07471 Abstracts Collection
Equilibrium Computation
— Dagstuhl Seminar —

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Abstract. From 18 to 23 November 2007, the Dagstuhl Seminar 07471 "Equilibrium Computation" was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Equilibrium, algorithm, polynomial time, game theory, economics

The purpose of this Dagstuhl seminar was to bring together researchers from different disciplines working on algorithmic problems of finding equilibria. As explained in the Motivation for this seminar, the different topics were (with talks given by)

- *Parity Games,* Andersson (junior researcher), Gimbert (junior researcher), Grädel (survey talk), Svensson (junior researcher), Zwick (survey talk),
- *Complexity of Finding Equilibria,* Elkind (junior researcher), Etessami (survey talk), Goldberg (introductory survey talk on the class PPAD, not listed below), Fabrikant (junior researcher), Hoefer (junior researcher), Monien, Vöcking (survey talk),
- *Mathematical Programming,* Halman, Morris, Theobald,
- *Economic Equilibria,* Heydenreich (junior researcher), Jain, Peeters (survey talk),
- *Game Theory,* Balthasar (junior researcher), Goldberg, Jiang (junior researcher), Sørensen (junior researcher), Turocy (survey talk), Vermeulen, von Stengel.

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The abstracts of 25 talks are listed below. In addition to his talk, Paul Goldberg gave an introduction to the complexity class PPAD which is central for defining the complexity of finding one equilibrium (for example, a Nash equilibrium in a game of n players). This talk is accessible through the following URL: http://www.csc.liv.ac.uk/∼pwg/PPADintro/PPADintro.html. This colorful talk was prepared by Paul Goldberg in Dagstuhl in immediate response to requests for such an introduction, and is described as such on the mentioned webpage.

Furthermore, Mike Paterson gave a popular evening talk on the entertaining topic of piling bricks so that they can "stick out" as far as possible, which can be considered as an "equilibrium problem" in physics but not in the computational sense studied in the seminar.

Overall, the seminar talks represented a good balance of the topics, and were not too numerous so as to make listening tiresome. A significant time was spent in discussions, drawing on the expertise of experienced scholars in the field (for example, Nimrod Megiddo). We encouraged, with success, to let junior researchers present their work as much as possible. The survey talks, often given by senior researchers, gave introductions and overviews.

Many of the topics of the seminar are in areas with very hard and longstanding open questions. In particular, solving parity games, or the related mean-payoff and simple stochastic games, in polynomial time is an intriguing open problem. It is a plausible conjecture that this can be done, given that the problem is in the intersection of the complexity classes NP and co-NP. The most famous problem with that property is linear programming (LP), which is equivalent to finding an equilibrium in a zero-sum game, and thus closely related to the problems discussed in the workshop. The polynomial-time algorithms for linear programming, such as the ellipsoid method, were hailed as breakthroughs at the time. To this day, the understanding even of the linear programming problem is limited; for example, we do not have a "combinatorial", simplex-type algorithm that would solve this problem in polynomial time.

In this context, the results by Nir Halman on an abstract view of LP-type problems and their connection to the parity games and their relatives, provided one example of a "bridge" across several fields, Mathematical Programming, Parity Games, and the Complexity of Finding Equilibria. Some discussions started in how this could be extended to the computation of Nash equilibria of bimatrix games.

The equilibrium problems discussed in this workshop are much harder than linear programming, which by itself is an important interesting case. None of the main problems were solved – this would have been close to sensational –, but we could observe some (necessarily partial and incremental) progress.

The hard problems mentioned above are concerned with computing Nash equilibria. Another focus of the workshop was the computation of other, more refined solution concepts. Here, the interaction between the participants from the computer science community and the participants from the economics community was extremely fruitful. In particular, a number of computational problems were jointly formulated which together form an interesting research program.

