certain level ($K \geq 2.5$, for the settings in Fig. 3). Such thresholds can be explained as follows. If the synergy buyer currently owns nothing, then she will only bid on a good

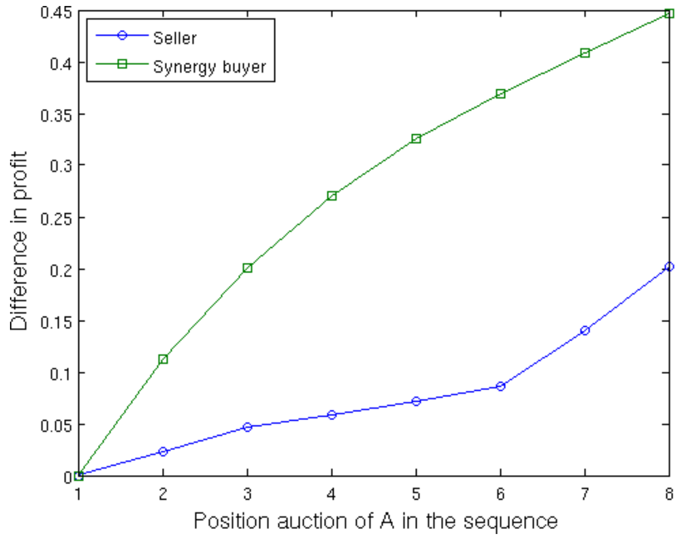

Fig. 5. Influence of the position in an auction queue of an item on the seller's expected profit. Settings are the same as in Fig. 2, but with one important difference: the rarity of the goods is no longer symmetric. There is now only 1 auction for a good of type A, but 7 auctions for a good of type B. What is varied along the horizontal axis is the position in the auction queue of the sale of the rarer item (of type A). The graph shows the absolute difference in profit for a seller of an item *of type B* and for the synergy buyer (i.e. the difference in profit between an options and direct auctions model). Note that, if the rare item of type A is sold at the end of the auction sequence, the benefit of selling item B through an option increases, because the exposure risk of not acquiring item of type A increases.

if the number of remaining auctions and their exercise prices give her a prior expectation of a positive profit. Conversely, if the synergy buyer is not offered a sequence of option sales from which she derives a positive expected profit, she has the incentive to leave the market altogether. There are two main factors that increase a synergy buyer's expected profit in a sequence of auctions (sold as options):

- The number of remaining future auctions of the other good, necessary to complete her bundle.
- The exercise price of the options (that only needs to be paid at the end). This should be high enough to cover the risk, given her valuation for the bundle.

Note that in some market setting (such as the one in Fig. 2), no participation effects (ie.e. thresholds) occur, because the value the synergy buyer assigns to her desired bundle is already high enough, so she would participate in the market anyway (i.e. regardless of whether she gets offered options or not), and at any point in the sequence that there is still a chance of completing her bundle.

However, in the valuation settings in Fig. 3, the synergy buyer will only bid on a good if there are two remaining auctions for the other good. So she places a bid for A if the auctions are $[A, B, B]$, but not if they are $[A, B]$. This is because with a single auction for B, the risk of ending up with a only a worthless A is too great. But in a market with exercise prices of at least 2.5, the risk is reduced and one remaining auction is already enough for the synergy buyer to stay in the market. So a higher exercise price enables the synergy buyer to stay the market, even if she owns nothing and there are only a few auctions left, which increases the seller's expected profit. This increase in participation is beneficial to the seller, who thus has an incentive to fix the exercise prices $K_A = K_B = 2.5$.

5 Settings with longer sequences of auctions and effect of auction order

In the previous Section, we examined a sequence of auctions of a spefici length of $n_A = 1, n_B = 2$. We now look at whether we can observe similar effects in the case when the number of opportunities to buy goods A and B increases. With the exception of auction lengths, the parameters are kept the same as in the previous case. First, we keep the relative rarity of both goods symmetrical, but increase the number of auctions available for each to 4, i.e. $n_A = n_B = 4$. Results are shown in Fig. 4.

