# Optimization Models in WindFarm Design: The case of a routing-location problem

Author: Demetra Orthodoxou
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Universidad de Málaga in co-operation with Wageningen University
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*Supervisors:* Dr. E.M.T. Hendrix and Dr.G.D.H Claassen

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**Abstract**

Wind energy can lead to a great socio-economic impact in Europe nowadays, while the installed capacity of wind power is estimated to be double by 2030. Therefore, it has gained more attention lately. In order to generate power from the wind, it is needed to have a set of wind turbines which is known as wind farm. The wind turbines have to be connected via cables into a power grid and the generated power is collected at the main power station. Simultaneously, wind farm researchers need to exploit supercomputers, by having the ultimate aim of developing accurate models. Computational models on wind farm design vary in the existing literature and they have been developed by engineers and scientists with different backgrounds in order to meet the requirements. This thesis report investigates the contribution of operations research scientists associated to these mathematical models. A more precise examination goes through a selected Integer Linear Model which aims to minimize the cabling costs of offshore farms. The model follows the capacitated minimum spanning tree structure, which allows branching and prevents crossing paths. The existing model has been modified in order to understand the complexity while more constraints are taken into account. Besides the fact to satisfy the objective of optimizing the cable routing, a sub-problem arose to facilitate the model by relaxing the planarity constraint. The addition of a node in the existing grid has been proven to be beneficial for specific cases, taking into account the fixed costs arising from the installation and maintenance.

# Contents

# Chapter 1

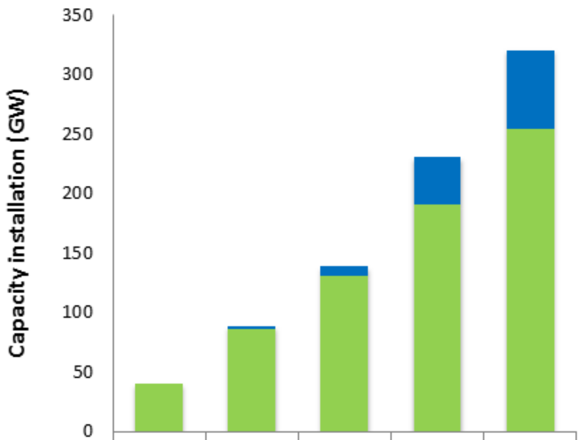
# Introduction

"The best way to predict your future is to create it"(Covey S.) Fashion always comes and goes, what really remains untouched is the meeting of needs. Thanks to latter generations and the consequences of their reckless actions concerning the planet safety, it seems that the current generation has realized the importance of being proactive and ensuring their future. A perpetual desire for saving costs, has lately forced people to seek creative ways to achieve it without harming the nature.

So, the anxiety is decreasing when we realize the connection between economic growth and natural resources from earth. The fastest renewable energy production is wind energy and it is a key for social well-being and a source for cost reductions. The real challenge, that intrigues many researchers to innovate and exchange ideas, is how to balance this usage in a social, economic and environmental way by the pursuit of a common ideal. And this is how a trend of lifestyle, sustainability has started spreading over the world.

Apparently, nowadays more people are becoming educated and they are looking for jobs and opportunities to improve products/services that thus can lead to a great socio-economic impact. It is important to mention the European position on project foundation and their funding to contribute towards a sustainable world development [1]. According to the EWEA scenario (European Wind Energy Association), the wind energy industry will provide around 334,000 jobs in Europe by 2030 and the installed wind power capacity could reach 320GW.

Europe is required to provide at least 20% of European electricity consumption from wind by 2020 and even better 27% by 2030 (European Wind Energy Association, 2015). This can be achieved by exploiting offshore wind farms (European Commission, 2015). Aeolic energy can commit to secure European energy independence [1] since Europe is the leader of the global wind market [1].




| | 2005 | 2010 | 2014 | 2020 | 2030 |
|---|---|---|---|---|---|
| Offshore | 0 | 3 | 8 | 40 | 66 |
| Onshore | 40 | 85 | 130 | 190 | 254 |

Figure 1.1: Total capacity of EU wind energy installation

At the same time, computational science is exploding while the supercomputers can perform to a quadrillion of FLOPS (floating-point operations per second) and handle multiple components. Although computers will never be able to think at the same level as human brain [10], people can hardly understand the advantage of the performance outcome.

## 1.1 Background

Wind energy researchers create opportunities for potential investigations and individual contributions from conventional technologies by optimising wind farm design and developing accurate models. In order to achieve an integrated tool, aspects from multiple disciplines should be taken into account. Every discipline has a different contribution and adds data which can lead research to a highly complex level. The proper use of computers can promote the development of those models.

Wind energy cannot be stored [14] and therefore we are looking for more accurate and predictable models. In the wind farms case, the computational procedure can be based on a very large number of design objectives and constraints. With regards to objectives, the main goal of a wind farm is to maximize the benefits by maximizing energy generation and minimizing the costs [12]. Constraints are related to environmental aspects, technical and electrical specifications, design effects and setback restrictions ([8], p.372). Furthermore, it is worth to mention the distinction between on-shore and off-shore wind farms which can diversify the constraints of the models.

A large number of software programs is available to facilitate these models, but human intervention is still required in order to solve complex optimization problems using approximation algorithms ([8], p.371). And here is where my interest starts. As a computer scientist, who has dealt with NP-hard problems in the past, I have been attracted by the complexity of wind farm layout design problems since they need computational intelligence techniques to be solved. Furthermore, my interest has been increased due to my postgraduate knowledge on heuristic algorithms, which can solve these problems more efficiently.

In order to capture physical phenomena, mathematical models involved in the design of a wind model could rely on dynamical numerical representation for a better understanding of the behaviour of wind farm effects. With a stochastic model, we can repeatedly simulate outcomes using suitable approaches to generate random numbers to measure the probability that they will occur for any choice. However, these simulations are based on large computers due to the complexity of the model and further work should focus on creating accurate and comprehensive models [7], p.14).

## 1.2 Research Questions

### 1.2.1 General Research Questions

Why is Wind Farm Design interesting from an operations research perspective?

### 1.2.2 Specific Research Questions

1. What kind of investigation has already been done? And at what point?

2.Which models requires higher computational effort? How can we deal with this type problems?

#### Approach

Section 1.4 gives an overview of different computational models involved in wind farm design based on existing literature. Each researcher has presented the wind farm design from a different point of view. Indeed, they mentioned some similar important issues needed to be taken into account, according to the results. At the end of this section, Table 1.1 presents a short overview in order to answer the first specific research question.

After the examination of existing literature, I have selected an integer programming model for minimizing cable lengths. The model is presented in the paper "An Integer Programming Model for Branching Cable Layouts in Offshore Wind Farms" [9]. I found this model extremely interesting because of its complexity level. Section 2.1 introduces the notations of the original model using graph theory. The researchers, who developed this model, achieve to generate the minimum total cable length in a wind farm by taking the required constraints into account.

In order to understand the above model and prove the high computational effort that is required to answer the second specific question, I implemented the model in GAMS: General Algebraic Modelling System (Appendix B1). During the implementation, a modification of the original model has been achieved and it is presented in Section 2.2, in the same graph theory form as the original. Next to this, three numerical examples are presented to show the efficiency of the model.

Through the examination of the above problem, a bi-level problem arose. Bi-level optimization is a special kind of optimization where one problem is embedded (nested) within another. This problem has been considered complex so it has been investigated as a subproblem. The configuration of the model is given in details in Chapter 3 and it refers to adding a node on the power grid which is explained in detail in Section 3.1.

**Questions of the additional node problem:**

1. Is the addition of a node beneficial? If yes, for which instances?
2. How can continuous optimization give the optimal location of the additional node in the grid?

The adjustment of the original model is embodied to answer the first question. In order to compare the results of the potential usage of an additional node, more calculations are required. The second question has been answered in a more analytical way. Firstly, the case of two wind turbines, has been implemented by studying the corresponding Weber problem [16] (Appendix A.4). The above case has been examined twofold, i.e. in a symmetric and a non-symmetric way (Section 3.1.1/3.1.2). In addition, the case of three wind turbines has been proven more complex since it alters the results of the two wind turbines case (Section 3.1.3). As a result, a complete figure has been added for visual illustration and better understanding. The investigation has been ended with the symmetric case of four wind turbines, which has the same results as two wind turbines symmetric case (Section 3.1.4). Section 3.2 includes the consideration of planarity constraint. The updated model embedding Weber problem can be found in Section 3.3 and the implementation of the model in GAMS is presented in Appendix B2-B3.

Chapter 4 discusses the findings of the thesis research as a conclusion chapter, based on reasoning and evidence. It aims to carry the reader to a new level of perception while summarizing the overall content, and it offers answers to the questions raised in the research.

## 1.3 What the Reader can Expect

This thesis report aims to intrigue the operations research scientists to explore the opportunity of combining multiple aspects in wind farm design problems. The short description of the existing literature in wind farm field, Section 1.4, gives a clear idea of how different researchers have tried to contribute to the development of accurate and precise models.

In my turn, I tried to prove the complexity of an existing model by analyzing and breaking down every single equation. In addition I have simply visualized the constraints, in order to give the picture to readers and potential wind energy field researchers and show the purpose of each equation. Furthermore, I gave some graphical representations as examples, by comparing the expected with the real result.

As an additional section in this report, the reader can find a description of an optimization subproblem, which arose during the examination of the modified model. The introduction of an additional node to an existing model has facilitated the formation of the problem and the solution method. In order to prove and illustrate the extended model, I have followed a more analytical method by creating simple and small cases with different graphical examples.

Finally, I end the report with a general conclusion arguing that wind farm design can be interesting from an operations research perspective, due to the complexity of models. Also the discussion includes a summary of the examined model, interpretation of findings and recommendations for further research. At the end of this report, the reader can find information about additional general knowledge and the GAMS code.

## 1.4 Existing Literature

My research questions have been raised by examining existing publications and articles. As I mentioned before, there are multiple aspects that should be taken into account in order to develop a comprehensive model. According to the existing literature, a lot of researchers with different backgrounds have achieved to develop and/or contribute to computational models for wind farm design up to a level so far.

First of all, it would be wise to introduce the definition of the popular wake effect. This phenomenon is relevant to the power loss caused by the reduction of wind velocity behind the turbine. The velocity deficit depends mainly on the closest turbine and this is the reason why it is important to position the turbines in a way of minimizing the effect [13]. In other words, the optimization of turbine location can be considered as the most important aspect.

Currently, according to the WFLOP-Wind Farm Layout Optimization

Problem paper in offshore design, wind turbines are organized in identical rows with large distance and a feasible solution is given by heuristic methods such as genetic algorithms and simulated annealing [13]. Heuristic methods give a feasible solution, but do not guarantee the solution to be optimal, because they prefer to use a reasonable time to find a solution, instead of providing the optimal solution in an increasing amount of time [8].

Unfortunately, the extension of this problem, which includes the use of mixed-integer, dynamic and stochastic programming, has been disregarded by the operations research groups due to non-linearity and the lack of obtaining data on how to implement models related to topography. The above methods give the optimal solution either by enumerative search or by decomposing the problem into more tractable subproblems or by estimating the probabilities of data [13].

Furthermore, an important constraint that can be introduced is the constraint, which allows positioning wind turbines in any intermediate point in order to produce more energy than a normal grid with predefined positions [6]. Wind farm developers have to take into account that each turbine location must be connected to a road network, not too close to houses, military facilities and airport and not along the migration path of birds [13].

The UWFLO- Unrestricted Wind Farm Layout Optimization method avoids limitation of the grid layout pattern, enabling to optimize the placement of turbines. Moreover, it allows to explore the opportunity of multiple types of turbines based on different joint distributions of wind speed and wind directions. The results of this method show the importance of the turbine type selection [3].

