04172 Abstracts Collection
Perspectives Workshop: Visualization and Image Processing of Tensor Fields
— Dagstuhl Seminar —

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Abstract. From 18.04.04 to 23.04.04, the Dagstuhl Seminar 04172 "Perspectives Workshop: Visualization and Image Processing of Tensor Fields" was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Warping and Registration of Diffusion Tensor MR Images

Daniel Alexander (University College London, GB)

I will discuss the problems of warping and registration of diffusion-tensor MR images. The orientational information complicates the process of warping tensor fields, since the warp affects both the location and orientation of the data. I will review the literature on strategies for data reorientation during tensor-field warping.

Registration and matching of tensor fields has similar complications, since we must reorient tensors in a warped image before comparing it to the target. However, the orientational information in tensor fields potentially enhances the matching process. Homogeneous regions of scalar data are very difficult to match accurately, since a wide variety of transformations provide the same similarity score. The variety of transformations that can match homogeneous regions of tensor data, however, is much smaller since the orientations must match after warping. I will describe some early attempts to exploit orientation matching to improve image registration.

Keywords: Diffusion-tensor MR images, Warping, Registration

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Means and Interpolations of Positive Definite Tensors using Riemannian Geometry

Philipp Batchelor (King's College London, GB)

In DT-MRI, the tensors acquired are positive definite: they have all positive eigenvalues.

It is often felt that this should be taken into account for different operations on tensors, such as interpolation, but there doesn't seem to be a clear framework for doing so. I will describe methods inspired from Riemannian Geoemtry which provide sucha framework.

Keywords: Positive definite tensors, DT-MRI, symmetric spaces

The Nonlinear Structure Tensor

Thomas Brox (Universität Saarbrücken, D)

The structure tensor is a matrix field that is very popular in image processing and computer vision. Holding the gradient information of a local neighborhood, it can be used to estimate the dominant orientation, magnitude, or coherence of the local structure.

With the findings of matrix-valued nonlinear diffusion, it is possible to introduce a nonlinear structure tensor by replacing the Gaussian convolution of the classic structure tensor.

The performance of the nonlinear structure tensor is demonstrated in three computer vision tasks where the classic structure tensor has already been successfully applied: optic flow estimation, texture discrimination, and corner detection. In a direct comparison, the nonlinear structure tensor shows its superiority towards its linear counterpart.

Keywords: Structure Tensor, Nonlinear Diffusion, Optic Flow Estimation, Texture Discrimination, Corner Detection

Joint work of: Brox, Thomas; Weickert, Joachim; Mrazek, Pavel

Mathematical Morphology on Tensor Data

Bernhard Burgeth (Universität Saarbrücken, D)

Mathematical morphology provides well established and successful techniques for image processing and analysis. It would be very desirable to have these tools at our disposal for the processing of tensor-valued data. The notion of supremum and infimum is essential for the definition of morphological operations. In this talk three different novel definitions of maximum/minimum for tensors are introduced and discussed. The definitions are based on algebraic, geometric or ordering (Loewner ordering) properties of symmetric matrices. We investigate the resulting morphological operations and gradients theoretically and by experiments on positive semidefinite as well as indefinite tensor fields.

Keywords: Morphological operations, morphological gradients, Loewner ordering, tensor fields

Joint work of: Burgeth, Bernhard; Welk, Martin; Feddern, Christian; Weickert, Joachim

Tensor Valued Level Set Methods

Christian Feddern (Universität Saarbrücken, D)

Matrix valued data appears in a number of applications, for example in magentic resonance imaging (DT-MRI). Therefore methods to filter and segment such data sets are needed. In order to design such filters, we have extended the concept of Di Zenzo's structure tensor for vector valued data to tensor valued data. This structure tensor then allows us to extend scalar valued mean curvature motion, self snakes and active contour models to the tensor valued case. Due to using information from all tensor channels, these extended filter versions are highly robust under noise, which will be shown by experiments on DT-MRI data.