A representative example is the following: Given a three-player game in normal form *and* a strategy profile of the game, can it be decided in polynomial time if the strategy profile is a (trembling hand) perfect equilibrium of the game? The corresponding computational problem for Nash equilibria is trivial. One can ask the same question for other refinement notions, and we believe it would be very interesting to classify refinement notions by hardness or easiness of their verification problem, for two reasons. First, an easiness result would be useful in practice for studying equilibria computationally. Secondly, a refinement notion where even the verification problem is computationally intractable may arguably be considered an inferior solution concept to one where it is tractable to determine if a given profile is in equilibrium.

The workshop demonstrated that the topic of "equilibrium computation" is of great current interest. The stimulating discussions showed that it was very well worth bringing researchers together who normally operate in different communities.

Improved Algorithms for Discounted Payoff Games

Daniel Andersson (Aarhus University, DK)

We present the first strongly polynomial algorithms for solving single-player Discounted Payoff Games. We show that an optimal counter strategy against a fixed positional strategy in a generalized two-player discounted payoff game, where edges have individual discounts, can be computed in $O(mn^2 \log m)$ strongly polynomial time, where n and m are the number of vertices and edges in the game graph. This results in the best known strongly subexponential time bound for solving two-player zero-sum generalized discounted payoff games.

Equilibrium Tracing in Bimatrix Games

Anne Balthasar (London School of Economics, GB)

We analyze the relations of the van den Elzen-Talman algorithm, the Lemke-Howson algorithm and the global Newton method introduced by Govindan and Wilson. It is known that the global Newton method encompasses the Lemke-Howson algorithm; we prove that it also comprises the van den Elzen-Talman algorithm, and more generally, the linear tracing procedure, as a special case. This will lead us to a discussion of traceability of equilibria of index +1. We answer negatively the open question of whether, generically, the van den Elzen-Talman algorithm is flexible enough to trace all equilibria of index +1.

Keywords: Bimatrix games, Equilibrium computation, Homotopy methods, Index
Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1526

Nash Equilibria in Graphical Games on Trees

Edith Elkind (Univ. of Southampton, GB)

Graphical games are a natural and compact representation formalism for a large class of multi-player games. They have been introduced by Kearns, Littman and Singh in UAI'01, and have recently been used in the proof of the seminal PPAD-hardness result for finding a Nash equilibrium in 2-player, n-action games. This proof shows that finding a Nash equilibrium in general bounded-degree graphical games in PPAD-hard as well. In this talk, we discuss polynomial-time algorithms for finding Nash equilibria in graphical games on bounded-degree trees. We show that an algorithm for this problem that was proposed by Littman, Kearns and Singh in NIPS'01 is incorrect and describe how to fix it. Our version of the algorithm runs in polynomial time if the underlying graph is a path, but may require exponential time even on very simple trees. We show that this is, in some sense, inevitable: any algorithm that is based on the general approach of Littman et al. will have to store an exponentially large data structure. Moreover, the problem remains PPAD-complete for graphs with bounded pathwidth.

We also discuss the problem of finding Nash equilibria with special properties, such as the ones that maximize the total payoff or guarantee certain payoffs to all players.

Joint work of: Elkind, Edith; Goldberg, Leslie Ann; Goldberg, Paul

On the Complexity of Nash Equilibria and Other Fixed Points

Kousha Etessami (Univ. of Edinburgh, GB)

We reexamine what it means to compute Nash equilibria and, more generally, what it means to compute a fixed point of a given Brouwer function, and we investigate the complexity of the associated problems. Specifically, we study the complexity of the following problem: given a finite game, Γ, with 3 or more players, and given $\epsilon > 0$, compute a vector x' (a mixed strategy profile) that is within distance ϵ (say, in l_∞) of some (exact) Nash equilibrium.

We show that approximation of an (actual) Nash equilibrium for games with 3 players, even to within any non-trivial constant additive factor $\epsilon < 1/2$ in just one desired coordinate, is at least as hard as the long standing square-root sum problem, as well as more general arithmetic circuit decision problems, and thus that even placing the approximation problem in NP would resolve a major open problem in the complexity of numerical computation. Furthermore, we show that the (exact or approximate) computation of Nash equilibria for 3 or more players is complete for the class of search problems, which we call FIXP, that can be cast as fixed point computation problems for functions represented by algebraic circuits (straight line programs) over basis $\{+, *, -, /, \max, \min\}$, with rational constants. We show that the linear fragment of FIXP equals PPAD.