VRP-Vehicle routing problems aim to minimize the cost of cable layout and introduce the planarity constraint, which specifies that every wind turbine is connected to a depot while the cable routes cannot cross each other and the cable capacity should not be exceeded. The optimal solution on cable layout is given either by exact methods and a feasible solution on cable cost is given by heuristic methods.

An additional point of discussion could be the choice of cable types based on capacity and cost and the turbine types according to wind conditions. Next to this, the branching method could cause cost reduction while the merge of cables is allowed at a turbine node [2].

Beside the layout of the turbines, it is significant to consider the topography of the territory for the computation of wind speed and wake effect. The land area must be proven profitable in order to convince the land owner and start the design of the wind farm. Last but not least, the wind distribution, the turbine types and size increase the turbulence of the wake effect, because it may cause damage to wind blades [13].

| **Paper** | **Onshore/ Offshore** | **Decision Parameters** | **Performance Indicators** | **Further Research** |
|---|---|---|---|---|
| Samorani, 2010 [13] | Both | Turbine Location | Energy production & wake effect | Proximity constraint, topography |
| Chowdhury et al., 2012 [3] | Both | Turbine Location, number and type (rotor diameters, hub-height, etc.) | Energy production & cost | Land area per KW, nameplate (installed)capacity |
| Haugland J.K and Haugland D., 2012 [6] | Both | Turbine Location<br>a)X: coordinates on the grid<br>b)X: 0/1 binary matrix whether to install a turbine or not | Power production and net profit | N/A |
| Khan S. and Rehman S., 2012 [8] | On shore | Turbine Location | Energy production wake effect & cost | Benchmark |
| Bauer J. and Lysgraard J., 2014 [2] | Offshore | Turbine Location<br>X: 0,1 binary matrix whether a turbine belongs to the route or not | Cable layout cost | Cable types |
| Klein et al., 2015 [9] | Offshore | Grid Layout<br>X: turbines connect to power station<br>• Turbine Location, number, position<br>• Power station Location<br>Distance between turbines and main power stations | Cable layout cost & distance | Cable types, length, location Non-straight routes |

Table 1.1: Investigated Literature and its Optimization Characteristics

# Chapter 2

# The Model

## 2.1 Original Model

We start studying the model of Klein et al., which can be found in the paper "An Integer Programming Model for Branching Cable Layouts in Offshore Wind Farms" [9]. This model refers to the minimum spanning tree ($MST$) problem. MST is an undirected graph which connects all the nodes with the minimal total weight on its edges, see $Appendix A$.3. The authors of the paper refer to the MST with the terms of degree-constrained and capacitated MST, which are explained later in this section. The first term addresses the issue of branching capacities and the term capacitated MST refers to the number of nodes in each subtree.

Consider a graph with nodes set V=($V_c \cup V_d$), where $V_c$ represents a given set of wind turbine locations and $V_d$ represents power station locations. The edge set E $\subseteq V^2$ represents the possible connections between the wind turbines or between the turbine and the station. In this case, the turbines and the stations can be considered as nodes/vertices and the connection between these two nodes can be completed with an edge. An edge can be directed or undirected; directed edges are also called arcs [17]. The arc set $A_E = \{(i,j) : \{i,j\} \in E\}$ has an associated symmetric cost which is proportional to the Euclidean distance between the locations of the end nodes of the edge.

Despite the fact that the arc set is symmetric in the sense of distance cost, it differs from the edge set because the direction of connection between the wind turbines matters. Figure 2.1 shows the connection between the turbine node $i$ and $j$, and subsequently the potential connection of $j$ to the next turbine node $u$. The second path in the figure shows that the turbine nodes $i$ and $u$ are connected to turbine node $j$ and this differs from the first path, because the connection with the main power station $d$ launches from wind turbine node $j$ instead of $u$.

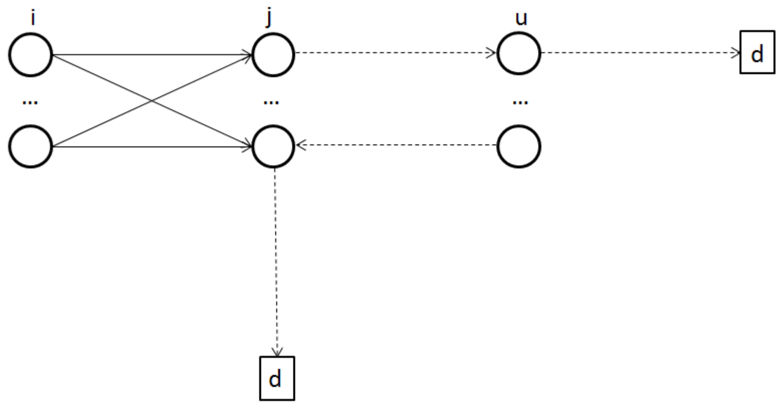



Figure 2.1: Single Commodity Multiple Echelon Discrete Location Model

Figure 2.1 shows a single commodity, multiple echelon discrete location model. This term is used in supply chain distribution models and occurs when the transportation commodity is unique and it moves between different layers in the supply chain [5].

The following notation is used:
*Indices*

| | | |
|---|---|---|
| $h$ | number of sources connected to an arc | $h \in \{1, 2, .., \|V_c\|\}$ |
| $i$ | source index | $i \in V$ |
| $j,u,v$ | alias indices for nodes | |

*Data*

| | | |
|---|---|---|
| $c_{ij}$ | symmetric cost/distance on arc | $(i, j) \in A_E$ |
| $C$ | maximum number of turbines | $C = \|V_c\|$ |
| $m$ | upper bound on number of branches | $m \in \mathbb{N}$ |
| $\chi$ | set of crossing routes | $\chi \in E^2$ |

*Variables*

| | | |
|---|---|---|
| $x^h_{ij}$ | connection between $i$ and $j$ with intensity of cables $h$ | $x^h_{ij} \in \{0, 1\}$ |

**Objective function:**

Minimize the total cost

$$min \qquad COST = \qquad \sum_{(i,j)\in A_E} \sum_{h=1}^{C} c_{ij} x_{ij}^h \tag{1}$$

The objective function of the model is to minimize the total cable cost/distance over all used routes taking into account the following restrictions:

**Subject to:**

Connection

$$\sum_{(i,j)\in A_E} \sum_{h=1}^{C} x_{ij}^h = 1 \qquad \forall i \in V_c \tag{2}$$

Each turbine should be connected to the network with one outgoing cable directed towards some substation.

Degree-constraint

$$\sum_{(i,j)\in A_E} \sum_{h=1}^{C-1} h x_{ij}^h - \sum_{(j,k)\in A_E} \sum_{h=1}^{C} h x_{jk}^h = -1 \qquad \forall j \in V_c \tag{3}$$

The number of connected turbines to a turbine node is one less than that of the outgoing cable.

Branching Capacity

$$\sum_{(i,j)\in A_E} \sum_{h=1}^{C-1} x_{ij}^h \leq m \qquad \forall j \in V_c \tag{4}$$

The number of incoming cables in a turbine node is less than or equal to $m$.

Crossing Routes

$$\sum_{h=1}^{C} (x_{ij}^h + x_{ji}^h + x_{uv}^h + x_{vu}^h) \leq 1 \qquad \forall \{\{i,j\},\{u,v\}\} \in \chi \tag{5}$$

No cables cross each other over all used connections.

Bound constraint

$$x_{ij}^h \in \{0,1\} \qquad \forall (i,j) \in A_E, h = 1, ..., C \tag{6}$$

$$x_{ij}^C = 0 \qquad \forall (i,j) \in A_E \cap \{V \times V_c\} \tag{7}$$

## 2.2 Modified Model

The model presented in this section aims to satisfy the same objective function as in the original model in Section 2.1. The original model has been modified due to its complexity.

Consider a graph with node set V=($V_c \cup V_d$), where $V_c$ represents a given set of wind turbine locations and $V_d$ represents power station locations, $|V_d| = 1$. The edge set E $\subseteq V^2$ represents the possible connections between the turbines and the power station. The arc set $A_E = \{(i,j) : \{i,j\} \in E\}$ has an associated symmetric cost which is relevant to the distance between the end nodes of the edge.

Recall, an edge is the connection between two nodes/vertices and a directed edge is called arc. An edge differs from an arc in the case of a wind farm, because the direction of a turbine connection matters.

The following notation is used:
*Indices*

| | | |
|---|---|---|
| $h$ | number of sources connected to an arc | $h \in \{1, 2, .., \lvert V_c \rvert\}$ |
| $i$ | source index | $i \in V$ |
| $j$,$k$ | alias indices for nodes | |

*Data*

| | | |
|---|---|---|
| $ca_{ij}$ | symmetric cost/distance on arc | $(i,j) \in A_E$ |
| $cb_i$ | cost/distance on arc (i,1) | $i \in V_d$ |
| $C$ | upper number of turbines | $C \in \mathbb{N}$ |
| $\chi$ | set of crossing routes | $\chi \in E^2$ |

*Variables*

| | | |
|---|---|---|
| $x_{ij}^h$ | connection between $i$ and $j$ with intensity of cables $h$ | $x_{ij} \geq 0$ |
| $y_i$ | connection between i and final destination | $y_i \in \{0,1\}$ |
| $COST$ | total cost | $COST \in \mathbb{R}^+$ |

**Objective function:**

Minimize the total cost

$$COST = \sum_{(i,j) \in A_E} \sum_{h=1}^{C-1} ca_{ij} x_{ij}^h + \sum_i cb_i y_i \tag{8}$$

The objective function of the model is to minimize the total cost/distance over all used routes.

Minimize the total cost plus a penalty to avoid non-integer alternatives

$$min \qquad pCOST = COST + 0.0001 \sum_{(i,j)\in A_E} \sum_{h=1}^{C-1} h^2 x_{ij}^h \tag{9}$$

The penalty cost has been added to the objective function. The penalty weight aims to force integer solutions for $x$ in the linear programming relaxation and prioritizes the sequence of nodes and the intensity of cables consequently. Every time that a single connection is created, the cost of the intensity of cables is considered. In order to minimize the total cost, an arbitrary value of **0.0001** is multiplied by the squared number of $h$. A small penalty is included to prevent alternative solutions that do not represent an adjacent choice for the intensity of cables.

**Convex quadratic minimization penalty method:** In the right hand side of the Equation 9, the intensity of cables is squared and it creates a convex quadratic function which can be divided into four pieces called breakpoints where $h$ addressed to (1,1),(2,4),(3,9),(4,16) points as is shown in Figure 2.2. The secant line lies above the graph of the function between the $h_1^2 = 1$ and $h_4^2 = 16$ and it forces to give a solution in between due to the weight of the function.

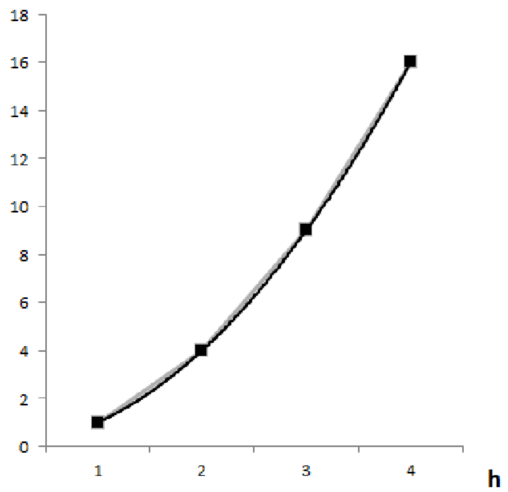



Figure 2.2: Penalty Weight

**Subject to:**

Connection

$$\sum_{(i,j)} \sum_{h=1}^{C-1} x_{ij}^h + y_i = 1 \qquad \forall i \in V_c \tag{10}$$

The first constraint which defines the creation of a connection between a turbine with another turbine or substation, remains the same as defined in the original model (Section 2.1). It ensures that each turbine has exactly one outgoing cable which ends in another node.