Keywords: DT-MRI, tensor valued, mean curvature moition, self snakes, active contours.

Joint work of: Feddern, Christian; Weickert, Joachim; Burgeth, Bernhard; Welk, Martin

Some developments in DT-MRI registration and validation

James Gee (University of Pennsylvania, USA)

In this talk, I will sketch three ideas related to the registration and visualization of DT-MRI brain data. First, a novel similarity metric with which to drive diffusion tensor registration is described that exploits the unique shape and orientation characteristics of our diffusion ellipsoids. Specifically, the pattern of pairwise orientation differences between the voxel of interest located at x and every voxel within a neighborhood centered at x is proposed as a more robust and accurate replacement of the usual voxelwise comparison of orientation information (either at a voxel or over a region).

Preliminary results indicate this new metric may reduce the number of local minima typically observed with standard applications of diffusion orientation in tensor registration. The second part of the presentation will consider the diffusion MRI registration problem from the more general perspective of abitrary diffusion profiles as opposed to the Gaussian distributions assumed in DT-MRI. In practice, non-Gaussian diffusion profiles occur whenever fibers cross in white matter, thus MRI reconstruction techniques that can accommodate multiple fiber orientations are an active area of research. The naturally induced L_2 distance between positive-valued spherical functions is specialized to the case of diffusion tensors,

and this is coupled with a non-standard affine parameterization that facilitates the finite strain-based reorientation of tensors adopted in this work.

Preliminary results demonstrating the piecewise affine extension to high dimensional non-rigid registration of DT-MRI data will be shown.

The final topic I will discuss leverages image registration to warp a labeled brain atlas to segment extracted fiber tracts from an individual, thus enabling an anatomical basis for the visualization of white matter tracts.

Keywords: Diffusion tensor, Diffusion profiles, Registration, Visualization, Tractography, MRI, Corpus callosum, Brain atlas

Spatial Smoothing for Diffusion Tensor Imaging with high noise level

Klaus Hahn (GSF - Neuherberg, D)

Very noisy or low signal to noise ratio (SNR) experiments in diffusion tensor imaging (DTI) give key information about tracking and anisotropy, e.g., by measurements with small voxel sizes or with high b values. However, due to the complicated and dominating impact of thermal noise such data are still seldom analysed. A method to overcome this situation is proposed, the following topics are addressed:

a) Noise, edges and curvature in DTI data
b) The random fields of the DTI variables
c) Negative Eigenvalues
d) A nonlinear spatial Gaussian filter
e) Bias correction and statistical properties of the smoothing method
f) Denoising of an experiment with very small voxels

The talk will stress statistical features of general interest, when modelling of the tensor fields of DTI data is intended.

Keywords: Random Fields, Statistics, nonlinear Spatial Smoothing, Experimental Applications

Joint work of: Hahn, Klaus; Prigarin, Sergei; Hasan, Khader

Tensor Splats in Medicine and General Relativity

Hans-Christian Hege (K. Zuse Zentrum Berlin, D)

First, the basic problems of tensor field visualization are outlined:

(1) the large number of degree of freedoms per space-time point which have to be encoded in color, texture or geometry, and

(2) the occlusion in $D>2$ which can be circumvented only partially by use of transparent, sparsely distributed graphical objects that encode higher level structural information.

Then a new general-purpose technique is described for the visualization of 3D time-dependent symmetric positive definite tensor fields of rank two. It is based on a splatting technique that builds on tiny transparent glyphs which incorporate the full orientational information content of a tensor. The result are information-rich images that allow to read off the preferred orientations in a tensor field. The technique is useful for analyzing slices or volumes of a 3D tensor field. It can be easily combined with standard volume rendering for display of an additional scalar field.

The application of the rendering technique is demonstrated on

- medical diffusion tensor imaging data (DT MRI)
- the spatial part of the space-time metric tensor arising as solution of the Einstein equations, e.g. in numerical simulations of black hole collisions.