Basically, there are two main effects to observe here. First, the benefits to the buyer of having options mechanism decreases (seen from comparing the percentage increases shown in the right-hand vertical axis of Figs. 3 and 4). The reason for this (as discussed in the earlier, risk-based bidding paper) is that, in sequential auctions, the number of available future opportunities plays a big role in how big the exposure problem the synergy buyer faces is. If there is less exposure, then the relative benefits of using options becomes smaller (although it is still quite considerable). The second effect to be observed from Fig. 4 is that there are more participation thresholds (denoted by peaks), but they are smaller. The reason is that, for a longer sequence of auctions, there are more possible sequences of remaining auction combinations. The synergy bidder will join in the bidding in some, but not in others, leading to multiple participation thresholds.

The second problem we look in this subsection at is what happens if the relative frequency of the two goods is more asymmetric. We keep the same total number of auctions in the sequence (8), but the relative frequency is highly asymmetric: $n_A = 1, n_B = 7$. As mentioned, in the previous graphs, results were averaged over all possible auction orders - while here, by contrast, we look at auction orders one by one.

For this setting, there are exactly 8 possible auction orders, corresponding to the point where the rarer good (type A) can be inserted in the auction queue. What is varied on the horizontal axis is this position of the type A good. The reason why we look at whether a seller of items of type B would use options is that the exposure of the synergy buyer exists for the other good in the sequence. For the single item of type A, the benefits of using options are limited, because the synergy buyer has 7 other auctions in which to acquire the second item anyway, hence she has much less of an exposure problem.

Clearly, we can see an important effect of the position of the rarer good in the auction queue, from the perspective of both parties. If the item of type A is sold at the very beginning of the auction sequence, then the synergy bidder has no exposure problem left for the rest of the sequence, hence there is no incentive to use options, for either party. However, it is at the very end of the auction sequence, the synergy buyer will not know whether she would need the item acquired until all auctions end. For this case, the benefits of using options are considerably greater.

6 Multiple synergy buyers

Finally, we consider market settings in which multiple synergy buyers are active simultaneously. Much of the experimental set-up and parameter choices are the same as described in the above Sections, for the case of one for the single synergy buyer. The only difference is that now multiple synergy buyers may enter and leave the market at different times and they have different valuations for the combination of A and B.

We have to emphasize that the results from this Section are still rather preliminary and are based on some restrictions on the reasoning capability of the synergy buyers in the market. Specifically, as in the single-bidder case, we assume the synergy bidders have some prior expectations about the closing prices in future auctions and compute their optimal strategy wrt. this expectation. In these results, this expectation is assumed the same for all synergy bidders, which is a reasonable choice in comparing their strategies. In a more realistic market, however, synergy bidders could be expected to be able to learn and adjust their expectations based on past interactions, as well as reason game-theoretically about the fact that another synergy bidder may present in the market at the same time. At this point, these more sophisticated forms of reasoning are left to future work.

As in the previous section all simulations of this section have reserve prices of 8 and local bidders following $\sim N(10, 2.5)$. The first two experiments also have two synergy buyers syn_1 and syn_2 with valuations for both goods of 21.5 and 22.5 respectively. The order the synergy bidders enter the market (and the number of auctions they can stay in) are given in Figs. 8 and 8, while results for all settings are shown in Fig. 6, respectively 7. In the following, we will discuss these in separate subsections.

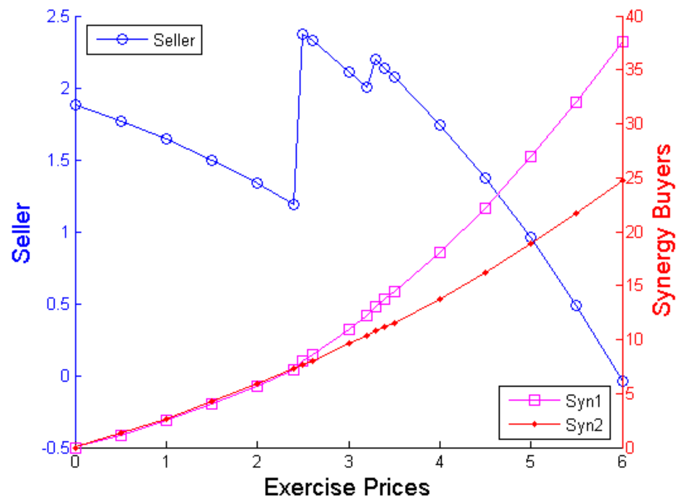

Fig. 6. Percentage increase in profits for a market with with 2 synergy bidders. There are 3 auctions for A and 3 for B, and for each one the bids from the competition formed by local bidders follows the distribution $N(10, 2.5)$. The valuations of the two synergy bidders for a bundle {A,B} are 21.1 for $syn1$, respectively 22.5 for $syn2$. The order the agents enter the market is described by Fig. 8 below (so the two agents do not compete directly against each other in this setting). Notice that, in this case, the average profit of $syn2$ does not decrease with the entry of $syn1$ in the market.

