Modification of recourse data for mixed-integer recourse models

(extended abstract)

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Abstract. We consider modification of the recourse data, consisting of the second-stage parameters and the underlying distribution, as an approximation technique for solving two-stage recourse problems. This approach is applied to several specific classes of mixed-integer recourse problems; in each case, the resulting recourse problem is much easier to solve.

Keywords. mixed-integer recourse, approximation

1 Introduction

Consider the two-stage recourse model with random right-hand side

$$\begin{aligned} &\min_x \; cx + Q(x) \\ &\text{s.t.} \;\; x \in X := \{x \in \mathbb{R}^{n_1}_+ : Ax = b\}, \end{aligned}$$

with recourse function Q,

$$Q(x) := \mathbb{E}_\omega \left[v(\omega - Tx) \right], \quad x \in \mathbb{R}^{n_1},$$

and second-stage value function v,

$$\begin{aligned} v(s) := \; &\min_y \; qy \\ &\text{s.t.} \;\; Wy = s, \qquad s \in \mathbb{R}^m. \\ &\qquad\; y \in Y \end{aligned}$$

The distribution of the random right-hand side parameter $\omega \in \mathbb{R}^m$ is assumed to be known; we will denote its cumulative distribution function (cdf) by F, and its probability density function by f (if it exists). The set $Y \subset \mathbb{R}^n$ specifies simple bounds and/or integrality restrictions on the second-stage variables y. The vectors and matrices c, A, b, T, q, and W, have conformable dimensions.

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Obviously, all characteristic difficulties of such a recourse model are captured by the recourse function Q. Depending on the *recourse structure*, represented by the triple (q, W, Y), and the distribution of ω given by its cdf F, the function Q may or may not have nice mathematical properties and be relatively easy or very difficult to evaluate. For example, if Y specifies integrality restrictions on (some of) the second-stage variables, the function Q is in general non-convex; it is precisely the convexity which underlies all efficient algorithms for solving recourse models with continuous variables.

All essential information about a recourse model can therefore be summarized by the tuple (q, W, Y, F), which we will call the *recourse data*.

If a given recourse problem is difficult to solve, a natural approach is to construct an approximating problem by modifying the recourse data,

$$(q, W, Y, F) \longrightarrow \left(\tilde{q}, \tilde{W}, \tilde{Y}, \tilde{F}\right),$$

such that

$$\min_{x \in X} cx + \tilde{Q}(x),$$

where $\tilde{Q}$ is specified by the recourse data $(\tilde{q}, \tilde{W}, \tilde{Y}, \tilde{F})$, is relatively easy to solve.

In combination with an algorithm for solving the approximating problem, modification of the recourse data constitutes an algorithm for solving the original recourse problem. We apply this conceptual algorithm to three model types, namely *simple integer recourse*, *complete integer recourse*, and a particular *mixed-integer recourse* problem. In all cases the modification of the problem data involves both the distribution of ω as well as the recourse structure (q, W, Y), and the resulting approximation proves to be a continuous recourse problem.

2 Results

For pure integer problems, modification of recourse data consists of the following two ingredients.

Perturbation of the distribution: Integer recourse functions Q are non-convex in general. However, Q is convex if the right-hand side parameter ω follows a continuous distribution belonging to a specific class, which includes distributions with densities that are constant on m-dimensional hypercubes

$$\prod_{i=1}^{m} (\alpha_i + k_i - 1, \alpha_i + k_i], \quad k \in \mathbb{Z}^m,$$

for a fixed $\alpha \in [0, 1)^m$. Thus, by replacing the given distribution of ω by a distribution of this type, a convex approximation of the function Q is obtained.

Representation as continuous recourse function: Such convex approximations of the function Q can be represented as recourse functions of a problem with continuous recourse variables. Essentially, this means that second-stage integrality restrictions can be dropped, at the same time applying another suitable transformation to the distribution of the right-hand side parameters. The resulting distribution, which is always discrete, can be computed directly from the original recourse data.

For simple integer recourse problems, the approach yields good approximations if the original distribution is continuous [1]. (See [2] for an alternative method for the case with discrete distribution.) If the recourse matrix W is totally unimodular, then the parameter α can be chosen such that one obtains the convex hull of Q. For the general case, the resulting convex approximation is often strictly better than the LP relaxation [3].

The results on mixed-integer recourse are so far limited to models with only a single recourse constraint, i.e., with $\omega \in \mathbb{R}$. Under some technical assumptions, it is shown that the mixed-integer value function v is equal to a (one-dimensional) simple integer recourse expected value function, where the expectation is with respect to a distribution which reflects the properties of a particular v. This allows to follow the same approach as described above, so that also in this case modification of recourse data yields the desired continuous recourse approximations [4].

The results for simple and complete integer recourse are summarized in [5], which also describes modification of recourse data for so-called multiple simple recourse models [6].

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