Keywords: Tensor field visualization, splatting, volume rendering, diffusion tensor imaging, metric tensor, space-time, Einstein equations

Joint work of: Hege, Hans-Christian; Benger, Werner

Tensor Field Visualization Using a Metric Interpretation

Ingrid Hotz (Univ. of California, Davis, USA)

The physical interpretation of mathematical features of tensor fields is highly application-specific. We present a visualization method tailored specifically to the class of tensor field exhibiting properties similar to stress and strain tensors, which are commonly encountered in geomechanics.

Our technique is a global method that represents the physical meaning of these tensor fields with their central features: regions of compression or expansion. The method is based on two steps: first, we define a positive definite metric, with the same topological structure as the tensor field; second, we visualize the resulting metric. The eigenvector fields are represented using a texture-based approach. Our method supports an intuitive distinction between positive and negative eigenvalues.

Tensor invariants, their gradients, and their failings

Gordon Kindlman (University of Utah, USA)

The visualization and analysis of DT-MRI is a challenging task due to the multivariate nature of the tensor data and complexity of the three-dimensional structures in question. Basic ingredients of the visualization and analysis are tensor invariants – tensor metrics independent of the coordinate frame in which the tensor is expressed.

The spatial-domain gradients of these invariants enable improved visualization by providing an approximate surface normal for shading purposes. On the other hand, the value-domain gradients of the invariants provide means of characterizing the degrees of freedom in tensor shape. An interesting aspect of the invariants is their failure to characterize tensor variations near points of isotropy, at which two or three eigenvalues are equal. A framework for overcoming this limitation is described, and is used in an application to PDE-based filtering of tensor values.

Integrated Edge and Junction Detection using Tensors

Ullrich Köthe (Universität Hamburg, D)

From topology it is well known that partitionings of the plane (i.e. image segmentations) can only be correctly described if one does not only consider edges but also junctions. Typically, in image analysis one uses independent detectors for these two feature types. However, it is then very difficult to subsequently combine these independent detector responses into a single coherent boundary representation: edge and junction resposes often don't fit together, and this leads to errors (such as gaps or wrong links) in the boundary representation.

Therefore I propose new algorithms that are able to detect edges and junctions simultaneously from a tensor representation of the image. I'll demonstrate two ways to derive suitable tensors: the first approach leads to an improved version of the well-known structure tensor that is based on non-linear smoothing. The other approach uses a new class of 2-dimensional quadrature filters. In both cases, the tensor trace is a boundary strength measure that assumes high values at both edges and junctions. By means of the tensor eigenvalues, the boundary information can be decomposed into its edge and junction portions if desired.

Experiments on real images demonstrate significant improvements over existing approaches.

Some Issues in Modeling and Applications of Diffusion Imaging

David H. Laidlaw (Brown Univ. - Providence, USA)

With the goal of spurring discussion, I will discuss progress and issues in five research areas relevant to the visualization and image processing of diffusion MRI datasets. First, I will describe several biomedical applications that drive our research illustrating that, while different applications all look at white matter structure, they have different visualization and analysis needs. Second, I will outline some of the issues surrounding image acquisition in the context of these applications. Third, I will give an overview of a series of exploratory visualization methods, pointing out some limitations and future directions. Fourth, I will touch on derivation of quantitative measures from diffusion imaging data. Fifth, I will briefly mention some of the ways to validate imaging methods, visualization methods, and quantitation.

About Some Optical Flow Methods from Structure Tensor: Review and Contribution.

Francois Bernard Lauze (The IT University of Copenhagen, DK)

This paper is about optical flow estimation from structure tensor field analysis of a sequence seen as a spatio-temporal volume. It tackles two issues. The first is to review related recent techniques showing their relations. We essentially discuss the stages of flow extraction from structure tensor fields and the obtention of regularized tensor fields. The second is to propose a nonlinear approach controlled by an intuitive corner measure. The overall approaches will be compared in details on several test sequences.

