06341 Abstracts Collection
Computational Structures for Modelling Space, Time and Causality
— Dagstuhl Seminar —

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Abstract. From 20.08.06 to 25.08.06, the Dagstuhl Seminar 06341 "Computational Structures for Modelling Space, Time and Causality" was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Keywords. Borel hierarchy, causets, Chu spaces, computations in higher types, computable analysis, constructive topology, differential calculus, digital topology, dihomotopy, domain theory, domain representation, formal topology, higher dimensional automata, mereotopology, partial metrics, quasi-metrics, quantum gravity, region geometry, space-time, Stone duality, Wadge reducibility

06341 Executive Summary – Computational Structures for Modelling Space, Time and Causality

Topological notions and methods have been successfully applied in various areas of computer science. Image processing, programming language semantics and exact computing with real numbers and vectors in Banach spaces are important examples. Computerized geometric constructions have many applications in engineering and physics. The seminar concentrated on computational structures for modelling space, time and causality, which are basic in these applications. Special emphasis was given to connections with physics.

Due to the digital nature of computation, such structures differ from the mathematical structures they model, based on the continuum, that are classically used in these fields. Their typical features include a graph-based digital framework useful in computing algorithms, and also feature asymmetry and partiality. The classical spaces contain only the ideal elements that are the result of a completed computation (approximation) process which involves algorithms based on moving between points (for which a graph structure is used). But spaces that also allow reasoning about such processes in a formal way must contain the partial (and finite) objects appearing during a computation as well, and must consider a limiting process. Only the partial and finite objects can be observed in finite time. The leading example of such a structure is the domain (in Scott's sense). Here, the finitely observable properties of a process are the primary objects of study. The ideal entities which are the only elements considered in classical mathematical structures are obtained as derived objects via the limiting relationship. By a continuous model of a classical space we usually mean a domain, perhaps with additional structure such as a measurement or partial metric to represent the original space, as the subspace of maximal points. This gives a handle on the computational aspects of the classical space.

This, from a computational perspective, is some of the motivation for developing alternative models, in which a partial ordering (of the approximation of ideal elements by partial or finite ones) is fundamental.

What is remarkable is that very similar order-theoretic models are being developed for (apparently) entirely different reasons in theoretical physics.

The singlarity theorems e.g. show that in classical general relativity the basic geometric assumptions break down (singularities develop) so one seeks alternatives: Ashtekar's quantum geometry, string theory or Sorkin's causets.

Rafael Sorkin and his collaborator in combinatorics, Graham Brightwell, working in a program towards quantum gravity where the causal structure is taken as fundamental, use causal sets as basic structure, which are nothing more than locally finite partial orders. Keye Martin and Prakash Panangaden showed that globally hyperbolic spacetimes (studied in Kronheimer and Penrose's 1967 classic, "On the structure of causal spaces", Proc. Camb. Phil. Soc. 63, 481–501) are special continuous domains.

There are several consequences of the work of Martin and Panangaden. The topology of the spacetime manifold can be reconstructed from the causal structure; indeed from a countable dense subset of the spacetime. The result relates the areas of domain theory in computing and causality in physics, and provides new tools for deriving results relevant to quantum gravity, but it is only a beginning and much needs to be done.

In most work in physics of the kind just mentioned, one views space (or space-time) as a continuous manifold. But by using domains, we gain a clearer view of ideas derived from computer science being applied in the direction of physics.

There are reasons for wanting to consider also discrete models of space and time. Philosophically quantum mechanics suggests that one should look to discrete structures rather than continuum structures. There are no experiments that can probe arbitrarily deeply into the structure of spacetime (as that would require unboundedly high energies) so there can never be any experimental support for a true continuum.

We can now compare causal sets and other event structures with process models in computer science, so that posets and graphs will figure extensively in"discrete" models.

After very successful predecessor seminars in 2000, 2002 and 2004, the seminar in 2006 was the fourth in this series of Dagstuhl seminars which aim to bring together people working in fields like domain theory, topology, geometry, formal topology, and now causal spaces in physics, and to foster interaction between them. A further goal has always been to encourage communication and, hopefully, collaboration between computer scientists and those mathematicians and now physicists who work on similar problems but from a different perspective and who, often, are not aware of what their work has in common.

We are actively looking for people in more fields that involve related ideas of digital approximation of continuous structures.

This time the seminar attracted 49 participants representing 16 countries and 5 continents, among them 8 young researchers working for their master or PhD. The atmosphere was very friendly, but discussions were most lively. During the breaks and until late at night, participants also gathered in smaller groups for continuing discussions, communicating new results and exchanging ideas. Again the seminar led to several new research contacts, collaborations, and at least one successful application for a new Dagstuhl seminar.

As with the seminars in 2000 and 2004, the participants are again invited to submit a full paper for a special issue of *Theoretical Computer Science*.

The great success of the seminar is not only due to the participants, but also to all the staff members, both in Saarbrücken and Dagstuhl, who always do a great job in making everything run in such an efficient and smooth way. Our thanks go to both groups!

Keywords: Borel hierarchy, causets, Chu spaces, computations in higher types, computable analysis, constructive topology, differential calculus, digital topology, dihomotopy, domain theory, domain representation, formal topology, higher dimensional automata, mereotopology, partial metrics, quasi-metrics, quantum gravity, region geometry, space-time, Stone duality, Wadge reducibility

Joint work of: Kopperman, Ralph; Panangaden, Prakash; Smyth, Michael B.; Spreen, Dieter

Markovian concurrency model

Samy Abbes (University of Maryland - College Park, USA)

Markov chains are an example of process where causality has an influence on probability. The main feature is that only the present, not the whole past of a process, has an influence on its future. We propose to further extend the notion of Markovian process to systems featuring concurrency properties. In this setting, the notion of stopping operator, generalizing stopping times, is introduced, a Strong Markov property is given as well as consequences on renewal theory for concurrent processes.

