

04131 Abstracts Collection
Geometric Properties from Incomplete Data
— **Dagstuhl Seminar** —

Reinhard Klette¹, Ryszard Kozera², Lyle Noakes³ and Joachim Weickert⁴

¹ Univ. of Auckland, NZ

`r.klette@auckland.ac.nz`

² Univ. of Western Australia, AU

`ryszard@cs.uwa.edu.au`

³ Univ. of Western Australia, AU

`lyle@maths.uwa.edu.au`

⁴ Saarland University, DE

`weickert@mia.uni-saarland.de`

Abstract. From 21.03.04 to 26.03.04, the Dagstuhl Seminar 04131 “Geometric Properties from Incomplete Data” was held in the International Conference and Research Center (IBFI), Schloss Dagstuhl. During the seminar, several participants presented their current research, and ongoing work and open problems were discussed. Abstracts of the presentations given during the seminar as well as abstracts of seminar results and ideas are put together in this paper. The first section describes the seminar topics and goals in general. Links to extended abstracts or full papers are provided, if available.

Confidence Measures for Variational Optic Flow Methods

Andrés Bruhn (Universität Saarbrücken, D)

It has often been criticised that there is no reliable confidence measure that allows to sparsify the result of a dense flow field such that the remaining flow is more reliable. In this way it would be possible to compare the real quality of dense methods with the characteristics of local, nondense approaches. In our paper we investigate the usefulness of local image based confidence measures for global variational methods and present a flow based alternative that is simple and applicable to the entire class of energy minimising global optic flow techniques. Experimental evaluation on various images sequences will show that this confidence measure can give excellent results.

Joint work of: Bruhn, Andrés; Weickert, Joachim

Shape Recovery of Strictly Convex Solid from N-Views

Simon Collings (Univ. of Western Australia - Nedlands, AU)

State-of-the-art methods for N-view shape recovery generally involve formulating it as constraint satisfaction problem in the discrete domain. In this talk it is shown that in the case where the scene is a smooth, strictly convex solid, this approach can be avoided.

As an introduction to the ideas in this talk, some neat results are presented in the two dimensional case. In the three dimensional case it is shown that two photographs will yield two calculable points on the surface of the solid. A set of cameras will then produce a point cloud that can be interpolated to give an estimate of the original shape.

Keywords: N-View Shape Recovery

Joint work of: Collings, Simon; Kozera, Ryszard; Noakes, Lyle

Estimation of tangents to a noisy discrete curve

Isabelle Debled-Rennesson (INRIA Lorraine & LORIA - Nancy, F)

A new notion of discrete tangent, called order d discrete tangent, adapted to noisy curves, is proposed. It is based on the definition of discrete tangents given by A. Vialard in 1996, on the definition of fuzzy segments and on a linear algorithm of fuzzy segments recognition. The algorithm calculating the order d discrete tangent at a point of a curve relies on simple calculations and is linear according to the number of points of the obtained tangent. From the definition of an order d discrete tangent, we deduced an estimation of the normal vector and of the curvature at a point of a discrete curve for a given order d .

Keywords: Fuzzy Segment, Discrete Tangents, Noisy Discrete Curves

Estimating Rigid Body Motion of Point Clouds

Leo Dorst (University of Amsterdam, NL)

Orientation measurements estimate relative rotations of objects. Such estimates may need to be averaged according to their covariances, e.g. for a Kalman filter. The non-commutative algebra of rotations makes transference of techniques inspired by the usual vector-based approaches for translations non-trivial. This talk shows how the rotor representation of rotations (which is an embedded form of the quaternion representation) permits straightforward computations

with rotation estimates, from averaging and interpolation to filtering. We characterise rotational noise and present a way to combine estimates of orientations which minimizes error covariance. For the Procrustes method of estimating the relative motion of point clouds we give the precise error propagation of point localization errors to the rigid body motion parameters and their covariances.