Keywords: Optical Flow Structure Tensors Non-linear PDE Corner Measure

Joint work of: Lauze, Francois Bernard; Kornprobst, Pierre; Lenglet, Christophe; Deriche, Rachid; Nielsen, Mads

On the Concept of a Local Greyvalue Distribution and the Adaptive Estimation of a Structure Tensor

Hans-Hellmut Nagel (Universität Karlsruhe, D)

As a step towards a local analysis of local image features, the position, peak value, and covariance matrix of an isolated, noise-free multivariate Gaussian are determined in closed form from four 'observables', computed by gaussian-weighted averaging first and second powers of (up to second order) partial derivatives of a digitized greyvalue distribution.

Keywords: Structure tensor, local image features, optical flow, local estimation procedures, Gaussian-blob-defined local environments

Higher Rank Tensors in Diffusion MRI

Evren Ozarslan (University of Florida, USA)

Diffusion tensor magnetic resonance imaging (DT-MRI) has made it possible to map neuronal connections betweeen different structures in the brain. The underlying model used in DT-MRI employs a rank-2 positive definite symmetric tensor and assumes a Gaussian displacement profile for water molecules. Despite its apparent success in relatively simple geometries, DT-MRI has been known to have shortcomings especially in voxels with orientational heterogeneity. This problem may manifest itself by producing incorrect estimates of the fiber directions as well as inaccurate values for anisotropy. In this talk, I will be presenting a new approach called "generalized diffusion tensor imaging" that uses Cartesian

diffusion tensors of rank higher than two. Starting from the phenomenological generalization of the fundamental equation describing the transport of magnetization, it is possible to relate the observed signal values to the components of higher rank tensors. I will discuss the symmetry and multiplicity properties of these tensors. Using generalized diffusion tensor imaging, it is possible to calculate more accurate anisotropy maps and fiber orientations. Results from simulations and magnetic resonance images performed on excised rat brains and spinal cords will be presented.

Keywords: Diffusion, DT-MRI, anisotropy, higher-rank

Joint work of: Ozarslan, Evren; Vemuri, Baba C.; Mareci, Thomas H.

Continuous Approximation of DT-MRI Data

Sinisa Pajevic (National Institut of Health, USA)

The measured effective diffusion tensor of water is inherently a discrete, noisy, voxel-averaged sample of the underlying diffusion tensor field. We use and compare different continuous tensor field models to construct the approximated tensor field that best fits the data in the least square sense. The ultimate goal is to construct a model which enables one to apply differential geometric operations on the diffusion tensor field, which are notoriously unreliable when only interpolated data is used.

Keywords: Diffusion tensor MRI continuous approximation B-Spline wavelets nurbs

Degenerate 3D Tensors

Alex T. Pang (Univ. California - Santa Cruz, USA)

There are 2 types of degenerate points in 3D tensors: doubly and triply degenerate, where 2 or 3 eigenvalues are the same, respectively. We find that in typical non-degenerate tensors that are real symmetric 2nd order tensors the doubly degenerate points form lines. Furthermore, these doubly denerate topological lines are stable. Other forms exists, eg. degenerate points, surfaces and volumes.

To find these doubly degenerate points, we present a reformulation of the cubic discriminant function that is based on tensor components and does not require eigenvalues; results in cubic polynomials that are more stable to compute.

The new formulation consists of 7 polynomials that are the sum of squares of tensor components. An algorithm is then presented that connects these extracted points to form topological lines.

Joint work of: Pang, Alex T.; Zheng, Xiaoqiang

Tensor Valued Image Registration - What can we learn from Optical Flow?

Nils Papenberg (Universität Saarbrücken, D)

Optical Flow Estimation and Image Registration are two related problems in Computer Vision. So it is obvious to transfer approaches of one topic into the context of the other. In this talk I will show a way formulate both problems in an identical way and motivate the usage of several ideas from optical flow in image registration. This will not be shrinked to the registration of scalar valued imagas, a extension to tensor valued images is also given.