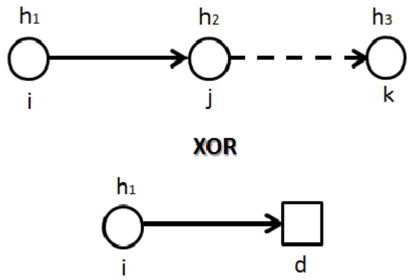



Figure 2.3: Connection of nodes

Direction

$$\sum_{(i,j)} \sum_{h=1}^{C-1} x_{ij}^h + \sum_{(j,i)} \sum_{h=1}^{C-1} x_{ji}^h \leq 1 \qquad \forall (i,j) \in V_c \tag{11}$$

The next constraint has been developed in order to avoid multi directional connections. Once a connection has been created, then it should be saved as one direction connection, preventing the creation of the same connection backwards.

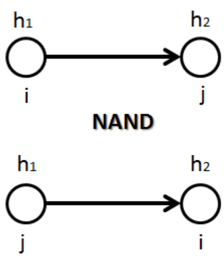



Figure 2.4: Single direction of two node connection

Origin

$$\sum_{(k,i)}\sum_{h=1}^{C-1} x_{ki}^h + \sum_{(i,j)} x_{ij}^1 + y_i \geq 1 \qquad \forall i \in V_c \tag{12}$$

An important consideration that became clear while testing the model was the definition of the origin turbine. An origin turbine is the first turbine on the arc, but in reality, it is the one which sends the electrical power last, since it is the farthest turbine starting from the power station. So, it is the first node considering the intensity of cables and this number is increasing while the turbines are approaching the station.

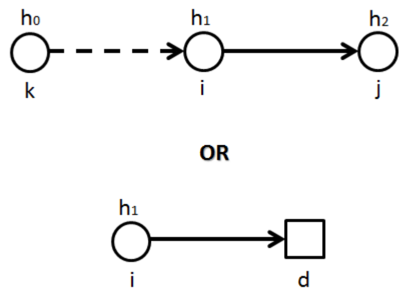



Figure 2.5: Definition of the first node of route

The importance of this constraint is based on the next inequality.

Inequality

$$Cy_j + \sum_{(j,k)\in A_E}\sum_{h=1}^{C-1} hx_{jk}^h - \sum_{(i,j)\in A_E}\sum_{h=1}^{C-1} hx_{ij}^h \geq 1 \qquad \forall j \in V_c \tag{13}$$

The development of this certain inequality is the reason that makes this model differ from the original. The first part defines the possibility of the direct connections of turbines to the substation without any intermediate node turbine. Otherwise, if a turbine is connected with another turbine then the number of cables should be increased by one in order to afford the electrical transport through the cable.

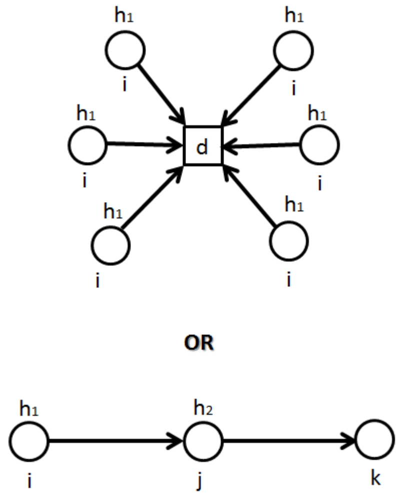



Figure 2.6: Intensity of turbine cables

### 2.2.1 Missing constraints

In the original model, the following constraints have been introduced:

<u>Branching</u>

Branching defines the maximum number of branches per turbine. In

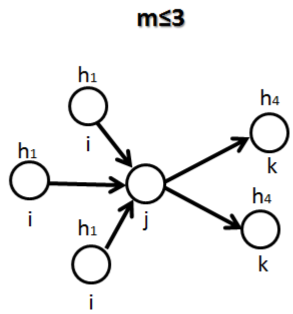



Figure 2.7: Number of turbine branches

this computational model the branching has not been taken into account, because of the lack of data. Reasons which determine the maximum number of branches could be relevant to turbines and cable types (size and capacity) or to topography of territory. However, as can be seen in the numerical examples 2 and 3 in Section 2.2.2, the branching is applied, because the specific constraint is relaxed.

Crossing Routes

It is very important to prevent the crossing of cables. Crossing cables can cause high voltage power that leads to generate heat, so they may burn. Also, in order to protect the cables, the wind farm developers bury the cables into the seabed. In the case where the lowest one fails, then both have to be dug up in order to replace the deeper one. Both cases, the cost is really high, so it is more profitable to avoid crossing cables.

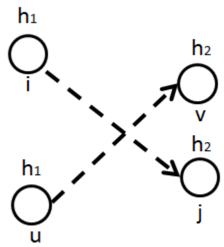



Figure 2.8: Intersection/crossing points

In this computational model, the crossing routes have been developed in an explicit way for reasons of convenience.

### 2.2.2 Numerical Examples

**Example 1**

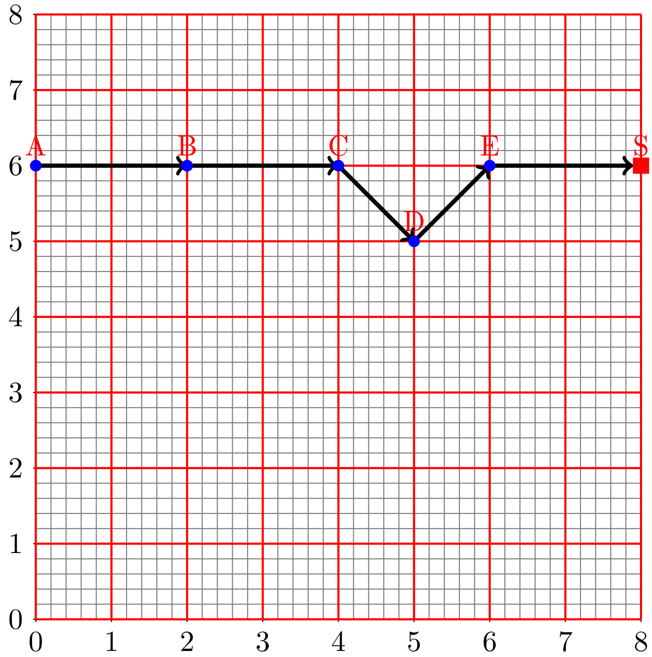



Figure 2.9: Graphical Example 1

The wind turbines in Figure 2.9 are located in a continuous path and the power station appears at the end of this route. Before the execution of the computational model, we can assume that the wind turbines which are located at the same row would be connected by a single arc. There is an uncertainty about turbine D, whether it would be connected on the same arc or it would connect directly to the power station.

Coordinates of elements in the grid:

Wind turbine locations:
A: (0.0,6.0), B: (2.0,6.0), C: (4.0,6.0), D: (5.0,5.0), E: (6.0,6.0)
Power Station: S: (8.0,6.0)

As shown below, the solution of GAMS appears as follow:
$x^1_{AB} = 1, x^2_{BC} = 1, x^3_{CD} = 1, x^4_{DE} = 1, y_E = 1$
and all turbines are connected to the power station with a single arc. Due to the intensity of cables, wind turbine D is connected to the same arc instead of directly to the power station.

**Example 2**

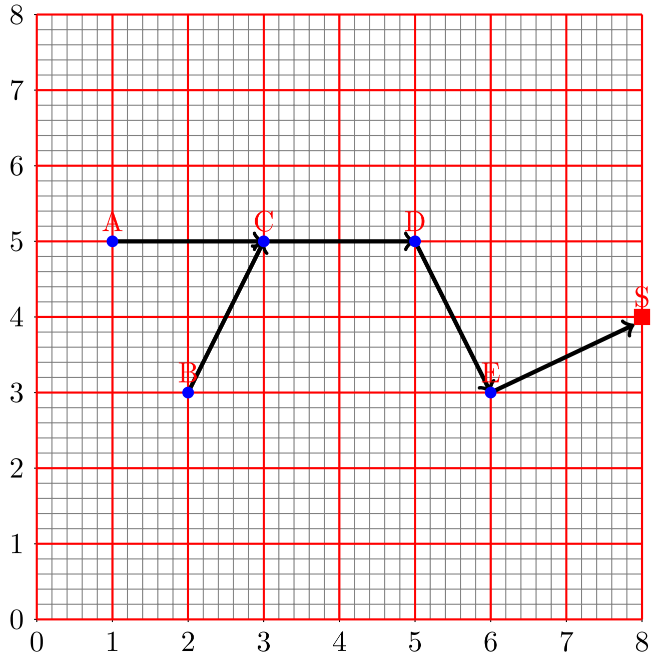



Figure 2.10: Graphical Example 2

The wind turbines in Figure 2.10 are located in two continuous paths and the power station appears in between the two route endpoints. Before the execution of the computational model, we can assume that the wind turbines which are located at the same row would be connected by a single arc. So, two arcs would be created. However, we need to take the branching capacity into consideration, where the model allows branching of the cables at the wind turbine locations.

Coordinates of elements in the grid:

Wind turbine locations:
A: (1.0,5.0), B: (2.0,3.0), C: (3.0,5.0), D: (5.0,5.0), E: (6.0,3.0)
Power Station: S: (8.0,4.0)

As shown below, the solution of GAMS appears as follows:
$x^1_{AC} = 1, x^1_{BC} = 1, x^3_{CD} = 1, x^4_{DE} = 1, y_E = 1$
and there is a branching at turbine C.

**Example 3**

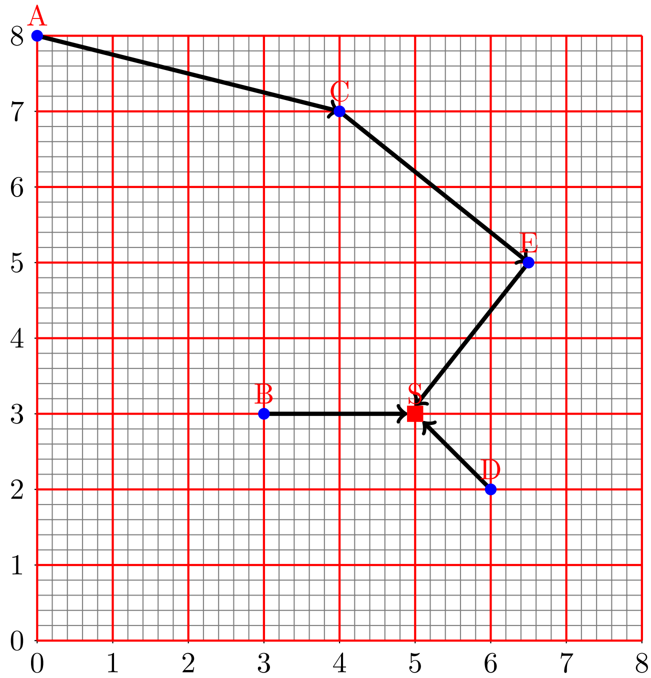



Figure 2.11: Graphical Example 3

The wind turbines in Figure 2.11 are located around the power station. Before the execution of the computational model, we can assume that the wind turbines would connect directly to the station by creating a star shape. So, each individual wind turbine would send energy directly to the station and no penalty weight would be added for the intensity of cables. However the distances between the wind turbines and the power station vary and this is crucial reason to falsify the above assumption.