Keywords: Concurrency, Markov operator, Petri net, event structures, partial order

Classical Domain Theory lives faithfully in Topological Domain Theory

Ingo D. Battenfeld (University of Edinburgh, GB)

Classical Domain Theory, in form of countably-based continuous dcpos (ωCONT), embedds into the topological domain theoretic framework, developed by Schröder and Simpson, and represented by the category of quotients of countably-based topological spaces (QCB) and its reflective subcategory of topological domains (TP). Both of these domain-theoretic frameworks provide useful constructions for denotational semantics, and in this talk, we give an overview how they compare. In particular, we outline that the free algebra construction for finitary inequational theories, which can be used to model computational effects, coincides in both frameworks. Thus, one can construct the classical powerdomains over countably-based continuous dcpos as free algebras in TP. Furthermore, the function space construction coincides for certain classes of dcpos, including the ωFS-domains, which form the largest cartesian closed subcategory of ωCont.

Keywords: Denotational semantics, domain theory, domain constructions

Domains and computational complexity in higher types

Ulrich Berger (University of Wales - Swansea, GB)

We apply logical relations to define two hierarchies of functionals in all finite types as new candidates for higher type feasibility. The first hierarchy uses logical relations to generalize Cobham's Limited Recursion on Notation to all finite types. The hierarchy coincides with Cook and Kapron's Basic feasible Functionals (BFF) up to type level two, but at higher types our hierarchy might be strictly larger than BFF. The second hierarchy is defined by a generalized Kripke logical relation. The new point here is that the starting relations are not defined at base types, i.e. type level 0, but at types of any chosen level k. This requires a new construction of the relation for function types that in general differs from the usual one. The second hierarchy coincides with BFF up to type level k and is probably strictly larger at type level $k+1$. We prove that if we work in the model of partial continuous functionals, all elements of our hierarchies are computable (which is not obvious for the second hierarchy). Both our hierarchies are closed under λ-definability. We neither know whether our hierarchies can be effectively generated nor how they are related in general.

Keywords: Domain theory, computational complexity, higher types, lambda calculus, logical relations

Elementary Differential Calculus on Discrete, Continuous and Hybrid Spaces

Howard Blair (Syracuse University, USA)

We unify a variety of continuous and discrete types of change of state phenomena using a scheme whose instances are differential calculi on structures that embrace both topological spaces and graphs as well as hybrid ramifications of such structures. These calculi include the elementary differential calculus on real and complex vector spaces.

One class of spaces that has been increasingly receiving attention in recent years is the class of convergence spaces [cf. Heckmann, R., TCS v.305, (159–186)(2003)] The class of convergence spaces together with the continuous functions among convergence spaces forms a Cartesian-closed category CONV that contains as full subcategories both the category TOP of topological spaces and an embedding of the category DIGRAPH of reflexive directed graphs.

(More can importantly be said about these embeddings.) These properties of CONV serve to assure that we can construct continuous products of continuous functions, and that there is always at least one convergence structure available in function spaces with respect to which the operations of function application and composition are continuous. The containment of TOP and DIGRAPH in CONV allows to combine arbitrary topological spaces with discrete structures (as represented by digraphs) to obtain hybrid structures, which generally are not topological spaces.

We give a differential calculus scheme in CONV that addresses three issues in particular.

1. For convergence spaces X and Y and function $f : X \longrightarrow Y$, the scheme gives necessary and sufficient conditions for a candidate differential $df : X \longrightarrow Y$ to be a (not necessarily "the", depending on the spaces involved) differential of f at x_0.

2. The chain rule holds and the differential relation between functions distributes over Cartesian products: e.g. if Df, Dg and Dh are, respectively, differentials of f at $(g(x_0), h(x_0))$ and g and h at x_0, then $Df \circ (Dg \times Dh)$ is a differential of $f \circ (g \times h)$ at x_0.

3. When specialized to real and complex vector spaces, the scheme is in agreement with ordinary elementary differential calculus on these spaces.

Moreover, with two additional constraints having to do with self-differentiation of differentials and translation invariance (for example, a linear operator on, say, $\mathbf{C}^2$, is its own differential everywhere) there is a (unique) maximum differential calculus in CONV.

Keywords: Hybrid space, convergence space, differential calculus, chain rule, hybrid dynamical system, discrete structure, topological space

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2007/895

Reducibility between domain representations

Jens Blanck (University of Wales - Swansea, GB)

A notion of (continuous) reducibility of representations of topological spaces is introduced and basic properties of this notion are studied for domain representations.

A representation reduces to another if its representing map factors through the other representation. Reductions form a pre-order on representations. A spectrum is a class of representations divided by the equivalence relation induced by reductions. Representations belonging to the top element of a spectrum are said to be universal and these representations are the ones most closely capturing the structure of the represented space. Notion of admissibility considered both for domains and within Weihrauch's TTE are shown to be universality concepts in the appropriate spectra. A real number structure is shown to be computably stable, i.e., all effective domain representations of that structure are computably equivalent.

Keywords: Domain representation, reducibility

Computability of Compact Operators

Vasco Brattka (University of Cape Town, ZA)

We develop some parts of the theory of compact operators from the point of view of computable analysis. While computable compact operators on Hilbert spaces are easy to understand, it turns out that these operators on Banach spaces are harder to handle. Classically, the theory of compact operators on Banach spaces is developed with the help of the non-constructive tool of sequential compactness.

