# A MATLAB™ program for the computation of the confluent hypergeometric function $\Phi_2$

E. Martos-Naya, J. M. Romero-Jerez, F. J. Lopez-Martinez and J. F. Paris

**Abstract**

We here present a sample MATLAB™ program for the numerical evaluation of the confluent hypergeometric function $\Phi_2$. This program is based on the calculation of the inverse Laplace transform using the algorithm suggested by Simon and Alouini in their reference textbook [1].

## I. THE MULTIVARIATE $\Phi_2$ FUNCTION

The following confluent form of the generalized Lauricella series is defined in [2, eq. 7.2, pp. 446]

$$\Phi_2^{(n)}\left(b_1,\ldots,b_n;c;x_1,\ldots,x_n\right) \triangleq \sum_{m_1\ldots m_n}^{\infty} \frac{(b_1)_{m_1}\ldots(b_n)_{m_n}}{(c)_{m_1+\ldots+m_n}} \frac{x_1^{m_1}}{m_1!}\cdots\frac{x_n^{m_n}}{m_n!} \tag{1}$$

where $(\cdot)_m$ denotes the Pochhammer symbol. This function, also regarded as confluent hypergeometric function of $n$ variables [3], makes appearances in numerous problems in communication theory [4–8], either in a bivariate form ($n=2$) or in a multivariate fashion.

Because it is defined as an $n$-fold infinite summation, its numerical evaluation poses some challenges from a computational point of view. However, the Laplace transform of the $\Phi_2$ function has a comparatively simpler form, in terms of a finite productory of elementary functions. Specifically, in [9, 4.24.5], the following Laplace transform pair is listed:

$$\mathcal{L}\left\{t^{c-1}\Phi_2^{(n)}\left(b_1,\ldots,b_n;c;x_1t,\ldots,x_nt\right);t,s\right\} = \frac{\Gamma(c)}{s^c}\left(1-\frac{x_1}{s}\right)^{-b_1}\cdots\left(1-\frac{x_n}{s}\right)^{-b_n}, \tag{2}$$

which is valid for $\Re(c)>0$, $\Re(s)>0$, $b_i\in\mathbb{R}$, $i=1\ldots n$.

Thus, we have that

$$\begin{aligned}\Phi_2^{(n)}\left(b_1,\ldots,b_n;c;x_1,\ldots,x_n\right) &= t^{c-1}\Phi_2^{(n)}\left(b_1,\ldots,b_n;c;x_1t,\ldots,x_nt\right)|_{t=1}\\ &= \mathcal{L}^{-1}\left\{\frac{\Gamma(c)}{s^c}\left(1-\frac{x_1}{s}\right)^{-b_1}\cdots\left(1-\frac{x_n}{s}\right)^{-b_n};s;t\right\}\end{aligned} \tag{3}$$

Therefore, the $\Phi_2$ function can be evaluated by means of an inverse Laplace transform. Due to numerous requests, we here provide a MATLAB™ sample code for the evaluation of the $\Phi_2$ function. This code implements an Euler summation-based technique described in detail in the appendix 9B of the reference textbook by Simon and Alouini [1], inspired in [10].

## II. MATLAB™ CODE

```
%% y=Phi2(bvector,c,xvector,N)
%
% bvector      : [b1 ... bn]
% c            : scalar parameter Re(c)>0
% xvector      : [x1 ... xn]
% N            : Truncation of infinite summation
%
function y=Phi2(bvector,c,xvector,N)
% Heuristically adjusted so that the discretization error term
% abs{E(A)}<exp(-A)
A=15;
% Inverse Laplace Transform
K=exp(A/2);
alphainv=[0.5,ones(1,N-1)];
n=0:N-1;
y1=(-1).^n.*alphainv.*real(LaplacePhi2((A+2*pi*1i*n)/2,c,xvector,
    bvector));
y=K.*sum(y1);

function y=LaplacePhi2(s,c,x,b)
P=ones(1,length(s));
for k=1:length(b)
    P=P.*((1-x(k)*(s.^(-1))).^(-b(k)));
end;
y=gamma(c)*(s.^(-c)).*P;
```

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