Coordinates of elements in the grid:

<u>Wind turbine locations:</u>
A: (0.0,8.0), B: (3.0,3.0), C: (4.0,7.0), D: (5.0,3.0), E: (6.0,2.0)
<u>Power Station:</u> S: (6.5,5.0)

As shown below, the solution of GAMS appears as follows:
$x^1_{AC} = 1, x^2_{CE} = 1, y_B = 1, y_D = 1, y_E = 1$
and the connection of turbines A, C and E is merged in a single arc while it creates a star shape with the rest of the arcs.

## 2.3 Conclusion

In the paper "An Integer Programming Model for Branching Cable Layouts in Offshore Wind Farms" [9], the researchers refer to the terminology of capacitated Minimum Spanning Tree($MST$) under the condition of satisfying some specific restrictions.

Firstly, the definition of capacitated refers to the cable capacity of each subtree in the graph and the restriction requires not to be exceeded by the number of connected nodes as Equation 3 implies. For each subtree the cable installation can be calculated by choosing the optimal cable connection from the origin to terminal node in order to transport the power along its edge.

In graph theory, there is a possibility where every spanning tree can be minimum if the edges have the same weight. On the other hand, when each edge has a distinct weight, then there is only one unique spanning tree, such as in the numerical examples of Section 2.2.2. Both cases are likely to occur in the case of a wind farm.

In addition, the branching property of MST which is described in Equation 4, is included in this optimization problem and it implies to predefine the maximum number of branches at each node. The modified model in Section 2.2 relaxes this restriction by allowing any number of branches at the nodes, as shown in Figures 2.10 and 2.11. However, Figure 2.9 seems to be unsuitable for branching since the result shows that it is optimal to connect the turbines sequentially.

Minimum Spanning Trees were first invented for the design of networks such as computer, telecommunication transportation, electrical grid, such as this, and more [18]. So another important property of MST and the wind farm case is to prevent creating cycles/loops in the network by connecting each node only once.

The model has been implemented in GAMS, see $Appendix B.1$, and Equation 13 which is presented as unique in Section 2.2 has been developed due to the consideration of the intensity of cables to ensure the electrical transportation. The modified model has been solved using Combinatorial Optimization techniques (Appendix A.1) and Branch and Bound method has been elaborated to eliminate continuous solutions (Appendix A.2).

# Chapter 3

# Additional node

The title of this chapter refers to an additional node that can be added to the network as a power substation. It is very likely that an additional node into the existing grid network can relax other limitations, such as crossing routes and it can also reduce the intensity of cables. We assume that this can lead to a reduction of cable cost significantly.

The initial assumption was that the additional node should not necessarily be directly connected with the main power station in order to transport energy. This implies that it can also be connected with an intermediate turbine which must be connected to the main power station later. But this is not the case, because if we use an additional node to connect some wind turbines which are close to it, then the power substation works/acts exactly like an additional turbine which needs to transfer the receiving energy to the next turbine. After several trials, it turns out that is more wise to connect the additional node directly to the main power station.

Needless to mention that some wind turbines can be connected directly to the main power station. The questions are, for which instances it is optimal to add a node, and if yes, where is the optimal location for this power substation. Using the Weber problem property [16], it is possible to find an optimal point by minimizing the distances of the elements in the grid in order to tackle the problem. This point can be considered as the location of the additional node.

Section 3.1 examines the potential cost reduction for four different cases while a node is added in the grid. Crossing routes have been developed in an explicit way, as in Section 2.2. Section 3.2 includes the description of possible occurrence of crossing routes based on scenarios. In addition, the idea of adding a node on existing crossing points is introduced, when the planarity constraint is disregarded. The updated model which includes the Weber problem presented in Section 3.3 and it has been used for the following analytical examination.

## 3.1 Analytical examination

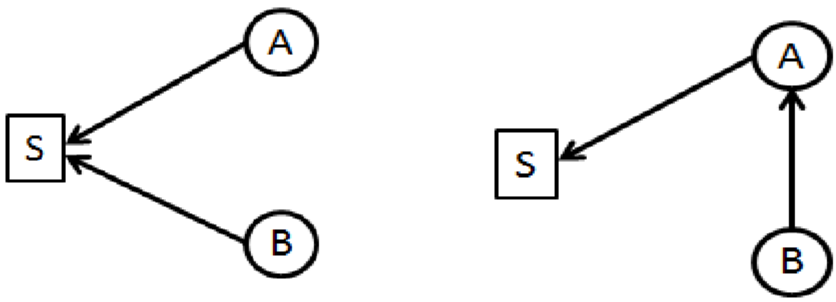
At the beginning, the Weber problem has been applied only for the case of two wind turbines to check the possible relation of the distances between the turbines and the main power station. The initial observation after this examination is the relation of distance between the elements into the grid. The additional node is always placed at a specific point which depends on the location of the rest of the elements. Later on, the cases of three and four wind turbines have been examined. So, we have started doing some more analytical investigation by designing four specific cases.

### 3.1.1 Symmetric case-2 wind turbines

Firstly, a case is studied where two wind turbines are located at equal distance from the main power station.

Let S be the location of the main power station, A is the first wind turbine and B is the second wind turbine:
$S = (S_1, S_2), A = (\alpha_1, \alpha_2), B = (\beta_1, \beta_2)$ at some distance of S without loss of generality, the problem is equivalent by shifting, rotation, scaling to the following instance.
$S = (0, 0), A = (1, c), B = (1, -c)$

<u>Without the addition of a node</u>




(a) $g = 2\sqrt{1 + c^2}$ (b) $h = \sqrt{1 + c^2} + 2c$

Figure 3.1: Configuration of alternatives for the lay-out of the cables without the additional node

Figure 3.1 shows the two alternative configurations for the lay-out of the cables without the additional node. The functions $g$ and $h$ estimate the total cable length when two wind turbines transport the generated power into a single station.

($a$)The alternative where wind turbines A and B are connected directly to the main power station S has a total cable length g:

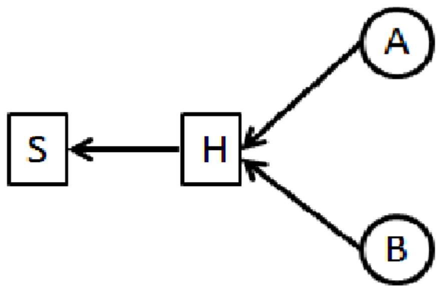
$$g = 2\sqrt{1 + c^2} \tag{1}$$

($b$)The alternative where wind turbines A and B are interconnected has a total cable length h:

$$h = \sqrt{1 + c^2} + 2c \tag{2}$$

With the addition of a node

The optimal location of the additional node $H = (x, 0)$ given $A = (0, c)$ and $B = (0, -c)$ is determined by the optimum of the Weber problem taking into account the ratio between the distance from the power station and the distance between the two turbines.




($c$)$f(x, c) = x + 2\sqrt{(1 - x)^2 + c^2}$

Figure 3.2: Configuration of alternative for the lay-out of the cables with the additional node

Figure 3.2 shows the alternative configuration for the lay-out of the cables with the additional node. The function $f(x, c)$ estimates the total cable length when two wind turbines transport the generated power via a substation.

($c$)The alternative where wind turbines A and B are connected directly to the additional node at $H = (x, 0)$ position and this node is connected to main power station S implies a cable length f:

$$f(x, c) = x + 2\sqrt{(1 - x)^2 + c^2} \tag{3}$$

The value $x$ determines the distance of the additional node from the main power station. In order to calculate the optimal value $x$, we use the first derivative of $f(x,c)$ with the respect to $x$.

*First derivative:*

In graphical terms, the first derivative is the slope of the tangent line to the function at the point $x$. It shows whether a function is increasing or decreasing. In this case, where we are looking to minimize the distance which depends on point c, the positive slope tells us that, as $x$ increases,$f(x,c)$ also increases.

$$\begin{aligned}
\frac{\partial f}{\partial(x,c)} &= 1-\frac{2(1-x)}{\sqrt{((1-x)^2)+c^2}} &&= 0\\
&2(1-x) &&= \sqrt{((1-x)^2)+c^2}\\
&(1-x)^2 &&= \frac{c^2}{3}\\
&1-x &&= \frac{c}{\sqrt{3}}\\
x^*(c) &= 1-\frac{c}{\sqrt{3}} && \text{for } c \leq \sqrt{3}
\end{aligned}$$

$$f(x^*(c),c) = 1-\frac{c}{\sqrt{3}}+2\frac{4c^2}{3} = 1+c\sqrt{3} \tag{4}$$

The addition of an additional node is optimal if $1+c\sqrt{3} \leq 2\sqrt{1+c^2}$.

**Explanation:**
As mentioned before, the pre-defined location of the elements in the network defines the exact location of the additional node.

Equation 1 gives as optimal the option to connect both wind turbines to the main station. This is the case where the two wind turbines are far apart from each other, meaning that the value of $c$ is larger, but close enough to the main power station.

Equation 2 corresponds to a connection between the two wind turbines, when the turbines are close enough, which implies that the value of $c$ is smaller. The last wind turbine which is connected to the main power station, is the one who is closest. In the symmetric case, where the two turbines have equal distance from the main power station, any turbine can be considered as the last node.

In both cases the addition of a node has been proven as expensive and therefore the Weber problem gives as optimal the coordinate position of the main power station $S = (0, 0)$.

On the other hand, Equation 3 estimates that the minimum total cost can be found in the case where both wind turbines are connected to the additional node at the location $x$. Equation 4 shows that the optimal value of $x$ depends on the value of $c$.

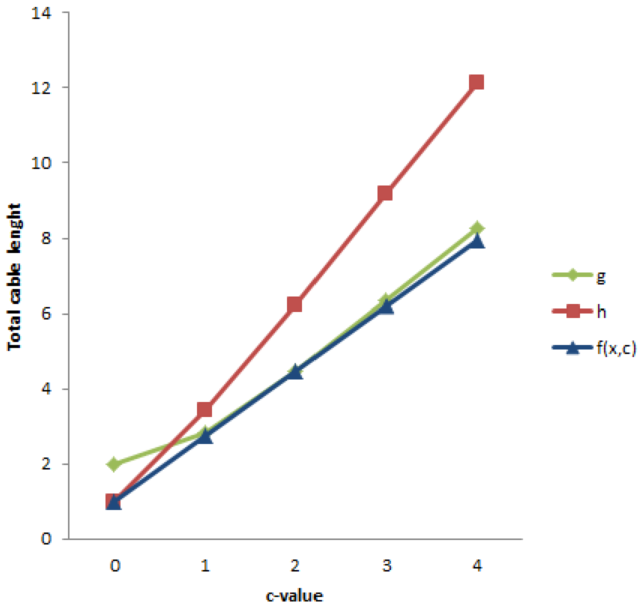



Figure 3.3: Total cable length for the three alternative configurations as function of the parameter c

Figure 3.3 shows the total value of functions according to the price of $c$ and as has been mentioned, the aim is to minimize the total value which represents the total length of cable distance. In terms of dominance, this means that less is preferred to more.

The dominant linear function of f$(x, c)$ for the values of $c$ smaller than 1, always lie below and proves the case where the connection of turbines to the additional node gives a smaller total cost. Therefore, the alternative $(c)$ shown in Figure 3.2, where function f is minimum, is selected.

Function h seems to be dominated by both functions g and f for the greater values of c. This proves that as far as the turbines are located from each other, alternative $(b)$ shown in Figure 3.1 is not chosen.

Function g is mainly dependent on the distance between the wind turbine and the main power station. Therefore, at certain values of $c$, function g compared to f, to find the minimum total length and the alternative $(a)$ shown in Figure 3.1 or $(c)$ shown in Figure 3.2, is chosen.