We demonstrate that a substantial amount of this theory can be developed computably on Banach spaces with computable Schauder bases that are well-behaved. The conditions imposed on the bases are such that they generalize the Hilbert space case. In particular, we prove that the space of compact operators on Banach spaces with monotone, computably shrinking and computable bases is a computable Banach space itself and operations such as composition with bounded linear operators from left are computable. Moreover, we provide a computable version of the Theorem of Schauder on adjoints in this framework and we discuss a non-uniform result on composition with bounded linear operators from right.

Keywords: Computable functional analysis

Joint work of: Brattka, Vasco; Dillhage, Ruth

Apartness in Lattices

Douglas Bridges (University of Canterbury - Christchurch, NZ)

The theory of apartness spaces is lifted to the more abstract context of frames, thereby providing another point-free constructive approach to topology.

Keywords: Apartness, lattice, constructive

Joint work of: Bridges, Douglas; Vita, Luminita

When is the Standard Domain Representation of a Quotient Space Admissible?

Fredrik Dahlgren (Uppsala University, S)

Effective domain theory and effective domain representations can be used to study computable processes on uncountable spaces. To be able to represent continuous functions, and to construct type structures over representable spaces, we need to require that the domain representations of the spaces in question are admissible.

We give a result characterising the quotient constructions which preserve admissibility of the representation, and say something about the consequences of this theorem.

Keywords: Computability theory, computable analysis, domain theory, topology

Random constructions of universal causal sets and universal Scott-domains

Manfred Droste (Universität Leipzig, D)

Causal sets are particular partially ordered sets which have been proposed as a basic model for discrete space-time in quantum gravity. We show that the class C of all countable past-finite causal sets contains a unique causal set (U,<) which is universal (i.e., any member of C can be embedded into (U,<)) and homogeneous (i.e., (U,<) has maximal degree of symmetry). Moreover, (U,<) can be constructed probabilistically.

In joint work with Dietrich Kuske, we also present probabilistic constructions of bifinite domains and of Scott-domains which produce, each with probability 1, the universal homogeneous bifinite domain respectively the universal homogeneous Scott-domain.

Keywords: Causal sets, quantum gravity, domain theory, universal Scott-domain, universal bifinite domain

Full Paper:
http://arxiv.org/abs/gr-qc/0510118

See also: Journal of Mathematical Physics 46 (2005) 122503

Infinite spaces that admit fast exhaustive search

Martin H. Escardo (University of Birmingham, GB)

Perhaps surprisingly, there are infinite spaces that admit exhaustive search: given any decidable property, it is possible to uniformly decide, mechanically and in finite time, whether or not the property holds for all elements of the space. An example has been given by Gandy and Berger independently. We investigate exhaustible spaces in higher-type computation over the natural numbers. Our main contributions are a characterization of them and tools for systematically building them. This is work on higher-type computability theory (specifically and technically, on the Kleene–Kreisel continuous functionals). Our main mathematical tool is topology, but our characterizations and constructions are purely computational, and their formulations can be understood without any background on topology, which is needed only for proofs. Regarding efficiency and complexity, we have compelling experimental results and preliminary theoretical explanations — we hope they will stimulate further theoretical work on higher-type complexity theory, and applied work on efficiency of exhaustive search over infinite spaces.

Keywords: Topology, domain theory, higher-type computation, PCF, simply-typed lambda-calculus, Haskell

Universal dicoverings and Equivalence of Higher Dimensional Automata

Lisbeth Fajstrup (Aalborg University, DK)

In his thesis, U. Fahrenberg showed that VanGlabbeeks equivalence relation between Higher Dimensional Automata is in fact a path lifting property in a directed setting. Two HDA, modelled as cubical sets A and B, are equivalent if there is an HDA C and a diagram $A \leftarrow C \rightarrow B$ where directed paths, dipaths, and directed homotopies, dihomotopies, lift along both maps.

A dicovering wrt. $x_0 \in X$ is a map $p : Y \rightarrow X$ such that all dipaths and dihomotopies initiating in x_0 lift uniquely. The universal dicovering $\pi : \tilde{X} \rightarrow X$ has been constructed earlier, and has the following property: Given a dicovering $p : Y \rightarrow X$, there is a map $\phi(p) : \tilde{X} \rightarrow Y$, which with mild assumptions is a dicovering. In a dicovering, dipaths and dihomotopies are required to lift uniquely, whereas Van Glabbeek only asks that they lift, i.e., a fibration property. However, if A and B, are equivalent HDA, then their universal dicoverings are equivalent and vice versa.

The universal dicovering of a directed space is the space of directed paths up to directed homotopy, so the universal dicovering of an HDA is the space of all executions up to equivalence. Similarly, the universal dicovering could perhaps be of interest in the study of the topology of space time. It does give ditopological information, and if one would try to interpret the physical meaning, the universal dicovering encodes "all time-like paths" up to equivalence (!) Whether the equivalence relation between dipaths induced by dihomotopy has a physical meaning, has to be discussed.

We give a combinatorial construction of the universal dicovering and see how dicoverings are constructed from the universal dicovering as quotients, hence providing a zoo of equivalent HDA.

Keywords: Higher dimensional automata, dicovering, dihomotopy

An Operational Domain-Theoretic Treatment for Recursive Types

Weng Kin Ho (University of Birmingham, GB)

An operational domain theory is developed for treating recursive types.

The principal approach taken here deviates from classical domain theory in that we do not produce recursive types via inverse limit constructions - we have it for free by working directlly with the operational semantics of FPC. The important step taken in this work is to extend type expressions to legitimate n-ary functors on suitable 'syntactic' categories. To achieve this, we rely on operational versions of the Plotkin's uniformity principle and the minimal invariance property. This provides a basis for us to introduce the operational notion of algebraic compactness. We then establish algebraic compactness results in this operational setting. In addition, a "pre-deflationary" structure is derived on closed FPC types and this is used to generalise the "Generic Approximation Lemma" recently developed by Huttons and Gibbons. This lemma provided a powerful tool for proving program equivalence by simple inductions, where previously various other more complex methods had been employed.

