Ranking Revision Reloaded

A semi-extended abstract

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Ranking measures are coarse-grained, quasi-probabilistic plausibility valuations measuring the degree of surprise. Since Spohn's work on the subclass of natural conditional functions, they have been used to model epistemic states and their dynamics. The simplest instance may be elementary ranking revision, which is a non-parametrized version of Spohn-style revision based on minimal Jeffrey-conditionalization for ranking measures [Wey 05].

The restricted expressiveness of ranking valuations, at least when compared to probability measures, is a lesser issue if one sees them as auxiliary ingredients of a more sophisticated dynamic epistemic framework. For instance, if we seek more flexible revision strategies, but want to conserve the semi-qualitative flavour of the initial account, we may consider *Sequential Ranking Revision* (SMS-revision).

In this approach, the epistemic states are sequences $(R, \Delta_1, \ldots, \Delta_n)$ consisting of a prior ranking measure R and a sequence of sets Δ_i of ranking constraints of the form $R(A_i) \geq \alpha_i$. Revision here corresponds to concatenation with appropriate ranking constraints, derived from the informational inputs (e.g. $R(\neg A) \geq 1$ for an incoming fact A). This is actually a generalization of the sequential philosophy proposed by Lehmann.

The non-trivial part is to determine the epistemic ranking measure R^* which reflects best the epistemic state $(R, \Delta_1, \ldots, \Delta_n)$. The idea of SMS-revision is, first, to split the sequence $(\Delta_1, \ldots, \Delta_n)$ into maximally consistent segments $(\Delta_{n_i+1}, \ldots, \Delta_{n_{i+1}})$, starting with the most recent constraints $\Delta_n = \Delta_{n_m}$, and setting $n_0 = 0$. The resulting ranking projection R^*, i.e. the actual epistemic valuation, is then obtained through iterated, multiple constraint ranking revision based on an extension of the JLZ-shifting strategy [Wey 03], which replaces entropy maximization at the ranking-level.

$$R \star \bigcup_{i \leq n_1} \Delta_i \star \ldots \star \bigcup_{n_{m-1} < i \leq n_m} \Delta_i$$

The postulates of Darwiche and Pearl, which have been inspired by standard ranking revision, however fail for this account.

Wey 03 System JLZ - Rational Default Reasoning by Minimal Ranking Constructions. In *Journal of Applied Logic*, 1:273-308, Elsevier, 2003.

Wey 05 Projective default epistemology - a first look. In Conditionals, Information, and Inference, Springer LNAI 3301, p.65-85, Springer, 2005.

Dagstuhl Seminar Proceedings 07351
Formal Models of Belief Change in Rational Agents
http://drops.dagstuhl.de/opus/volltexte/2007/1202

1 Talk - Formal Models of Belief Change, Dagstuhl 07

1.1 Prologue

Two "epistemic" modeling traditions (static + dynamic)

1. Qualitative : e.g. epistemic entrenchment + AGM
→ coarse-grained, no independence, revisable full belief

2. Quantitative : e.g. subjective probability + conditioning
→ fine-grained, independence, no revisable full belief

Hyper-quantitative extension : full/nonstandard measures
→ a bit cumbersome

Semi-qualitative synthesis : [Spohn 88, 90]
Natural conditional functions/κ-rankings + J-conditionalization
→ intermediate granularity, independence, revisable full belief

More generally : dynamics of ranking measures [Wey 90, 94,...]

1.2 Roadmap

1. General dynamic epistemology

2. Ranking measure model

3. Elementary ranking revision

4. General ranking revision

5. Sequential ranking revision

6. SMS revision

7. Examples

1.3 Ranking measures

Coarse-grained quasi-probabilistic implausibility/surprise valuations

Ranking measures = total pre-orders + reasonable dependence [Wey 94]

$R : \mathcal{B} \rightarrow \mathcal{V}$ is a **ranking measure** iff $\mathcal{B}$ is a boolean set system,

$\mathcal{V} = (G^+_\infty, 0, \infty, +, <)$ is the positive part of an ordered commutative group topped by infinity,

$R(T) = 0$, $R(\emptyset) = \infty$, and $R(A \cup B) = min(R(A), R(B))$

Conditional ranking measure $R(|) : R(A|B) = R(A \cap B) - R(B)$.

R is a $\kappa\pi$**-measure** iff $\mathcal{V}$ is divisible ("x/n exists")
$\rightarrow$ homogeneity, extendibility, inference specification

Only rankings : NCF, rat π – also $\kappa\pi$-measures : $\mathcal{V}_0$, real π

Our choice : minimal $\kappa\pi$-structure $\mathcal{V}_0 = ([0, \infty]_{rat}, 0, \infty, +, <)$

1.4 Ranking epistemology

Ranking measures : main/auxiliary epistemic valuations

Idea : A is believed iff $\neg A$ is sufficiently surprising

$Bel_R(A)$ iff $R(\neg A) \geq \alpha > 0$ or $R(\neg A) > \alpha$

$A \leq_R B$ iff not $Bel_R(\neg B | A \cup B)$

Closure under conjunction, weakening $\rightarrow$ full belief

Canonical ignorant prior : $R_0 = 0$

Shifting by $\alpha \geq 0 : R \rightarrow R + \alpha A$

= uniformly shifting the A-worlds by α, then normalizing

1.5 General dynamic epistemology

Projection paradigm for epistemic frameworks : Epistemic states e can be arbitrarily complex entities, only partially modelable through epistemic projection functions, e.g. Bel_e, $\leq_e$, P_e, ...