Consider the following instance:

Let $c = 0.6$ and the coordinates of elements in the grid are:
$A = (1.0, 0.6), B = (1.0, -0.6), S = (0.0, 0.0)$
Let H denote the optimal location of the additional point. Then it follows from Equation 4 that $H = (0.65, 0.0)$

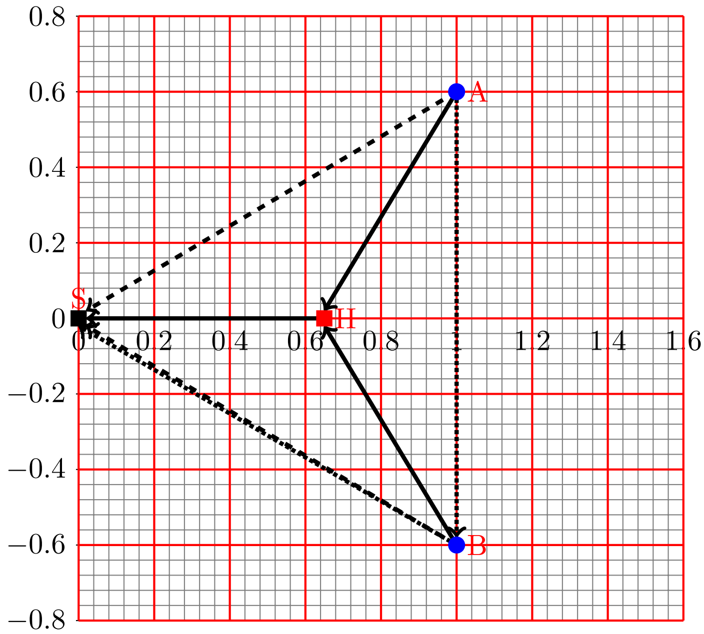



Figure 3.4: Graphical representation-Symmetric case

The wind turbines in Figure 3.4 are located at equal distance from the power station and the total distance/cost value of the three alternative cases is given below:
*dashed lines*: g=2.33, *dotted lines*: h=3.36, *straight lines*: f(x,c)=2.03

As is shown, the minimum total value requires the addition of a node. The Weber problem gives the optimal location of the additional node at point H and the optimal cable connection requires the direct connection of wind turbines to the additional node and the statutory connection between the additional node and power station. Apparently, the addition of a node reduces the total cable length.

**Geometric Statement**

From the geometric point of view, in the symmetric case, the optimal location of the additional node is determined by the angle 120° between the wind turbine located at $(1, c)$ and the main power station $S = (0, 0)$. This observation coincides with earlier observations with respect to the optimum of the Weber problem having three points.

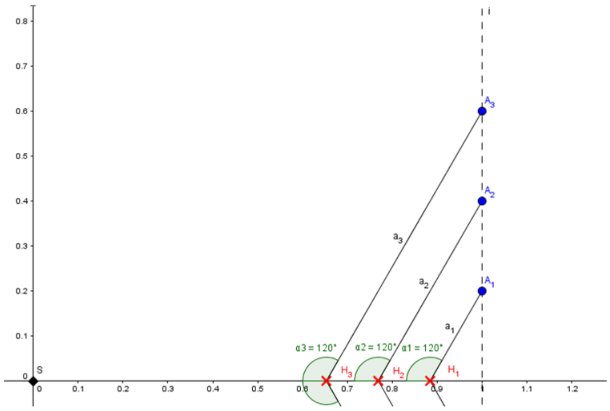



Figure 3.5: Geometric position of the additional node on positive grid while increasing $c$

Figure 3.5 shows that the position of the additional node H creates an angle of 120° between the main power station S and the wind turbine A. While $c$ is increasing, the position of the node is changing accordingly. The value of $c$ defines the position of the turbine on vertical axis y. Recall, the addition of a node can reduce the total cable length based on value of $c$, according to Figure 3.3.

### 3.1.2 Non-Symmetric case-2 wind turbines

Any non-symmetric instance with two turbines can be transformed to an instance in which the main power station and the first turbine have a fixed location into the grid while the wind turbine B can be located at any place. In that case, the addition of a node has been proven optimal if and only if the wind turbine B is located anywhere in between the lines which provide an angle of 120° with the turbine A or the main power station. Recall, the addition of a node is optimal when the total cable length is reduced.

Assume again that S is the main power station, A is the first wind turbine and B is the second wind turbine:
$S = (S_1, S_2), A = (\alpha_1, \alpha_2), B = (\beta_1, \beta_2)$

**Geometric Statement**
Any instance can be transformed to the following standard instance:

$S = (0,0), A = (1,0), B = (\beta_1, \beta_2)$
Again, from the geometric point of view, in the non-symmetric case, the optimal location of the additional node is determined by the angle 120° between the wind turbine A located at $(1,0)$ and the main power station $S = (0,0)$.

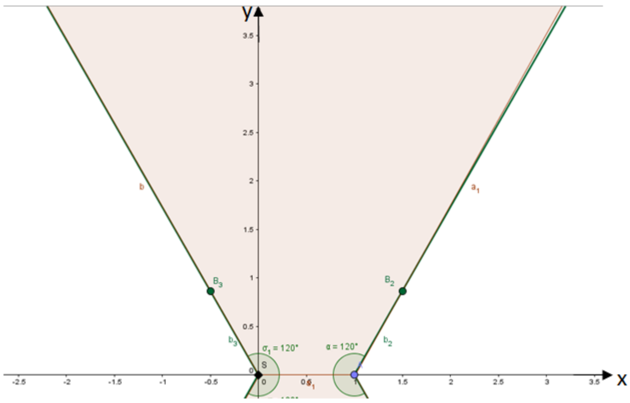



Figure 3.6: Set of locations of wind turbine B, where the additional node is optimal

The shaded area in Figure 3.6 shows the potential location of the wind turbine B, where the addition of a node is optimal. Otherwise, the wind turbines are connected directly to turbine A or to the main power station. Equation 4 is modifies as follow:

$$f(x^*, (\beta_1, \beta_2)) \tag{5}$$

where

$$x^*(\beta_1, \beta_2) = \arg_x min\{d(A,x) + d(B,x) + d(S,x)\}$$

and $d$ represents the Euclidean distance between the two points.

An important issue to consider are the fixed costs of adding a node. In this paper, these costs have been considered as negligible. But, in the real case field, this may not be optimal. This can be investigated by wind farm developers and engineers.
The advantage of adding a substation is the difference between the installation and maintenance cost of the additional node and the saving costs of using it.

Figure 3.7 incorporates the contour lines to emphasize the advantage of the added node in the grid, if and only if the wind turbine B is placed inside the white area. The label number shows the benefit of using the

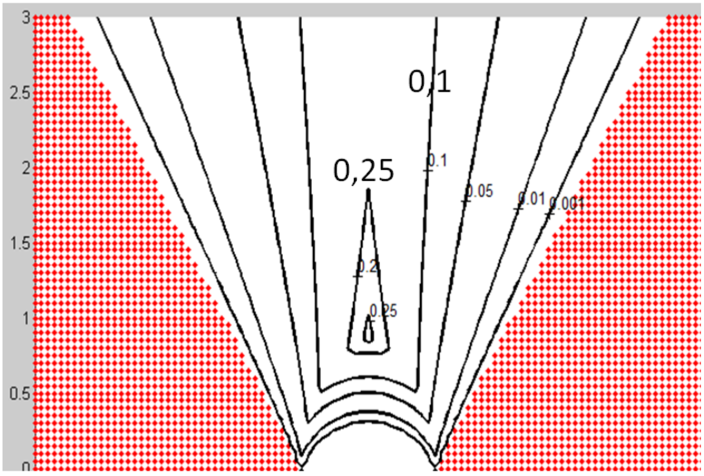



Figure 3.7: Contour lines showing the advantage of adding a node given the location of the wind turbine B= $(\beta_1, \beta_2)$

additional node. The addition of a substation involves the installation and maintenance costs which can be high in relation with the benefit that can be reached. Therefore the decision can be made based on realistic numbers. When the fixed costs are less than the advantage, the number shown in the label, then the installation of a node is beneficial. Recall, when the wind turbine B is placed inside the red area then the addition of a node is not optimal anymore.

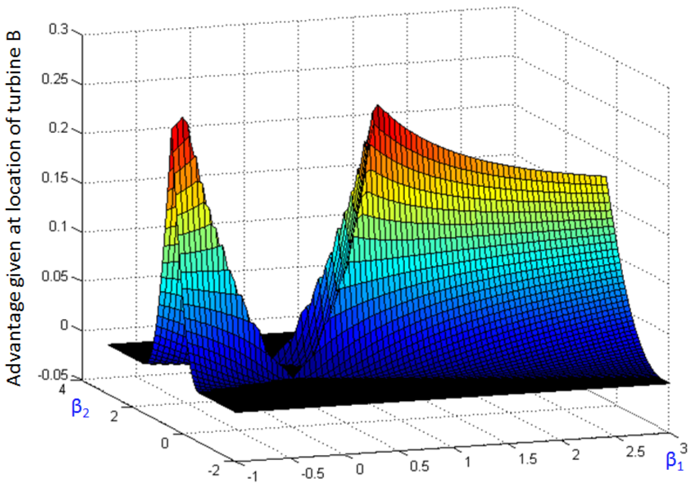



Figure 3.8: 3D-Surface visualisation of the advantage of adding a node

### 3.1.3 Semi-Symmetric case-3 wind turbines

The semi-symmetric case can be considered as an extended case of the symmetric case, where the two wind turbines have been set in symmetric position and the location of the third turbine is varied in a systematic way again. Depending on the third wind turbine position, the optimal configuration and location of a possible additional node varies in order to satisfy the general objective.

Assume that S is the main power station, A is the first wind turbine, B is the second wind turbine and now C is the third turbine with the undetermined position:
$S = (S_1, S_2), A = (\alpha_1, \alpha_2), B = (\beta_1, \beta_2), C = (\gamma_1, \gamma_2)$

Consider that the coordinates of elements are as follow:
$S = (0, 0), A = (1, 1), B = (\alpha_1, -\alpha_2)), C = (\gamma_1, \gamma_2)$

The question is now for which instances of the location of C it is optimal to have an additional node $H = (\eta_1, \eta_2)$ and which configurations are optimal. By the meaning of that, the addition of a node can be considered as optimal, when the total length of the cable is reduced. Recall, the additional node should be connected directly towards the main power station and the location of the additional node between the symmetric wind turbines A and B is $H = (\sqrt{2} - 1, 0)$, as has been proved in Section 3.1.1.

1. The alternative where the $C = (\gamma_1, 0)$:
   a) $0 < \gamma_1 \leq (\sqrt{2} - 1)$
   The addition of a node is not considered as optimal. Both symmetric wind turbines are connected to wind turbine C.
   b) $(\sqrt{2} - 1) < \gamma_1 \leq 1$
   The addition of a node is optimal. All of the wind turbines are connected to it.
   c) $\gamma_1 \geq 1$
   The addition of a node is optimal and it is always placed at point $H = (1, 0)$.
   d) $\gamma_1 < 0$
   The addition of a node is optimal at point $H = (\sqrt{2} - 1, 0)$ and the third turbine is now connected directly to the main power station.

2. The alternative where $C = (\gamma_1, \gamma_2)$.
   a) $\gamma_1 > 1$
   The additional node is always optimal and there are two possible configurations:
   • In the case where wind turbine C is closer to one of the fixed tur-

bines, C connects to the closest one and the two symmetric turbines are connected to the additional node $H = (\sqrt{2} - 1, 0)$.
• In the case where wind turbine C is located somewhere in between the two fixed turbines then a new position for the additional node $H = (\eta_1, \eta_2)$ is defined and all the turbines are connected to it.

b) $0 < \gamma_1 < 1$
The addition of a node is optimal for two different configurations:
• In the case where wind turbine C is closer to one of the fixed turbines, C connects to the closest one and the two symmetric turbines remain connected at $H = (\sqrt{2} - 1, 0)$, such as in first case of 2a.
• In the case where third turbine is located somewhere in between the two turbines then a new position for the additional node $H = (\eta_1, \eta_2)$ is defined and all the turbines are connected to it, such as in second case of 2a.