Keywords: Operational domain theory, recursive types, FPC, realisable functor, algebraic compactness, generic approximation lemma, denotational semantics

Full Paper:
http://www.sciencedirect.com/science/journal/15710661

See also: See bibliography in paper

Free Lunch in the Borel Hierarchy for Partial Spaces?

Michael Huth (Imperial College London, GB)

In this talk we present a domain-theoretic model for abstraction and refinement of partial transition systems. The model is fully abstract, universal, and contains transition systems up to bisimulation as maximal-points space. We study the descriptive complexity of sets of points that satisfy a fixed property of temporal logic. Such a satisfaction notion comes in two guises: an efficient one, that treats conjunction and disjunction as having "independent" clauses; and an expensive one, that retains all dependencies for these logical operators. We then study how much precision is actually lost for important properties when relying on the efficient analysis and demonstrate that, often, there is no loss of precision or the formula can be transformed with a linear blow-up so that the efficient analysis on the transformed formula renders the result of the expensive analysis for the original formula. We then report results on the complexity for deciding whether formulas of propositional logic lose precision if evaluated under the efficient analysis.

Joint work of: Antonik, Adam; Godefroid, Patrice; Jagadeesan, Radha; Schmidt, David

Acknowledgment: Victor Selivanov.

Keywords: Partial transition system, abstraction, model checking, precision, complexity

On the bitopological nature of Stone duality

Achim Jung (University of Birmingham, GB)

In 1936, Stone published representation theorem for Boolean algebras which associates with every Boolean algebra a topological space.

A year later, he published a similar theorem for distributive lattices. More general still, Jung and Suenderhauf established a duality between proximity lattices and stably compact spaces in 1995. All these dualities are bitopological in nature, although the spaces are very special and one topology suffices for their description. If one wants to prove a more general duality, then the bitopological point of view must be made explicit.

We present a general framework for Stone duality for bitopological spaces based on frame-theoretic tools.

There is also a logical reading of this work. Pairs of opens from the two topologies can be read as the extents of partial predicates. This is in line with Smyth's dictum that observable properties are modelled by topologies, and with three and four-valued logics. One of our main results is a compact and precise algebraisation of three valued logics, similar to Boolean algebras for classical propositional logic; we propose the name 'partial frame' for these.

We also show a Hofmann-Mislove Theorem for d-sober spaces in the regular setting, supporting Martin Escardo's postulate that this is a statement about universal quantification.

Keywords: Bitopological spaces, Stone duality, d-frames, d-sober spaces, Hofmann-Mislove Theorem, three-valued logic

Joint work of: Jung, Achim; Moshier, M. Andrew

Completions of partial metric spaces

Ralph Kopperman (City University of New York, USA)

In this paper we investigate some notions of completion of partial metric spaces, including the bicompletion, to some extent the Smyth completion, and a new "spherical completion". All these completions are dcpo's, and given a poset with auxiliary relation, we find a partial metric whose spherical completion is its round ideal completion.

We give an example of a partial metric space which is a continuous dcpo with respect to the order it induces, but its bicompletion and Smyth completion are not continuous posets. We also give an example of a poset in which each element is the directed sup of those way-below it, but whose spherical completion is not a continuous dcpo.

Keywords: Partial metric space, continuous dcpo, bicompletion, Smyth completion, spherical completion

Joint work of: Kopperman, R. D.; Matthews, S. G. ; Pajoohesh, H.

Instant topological relationships hidden in the reality

Martin Kovar (Techn. University - Brno, CZ)

In most applications of general topology, topology usually is not the first, primary structure, but the information which finally leads to the construction of the certain, for some purpose required topology, is filtered by more or less thick filter of the other mathematical structures. This fact has two main consequences:

(1) Most important applied constructions may be done in the primary structure, bypassing the topology.

(2) Some topologically important information from the reality may be lost (filtered out by the other, front-end mathematical structures).

Thus some natural and direct connection between topology and the reality could be useful. In this contribution we will discuss a pointless topological structure which directly reflects relationship between various locations which are glued together by possible presence of a physical object or a virtual "observer".

Keywords: Pointless topology, reality

Extended Abstract: http://drops.dagstuhl.de/opus/volltexte/2007/896

How to make predictions-related computations feasible: From domain-like computational structures for modeling space, time, and causality to the use of non-trivial space-time-causality processes in computations

Vladik Kreinovich (University of Texas - El Paso, USA)

One of the main objectives of physics is to predict the future behavior of real-world systems. To describe space, time, causality, and physical processes in general, modern physics uses complex mathematical models: e.g., a simple quantum description of a curved space-time is a wave function defined on the class of all pseudo-Riemannian manifolds. The complexity of these models makes the prediction-related computational problems very difficult: either algorithmically unsolvable or extremely time-consuming – up to the point where by the time we finish computations, the predicted event has already occurred. How can we speed up these computations?

One possible solution to this problem is to take into account that in modern physics, many quantities used in the corresponding equations (e.g., the wave function) are not directly observable.

Hopefully, we can save time if we restrict ourselves to only computing directly observable quantities. The efficiency of such an operationalistic approach is well known – it helped Einstein and Heisenberg in the design of modern physics.

If we restrict ourselves to directly observable results, then each measuring instrument can be represented as a finite graph (or, more generally, a simplicial complex) in which vertices are possible measurement results and vertices a and b are connected by an edge iff a and b can come from measuring the same quantity. A physical quantity can then be naturally represented as a sequence of more and more accurate measuring instruments – i.e., as a projective limits of such graphs (or complexes). The resulting domain-like theory [3] has a clear computational advantage – that higher order objects like functions or operators can be described by similar graphs (complexes) and are, thus, algorithmically computable.