Keywords: Orientation estimation, rigid body motion, Kalman filtering, Procrustes, quaternions, geometric algebra

Convex and Concave Parts of Digital Curves

Helene Dörksen (Universität Hamburg, D)

Decomposition of a digital curve into convex and concave parts is of relevance in several scopes of Image Processing. In digital plane convexity cannot be observed locally. It becomes an interesting question, how far one can decide whether a part of a digital curve is convex or concave by a method which is "as local as possible". In [1] it was proposed to define the meaningful parts of a digital curve as meaningful parts of the corresponding polygonal representation. This technique has an approximative character. In our considerations we use geometry of discrete line segments. We will introduce an exact method how to define convex and concave parts of a digital curve. [1] Eckhardt U. and Reiter, H.: Polygonal representations of digital sets. *Algorithmica* 38, vol.1, 2004.

Keywords: Digital geometry, digital convexity, discrete line, convex and concave curves, polygonal representation

Shape from incomplete boundary data

Ulrich Eckhardt (Universität Hamburg, D)

A set in the plane (or in 3-space) can be described efficiently by means of its boundary (or surface). Generally, image processing can be described as accumulation of local boundary information in order to get global informations about the objects in an image. In this presentation the following question is investigated: What can be said about a set if its boundary is probed in a finite number of points?

When investigating the boundary of a set, it is appropriate to impose 'tameness' hypotheses on it. In the context of image processing, polygonal or even discrete sets are considered, so the usual tools of differential geometry are not adequate. Consequently, a 'differentialless' geometry should be adapted (Latecki and Rosenfeld, 1998). A very efficient description of a set can be found by attributing its boundary with predicates as convex or concave parts (Latecki, Lakamper, 1999).

The mapping associating to each linear functional the set of its local maximizers on a nonempty compact set is upper semi-continuous. Moreover, an upper semi-continuous inverse for this map can be found. The study of this map yields insights into the structure of the boundary of a given set. First, as a very ‘nice’ example, the case of a convex set is considered. Here, all boundary points are convex points and a convex set is characterized by this fact (Tietze, 1927-29). The mapping mentioned above can be deformed to yield a homeomorphism from the unit sphere onto the boundary of the set. Moreover, by a finite number of boundary points and tangent directions one can find a directionally convex set containing the given set.

In the general case, things become very complicated. It is necessary to rule out ‘wild’ boundary parts. A minimal requirement is that the set under consideration should have the Jordan property. However, some more structure has to be imposed in order to get practical results. A certain regularity condition is stated which is theoretically tractable and practically acceptable. For sets with regular boundaries one can derive results concerning reconstruction of certain properties from incomplete data.

Keywords: Boundaries of sets, convex and concave parts, incomplete data

Greedy Kernel Principal Component Analysis

Vojtech Franc (Czech Technical University, CZ)

A technique allowing to approximate a huge training set is proposed. The approach aims to represent data in a low dimensional space with possibly minimal representation error which is similar to the Principal Component Analysis (PCA). In contrast to the PCA, the basis vectors of the low dimensional space used for data representation are properly selected vectors from the training set and not as their linear combinations. The basis vectors can be selected by a simple algorithm which has low computational requirements and allows processing of huge data sets. Moreover, the algorithm can be simply extended to its non-linear version by using the kernel functions.

The proposed method was tested to approximate training sets of the Support Vector Machines and Kernel Fisher Discriminant which are known methods for learning classifiers. The experiments show that the proposed approximation can significantly reduce the complexity of the found classifiers while retaining their accuracy.

Keywords: Kernel Principal Component Analysis

On instability, error correction and noise compensation in discrete tomography

Peter Gritzmann (TU München, D)

The task of reconstructing binary images from the knowledge of their line sums (discrete X-rays) in a given number m of directions is highly ill-posed. Even some small noise of cardinality at most $2(m-1)$ can lead to dramatically different (in fact disjoint), yet still unique solutions. However, the bound $2(m-1)$ is sharp: the Hamming distance between any two different sets of X-ray images is at least $2(m-1)$. Besides providing a completely unconditioned and best possible stability result, this theorem implies stable versions of various uniqueness results including the classical one of Renyi and those for convex lattice sets by Gardner and the author.