The addition of a node is not optimal for two different configurations:
• In the case where either the turbine A or B, depending on which one is closest, is connected directly to turbine C. Wind turbines C and the remaining one are now connected to main station S.
• In the case where the third turbine is located between the main power station and the additional node at $H = (\sqrt{2} - 1, 0)$, then the fixed wind turbines are connected directly to the third turbine and towards the main power station.

c) $\gamma_1 < 0$
The addition of a node is optimal for two different configurations:
• In the case where the third turbine is connected either to turbine A or B, depending on which one is closest, and the two symmetric wind turbines are connected to the additional node $H = (\sqrt{2} - 1, 0)$, such as in first case of 2a.
• In the case where the wind turbine C is connected directly to the main power station S and the connection of the symmetric turbines to additional node at $H = (\sqrt{2} - 1, 0)$.

Furthermore, there is a case where the addition of a node is not optimal, such as in third case of 2b. The connection of the closest fixed turbine to turbine C and the connection of turbine C and the remaining turbine to main power station has been proven more beneficial, such as in third case of 2b.

In order to prove the above statement analytically, we have tried to define the boundaries of the position of the third turbine, as is shown with dots in Figure 3.9.

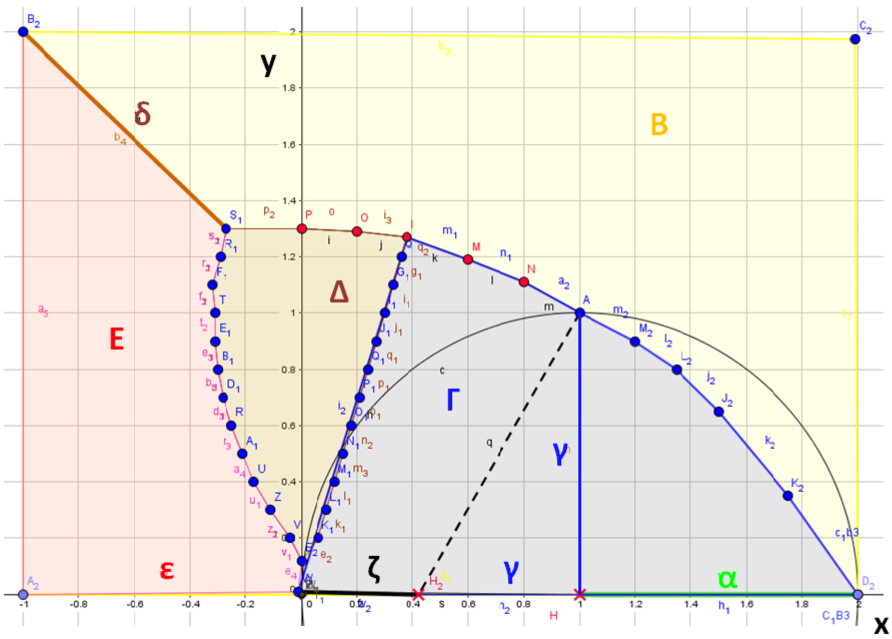



Figure 3.9: Positive side of grid where different scenarios are optimal depending on the third turbine location

In general, Figure 3.9 defines whether the additional node is optimal or not, depending on location of turbine C in the positive area of grid based on the following scenarios:

- The green line $\alpha$ shows that if the third turbine is located at this line, then the additional node is always optimal at point $H = (1, 0)$, where the rest of the turbines are connected there as well.

- The yellow shadow area $B$ defines that the addition of a node is optimal as in the symmetric case (Section 3.1.1), at point $H = (\sqrt{2}-1, 0)$, if the wind turbine C is located inside there. Although, the third turbine is connected to the closest turbine, which is turbine A, the wind turbines A and B remain connected to the additional node.

- The additional node can be optimal and vary when the third wind turbine is placed in the blue area $\Gamma$ or lines $\gamma$ and all of the wind turbines are connected to it.

- The orange area $\Delta$ shows the area where the additional node is not optimal, while the closest fixed turbine A is connected to turbine C and then, turbine C and B are connected to the main station.

- At the orange line $\delta$ the distance between the wind turbines C and A and the distance between the turbine C and the main power station are equal. So, the turbine C can either connected to turbines A or to the main power station, while the symmetric turbines remain connected to additional node at $H = (\sqrt{2} - 1, 0)$.
- The red area $E$ and the line $\varepsilon$ define the area where the third turbine can be connected directly to main power station while the symmetric turbines remain connected to additional node at $H = (\sqrt{2} - 1, 0)$.
- The black line $\zeta$ shows that if turbine C is located at this line, then the symmetric turbines are connected to third turbine and towards the main power station. So, the addition of the node is not optimal.

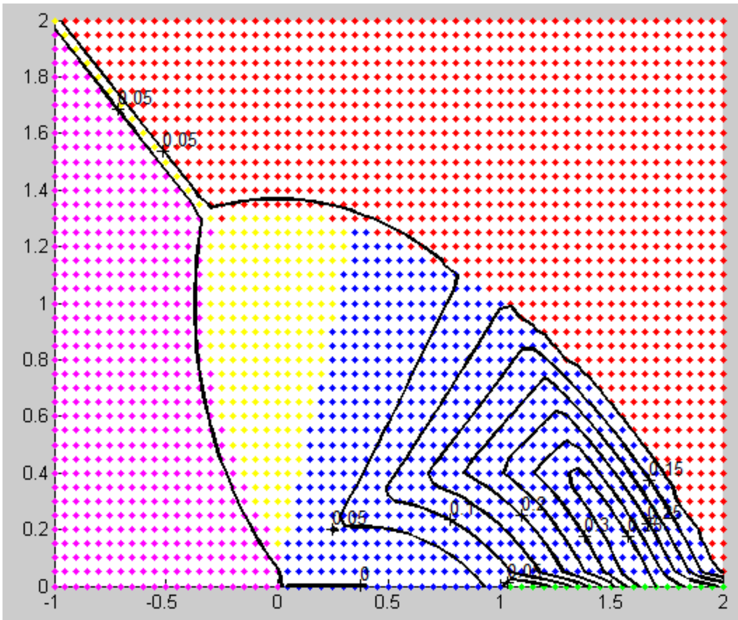


Figure 3.10: Contour lines showing the advantage of adding a node by given the location of the third wind turbine

Figure 3.10 incorporates the contour lines, such as Figure 3.7, to emphasize the advantage of the added node in the grid. Recall, the addition of a substation involves the fixed costs which can be higher than the benefit that it provides. So, the decision will be made based on realistic numbers.

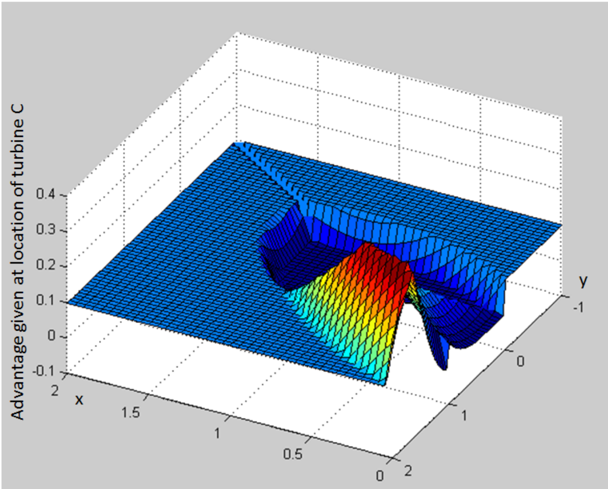



Figure 3.11: 3D-Surface visualisation of the advantage of adding a node varying the location of the third turbine C

### 3.1.4 Symmetric case-4 wind turbines

In the case of two symmetric turbines the addition of a node have been proven optimal while the location of the additional node has been placed towards the main power station. An interesting extended case arose, which is based on the symmetric case (Section 3.1.1). In this case, the total number of wind turbines are four and the last two turbines are symmetric with the two initial wind turbines, in a sense of the vertical axis y.

Assume again that S is the main power station, A is the first wind turbine and B is the second wind turbine:
$S = (S_1, S_2), A = (\alpha_1, \alpha_2), B = (\beta_1, \beta_2), C = (\gamma_1, \gamma_2), D = (\delta_1, \delta_2)$

Now consider that the coordinates of elements are as follow:
$S = (0, 0), A = (1, 1), B = (\alpha_1, -\alpha_2)), C = (\gamma_1, \alpha_2), D = (\gamma_1, -\alpha_2)$

$(a)$ The alternative where the last two turbines are connected to the two turbines which are already connected to the optimal additional node $H = (\sqrt{2} - 1, 0)$ as has been found for the symmetric case of two turbines (Section 3.1.1).

$(b)$ The alternative where the last two turbines have been set as the only ones in the grid in order to define a new position for the additional node $H2 = (\gamma_1 - 1/\sqrt{3}, 0)$. Thanks to the analytical expression in Section 3.1.1 the location of the additional node is now known and all the turbines are connected to it.

($c$) The alternative where all of the four turbines are entered. So, the Weber problem now tries to minimize the distances between the 5 total elements in the grid and defines a new location for the additional node $H = (\eta_1, 0)$.

Consider the following graphical example:

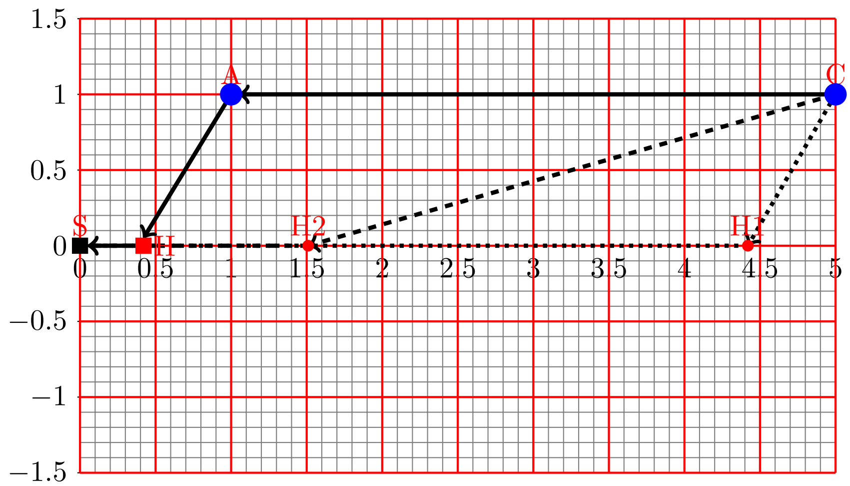



Figure 3.12: Three scenarios of the case of 4 symmetric wind turbines

Assume again that S is the main power station, A is the first wind turbine, C is the third wind turbine, while Hs is the additional node:
$S = (S_1, S_2), A = (\alpha_1, \alpha_2), C = (\gamma_1, \gamma_2), H = (\eta_1, \eta_2)$

Now consider that the coordinates of elements are as follow
$S = (0, 0), A = (1, 1), C = (5, \alpha_2), H = (\sqrt{2} - 1, 0), H1 = (1.5, 0), H2 = (5 - 1/\sqrt{3}, 0)$

Figure 3.12 represents graphically the potential connection of the elements in the grid. The alternative ($a$) describes the scenario where the wind turbine C can be connected direct to A and then to the main power station. Alternative ($b$) suggests the connection of wind turbines A and C are with the additional point H1 and then to the main power station, as shown in Figure 3.12 using dotted lines. The last alternative describes the connection of wind turbines A and C to the new position of the additional node H2 and towards the main station, as is shown using dashed lines.