6.1 Two synergy buyers interacting indirectly through the exercise price level

In the setting examined here, the two synergy buyers each have $n_A = 3$ and $n_B = 3$, without the other agent participating in these auctions. An example of such an auction sequence is shown in Fig. 8. However, these two synergy bidders do interact indirectly as follows. Since options are sold through open auctions based on the option price, the seller has to fix the exercise prices for the whole market. So while synergy buyers may not participate in the same auctions, their presence does influence the competition through the exercise prices set by the seller.

This effect can be seen in Fig. 6, in which the seller maximizes her expected profit at $K = K_A = K_B = 2.4$. In this case syn_2 is better off, because without the presence of syn_1 she would be offered options with lower exercise prices. But syn_1 is worse off, because if she were alone in the market the seller would choose $K = 3.2$, which gives her a higher expected profit. Yet, due to syn_2, the seller sets $K = 2.4$. In this case, due to the seller's choice of exercise prices, one synergy buyer (syn_1) gains, while syn_2 loses.

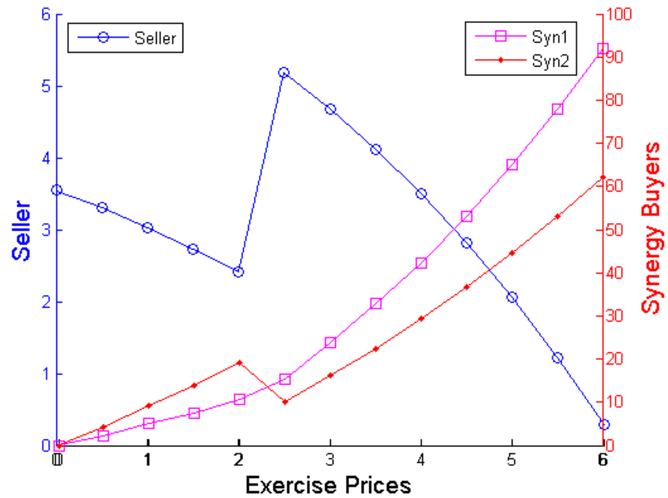

Fig. 7. Percentage increase in profits for a market with with 2 synergy bidders. The setting and valuations are the same as in Fig. 6 above. However, the order the agents enter the market is now described by Fig. 9 below (so the two agents *do* compete directly for the same goods). Notice that, in this case, the average profit of $syn2$ decreases due to the additional competition from $syn1$.

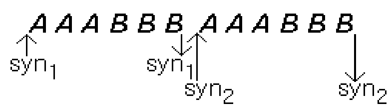

Fig. 8. An auction sequence for the case shown in Fig 6.

6.2 Direct synergy buyer competition in the same market

Next, we considered a setting in which synergy buyers compete directly for some of the goods. The entry points for such a setting are shown in Fig. 9, while simulation results are given in Fig. 7.

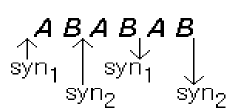

Fig. 9. An auction sequence for the case shown in Fig. 7.

As can be seen in Figure 7, the profit of syn_2 drops at 2.5. In previous figures the synergy buyers' profits were monotonically increasing in the exercise prices, because they then have a smaller loss when they fail to complete their bundle. But now this effect cannot immediately compensate the extra competition coming from syn_1, who participates in the same auctions more often after this threshold at 2.5. So, in this case, both synergy buyers lose from the presence of additional bidders. While one synergy buyer (i.e. syn_2) should benefit because she is offered better (higher) exercise prices than if she were alone in the market, this effect cannot immediately compensate the additional competition.

6.3 Larger simulation with random synergy buyers' market entry

In the final results we report in this paper, we conducted a larger scale simulation with multiple synergy buyers, which can enter the market randomly, with a certain probability.