The talk is based on joint work with A. Alpers, TU Munich.

Keywords: Discrete tomography, stability, instability, uniqueness, ill-posed, error correction

A Feature Sensitive Metric with Applications in Geometric Computing

Michael Hofer (TU Wien, A)

The isophotic metric is a new metric on surfaces, in which the length of a surface curve is not just dependent on the curve itself, but also on the variation of the surface normals along it.

A weak variation of the normals brings the isophotic length of a curve close to its Euclidean length, whereas a strong normal variation increases the isophotic length. The consideration of the normal variation alone is related to the concept of isophotes in classical geometry, and thus we speak of the isophotic metric.

The isophotic metric is sensitive to features such as smoothed edges, which are characterized by a significant deviation of the two principal curvatures: paths along features are close to geodesics in the isophotic metric, paths across features have high isophotic length. This shape effect makes the isophotic metric useful for a number of applications. We address feature sensitive image processing with mathematical morphology on surfaces and feature sensitive geometric design on surfaces.

Vectorization of 2 and 3D Digital Curves by Nonlinear Optimization

Atsushi Imiya (Chiba University, J)

In this talk, I present an algorithm for the vectorization of pixel- and hexel-curves on a plane and voxel-curves in a space using nonlinear optimization. Numerical examples to the evaluation asymptotic property of the algorithm are presented. The algorithm is robust comparing our old algorithm presented at DGCi 2003.

Keywords: Pixel, Voxel, Hexel, Rhombic-dodecahedral cell, supercover in non-square grids

Video Image Sequence Analysis: Estimating Missing Data and Segmenting Multiple Motions

Kenichi Kanatani (Okayama University, J)

Feature point tracking over a video sequence fails when the points go out of the field of view or behind other objects. We first show that we can extend such interrupted tracking by imposing a geometric constraint resulting from the affine approximation of camera imaging geometry, coupled with an outlier removal process based on a statistical model of image noise. Using real video images, we demonstrate that our method can restore a sufficient number of data for detailed 3-D reconstruction.

For video sequences consisting of multiple independent motions, many techniques have been proposed for segmenting them into individual motions. It has been found, however, that methods that perform very well in simulations perform very poorly for real video sequences. We resolve this mystery by analyzing the geometric structure of the degeneracy of the motion model. This leads to a new segmentation algorithm. We show by simulated and real video experiments that our method is superior to all existing methods in practical situations.

Variational Pairing of Image Segmentation and Blind Restoration

Nahum Kiryati (Tel Aviv University, IL)

Variational Pairing of Image Segmentation and Blind Restoration Leah Bar, Nir Sochen and Nahum Kiryati Tel Aviv University Segmentation and blind restoration are both classical problems, that are known to be difficult and have attracted major research efforts.

This paper shows that the two problems are tightly coupled and can be successfully solved together. Mutual support of the segmentation and blind restoration processes within a joint variational framework is theoretically motivated, and validated by successful experimental results. The proposed variational method integrates Mumford-Shah segmentation with parametric blur-kernel recovery and image deconvolution. The functional is formulated using the gamma-convergence approximation and is iteratively optimized via the alternate minimization method. While the major novelty of this work is in the unified solution of the segmentation and blind restoration problems, the important special case of known blur is also considered and promising results are obtained.

Full paper will appear in Proc. ECCV'04.

Multigrid Convergence of Property Estimators in Picture Analysis

Reinhard Klette (University of Auckland, NZ)

The talk starts with a brief review of digitization models (from Euclidean space into digital 2D or 3D space) and of multigrid convergence. The latter concept is in use for evaluating property estimators in digital picture analysis. It is illustrated by measuring the length of a curve in 2D or 3D digital space. The talk then informs about curvature estimation for curves and surfaces in digital space, with a special focus on results obtained within a student project by Simon Hermann (Göttingen University) during his time at Auckland University. The talk concludes with listing open problems in the context of curvature estimation.