Algorithmically computable may still mean requiring a lot of computation time – especially since it can be shown that computations with space-time models require much more time than similar computations with spatial models [1].

How can we decrease the computation time? In general, some prediction problems may require a long time, but in practice, we can actually predict. It is thus reasonable to treat this "predictability" as a new physical "law" (similar to anthropic principle) – and indeed, we show that many un-explained facts from physics (e.g., Dirac's relations between large numbers) can be thus explained [2].

Even with these restrictions on physical models, computations may take too long. One possible explanation is that traditional computational complexity estimates are based on traditional physics and traditional space-time. It is well known that quantum effects can drastically speed up computations. We show that space-time-causality processes – ranging from highly curved space-times to acausal processes – can also lead to a drastic computational speed up [2,4,5].

Keywords: domains, computational complexity, use of non-trivial space-time-causality in computations

See also: [1] V. Kreinovich, Space-time is 'square times' more difficult to approximate than Euclidean space, *Geombinatorics*, 1996, Vol. 6, No. 1, pp. 19–29.

[2] V. Kreinovich and A. M. Finkelstein, Towards Applying Computational Complexity to Foundations of Physics, *Notes of Mathematical Seminars of St. Petersburg Department of Steklov Institute of Mathematics*, 2004, Vol. 316, pp. 63–110.

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[4] V. Kreinovich and L. Longpré, Fast Quantum Algorithms for Handling Probabilistic and Interval Uncertainty, *Mathematical Logic Quarterly*, 2004, Vol. 50, No. 4/5, pp. 507–518.

[5] D. Morgenstein and V. Kreinovich, Which algorithms are feasible and which are not depends on the geometry of space-time", *Geombinatorics*, 1995, Vol. 4, No. 3, pp. 80–97.

Balanced quasi-metrics revisited

Hans-Peter Albert Künzi (University of Cape Town, ZA)

It is well known that D. Doitchinov has found an interesting and highly original method to complete quasi-metric spaces. Unfortunately his theory only applies to balanced quasi-metrics which makes it useless for many applications.

In our talk we shall investigate the question whether the underlying construction can be amended so that the underlying completion theory becomes useful in some broader context.

Keywords: Quasi-metric, completion, balanced quasi-metric, quiet quasi-uniform space

Joint work of: Künzi, Hans-Peter; Makitu, Charly

Quasicontinuous Functions, USCO maps, and a Generalized Calculus

Jimmie D. Lawson (Louisiana State University, USA)

A subset of a topological space is said to be quasiopen if has a dense interior and a function is quasicontinuous if the inverse image of every open set is quasiopen. The topological theory of quasicontinuous functions on compact metric spaces has seen considerable recent development, motivated by connections with dynamical systems, upper semicontinuous multifunctions (USCO-maps), and viscosity solutions of certain types of pde's.

The latter is closely connected to the ability to extend the differential calculus to the quasicontinuous setting. We trace some of the recent developments of the topological theory of quasicontinuous functions and sketch their generalized differential calculus.

Keywords: Quasicontinuous map, USCO-map, domain, approximate functions, generalized calculus

Introducing complete points to topology

Steve G. Matthews (University of Warwick, GB)

W.W. Wadge once introduced the notions of 'complete' and 'partial' point for an element in a domain. A complete point is one that cannot be futher completed, and thus is maximal, although the reverse is not necessarily true. Thus the notion of 'complete point' is consistent with domain theory, but, may we say, supra domain theoretic. The argument here is that to reason domain theoretically about computer programs often requires additional information to be added to the domain. Stepping back into the foundations of domain theory raises the question of how the notion of 'complete point' can be understood in general topology? The experience we have so far is that this entails two topologies over the same set. A correspodence with this bi-topological work on complete points corresponds closely to work in fuzzy set theory. The talk will discuss this work using partial metric spaces.

Keywords: Domain, complete, partial-metric

Domains and the Causal Structure of Spacetime

Prakash Panangaden (McGill University - Montreal, CA)

In a recent paper (A Domain of Spacetime Intervals in General Relativity by Keye Martin and Prakash Panangaden, Communications in Mathematical Physics, Volume 267, Number 3, November, 2006 pp. 563-586, available online Aug 15.) Keye Martin and I showed how to reconstruct the spacetime topology for a globally hyperbolic spacetime from the causal structure using purely domain theoretic ideas. We gave an axiomatic notion of globally hyperbolic poset. We also showed that one can axiomatize the notion of interval domain and define a category of interval domains. The category of interval domains turns out to be equivalent to the category of globally hyperbolic posets.

The next most important remaining problem is understanding differential geometry on posets.

This is important for developing the theory of matter fields on causets.

Keywords: Spacetime, Relativity, Domains, Bicontinuity, Scott topology, interval topology, global hyperbolicity

Full Paper:
http://www.cs.mcgill.ca/~prakash/cmp.pdf

Closure and Causality

John L. Pfaltz (University of Virginia, USA)

We present a model of causality which is defined by the intersection of two distinct closure systems, $\mathcal{I}$ and $\mathcal{T}$. Next we present empirical evidence to demonstrate that this model has practical validity by examining computer trace data to reveal causal dependencies between individual code modules. From over 498,000 events in the transaction manager of an open source system we tease out 66 apparent causal dependencies. Finally, we explore how to mathematically model the transformation of a causal topology resulting from unforlding events.