The experimental setup implies that each sequence of auctions (forming a test case) has 10 items of each type (i.e. $n_A = 10$ and $n_B = 10$). What differs from previous settings is the random entry of synergy buyers. For each auction, there is a 25% chance that a synergy buyer will enter the market. If she does, then her valuation is drawn from a uniform distribution between 20 and 22 and she will stay in the market for exactly four auctions. To simplify matters, the auction sequence is fixed at first selling A, then B, then A etc. so that each synergy buyer will face exactly two auctions for an item of type A and two for an item of type B. However, the general result of this section is also true for a random auction sequence, since the basic effects remain the same.

As shown in Figure 10, the seller's profit now only has one maximum at 5, because initially each increase in exercise prices causes, with some probability, a synergy buyer to participate more often. So each point is a threshold and the profit graph smooths out over those many local maxima, corresponding to a steady increase (on average) of the expected profit. This result shows why it can be rational for the seller to have the same exercise prices for all goods of the same type (e.g. the same K_A). In a market with random entry of synergy buyers, the seller does not know which buyers are participating in any particular auction. Her optimal policy is to set her exercise prices which maximize her overall expected profit (in this case, $K = 5$).

7 Discussion and further work

This paper examined, from a decision-theoretic perspective, the use of priced options as a solution to the exposure problem in sequential auctions. We consider a model in which the seller is free to fix the exercise price for options on the goods she has to offer, and then sell these options in the open market, through a regular auction mechanism.

For this setting, we derived analytically, for a market with a synergy buyer and under some assumptions, the expressions that provide the bounds on the

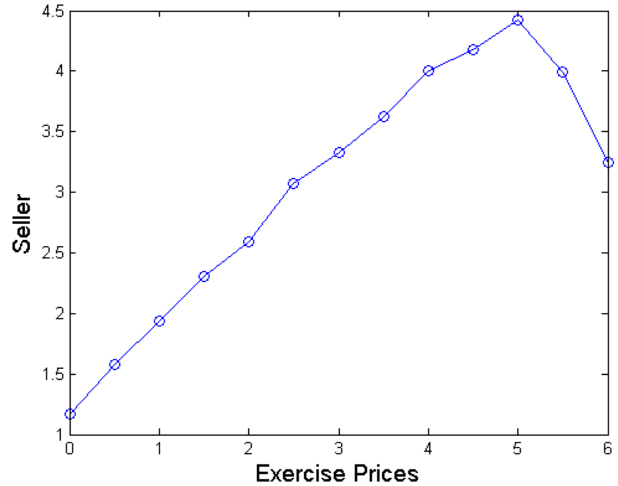

Fig. 10. Percentage increase in seller's profits in a larger experimental setting, with synergy buyers randomly entering the market.

option prices between which both synergy buyers and sellers have an incentive to use an option contract over direct auctions. Next, we performed an experimental analyses of several settings, where either one or multiple synergy bidders are active simultaneously in the market. We show that, if the exercise price is chosen correctly, selling items through priced options rather than direct sale can increase the expected profits of both parties.

The overall conclusion of our study is that the proposed priced options mechanism can considerably reduce the exposure problem that synergy bidders face when taking part in sequential auctions. Furthermore, and most important, *both* parties in the market have an incentive to prefer and use such a mechanism. We show that in many realistic market scenarios, sellers can fix the exercise prices at a level that both provides sufficient incentive for buyers to take part in the auctions, as well as cover their risk of remaining with the items unsold.

We should mention that, because sequential auction allocation is a highly complex and under-researched area, our study is still rather preliminary. Basically, we provide a full analysis and results for several realistic cases, but leave several, more complex issues to future work. These include more complex market settings, as well as more sophisticated reasoning abilities on the part of participating synergy bidders and sellers. For example, in a large market, synergy bidders could be expected to use learning strategies to adapt to changing market conditions, as well as the presence of other synergy bidders who want similar item combinations. However, the sellers of the items could also use learning to choose better levels of the exercise prices K with which to sell the options for their goods. Other possible issues open to future research include: markets where bidders have asymmetric or imperfect information, more complex preferences over bundles and different attitudes to risk of the involved parties.

To conclude, sequential auction bidding with complementary valuations is a problem that appears in many real-life settings, although no dominant strategies exist and bidders face a severe exposure problem. The main intuition of this work is that a simple options mechanism, where sellers auction options for their goods (with a pre-set exercise price), instead of the goods themselves can go a long way in solving the exposure problem, and can be beneficial to both sides of such a market.

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