Many problems in game theory, economics, and probability theory, can be cast as fixed point problems for such algebraic functions. We discuss several important such problems: computing the value of Shapley's stochastic games, and the simpler games of Condon, extinction probabilities of branching processes, termination probabilities of stochastic context-free grammars, and of Recursive Markov Chains. We show that for some of them, the approximation, or even exact computation, problem can be placed in PPAD, while for others, they are at least as hard as the square-root sum and arithmetic circuit decision problems.

(Joint work with Mihalis Yannakakis, Columbia U.)

The Complexity of Game Dynamics: Sink Equilibria, BGP Oscillations, and Beyond

Alex Fabrikant (Univ. of California – Berkeley, US)

We show that the concept of sink equilibria proposed recently by Goemans, Mirrokni, and Vetta is PSPACE-complete to analyze and approximate for graphical games. We also settle the complexity of a well-known problem in networking by establishing that it is PSPACE-complete to tell whether a system of path preferences in the BGP protocol can lead to oscillatory behavior; one key insight is that the BGP oscillation question is in fact one about Nash dynamics. Finally, we propose a new equilibrium concept inspired by game dynamics, unit recall equilibria, which we show to be close to universal (exists with high probability in a random game) and algorithmically promising. We also give a relaxation thereof, called component-wise unit recall equilibria, which we show to be both tractable and universal (guaranteed to exist in every game).

Keywords: Algorithmic game theory, sink equilibria, nash dynamics, best-response dynamics, unit recall, bounded recall, BGP, SPP, convergence, complexity

Joint work of: Fabrikant, Alex; Papadimitriou, Christos H.

Full Paper:
http://portal.acm.org/citation.cfm?id=1347082.1347175

See also: Fabrikant, A. and Papadimitriou, C. H. The complexity of game dynamics: BGP oscillations, sink equilibria, and beyond. In Proc. of the 19th Annual ACM-SIAM Symposium on Discrete Algorithms (SODA), San Francisco, CA, January 2008. SIAM, Philadelphia, PA, pages 844–853.

Fast-Converging Tatonnement Algorithms for the Market Problem

Lisa Fleischer (Dartmouth College – Hanover, US)

Why might markets tend toward and remain near equilibrium prices?

In an effort to shed light on this question from an algorithmic perspective, this paper formalizes the setting of *Ongoing Markets*, by contrast with the classic market scenario, which we term *One-Time Markets*. The Ongoing Market allows trade at non-equilibrium prices, and, as its name suggests, continues over time. As such, it appears to be a more plausible model of actual markets.

For both market settings, this paper defines and analyzes variants of a simple tatonnement algorithm that differs from previous algorithms that have been subject to asymptotic analysis in three significant respects: the price update for a good depends only on the price, demand, and supply for that good, and on no other information; the price update for each good occurs distributively and asynchronously; the algorithms work (and the analyses hold) from an arbitrary starting point.

Our algorithm introduces a new and natural update rule. We show that this update rule leads to fast convergence toward equilibrium prices in a broad class of markets that satisfy the weak gross substitutes property. These are the first analyses for computationally and informationally distributed algorithms that demonstrate polynomial convergence.

Our analysis identifies three parameters characterizing the markets, which govern the rate of convergence of our protocols. These parameters are, broadly speaking:

1. A bound on the fractional rate of change of demand for each good with respect to fractional changes in its price.
2. A bound on the fractional rate of change of demand for each good with respect to fractional changes in wealth.
3. The closeness of the market to a Fisher market (a market with buyers starting with money alone).

We give two types of protocols. The first type assumes global knowledge of only (an upper bound on) the first parameter. For this protocol, we also provide a matching lower bound in terms of these parameters for the One-Time Market. Our second protocol, which is analyzed for the One-Time Market only, assumes no global knowledge whatsoever.