Keywords: Digital geometry, multigrid convergence, length estimation, curvature estimation

Interpolation with Cumulative Chords for Length and Curve Estimation

Ryszard Kozera (Univ. of Western Australia - Nedlands, AU)

Cumulative chord piecewise-quadratics, piecewise-cubics and C^1 piecewise-cubics are studied in detail for unparameterized data from regular curves in R^n . Corresponding orders of approximation are established and verified through numerical experiments carried out for different families of samplings and curves.

Keywords: Unparameterized Data and Interpolation, Approximation's Orders, Length and Trajectory Estimation

Joint work of: Kozera, Ryszard; Noakes, Lyle

Approximation subject to nonlinear constraints

Wayne M. Lawton (National University of Singapore, SGP)

A classical result of Weierstrass ensures that any continuous finite length path in a vector space can be uniformly approximated by a path whose entries are trigonometric functions. We extend his result to the case where the paths lie in a sphere by developing a noncommutative approximation theory that exploits symmetry and incorporates global analysis and topology methods to deal with nonlinear constraints.

Interpolating Data in Cellular Arrays

Harry McLaughlin (Rensselaer Polytechnic, USA)

For a given convex sequence in a 2-d cellular array we ask: (1) Is it possible to adjoin one more cell to the sequence and preserve the convexity? and (2) If it is possible to adjoin such a cell, how can it be specified?

Joint work of: Geer, Panama; McLaughlin, Harry; Unsworth, Keith

On Polynomial Interpolation of Parametric Curves and Surfaces

Knut M. Morken (University of Oslo, N)

If we view parametric curves and surfaces as vector functions, it is relatively easy to extend approximation methods for ordinary functions to these geometric objects. However, this ignores one important aspect of curves and surfaces, namely the equivalence under reparametrisation. It turns out that this extra freedom can be exploited to improve the approximation properties of parametric polynomials, although this usually involves solving nonlinear equations. In this talk we are going to describe the general principle and consider some consequences for approximation of curves and surfaces by parametric polynomials.

On robust estimation and smoothing with spatial and tonal kernels

Pavel Mrazek (Universität Saarbrücken, D)

We study the connections between nonlinear filters for digital image processing. We show that a large class of methods can be cast as minimization of a single functional with nonlocal data and nonlocal smoothness terms. This class includes e.g. robust statistical estimation with (local) M-estimators, local mode filtering in image or histogram space, bilateral filtering, nonlinear diffusion, and regularization methods. This unification contributes to understanding of the individual methods, and makes the questions of the choice of the penalizer type or that of motivated parameter selection explicit.

Geometric Fitting with Uncertain and Correlated Data

Matthias Mühlich (Universität Frankfurt, D)

Recent research provided several new and fast approaches for the class of parameter estimation problems that are common in computer vision. Incorporation of complex noise model (mostly in form of covariance matrices) into errors-in-variables or total least squares models led to a considerable improvement of existing algorithms. However, most algorithms can only account for covariance of the same measurement – but many computer vision problems, e.g. gradient-based optical flow estimation, show correlations between different measurements.

In this talk, I will present a new method for improving TLS based estimation with suitably chosen weights. Additionally, it will be shown how to describe the error model up to second order with a generalized covariance tensor. Finally, I will use this tensor to compute the weights for a proper reweighting step that guarantees unbiased TLS estimates. For demonstration purposes, we examples for ellipse fitting with the new method and the FNS (fundamental numerical scheme) will be shown.

Curves in Spheres

Lyle Noakes (Univ. of Western Australia - Nedlands, AU)

The idea of replacing line segments by geodesics in classical constructions of Bézier polynomials has been around at least since the 1985 papers of Shoemake and Duff. However its practical use for interpolating spherical data has been impeded by mathematical difficulties in analysing generalised Bézier curves.