Keywords: Closure, causality, antimatroid, temporal, software engineering

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/897

Enriched categories and models for spaces of dipaths

Timothy Porter (University of Wales - Bangor, GB)

Partially ordered sets, causets, partially ordered spaces and their local counterparts are now often used to model systems in computer science and theoretical physics. The order models 'time' which is often not globally given. In this setting directed paths are important objects of study as they correspond to an evolving state or particle traversing the system. Many physical problems rely on the analysis of models of the path space of a space-time manifold. Many problems in concurrent systems use 'spaces' of paths in a system. Here we review some ideas from algebraic topology that suggest how to model the dipath space of a pospace by a simplicially enriched category.

Keywords: Enriched category

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/898

Mereotopology: A Survey

Ian Pratt-Hartmann (Manchester University, GB)

A mereotopology is a Boolean subalgebra of the regular open algebra of a topological space, satisfying a certain density condition. Any mereotopology can therefore be regarded as a structure interpreting a signature of mereological and topological predicates, and hence as a model of a theory of 'space' in which the basic objects are thought of as regions, rather than points. This talk surveys some recent results on the abstract characterization of various classes of mereotopologies, including mereotopologies over the Euclidean plane.

Keywords: Mereology, topology, mereotopology, first-order logic, space

Reparametrizations of Continuous Paths

Martin Raussen (Aalborg University, DK)

In elementary *differential geometry*, the most basic objects studied (after points perhaps) are *paths*, i.e., *differentiable* maps $p : I \to \mathbf{R}^n$ defined on the closed interval $I = [0,1]$. Such a path is called *regular* if $p'(t) \neq \mathbf{0}$ for all $t \in (0,1)$. A *reparametrization* of the unit interval I is a surjective differentiable map $\varphi : I \to I$ with $\varphi'(t) > 0$ for all $t \in (0,1)$, i.e. a (strictly increasing) self-diffeomorphism of the unit interval. Given a path $p : I \to \mathbf{R}^n$ and a reparametrization $\varphi : I \to I$, the paths p and $p \circ \varphi$ represent the same geometric object.

In differential geometry one investigates equivalence classes (identifying p with $p \circ \varphi$ for any reparametrization φ) and their invariants, like curvature and torsion.

Motivated by applications in *concurrency theory*, a branch of theoretical Computer Science trying to model and to understand the coordination between many different processors working on a common task, we are interested in *continuous* paths $p : I \to X$ in more general topological spaces up to more general reparametrizations $\varphi : I \to I$. When the *state space* of a concurrent program is viewed as a topological space (typically a cubical complex), "directed" paths in that space respecting certain "monotonicity" properties correspond to *executions*. A nice framework to handle *directed* topological spaces (with an eye to homotopy properties) is the concept of a d-space proposed and investigated by Marco Grandis. Essentially, a topological space comes equipped with a subset of preferred d-paths in the set of all paths in X. Note in particular, that the reverse of a directed path in general is *not* directed; the slogan is "breaking symmetries".

We do not try to capture the *quantitative* behaviour of executions, corresponding to particular parametrizations of paths, but merely the *qualitative* behaviour, such as the order of shared resources used, or the result of a computation. Hence the object of study are paths up to certain reparametrizations which

1. do not alter the *image* of a path, and
2. do not alter the *order of events*.

We are thus interested in general paths in topological spaces, up to *surjective* reparametrizations $\varphi : I \to I$ which are *increasing* (and thus continuous), but not necessarily strictly increasing. Two paths are considered to have the same behaviour if they are *reparametrization equivalent*. To understand this equivalence relation, we have to investigate the space of all reparametrizations which includes strange (e.g. nowhere differentiable) elements. Nevertheless, it enjoys remarkable properties: It is a monoid, in which compositions and factorizations can be completely analysed through an investigation of *stop intervals* and of *stop values*. The quotient space after dividing out the self-homeomorphisms has nice algebraic lattice properties.

A path is called *regular* if it does not "stop"; and we are able to show that the space of general paths *modulo reparametrizations* is homeomorphic to the space of *regular* paths *modulo increasing auto-homeomorphisms* of the interval. Hence to investigate properties of the former, it suffices to consider the latter. This is a starting point in the homotopy theoretical and categorical investigation of invariants of d-spaces.

This is essentially an elementary talk. Almost all concepts and proofs can be understood with an undergraduate mathematical background. There are certain parallels to the elementary theory of distribution functions in probability theory. The flavour is nevertheless different, since continuity (no jumps, i.e., surjectivity) is essential for us.

Keywords: Path, dipath, reparametrization, stop interval, stop value, lift, lattice, quotient

Joint work of: Raussen, Martin; Fahrenberg, Ulrich

Full Paper:
http://www.math.aau.dk/research/reports/R-2006-22.pdf

Algebraic structures arising from the basic picture

Giovanni Sambin (Universitá di Padova, I)

If one expresses the notion of interior and of closure in the setting of a basic pair (that is, a set of points and a set of observables, linked by an arbitrary forcing relation), then one can see that they are dual of each other in a precise logical sense. This is the beginning of the basic picture, a new theory which underlies and generalizes constructive topology. The duality extends to formal notions, a cover being dual of a positivity relation.

Also morphisms can be analyzed in a fully structural way, the essence of continuity being a commutative square of relations.

One obtains topology as a special case, by adding a postulate of convergence.

To deal properly with notions like closed, positivity, coinduction, which are dual of open, cover, induction, it is essential to use systematically the notion of overlap of two subsets: D overlaps E if D and E have inhabited intersection. Then, in the definition of the algebraic counterpart of the structure of subsets, it is natural to introduce a new primitive, corresponding to overlap in the same way as partial order corresponds to inclusion. This I call overlap algebras. The extra expressive power granted by the new primitive is sufficient to express all of the basic picture algebraically. One thus reaches an algebraic version of the basic picture, and hence of topology, which is a proper generalization of the set-theoretic approach.