Keywords: Tatonnement, market equilibria

Joint work of: Cole, Richard; Fleischer, Lisa

Full Paper:
http://www.cs.dartmouth.edu/~lkf/papers/fp299-rev1.pdf

See also: Fleischer, L. and Cole, R., Fast-Converging Tatonnement Algorithms for One-Time and Ongoing Market Problems. In: Proc. STOC 2008.

Solving Simple Stochastic Games with Few Random Vertices

Hugo Gimbert (LaBRI – Bordeaux, FR)

A Simple Stochastic Game is played by two players called Min and Max, moving turn by turn a pebble along edges of a graph. Player Max wants the pebble to reach a special vertex called the target vertex. On some special vertices called random vertices, the next vertex is chosen randomly according to some fixed transition probabilities. Solving a simple stochastic game consists in computing the maximal probability with which player Max can enforce the pebble to reach the target vertex. In this talk, we will present a new algorithm for solving stochastic games, especially efficient for games with few random vertices.

Keywords: Simple stochastic games

Joint work of: Gimbert, Hugo; Horn, Florian

The Price of Selfish Stackelberg Leadership in Network Games

Paul Goldberg (University of Liverpool, GB)

We study a well-known model of competitive routing through a network of "parallel links". In this setting there is typically a "price of anarchy" arising from the assumption that users behave selfishly and do not comply with any central control. We investigate the additional cost that arises when some user gets to choose his strategy and commit to it, ahead of the other users (this is what is meant by Stackelberg leadership).

All users are selfish, including the leader. We identify some upper and lower bound on the additional social cost that arises in this situation.

Keywords: Multicommodity routing, network, atomic splittable flow

Joint work of: Goldberg, Paul; Polpinit, Pattarawit

Infinite Games

Erich Grädel (RWTH Aachen, DE)

Graph games, in which two or more players take turns (or not) to move a token through a directed graph, tracing out a possibly infinite path, have numerous applications, for instance for the design and verification of reactive systems, for the efficient evaluation of logical formulae, for planning and so on.

The fundamental mathematical questions on such games concern the existence of optimal strategies for the players, the complexity and structural properties of such strategies, and their realization by efficient algorithms. Which games are determined, in the sense that from each position, one of the players has a winning strategy? How to compute winning positions and optimal strategies? How much knowledge on the past of a play is necessary to determine an optimal next action? Which games are determined by memoryless strategies? And so on.

These questions are not only mathematically interesting, but translate into questions of design and verification of computing systems. The answers strongly depend on the particular form of a graph game, such as the number of players, the form of interaction between players (turn based or concurrent), the information that is available to the players, the structure of the game graph, and the type of the objectives or winning conditions of the players. A well-understood case are two-player, zero-sum games with perfect information and omega-regular objectives.

This talk will be a survey on fundamental concepts and results in the algorithmic theory of infinite games, focussing on

- Positional determinacy: what kind of winning conditions guarantee that all games with that condition are determined via positional winning strategies.
- Solution concepts and algorithms for infinite multiplayer games:

Conditions that guarantee the existence of Nash equilibria and subgame perfect equilibria, algorithmic realisation of such solution concepts.

Simple Stochastic Games, Parity Games, Mean Payoff Games and Discounted Payoff Games are all LP-Type Problems

Nir Halman (MIT – Cambridge, US)

We show that a Simple Stochastic Game (SSG) can be formulated as an LP-type problem. Using this formulation, and the known algorithm of Sharir and Welzl for LP-type problems, we obtain the first strongly subexponential solution for SSGs (a strongly subexponential algorithm has only been known for binary SSGs).

Using known reductions between various games, we achieve the first trongly subexponential solutions for Discounted and Mean Payoff Games. We also give alternative simple proofs for the best known upper bounds for Parity Games and binary SSGs.

To the best of our knowledge, the LP-type framework has been used so far only in order to yield linear or close to linear time algorithms for various problems in computational geometry and location theory. Our approach demonstrates the applicability of the LP-type framework in other fields, and for achieving subexponential algorithms.

