Numerical Approximation of Parabolic Stochastic Partial Differential Equations

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Abstract. The topic of the talk were the time approximation of quasi linear stochastic partial differential equations of parabolic type. The framework were in the setting of stochastic evolution equations. An error bounds for the implicit Euler scheme was given and the stability of the scheme were considered.

Keywords: Stochastic Partial Differential Equations, Stochastic evolution Equations, Numerical Approximation, implicit Euler scheme

1 Introduction

In the talk I used the same notation as DaPrato and Zabyzcyk [1] and Pazy [2].

Let X be a separable Hilbert space. Let A be an infinitesimal generator of an analytic semigroup of negative type. Further, $W(t)$ is a Wiener process taking values in X with nuclear covariance operator Q. We consider the evolution equation

$$\begin{cases} du(t) = (Au(t) + f(t, u(t)))\ dt + \sigma(t)u(t)dW(t), \\ u(0) = u_0 \in D((-A)^\gamma), \end{cases}$$

where $D((-A)^\gamma)$ is the domain of $(-A)^\gamma$, equipped with norm $\|\cdot\|_\gamma = \|(-A)^\gamma \cdot\|$, $0 < \gamma < 1$. A typical example of such an evolution equation is a parabolic SPDEs defined on a smooth domain with Dirichlet or Neumann boundary condition.

If σ and f satisfy certain smoothness condition, existence and uniqueness is given. But there are only few evolution equations where the solution is explicitly given and one has to simulate it on computers. The main idea is to discretize the SPDE spatially obtaining a system of SDEs that can be solved by e.g. the implicit Euler scheme.

In the talk I considered the accuracy of approximation by the implicit Euler scheme, where I presented the order of convergence and gave a sketch of the proof. The talk were based on the work Hauseblas [3].

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