Keywords: Constructive topology, formal topology, basic picture, overlap algebras, open and closed subsets

Countable Pseudobases and Relative Compactness

Matthias Schröder (University of Edinburgh, GB)

In this talk, I investigate three notions of pseudobases for topological spaces and show that they are equivalent, if they are countable. Countable pseudobases play an important role in Computable Analysis: Exactly the T_0-spaces with a countable pseudobase have an admissible representation. Moreover, QCB-spaces (= quotients of countably-based spaces) are characterised as those sequential spaces that have a countable pseudobase. Based on a recent result by Peter Nyikos on compactness in spaces with a network of low cardinality, I show that relative compactness and relative sequential compactness agree for subsets of QCB-spaces.

Keywords: Computable Analysis, QCB-spaces, Relative Compactness

TCS-oriented variations on the Wadge reducibility

Victor Selivanov (Pedagogical University - Novosibirsk, RUS)

Wadge reducibility on the Baire and Cantor spaces is of primary importance for descriptive set theory. Some of its variations are also useful in several fields of theoretical computer science. We discuss some such variations including:

- effective versions of the Wadge reducibiility in the Cantor space used for characterizing infinite behaviour of computing devices;
- Wadge reducibility and its effective versions in some other spaces of interest for computer science, e.g. in the space of reals or in algebraic domains;
- extension of the Wadge reducibility to the case of finite partitions of the above-mentioned spaces.

Keywords: Wadge reducibility, hierarchy, space, domain, automaton

A convenient category of domains

Alex Simpson (University of Edinburgh, GB)

We motivate and define a category of "topological domains", whose objects are certain topological spaces, generalising the usual ω-continuous dcpos of domain theory.

Our category supports all the standard constructions of domain theory, including the solution of recursive domain equations. It also supports the construction of free algebras for (in)equational theories, provides a model of parametric polymorphism, and can be used as the basis for a theory of computability.

This answers a question of Gordon Plotkin, who asked whether it was possible to construct a category of domains combining such properties.

Keywords: Domain theory, topology of datatypes

Joint work of: Battenfeld, Ingo; Schroeder, Matthias; Simpson, Alex

Full Paper: http://drops.dagstuhl.de/opus/volltexte/2007/894

Convenient topologies on the digital plane

Josef Slapal (Techn. University - Brno, CZ)

We introduce and study a new topology on the digital plane Z^2. As a criterion of convenience for applications in digital topology, we prove an analogue of the Jordan curve theorem for the topology introduced. The new topology is discussed in relation to the two classical topologies used in digital topology, namely the Khalimsky and Marcus topologies, and its advantages over them are shown. We also investigate another convenient topology on Z^2 which is obtained as a quotient topology of the topology studied.

Keywords: Digital topology, digital plane, Khalimsky topology, Alexandroff topology, Jordan curve

Regions in discrete space.

Michael B. Smyth (Imperial College London, GB)

Space is viewed as a locally finite scattering of points. A relation of "overlap" (= connection, as in mereotopology) is defined over the cells of the space. (Formally, a cell is just a finite set of points.) Thus we have a graph G of cells. We argue that the ortholattice OG of (ortho)closed subsets of vertices of G is appropriately taken as the lattice of regions of the space. This lattice is not distributive in general. However in the geometric setting we can show, by invoking a theorem in quantum logic due to Foulis and Randell (1973), that the lattice OG is orthomodular. This means that orthomodular lattice theory can be applied in computational geometry. In particular, a triangulation of the space is in effect the choice of a maximal Boolean subalgebra (that is, a block) of the region lattice.

This talk is based on "Discrete Spatial Models", by M. Smyth and J. Webster, to appear in the Handbook on Spatial Logics, eds. Aiello, van Benthem, Pratt-Hartmann.

Keywords: Space, computational geometry, orthomodular lattices, triangulation, region lattice

A Construction Method for Partial Metrics

Dieter Spreen (Universität Siegen, D)

Partial metrics are weak metrics. They are still symmetric, but the distance of a point to itself may be zero. Partial metrics are used in spaces which beside of the total (ideal) objects that are usually studied in mathematics, contain partial objects appearing in the construction (computation) of the ideal ones. An important example of such spaces are domains in the sense of Scott. As has indeed been shown by M. B. Smyth, ω-continuous domains are partial metrizable. It is well known that they are not metrizable in the usual sense. As do metrics in the case of e.g. the real numbers, in spaces like domains partial metrics allow to measure how close an approximation is to the result it is approximating. The self-distance gives a measure of partialness.

In the talk a method for the construction of partial metrics out of a given quasi-uniformity with countable base is presented. It is shown that the quasi-uniformity induced by the partial metric thus obtained is the given one. Thus, the partial metric induces the same topology as the given quasi-uniformity. With respect to this partial metric the set of elements of self-distance zero (which can be considered as the final approximation results) is a G_δ set.

Keywords: Partial metric, domain theory, quasi-uniformity, asymmetric topology

Localising Topology and Geometry in a Manifold-like Causal Set

Sumati Surya (Raman Research Institute - Bangalore, IND)

In the causal set approach to quantising gravity, continuum spacetime emerges as an approximation to an underlying locally finite partial order.

In this talk I will discuss this approximation scheme and show how a localised topology on the causet encodes the homology of a continuum globally hyperbolic spacetime. This localised topology can also be used to obtain a distance function which approximates to the spacelike distance on a Cauchy hypersurface in the continuum.