The outcome of the above alternatives has been compared with each other. The minimum cable length is given for the first alternative, and the optimal location of the additional node remains the same in the symmetric case in Section 3.1.1.

## 3.2 More: Crossing Routes

In the previous section, the placement of an additional node into the electrical grid has been examined using the Weber property. The restriction/constraint that the Weber problem does not take into account is the crossing of cables. Although, it can give a point which indeed minimizes the distance, this point is the location of the additional node where more cable connection paths exist. So, it is very likely that for cases with more turbines, some crossing points will be created between the turbines which are connected to the substation and the turbines which are connected to the main power station. The connection paths can create the following possible scenarios:

1. Intersection: The intersection point is created when the paths that connect the elements are crossed. The intersection point can be found by the solving the linear functions.

2. Align:

    - Overlapping: This occurs when the edges are aligned. In the case of wind farm design, it may be assumed that this is not likely to occur because the aim is to minimize the distances between the elements. So, the element which is located in between two others, will not connect with an element which is further.
    - Non-overlapping: The possibility where the two connection paths are aligned, but not overlap.

3. Parallel: The possibility where the two connection paths are in parallel, then the creation of crossing points is unlikely.

4. Non of the above: The case where the elements are connected in a more random way and non of the above cases occur.

At the point, where the Weber problem has been proved inadequate, the investigation has been gone through the idea of adding a node, where the crossing point will be created. At the beginning, we can assume that the planarity constraint is not necessary anymore and we can compute the optimal cable layout connection. In that case should be checked, whether the optimal solution is creating paths with intersection points and it is necessary to repeat the calculation by placing the additional node on those crossing points. But after that, the planarity constraint should be added to prevent the crossing between the connection of the node and the power station with any other path. Otherwise, this can lead to a loop where we will have to add substations every time that a crossing point exists. The above hypothesis has not been examined and it can be considered as future research. In the next section, an updated version of the modified model (Section 2.2), is shown. The updated model which includes the Weber problem, has been used for the examination of the cases in Section 3.1.

## 3.3 Updated Model

Consider a graph with node set V=($V_c \cup V_d$), where $V_c$ represents a given set of wind turbine locations and $V_d$ represents the station locations, $V_d = |2|$. The edge set E $\subseteq V^2$ represents the possible connections between the turbines and the power stations. The arc set $A_E = \{(i,j) : \{i,j\} \in E\}$ which has an associated symmetric cost which is relevant to the distance between the end nodes of the edge.
As has been mention before, the difference between the arc and the edge is based on the direction of connection betweens the nodes.

The following notation is used:
*Indices*

| | | |
|---|---|---|
| $h$ | number of sources connected to an arc | $h \in \{1, 2, .., \|V_c\|\}$ |
| $i$ | source index | $i \in V$ |
| $s$ | station index | $s \in \{S\}$ |
| $j$,$k$ | alias indices for nodes | $V_d$ |

*Data*

| | | |
|---|---|---|
| $ca_{ij}$ | symmetric cost/distance between turbines on arc | $(i,j) \in A_E$ |
| $cb_i$ | cost/distance on arc from wind turbine to power station | $i \in V_c$ |
| $cd_i$ | cost/distance on arc from wind turbine to additional node | $i \in V_c$ |
| $dw_s$ | cost/distance on arc between power station and additional node | $s \in V$ |
| $\chi$ | set of crossing routes | $\chi \in E^2$ |

*Variables*

| | | |
|---|---|---|
| $x_{ij}^h$ | connection between $i$ and $j$ with intensity of cables h | $x_{ij} \in \mathbb{R}$ |
| $y_i^n$ | connection between $i$ and main power station | $y_i \in \{0,1\}$ |
| $z_i^n$ | connection between $i$ and additional node | $z_i \in \{0,1\}$ |
| $w$ | connection between main power station and additional node | $w \in \{0,1\}$ |
| $COST$ | total cost | $COST \in \mathbb{R}^+$ |

**Objective Function:**

Minimize the total cost

$$COST = \sum_{(i,j)\in A_E} \sum_{h=1}^{C-1} ca_{ij} x_{ij}^h + \sum_i cd_i z_i + \sum_i cd_{ij} y_i + \sum_s dw_s w \quad (6)$$

Minimize the total cost plus a penalty to avoid non-integer alternatives

$$min \qquad pCOST = COST + 0.0001 \sum_{(i,j)\in A_E} \sum_{h=1}^{C-1} h^2 x_{ij}^h + \sum_s dw_s w \quad (7)$$

**Subject to:**

Connection

$$\sum_{(j,h)} \sum_{h=1}^{C-1} x_{ij}^h + y_i + z_i \geq 1 \qquad \forall i \in V_c \tag{8}$$

Direction

$$\sum_{(i,j)} \sum_{h=1}^{C-1} x_{ij}^h + \sum_{(j,i)} \sum_{h=1}^{C-1} x_{ji}^h \leq 1 \qquad \forall (i,j) \in V_c \tag{9}$$

Origin

$$\sum_{(k,h)} \sum_{h=1}^{C-1} x_{ki}^h + \sum_{(j)} x_{ij}^1 + y_i + z_i \geq 1 \qquad \forall i \in V_c \tag{10}$$

Inequality

$$Cy_j + Cz_j + \sum_{(j,k)\in A_E} \sum_{h}^{C-1} hx_{jk}^h - \sum_{(i)\in A_E} \sum_{h=1}^{C-1} hx_{ij}^h \geq 1 \qquad \forall j \in V_c \tag{11}$$

Connection Main and Hub

$$z_i - w \leq 0 \qquad \forall i \in V_c \tag{12}$$

# Chapter 4

# Discussion

In this last chapter, a discussion about the examined model is included. Section 4.1 synthesizes the findings to answer the research questions, while Section 4.2 introduces recommendations for further research.

The aim of a cable routing model is to find a set of routes at a minimal cost. The objective function aims to optimize the cable route by maximizing the final performance of a single commodity flow network in an economic and environmental way. There are a lot of decisions to be taken at the design phase of a wind farm, such as the area of the wind farm, the number of wind turbines and power stations, which could be defined based on land size and the needs of energy. In addition, it is important to choose one or more cable types because it defines the cable capacity. More specific constraints necessary regarding the branching capacity, the proximity constraint which is depending on the turbine position, as well as the planarity constraint to prevent crossing routes.

The original model "An Integer Programming Model for Branching Cable Layouts in Offshore Wind Farms" [9] considers the number and the location of wind turbines and power stations as predefined, while the turbines are connected towards one of the stations. As a starting point, the paper refers that the elements could be connected by the chosen cable type taking into account the power losses which occur during the transportation. It also allows merging of the incoming cables and the spread of outgoing cables consequently known as branches. Later on, the authors described the usage of different cable types as an interesting extension in order to decrease the power loss. The original model has been modified accordingly. The numerical examples at the end of the paper show that the usage of multiple cable types and the branching capacity are significant with respect to cost savings. However, the cable capacity, the maximum number of turbines which can be connected in one arc, has been proved less important.

During the investigation of the existing model, we considered that the cable type is not a crucial point, so the cable capacity is negligible. The scope of the investigation was to create simple numerical examples in order to understand the model by examining the limitations. Therefore, the model has been modified in a way which allows an unlimited number of branches as long as it obeys the objective function. Although at the beginning the problem has been considered as route optimization, a subproblem arose, associated to location/allocation problems. A main decision to be made in a location problem, is the number of centres that should in used in order to receive the energy production [11]. In the case of wind farm, the power station represents the centre. The number of power stations depends on the number and the arrangement of wind turbines. In a real wind farm field, the number of the wind turbines can reach several hundreds [20] so the number of power stations may be dozens, based on the cable type and capacity. The question is whether it is efficient to add an additional station/node, taking into account the fixed costs of installing and maintenance of an extra station, and where it is in the grid.

At the beginning, the idea was that this node can work as a substation by connecting some wind turbines and transfer the energy to the next closest element, which could be a wind turbine or the main power station. Eventually, this did not simplify the problem. So, it has been decided that the additional node in the grid has to be directly connected to the main power station. We tried to prove the efficiency of its use by doing a more systematic analysis for specific cases.

The above cases have been examined by implementing the model in the GAMS solver using the Weber property to define the location of the additional node. In order to answer the question whether the addition of a node could be optimal according to its fixed costs, figures which contain contour lines have been created in the high level computing language, Matlab.

## 4.1 Conclusion

In general, wind farm design is interesting for operations research scientists to explore models and optimization techniques. The investigated problem has started as a cable layout problem with given location of turbines and a fixed location of power stations. The execution of the computational model computes the optimal cable route connections using Branch and Bound method. This creates a geometry shape of cabling which can be associated to a capacitated MST with respect to the planarity, proximity and branching capacity restrictions.

While an additional node is embedded to the grid, the symmetry of this connection is changed in order to minimize the cable cost and the problem became a location/allocation problem. The Weber problem has been used to define the optimal location(or not) of the additional node by minimizing the total distances between the existing elements in the grid and the new node. In order to conclude with a clear picture as an outcome, several cases have been examined by starting simple and creating scenarios subsequently.

- In the symmetric case of two wind turbines (Section 3.1.1) the additional node appears always in a deterministic position. Equation 4 in Section 3.1.1 shows that the location of the turbines define the exact location of the additional node as follow:
  $x^*(c) = 1 - \frac{c}{\sqrt{3}}$ for $c \leq \sqrt{3}$, where $x$ is the distance of the additional node from the main power station and $c$ value defines the position of the turbine.

- In the case of two non-symmetric wind turbines (Section 3.1.2) where the first turbine has a fixed position, as well as the power station, the additional node seems to be optimal if and only if the second turbine is placed inside the area where the two fixed elements create an angle of 120°.

- In the semi-symmetric case of three wind turbines (Section 3.1.3) the calculations have started to be more complex while the outcome varies. Recall that, two of the wind turbines remain at the fixed symmetric place and the additional node is obliged to connect to the main power station. The optimality of the additional node depends on the location of third turbine. Figure 3.9 in Section 3.1.3 shows the different cases which are based on the three scenarios:

  1. The additional node is optimal for the two symmetric turbines, while the third turbine is directly connected to its closest element(power station or turbine).
  2. The additional node is always optimal for the three wind turbines case.
  3. The additional node is not optimal and the two symmetric turbines are connected directly to the third turbine or the turbines are interconnected towards to main station.

- The last case of four symmetric wind turbines (Section 3.1.4) has the same outcome with the case of two symmetric turbines, where the additional node depends on the position of two turbines which are closer to the power station.

The additional node can be proven to be optimal when its fixed costs are smaller than the amount of saving costs while using it. For the extended case of more turbines, more than one additional node could be proved as optimal, while the Weber property can be used in different ways by combining group of elements each time. In that case, it is also important to consider the branching capacity in which branches are allowed to the nodes and the model would be more complex while the alternatives would increasing. The generalisation of a model for wind farms, requires further research, taking into account more aspects which are discussed below.

## 4.2 Further Research

Concerning a final comprehensive model there are multiple phases that need to be examined. First of all, finding the appropriate site to develop a wind farm can be crucial for the project. The land owner should be convinced that the project is profitable. After that, a testing phase can take a month at least, if the wind blows from the same direction, up to two years for seasonal winds. This can be done by installing lower height towers to access the wind distribution and extrapolate the wind speed at the actual hub height [13]. Last but not least, important decisions regarding the number, the position and the use of single or multiple types of turbines are necessary [3].