Epistemic space $(\mathcal{E}, \mathcal{E}_0, \mathcal{O}, \star, \Pi)$ **:**

$\mathcal{E}$: Epistemic states

$\mathcal{E}_0$: Default states

$\mathcal{O}$: Observations

$\star$: Revision function $\mathcal{E} \times \mathcal{O} \rightarrow \mathcal{E}$

Π : Epistemic projection functions on $\mathcal{E}$

$\pi_0 \in \Pi : \pi_0(e) = \leq_e$, a plausibility pre-order on a Boolean domain

Simple example : $(Prob_B, \{P_0\}, B, \star_{cond}, \{\leq_{pr}, Bel_{\geq 0.999}\})$

1.6 Elementary ranking revision

Idea : Spohn/Jeffrey-conditionalization as needed

Default belief strength : 1

$\rightarrow$ revision implicitly operationalizes ranking values

Elementary ranking revision space :

$\mathcal{E} = \mathcal{KP}$, $\mathcal{E}_0 = \{R_0\}$, $\mathcal{O} = \mathcal{B}$, $\Pi = \{\leq_{(.)}, Bel_{(.)}\}$

Minimal Spohn revision $\star = \star_{msp}$ **:**

$R \star_{msp} A = R - R(A)A + max(R(\neg A), 1)(\neg A)$ if $R(A) \neq \infty$

$\rightarrow$ ranking version of minimum cross-entropy for default constraints

Example : $R_0 \star_{msp} A \star_{msp} B \star_{msp} \neg A = R_0 \star_{msp} B$ for $A \perp B$

1.7 Open issues

Extension reasons :

Lack of granularity – order of magnitude view

Rudimentary inputs – single propositions

Monolithic shifting – just input proposition

Naive input evaluation – focus on target rank

Success and recency dogm – for possible inputs

History ignorance – information waste

Lack of learning – $R_0 \star \neg A^n \star A \star \neg A^n \star A = R_0 \star A$

Example : $R \star_{msp} (A \wedge B) \star_{msp} A \neq R \star_{msp} A \star_{msp} (A \wedge B)$

1.8 General ranking revision

– : ranking measures are not expressive enough to fully model uncertainty, or realistic epistemic states

+ : ranking measures may be a valuable ingredient of both

Idea : adding ranking measures to general epistemic spaces

$\kappa\pi$-ranking revision space $(\mathcal{E}, \mathcal{E}_0, \mathcal{O}, \star, \Pi)$ **:**

Ranking projection : $\mathcal{R} \in \Pi$ with $\mathcal{R}(e) \in \mathcal{KP}$

Plausibility coherence : $A <_{\mathcal{R}(e)} B$ implies $A <_e B$

$\mathcal{R}(e) = R_0$ for $e \in \mathcal{E}_0$

Reconstructibility : $\mathcal{R}(e \star o) = \mathcal{R}(e) + \Sigma \vec{\alpha}_{e,o} \vec{A}_{e,o}$

. . .

Our target : direct extensions of elementary ranking revision

1.9 Sequential ranking revision

Basic ideas :

keeping track of history

exploring set shifting

trade-off revision - projection

Simple sequential ranking revision spaces :

$\mathcal{E} = \bigcup_{n<\infty} \mathcal{KP} \times \mathcal{O}^n$ (prior R with observation sequences)

$\mathcal{E}_0 = \{(R_0)\}$

$\mathcal{O}$ = finite conditional constraint sets $\Delta = \{R(A|B) \geq \alpha \ldots\}$, e.g. $R(\neg A) \geq 1$

Ranking projection : $\mathcal{R} \in \Pi$ − through iterated set shifting

Epistemic order projection : $A \leq_e B$ iff $\mathcal{R}(e)(B) \leq \mathcal{R}(e)(A)$

Revision function : $(R, \vec{\Delta}) \star \Delta' = (R, \vec{\Delta}, \Delta')$

1.10 Sequential ranking projection

Sequential ranking projections $\mathcal{R}$:

$\rightarrow$ extraction procedures fitting the ranking revision philosophy

Our basic 3-step strategy :

Auxiliary non-sequential revision function $\star_0$ for sets : $R \star_0 \Delta$

Ordered partition $(\cup\vec{\Delta}_1, \ldots, \cup\vec{\Delta}_m)$ of the input sequence $\vec{\Delta}$

Projection by iterated non-sequential revision with $\Delta_i = \cup\vec{\Delta}_i$:

$\mathcal{R}(R, \vec{\Delta}) = R \star_0 \Delta_1 \star_0 \ldots \star_0 \Delta_m$

Our main specification task :

Non-sequential set revision + partition strategy

1.11 Sequential minimal shifting revision

Some guidelines for $\mathcal{R}$:

partitioning : order independence for consistent recent inputs

auxiliary revision : uniformly minimizing longest shifts

focus : $\mathcal{O} = \mathcal{B} \times [0, \infty]$ (non-conditional ranking constraints)

SMS Partitioning :

$\rightarrow$ backwards construction of maximal consistent connected $\vec{\Delta}_i$

SMS Non-sequential revision :

$\rightarrow$ minimal shifting following the JLZ strategy

i.e. ME-surrogate for ranking measures [W 03]

Example : $e = R_0 \star \neg A \star A \star (A \wedge B) = (R_0, \neg A, A, A \wedge B)$

$\mathcal{R}(e) = R_0 + 1\neg(A \wedge B)$

1.12 Conclusions

Ranking measures are alive and kicking

Interesting ingredient of richer epistemologies

Generalizing ranking revision in different ways

Synthesizing ranking revision and Lehmann's revision

Implementation based on the JLZ-shifting-paradigm