Keywords: Causal set, quantum gravity, hyperbolic spacetime

Joint work of: Surya, Sumati; Seth, Major; Rideout, David

On the Reaxiomatisation of General Topology

Paul Taylor (Manchester University, GB)

Abstract Stone Duality (ASD) started in 1993 by asking "what if" we take Paré's Theorem (that the contravariant powerset functor is monadic) as an axiom. I write $\Sigma^{(-)} \dashv \Sigma^{(-)}$.

That question began to get an answer in 1997 when I discovered another important, but easily overlooked, property of the powerset that I called the Euclidean principle, $F\sigma \wedge \sigma \iff F\top \wedge \sigma$, which led to an account of the quantifiers in a topos.

Remarkably, the only further component that was needed to turn this into a theory of locally compact topological spaces was the axiom that all maps are Scott-continuous.

Spaces that admit the quantifiers iff they are respectively compact and overt.

Recently, Andrej Bauer and I have specialised the theory to the real line. It worked like a dream, and there are very natural links to both elementary differential calculus and exact real computation. For the benefit of real analysts, I have even found a way of presenting the theory that hides its roots in category theory, lambda calculus and continuous lattices.

Unlike the real line in some other theories of recursive analysis, the one in ASD satisfies the Heine–Borel Theorem, that the closed interval is compact. This discovery vindicated my categorical intuition that the adjunction $\Sigma^{(-)} \dashv \Sigma^{(-)}$ should be monadic, whereas others had considered this idea bizarre and awkward, and had sought to omit from the theory. Unfortunately, working with this principle always seems to lead one back into the locally compact situation.

At Dagstuhl I propose to map out may escape plan, ie the generalisation of the theory beyond local compactness. I believe that the key is the subspace topology, ie how the functor $\Sigma^{(-)}$ interacts with equalisers. This time, my intuition

conflicts with the currently available models, because in cartesian closed categories of spaces, the Sierpiński space Σ cannot be injective in the usual external sense. I will explain this, and the examples that support my intuition, including the weak*-topology from functional analysis, and the relationship between compact subspaces and modal operators.

The mathematically formal theory, on the other hand, is able to reconstruct at least the category of spatial locales from those of my axioms that already have a model, including a non-computable "underlying set" axiom.

In conclusion, we now know what the picture is on the jig-saw puzzle, even though one or two of the pieces are still missing or disconnected.

The picture is complete enough for others to apply it to exact real computation, to begin to formulate functional analysis, and to look for new categorical constructions that might provide a model of the intuition.

Keywords: Stone duality, Heine–Borel, recursive analysis

Full Paper:
www.cs.man.ac.uk/∼pt/ASD/extension.html

Lawson Topology of the space of formal balls and the hyperbolic subbase

Hideki Tsuiki (Kyoto University, J)

The set B(X) of formal balls in a metric space X has two natural topologies; the Lawson topology and the product topology. We investigate the difference of these two topologies, in particular for the case X is a normed vector space. Then, we consider another topology of X which is induced by the metric of X, that is, generated by insides and outsides hyperbolic curves in X, and study its relation to the Lawson topology of B(X).

Keywords: Lawson Topology, Formal Balls, Domain Environment

Joint work of: Tsuiki, Hideki; Hattori, Yasunao

Only connect

Steven J. Vickers (University of Birmingham, GB)

The Intermediate Value Theorem is a fundamental theorem of analysis that follows from a topological fact, that intervals in the real line are connected. It is not straightforward to give a choice-free, constructive version for locales.

I shall describe an approach that uses a "logic of connected parts", based on the logic of parts implicit in the Vietoris hyperspace topology or Johnstone's Vietoris powerlocale or the Plotkin powerdomain.

In these, if U in open in X then $\Box U$ contains those parts of X included in U, and $\Diamond U$ contains those parts that meet U.

Then connectedness of parts can be enforced by an axiom

$$\Box(U \vee V) \to \Box U \vee \Box V \vee \Diamond(U \wedge V)$$

This gives rise to a "connected Vietoris powerlocale" $V^c X$, a sublocale of Johnstone's.

The parts are all required to be compact and overt.

For the real line $\mathbb{R}$, we have a homeomorphism between $\leq$ and $V^c\mathbb{R}$, where "$\leq$" denotes the locale of pairs (a, b) of reals with $a \leq b$. The pair (a, b) is taken to the closed interval $[a, b]$. The proof of this uses a localic treatment of complete metric spaces and their powerlocales.

A localic form of the Intermediate Value Theorem can now be stated.

For a map $f : \mathbb{R} \to \mathbb{R}$, $V^c(f)([a, b])$ is the localic image of $[a, b]$ under f and is itself a compact interval. Hence if $f(a) \leq 0 \leq f(b)$ then 0 is in the image.

I shall also mention preliminary findings on higher dimensional analogues (simple connectedness etc.) with a view to giving a similar treatment of Brouwer's fixed point theorem.

Keywords: Locale, connected Vietoris powerlocale

Bifinite Chu Spaces

Guo-Qiang Zhang (Case Western Reserve University, USA)

In the category of Chu spaces with mono morphisms, finite objects are distinct from finite structures (i.e., Chu spaces whose constituents are finite sets). Instead, finite objects are precisely the extensional finite structures. This motivates the consideration of a category whose objects are colimits of ω-sequences of finite, extensional Chu spaces (**SFCS**), in a similar fashion as Plotkin's SFP (i.e. bifinite domains). The finite objects of **SFCS** have the joint embedding property and the amalgamation property. Together with other properties, this category is shown to be algebroidal, and hence there exists a universal, homogeneous object using a result of Droste and Göbel. An internal characterization of objects in **SFCS** is provided, as finite branching, infinite trees with a unary predicate. Explicit random construction of an universal, homogeneous object is also provided.

Keywords: Domain theory, approximation, category theory, Chu spaces

Joint work of: Droste, Manfred; Zhang, Guo-Qiang