The model which has been examined in this report, gives a predefined number of turbines and predefined positions. A precedent step could be the investigation of the optimal position of turbines. Obviously, this would change the symmetry of wind turbines in an electrical grid and it can leads to alternatives. The optimal position has to be based on the minimization of the wake effect which has been introduce in Section 1.3. At this phase, further research could be done using a finer grid where the cells are shorter and the proximity constraint can be introduced [13].

In addition, an important restriction that has been examined in this paper is the planarity constraint which prevent the crossing of routes. Another interesting issue for further investigation could be the generation of intersecting edges from the set of nodes, since the wind turbines are denoted as nodes. Once the position of the turbines has been decided, the cable connection will be generated based on the distances between the elements, but it is not allowed to create crossing paths. So, how can the known coordinates denote whether there are potential intersection points between the elements? This has to remain as important consideration in any further research, taking into account the cable capacity as well, in order to afford the electrical transportation at the final node, which is the power station.

As mentioned before, a bi-level problem arose, but it turns out to be complex. The idea is to add of a node in the grid, which can represent an additional substation, in order to facilitate the problem since it could eliminate potential crossing paths and reduce the total cost. The bi-level problem has been changed into a similar subproblem where the additional node has to be connected directly to the main power station. However, an additional node could be connected with an intermediate element and reach the main power station. In that case, an additional constraint is needed which defines that the total number of incoming cables equals the number of outcoming cables. Assume, that the rest of the model remains as it is. In order to satisfy the objective function, it is essential to prove that saving costs occur.

In combination with the above aspects, intersection points and the additional node, another hypothesis could be investigated as has been mentioned in Section 3.2. The idea is to add a node at the crossing points which are generated if planarity constraint is not taken into account for the optimal cable layout connection. Once, the additional node has been placed at the location where the crossing point appears, then it is needed to repeat the computation and generate the optimal solution by adding the planarity constraint this time, in order to avoid a loop of adding substations every time.

With respect to the optimization methods, exact methods and approximation algorithms heuristic methods have been proposed in existing literature. Multiple recommendations are introduced on these papers such as the use of mixed-integer and dynamic stochastic programming [6]. A mixed-integer linear programming model has been examined in this report and the Branch and Bound method has been elaborated in order to deal with it.

An interesting approach would be the use of multi-objective optimization to satisfy more than one objective functions by balancing the weight of importance of each [8]. An efficient method to deal with multi-objective optimization could be the parallel computing which can carry many calculations simultaneously. Also, it would be wise to focus on the parameter which has the highest impact such as the turbulence intensity that the wake effect can cause.

Generally, the more precise the model, the more complex it is. The objectives and the parameters that can be added to a model in order to give more accurate results can vary. This is the main reason that the use of High Performance Computing(HPC) has to be introduced in further research in order to facilitate the computation of these problems in science and engineering.

# Appendix A

# Additional Knowledge

## A.1 Combinatorial optimization

Combinatorial optimization is a branch of optimization. Its domain is optimization problems where the set of feasible solutions is discrete or can be reduced to a discrete one, and the goal is to find the best possible solution. To deal with problems of combinatorial optimization, the objective is to find the best solution or optimal solution, one that minimizes a given cost function. There are some techniques to solve not complex problems, such as Branch and Bound [15].

## A.2 Branch and Bound

All the solutions of the wind farm cases studied in this report, have been declared automatically in an integer form due to the Branch and Bound method. All the variables are required to be 0 or 1 (binary), so the problem can be considered as MILP: Mixed Integer Linear Program. This problem is also classified as NP-hard problem and it is hard to solve. So, the elaboration method of B&B is used for MIP models in order to give an integral value as a solution. While the current solution is not integer then two new continuous LP problems are added as constraints. In fact, it calculates the bounds of the best integer solution to determine whether it is useful to branch the generated sub-problems [4].
It is worthwhile to mention, that at the beginning, before the penalty weight was added as convex quadratic method (Equation 9 in Section 2.2), the solver of GAMS was giving decimal solutions, by splitting the number of 1 to several solutions along the connections. The B&B method has been elaborated in order to eliminate the continuous solutions and now it creates a logical continuous cable solution.

## A.3 Minimum Spanning Tree

A minimum Spanning Tree ($MST$) is an undirected graph which connects all the nodes with the minimal total weight on its edges. A single graph can have different spanning trees. In the case of wind farm design, the weight of edges can be consider as the distance of the nodes. Since we aim to minimize the sum of the edge lengths and the total cost consequently, it is very important to include the minimum cable cost in the final route [18].

## A.4 Weber Problem

In general, the Weber Problem is a problem in location theory and it aims to find a point that minimizes the sum of transportation costs [16]. In the case of wind farm design, it minimizes the distances between the wind turbines and the main power station, in order to locate (or not) the additional node in the best possible position in the grid and minimize the total cable cost.

Given a set of points $\{P_i...P_n\}$

$$\min_x \sum_{i=1}^{n} d(x_i P_i) \tag{1}$$

where $d$ represents the Euclidean distance between the elements.

## A.5 Voronoi diagram

A Voronoi diagram consists of a set of points in a plane which define the region of the nearest point. The distances of these points can be found using the Euclidean and the Manhattan distance formulas. In this report, the Euclidean distance formula has been used in order to calculate the distances between the elements. The concept of Voronoi diagram has been proven useful for the creation of the geometric statement [19].

# Appendix B

# GAMS modelling

## B.1 My GAMS Model

```
Set
i               source index          /A*E/
h               arc states            /1*4/

Alias                                 (i,j,k);

Table a(i,j)    arc of connection
          A    B    C    D    E
     A         ab   ac   ad   ae
     B    ba        bc   bd   be
     C    ca   cb        cd   ce
     D    da   db   dc        de
     E    ea   eb   ec   ed

Parameters
b(i)                 arc of connection      /A as, B bs, C cs,D ds, E es/
nrcable(h)           number of cables       /1 1, 2 2, 3 3, 4 4 /
ca(i,j)              cost of connection
cb(i)                cost of connection;

ca(i,j)= a(i,j)*10;
cb(i)= b(i)*10;
```

```
Variables
x(i,j,h)            continuous variable
y(i)                binary variable
pCOST               penalty cost
COST                cost

Positive Variables  x;

Binary Variables    y(i);

Equations
Eq_COST             Total cost
Eq_pCOST            Penalty cost
Eq_Connection(i)    Connection constraint
Eq_Direction(i,j)   Direction constraint
Eq_OriginCon(i)     Origin Connection
Eq_Load(j)          Load

Eq_pCOST..
pCOST =e= sum((i,j), ca(i,j)*sum(h,x(i,j,h)))+sum(i, cb(i)*y(i))+0.0001*
*sum((i,j,h), nrcable(h)*nrcable(h)*x(i,j,h));

Eq_COST..
COST =e= sum((i,j), ca(i,j)*sum(h,x(i,j,h))) +sum(i, cb(i)*y(i));

Eq_Connection(i)..
sum((j,h), x(i,j,h))+y(i)=e=1;

Eq_Direction(i,j)..
sum((h), x(i,j,h)+x(j,i,h))=l=1;

Eq_OriginCon(i)..
sum((k,h), x(k,i,h)) +sum(j, x(i,j,'1'))+y(i) =g= 1;

Eq_Load(j)..
5*y(j)+sum((k,h), nrcable(h)*x(j,k,h))-sum((i,h),nrcable(h)*x(i,j,h)) =g= 1;

*Crossing Routes between turbines x*x example
Eq_ABCD..
sum(h,x('A','B',h)+ x('B','A',h)+x('C','D',h)+x('D','C',h)) =l= 1;
*Crossing Routes between depot x*y example
Eq_ABCS.. sum(h,x('A','B',h)+x('B','A',h))+ y('C')=l= 1;
```

```
x.fx(i,i,h)=0;

MODEL Branching /all/;
SOLVE Branching using mip minimizing pCOST;
Display COST.l;
```

## B.2 Weber Problem

```
Set i   turbines   /A,B/

Parameters
x(i)          x coordinate of turbine   /A xA,B xB/
y(i)          Y coordinate of turbine   /A yA,B yB/
Variables
a          x coordinate of turbine
b          Y coordinate of turbine
dist(i)    distance of turbines
di         distance to station
dis        total distance;
Equations
Eq_dis      Total distance
Eq_di       Distance to station
Eq_dis(i)   Distance

Eq_dis..dis =e= sum((i), dist(i)+di;

Eq_dist(i)..dis(i) =e= sqrt(sqr(x()i)-a)+sqr(y(i)-b);

Eq_di(i)..di=e= sqrt(sqr(a)+sqr(b));


    MODEL Branching /all/;
SOLVE Branching using nlp minimizing dis;
Display dis.l;
```

## B.3 My GAMS Model-The additional node

```
Set
i               source index         /A*E/
s               arc states           /S/
h               arc states           /1*4/

Alias                                (i,j,k);

Table a(i,j)    arc of connection
         A    B    C    D    E
    A         ab   ac   ad   ae
    B    ba        bc   bd   be
    C    ca   cb        cd   ce
    D    da   db   dc        de
    E    ea   eb   ec   ed

Parameters
b(i)                arc of connection from power station /A as, B bs, C cs,D ds, E es/
d(i)                arc of connection from hub /A as, B bs, C cs,D ds, E es/
cw(s)               arc of connection between power station and hub /S hs/
nrcable(h)          number of cables /1 1, 2 2, 3 3, 4 4 /
ca(i,j)             cost of connection between turbines
cb(i)               cost of connection between turbine and power station;
cd(i)               cost of connection between turbine and hub station;
dw(s)               cost of connection between power and hub station;

ca(i,j)= a(i,j)*10;
cb(i)= b(i)*10;
cd(i)= d(i)*10;
dw(s)= cw(s)*10;
```

| | |
|---|---|
| Variables | |
| x(i,j,h) | continuous variable |
| z(i) | binary variable |
| w | binary variable |
| y(i) | binary variable |
| pCOST | penalty cost |
| COST | cost |

| | |
|---|---|
| Positive Variables | x; |

| | |
|---|---|
| Binary Variables | z(i); |
| Binary Variables | w; |
| Binary Variables | y(i); |

```
Equations
Eq_COST
Eq_pCOST
Eq_Connection(i)
Eq_Direction(i,j)
Eq_OriginCon(i)
Eq_Load(j)
Eq_Connection_Hub(i)
Eq_pCOST..



Eq_COST..
COST =e= sum((i,j), ca(i,j)*sum(h,x(i,j,h)))+sum(i, cd(i)*z(i));+sum(i, cb(i)*y(i));

Eq_Connection(i)..
sum((j,h), x(i,j,h))+y(i)+z(i)=e=1;

Eq_Direction(i,j)..
sum((h), x(i,j,h)+x(j,i,h))=l=1;

Eq_OriginCon(i)..
sum((k,h), x(k,i,h)) +sum(j, x(i,j,'1'))+y(i)+z(i)=g= 1;

Eq_Load(j).. 5*y(j)+5*z(j)+sum((k,h), nrcable(h)*x(j,k,h))-sum((i,h),nrcable(h)*x(i,j,h)) =g= 1;;

Eq_Connection_Hub(i)..
z(i)-w =l= 0;

*Crossing Routes between turbines x*x example
Eq_ABCD..
sum(h,x('A','B',h)+x('B','A',h)+x('C','D',h)+x('D','C',h)) =l= 1;

*Crossing Routes between depot and hub h*y example
Eq_BHCS..
z('B')+ y('C')=l= 1;

x.fx(i,i,h)=0;

MODEL Branching /all/;
SOLVE Branching using mip minimizing pCOST;
Display COST.l;
```

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