We study two readily computable algorithms of this sort, giving theoretical results and numerical examples.

Keywords: Interpolation, spheres

An introduction to kernel methods in learning theory

Massimiliano Pontil (University College London, GB)

This talk provides a brief overview of kernel-based methods in learning theory, including results on the problem of classification and function approximation.

In this context we derive the form of the minimal norm interpolant to a finite set of data and apply it to study some regularization functionals which are important in learning theory. In particular, we discuss specific examples of such functionals corresponding to regularization networks and support vector machines.

Keywords: Learning theory, support vector machines, reproducing kernels, regularization, minimal norm interpolation

Graph-spectral Methods for Surface Height Recovery from Gauss Maps

Antonio Robles-Kelly (University of York, GB)

In this talk, I describe the use of graph-spectral methods for purposes of surface height recovery. Here, our input is a 2D field of surface normal estimates, delivered, for instance, by a shape-from-shading, shape-from-texture or stereo acquisition procedure. I will commence by showing how to use the surface normals to obtain a graph whose edge-weight matrix is related to the surface curvature. With this weight matrix at hand, I will proceed to find a path across the field of surface normals whose curvature is minimum. To do this, I will present two alternatives. The first one of these is to use a graph seriation method. The second one, consists of recovering the random walk making use of a probability matrix equivalent, by row-normalisation to the matrix of edge-weights. Then, the recovery of the integration path can be posed as that of finding the steady state random walk for the Markov chain defined by this matrix. For both methods, the solution is equivalent, up to scaling, to the leading eigenvector of the edge-weight matrix. Further, the height increments along the path are simply related to the traversed path length and the slope of the local tangent plane. I will show experiments on synthetic and real-world imagery whose fields of surface normals are delivered by a shape-from-shading algorithm.

Keywords: Surface reconstruction, Markov chain, graph seriation, curvature, surface normals

Feature Extraction by Cascade Architectures for Face Detection, Recognition, and Indexing

Wladislaw Skarbek (Warsaw Univ. of Technology, PL)

Two cascade architectures for extracting facial features are described. The first one, called the AdaBoost cascade was introduced by Viola and Jones in 2001 for face localisation. Applying an early and suboptimal version of Shapire and Freund's training algorithm, it learns the face specific local region contrasts and their thresholds producing a strong classifier by the cascade of weak classifiers. In this research, a novel training scheme has been proposed which defines a less complex cascade with real time performance on video sequences, gives a better control on false rejection rate, and improves the classifier generalisation property. In face detection, we reach on the testing data collection (above 12 million of negative examples, about four thousands of positive examples) the false acceptance rate below 0.00001 while the false rejection rate is kept below 0.05. The second cascade, called here as DLDA cascade, performs the feature extraction for face recognition and indexing. It consists of the core feature extraction stage

(here Fourier operator is applied for several image parts independently), three Dual LDA stages, one normalisation and one quantisation stage.

In this work, a novel optimisation technique for the cascade parameters and a new procedure for the face similarity function, are elaborated. The face recognition and indexing scheme, based on DLDA cascade concept, outperforms Samsung and NEC scheme (the winner in MPEG-7 version 2 competition) and has shown its superiority in Mitsubishi Electric internal competition (held in November 2003).

Keywords: Face detection, face recognition, face indexing, feature extraction, local region contrasts, AdaBoost multiclassifier, Fourier operator, dual linear discriminant analysis, cascade architecture

The Beltrami flow over Manifolds: Intinsic vs. Implicit

Nir Sochen (Tel Aviv University, IL)

In several image processing applications one has to deal with noisy images defined on surfaces, like electric impulsions or diffusion tensors on the cortex. We propose the Beltrami flow as a regularization method. This technique overcomes the over-smoothing of the L_2 and the stair-casing effects of the L_1 flow for strongly noised images. The surface and data are given in two main forms: intrinsic and implicit. We demonstrate how the Beltrami flow is formulated in each one of these forms. We show the relations between the approaches and demonstrate the flows on synthetic and real data.