Keywords: Subexponential algorithm, LP-type framework

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2008/1527

Full Paper:
http://www.springerlink.com/content/j771w6ku0141n5pr/?p=6d6dc8caf83e4c3fabe6a6320fdd4b0e&pi=0

See also: Algorithmica, volume 49 (September 2007), pages 37–50.

Graph Theoretic Characterization of Revenue Equivalence

Birgit Heydenreich (Maastricht University, NL)

The property of an allocation rule to be implementable in dominant strategies by a unique payment scheme is called revenue equivalence. In this paper we give a characterization of revenue equivalence based on a graph theoretic interpretation of the incentive compatibility constraints. The characterization holds for any (possibly infinite) outcome space and many of the known results are immediate consequences. Moreover, revenue equivalence can be identified in cases, where the existing literature will be silent.

Joint work of: Heydenreich, Birgit; Müller, Rudolf; Uetz, Marc; Vohra, Rakesh

Stackelberg Network Pricing Games

Martin Hoefer (RWTH Aachen, DE)

We study a multi-player one-round game in which a leader can set prices for a subset of m pricable edges in a graph. The other edges have a fixed cost.

Based on the leader's decision one or more followers optimize a polynomial-time solvable combinatorial minimization problem and choose a minimum cost solution satisfying their requirements. The leader receives as revenue the total amount of prices paid by the followers for pricable edges in their solutions. Our first result is a tight analysis of a single-price algorithm for revenue maximization with a single follower, which provides a $(1+\epsilon)\log m$ approximation for any constant $\epsilon > 0$. This can be extended to a $(1+\epsilon)(\log k + \log m)$-approximation for k followers. For the latter we show almost matching hardness results. Our second result is that in case of a single follower in Stackelberg bipartite vertex cover, there is an efficient algorithm to compute optimum pricies using LP-duality techniques. It can be extended to provide constant-factor approximations for any constant number of followers.

Joint work of: Hoefer, Martin; Briest, Patrick; Krysta, Piotr

Equilibria in the Presence of a Broker

Kamal Jain (Microsoft Corp. – Redmond, US)

Several equilibria situations have broker in between. For an example, search ad and product shelves at retailers. At a fundamental level, if we ignore the fine tuned pricing mechanism, we observe that these two brokering situation are similar. Of course, economic theory suggests that pricing mechanism is only a way to reach equilibria. In our model we notice that search ads and product shelves at retailers reach totally different market equilibria.

Keywords: Paid search, sponsored search, search, equilibrium, equilibria, retail

Full Paper:
http://search.live.com/results.aspx?mkt=en-us&q=kamal+jain&FORM=TOOLBR

See also:
http://www.eng.auburn.edu/~jvalenz/Colloquia2007/Kamal.htm

Action-Graph Games

Albert Xin Jiang (University of British Columbia – Vancouver, CA)

Action-Graph Games (AGGs) are a fully expressive game representation which can compactly express strict and context-specific independence and anonymity structure in players' utility functions. We present an efficient algorithm for computing expected payoffs under mixed strategy profiles. This algorithm runs in time polynomial in the size of the AGG representation (which is itself polynomial in the number of players when the in-degree of the action graph is bounded).

Keywords: Computational game theory, graphical models, compact representations of games

Joint work of: Jiang, Albert Xin; Leyton-Brown, Kevin

Multiprocessor Scheduling is PLS complete

Burkhard Monien (Universität Paderborn, DE)

We show that the problem of scheduling weighted jobs on identical machines is PLS complete for a sufficiently large neighbourhood. We show this result for two models of local improvement.

In the first model, in an improvement step either the makespan decreases or the makespan remains unchanged and the number of makespan machines decreases. In the second model, we consider the selfish version of the problem where the jobs are viewed as selfish agents. The cost of an agent is the load of the machine to which it is assigned. The cost of a coalition of agents is defined to be the maximal evaluation of its members. In an improvement step, the cost of the coalition of reallocating agents decreases. Both these problems are PLS-complete when 36 jobs/agents are allowed to be reallocated.