Keywords: Image Registration, Tensor Valued Images, Robust Estimation, Piecewise Smoothness, Multiresolution Approach, Warping

Joint work of: Papenberg, Nils; Bruh, Andres; Weickert, Joachim

Variational method for the separation of multiple tensor orientations

Ofer Pasternak (Tel Aviv University, IL)

Using DTI for modeling diffusion in voxels containing multiple fiber orientations usually provides tensors which are not aligned with the fiber orientations. This misalignment effects imaging techniques based on DTI such as fiber tracking. We offer a variational method for finding neighborly smoothed multiple tensor representation for such voxels. This method aims to allow the separation of differently oriented fiber compartment residing in the same voxel, while reducing noise effects.

Pattern Matching and Visualization on Tensor Fields

Gerik Scheuermann (TU Kaiserslautern, D)

In this talk, we will propose to transfer convolution, correlation and Fourier transform to second-order tensor fields using matrix multiplication. Since scalars can be interpreted as multiples of the identity, we get a natural extension of the usual convolution. It is shown that for this convolution and Fourier transform, the well known convolution theorem holds. Two basic ideas for pattern matching in tensor fields are discussed: matching eigenvectors and matching second-order tensors using the defined convolution.

Keywords: Tensor Visualization, Feature Detection, Convolution, Fourier transform

Joint work of: Scheuermann, Gerik; Hlawitschka, Mario; Ebling, Jila

Tensor Signal Processing

Eduardo Suárez-Santana (Univ. de Las Palmas de Gran Canaria, E)

Abstract: Classical signal processing is focused on vector spaces for signals. Edge-preserving and assuring the positive-semidefinite constraint in tensor filtering and interpolation is a new area of research in image processing, because tensors, in this sense, do not form a vector field. An approach to export signal processing linear filters to tensors based on homomorphic filtering is presented, as well as an interpolation scheme that preserves edges and semidefinite property. Different approaches to structure tensors will also be addressed. They are a key step in the interpolation approach and in some schemes of landmark extraction and registration.

Keywords: Tensors, signal processing, homomorphic, landmarks

Joint work of: Suarez-Santana, Eduardo; Ruiz-Alzola, Juan

Visualizing the Topology of Tensor Fields

Xavier Tricoche (University of Utah, USA)

In this presentation I will describe and discuss the topological approach to tensor field visualization.

This technique developed over the last decade has proved successful in permitting an efficient reduction of the rich amount of information contained in tensor quantities while allowing for insightful and accurate analysis of complex data sets. Its mathematical foundations lead back to both the qualitative theory of dynamical systems and differential geometry.

The essential notions of tensor topology will be introduced. In particular, I will explain the fundamental relationship between vector and tensor topology which is key for visualization purposes. The basic technique will be described along with improvements designed to meet the requirements of practical applications, including time-dependency and intricacy of typical data sets.

Visualization and Applications of Diffusion Tensor Imaging

Anna Vilanova i Bartroli (Eindhoven University of Technology, NL)

Diffusion Tensor Imaging (DTI) is a non invasive MR technique that measures water diffusion. DTI is used to visualize linear structures such as fibers in white matter or muscles. A visualization tool for DTI data (i.e., DTITool) has been developed in our group. This tool includes basic visualization techniques for DTI data. The DTItool also includes an algorithm to visualize planar structures which usually appear in areas of crossing or converging fibers. Most visualization techniques require the user to define an area from where the fibers are generated. In this way, the user can miss part of the information, if the area is not correctly defined. Furthermore the results are user dependent which makes difficult any comparison. In order to prevent user dependency, we present a method to visualize the structures in the whole volume with evenly-spaced distance between them. The applications and research that motivated this work and are using DTItool will be introduced.