Joint work of: Sochen, Nir; Deriche, Rachid; Lopez-Perez, Lucero

The twist representation of shape

Gerald Sommer (Universität Kiel, D)

We are presenting a new scheme of generating free-form curves and surfaces based on the Lie algebra $se(3)$, embedded into the conformal geometric algebra (CGA) $R_4, 1$. The starting point for this approach to the representation problem of shape is getting a highly efficient and generic way of defining free-form objects in Euclidean space. Interpreting curves and surfaces as orbits of the group $SE(3)$, we are describing a kinematic (or operational) shape definition. By embedding the problem into the CGA several advantages will result. For instance, the so generated objects are algebraic entities which themselves can be efficiently transformed. Second, they can be considered with respect to their Euclidean, projective or kinematic properties.

We will demonstrate the equivalence of the twist representation with Fourier representation of shape. This enables to parametrize a twist model to a given curve or surface, to apply Fourier based approximations of the representation,

and to regularize matching processes between objects in different pose. We finally demonstrate some applications of the presented shape representation for pose estimation.

Joint work of: Sommer, Gerald; Rosenhahn, Bodo; Perwass, Christian

Relations between nonlinear filters in signal and image processing

Gabriele Steidl (Universität Mannheim, D)

Wavelet shrinkage, nonlinear diffusion filtering, and nonquadratic regularisation methods are three useful techniques for edge-preserving denoising of signals and images.

In the first part of the talk we analyse relations between the important 1-D prototypes soft Haar wavelet shrinkage, space-discrete total variation (TV) diffusion and discrete total variation (TV) regularisation: we prove that space-discrete TV diffusion and TV regularisation are identical. We show that applying translationally invariant soft Haar wavelet shrinkage on a single scale can be regarded as an absolutely stable explicit discretisation of TV diffusion. Afterwards we show that wavelet shrinkage on multiple scales can be regarded as a single step diffusion filtering of the Laplacian pyramid of the signal.

In the second part of the talk we study general relations between arbitrary shrinkage functions and diffusivities: we show that one step of a (stabilised) explicit discretisation of nonlinear diffusion can be expressed in terms of wavelet shrinkage on a single spatial level. This equivalence allows a fruitful exchange of ideas between the two fields. For example, we derive new wavelet shrinkage functions from existing diffusivity functions, and identify some previously used shrinkage functions as corresponding to well known diffusivities.

Moreover, by transferring stability notions from diffusion filtering to wavelet shrinkage, we derive conditions on the shrinkage function that ensure that shift invariant single-level Haar wavelet shrinkage is maximum-minimum stable, monotonicity preserving, and variation diminishing. Finally, we present some novel diffusion-inspired wavelet shrinkage methods with improved rotation invariance in 2D.

Optic Flow Computation with High Accuracy

Joachim Weickert (Universität Saarbrücken, D)

In spite of two decades of intensive research, the reliable estimation of optic flow continues to be a key problem in computer vision. In the present talk we study an energy functional for computing optic flow that combines three assumptions: a brightness constancy assumption, a gradient constancy assumption, and a discontinuity-preserving spatio-temporal smoothness constraint.

In order to allow for large displacements, linearisations in the two data terms are strictly avoided. We present a consistent numerical scheme based on two nested fixed point iterations. By proving that this scheme implements a coarse-to-fine warping strategy, we give a theoretical foundation for warping which has been used on a mainly experimental basis so far.

Our evaluation demonstrates that the novel method gives significantly smaller angular errors than previous techniques for optic flow estimation. We show that it is fairly insensitive to parameter variations, and we demonstrate its excellent robustness under noise.

Keywords: Computer vision, optical flow, variational methods, performance evaluation, warping

Joint work of: Weickert, Joachim; Brox, Thomas; Bruhn, Andrés; Papenberg, Nils