We show these results by reduction from some Multi Constraint Assignment problem (p, q, r)-MCA which is an extension of the generalized MAXSAT problem to higher valued variables. Here, p is the maximal length of a constraint, q is the maximal number of appearances of a variable and r is the valuedness of the variables. We show that $(3, 2, r)$-MCA and $(2, 3, r)$-$\text{MCA}_{\text{bipartite}}$ are PLS complete for sufficiently large r.

We also show that the restricted scheduling problem, where for each job/agent there exists a set of forbidden machines, is PLS complete already for a 13-neighbourhood.

Good Hidden P-matrix Sandwiches

Walter Morris (George Mason Univ. – Fairfax, US)

A square matrix is a P-matrix if all of its principal minors are positive.

The complexity of solving the linear complementarity problem with data (M, q) where M is a P-matrix is unknown. Recently, Gärtner and Rüst, and also Svensson and Vorobyov, showed that the problem of solving a simple stochastic game can be reduced to solving a generalized LCP with a block P-matrix. A subclass of the class of P-matrices is that of hidden Minkowski matrices. (Generalized) LCPs with matrices in this class are known to be solvable in polynomial time. We briefly describe geometrically why this is so. We study some generalizations of the class of hidden Minkowski matrices. Some of these generalizations contain the class of P-matrices, and some are contained in the class of P-matrices. Thus the class of P-matrices is "sandwiched" between these generalized classes, many of which can be recognized in polynomial time.

Joint work of: Morris, Walter; Namiki, Makoto

See also: Linear Algebra and its Applications 426 (2007) 325–341.

Homotopy Methods to Compute Equilibria in Game Theory

Ronald Peeters (Maastricht University, NL)

This paper presents a survey of the use of homotopy methods in game theory. Homotopies allow for a robust computation of game-theoretic equilibria and their refinements.

Homotopies are also suitable to compute equilibria that are selected by various selection theories. We present the relevant techniques underlying homotopy algorithms. We give detailed expositions of the Lemke-Howson algorithm and the van den Elzen-Talman algorithm to compute Nash equilibria in 2-person games, and the Herings-van den Elzen, Herings-Peeters, and McKelvey-Palfrey algorithms to compute Nash equilibria in general n-person games. We explain how the main ideas can be extended to compute equilibria in extensive form and dynamic games, and how homotopies can be used to compute all Nash equilibria.

Keywords: Homotopy, Equilibrium computation, Non-cooperative games, Nash Equilibrium

Joint work of: Herings, P. Jean-Jacques; Peeters, Ronald

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2008/1525

Fast Algorithms for Finding Proper Strategies in Game Trees

Troels Bjerre Sørensen (Univ. of Aarhus, DK)

We show how to find a *normal form proper* equilibrium in behavior strategies of a given two-player zero-sum extensive form game with imperfect information but perfect recall. Our algorithm solves a finite sequence of linear programs and runs in polynomial time. For the case of a perfect information game, we show how to find a normal form proper equilibrium in linear time by a simple backwards induction procedure.

Keywords: Equilibrium refinements

Joint work of: Miltersen, Peter Bro; Sørensen, Troels Bjerre

Full Paper:
http://www.daimi.au.dk/~bromille/Papers/extproper2.pdf

Simple Stochastic Games and Linear Complementarity

Ola Svensson (IDSIA – Lugano, CH)

We show how to represent Simple Stochastic Games (SSG) as the (Generalized) Linear Complementarity Problem (GLCP). More specifically, we show that SSGs reduce to a nontrivial subclass of P-matrix GLCPs, which we call D-matrix GLCPs. This relates two problems with similar complexity status: NP-hardness for any of the two problems would imply NP=coNP but at the same time they are not known to be in P. P-matrix LCPs and GLCPs are not even known to be solvable in subexponential time.

Restricting ourself to D-matrix GLCPs allows us to present a randomized algorithm with subexponential analysis, matching the best known algorithms for SSGs.