Keywords: Crossing fibers, DTI Applications, DTI Visualization Tool

Tensor Field Interpolation

Joachim Weickert (Universität Saarbrücken, D)

Interpolation belongs to the most important operations of any image processing and visualisation system. While interpolation on scalar data fields is well understood, there is relatively little work on tensor field interpolation so far.

In this talk we will discuss interpolation methods that are based on partial differential equations (PDE) and that are closely related to so-called impainting methods. After explaining the basic ideas in the scalar-valued setting, extensions to the tensor framework will be proposed and illustrated by examples.

Keywords: Interpolation, partial differential equations, impaiting, filling-in effect, tensor fields

Tensor-Valued Median Filtering

Martin Welk (Universität Saarbrücken, D)

Median filters for scalar-valued images display a number of excellent properties that make them powerful tools for image denoising and analysis. They preserve discontinuities, they are robust under even extreme types of noise, and they stand in close relation to PDE-based image filters (mean curvature motion).

We generalise median filtering to matrix-valued data. Our generalisation is based on an energy-minimising approach; it is equally suitable for sets of positive semidefinite matrices, then preserving this essential property, and others which include indefinite tensors. The approach can even be extended to non-square matrices, vectors etc. Our experiments on DT-MRI (positive semidefinite) and fluid dynamics (including indefinite) tensor data sets demonstrate that tensor-valued median filtering shares important properties of its scalar-valued counterpart.

Finally we show that in a completely analogous way mid-range filtering can be generalised to matrix-valued data. This can serve as a building block in constructing other (e.g. supremum-based) tensor image filters.

Joint work of: Welk, Martin; Feddern, Christian; Burgeth, Bernhard; Weickert, Joachim; Schnörr, Christoph

Tensor Field Filtering: Theory and Medical Applications

Carl-Fredrik Westin (Harvard Medical School, USA)

Director, Laboratory of Mathematics in Imaging (LMI) Assistant Professor of Radiology, Harvard Medical School

In this talk we discuss various ways of filtering tensor field data and relate the methods to real world medical applications. At Harvard Medical School and Brigham and Women's Hospital we have a large image guided therapy program. Here we detail some of our applications using Diffusion Tensor MRI data for surgical planning and intraoperative guidance in brain surgery.

We further outline our work in distance-connectivity where the inverse of the diffusion tensors define the local Riemannian metric, we describe filtering of tensor fields using normalized convolution, and contrast it to a stochastic multiivariate MRF regularization method. We conclude with describing a novel representation of rotations in 3D using outer products of quaternions.

Keywords: DTMRI, tensor signal processing

Locating Closed Hyperstreamlines in Second Order Tensor Fields

Thomas Wischgoll (Univ. California - Irvine, USA)

The analysis and visualization of tensor fields is a central problem in visualization. Topology based methods based on investigating the eigenvector fields of second order tensor fields have gained increasing interest in recent years. Most algorithms focus on features known from vector fields, such as saddle points and attracting or repelling nodes, for instance. But, more complex features, such as closed hyperstreamlines are usually neglected. In this presentation, a method for detecting closed hyperstreamlines in tensor fields as a topological feature will be presented. It is based on a special treatment of cases where a hyperstreamline reenters a cell to prevent infinite cycling during hyperstreamline calculation. The algorithm checks for possible exits of a loop of crossed edges and detects structurally stable closed hyperstreamlines. These global features are not detected by conventional topology and feature detection algorithms used for the visualization of second order tensor fields.

Robust Structure Tensors

Rein van den Boomgaard (University of Amsterdam, NL)

Structure tensors can be used to estimate the dominant orientation of laminar patterns in images. The classical structure tensor suffers from the fact the smoothing occurs on the edges where differently oriented patterns meet. In this talk we present a robust estimator of the dominant orientation. The resulting orientation estimation based on the 'robustified' structure tensor proves to suffer much less from smoothing.

Joint work of: van den Boomgaard, Rein; van de Weijer, Joost