Keywords: Simple Stochastic Games, Linear Complementarity, P-matrices

Enumerating the Nash Equilibria of Rank-1 Games

Thorsten Theobald (Universität Frankfurt, DE)

A bimatrix game (A, B) is called a game of rank k if the rank of the matrix $A + B$ is at most k. We consider the problem of enumerating the Nash equilibria in (non-degenerate) games of rank 1.

In particular, we show that even for games of rank 1 not all equilibria can be reached by a Lemke-Howson path and present a parametric simplex-type algorithm for enumerating all Nash equilibria of a non-degenerate game of rank 1.

Keywords: Bimatrix game, Nash equilibrium, enumeration, games of fixed rank

Full Paper:
http://arxiv.org/abs/0709.1263

The Gambit Project: Current State and Future Directions

Ted Turocy (Texas A&M University, US)

The Gambit project is an Open Source software project providing tools for analyzing finite N-player games. This talk outlines the objectives of the project, enumerates the tools currently provided, and proposes some directions for future development and collaboration.

Keywords: Game theory, Nash equilibrium

Where Strategic and Evolutionary Stability Depart – A Study of Minimal Diversity Games

Dries Vermeulen (Maastricht University, NL)

A minimum diversity game is a strategic form game in which the players have the same pure strategy set. When playing a pure strategy profile, each player always receives 1, unless all players use the same pure strategy, in which case they all receive 0 payoff.

Such a game has an isolated completely mixed Nash equilibrium. The other equilibria form the component of Pareto optimal strategy profiles where all players receive 1.

The completely mixed Nash equilibrium is essential and strategically stable in the sense of Mertens. It is however a repellent rest point of the replicator dynamics. On the other hand, the set of Pareto optimal Nash equilibria is a topological spere that is asymptotically stable under the replicator dynamics. When the dimension of the sphere is even, this component of Nash equilibria contains a stable set in the sense of Mertens. However, for two player games where the dimension of the sphere is odd, that is when both players have an odd number of pure strategies, the sphere is no longer stable in the sense of Mertens. The same result is obtained for the three player minimum diversity game where each player has two pure strategies.

Keywords: Strategic stability, evolutionary stability

Joint work of: Vermeulen, Dries; Balkenborg, Dieter

Complexity of Congestion Games

Berthold Vöcking (RWTH Aachen, DE)

In a congestion game, several players simultaneously aim at allocating sets of resources, e.g., each player aims at allocating a shortest path between a source/destination pair in a given network or, to give another example, each player aims at allocating a minimum weight spanning tree in a given graph. The cost (length, delay, weight) of a resource (edge) is a function of the congestion, i.e., the number of players allocating the resource. In our talk, we survey recent results about the complexity of computing Nash equilibria and approximate equilibria for congestion games and the convergence time towards Nash equilibria.

Keywords: Congestion games, Nash equilibria, complexity, approximability

Algorithms for Finding Equilibria in Games: Things to Do

Bernhard von Stengel (London School of Economics, GB)

This is a short presentation arguing that we have to put existing algorithms for finding equilibria in games to use. This requires to implement them, and, most importantly, to give them interfaces. These should be back-end interfaces, using specific file formats for games, strategies, algorithmic problems, equilibria, etc., that allows these algorithms to be used by other programs. For the user, we need front-end interfaces to input and display games, and to generate large games from rules. Finally, we should care more to find, and listen to, users of our algorithms.

Keywords: Equilibrium computation, programming

Simple stochastic games, mean payoff games and parity games – a survey

Uri Zwick (Tel Aviv University, IL)

In this survey talk I will define the three classes of games mentioned in the title and summarize what is known about them.

It is a major open problem whether these games can be solved in polynomial time. The "highlights" of the talk include a randomized subexponential time algorithm for Simple Stochastic Games obtained in 1995 by Ludwig, and a deterministic subexponential time algorithm for Parity Games obtained in 2006 by Jurdzinski, Paterson and Zwick.

Keywords: Simple stochastic games, mean payoff games, parity games

Full Paper:
http://doi.acm.org/10.1145/1109557.1109571

See also: Proc. of SODA'06, pages 117–123.