

A NUMERICAL ANALYSIS OF THE RELATION BETWEEN CTOD AND FATIGUE CRACK GROWTH

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RESUMEN

El cálculo de la vida a fatiga durante la etapa de propagación de la grieta se suele realizar relacionando da/dN con ΔK . Sin embargo, la influencia que en estos análisis tiene el parámetro ΔK debe recaer en algún otro parámetro no lineal cercano al frente de la grieta, ya que estos son los que realmente controlan la velocidad de crecimiento de la grieta. El principal objetivo del presente artículo es intentar mejorar la comprensión del crecimiento de grieta en fatiga empleando el desplazamiento de apertura de la punta de la grieta (CTOD). Este parámetro no ha sido muy empleado en el problema del cierre de grieta y su propagación en fatiga, teniendo un gran potencial. Por este motivo, se ha realizado un análisis numérico para un amplio rango de cargas de amplitud constante en dos aleaciones de aluminio (6016-T4 y 6082-T6). Cuando no se considera el contacto entre los flancos de la grieta se puede observar una relación bien definida entre el valor máximo de CTOD y ΔK , lo cual indica que no existe influencia de la relación de cargas y valida la mecánica de la fractura elástica lineal. Se ha encontrado una relación lineal entre los valores de CTOD y ΔK al considerar una doble escala logarítmica. Los valores de CTOD cuando se considera el contacto se superpone a los resultados sin contacto, únicamente cuando se emplea el ΔK efectivo, lo cual valida el concepto del cierre de grieta. Se ha encontrado una relación lineal entre da/dN y CTOD cuando se considera una doble escala logarítmica para la aleación de aluminio 6082-T6.

PALABRAS CLAVE: Crecimiento de grieta en fatiga, parámetros no lineales en el frente de la grieta, CTOD

ABSTRACT

Engineering analysis of fatigue crack propagation is usually performed by relating da/dN to ΔK . However, the emphasis on ΔK parameter must be replaced by a close look to non-linear crack tip parameters, because these effectively control fatigue crack growth rate. The main objective here is to improve the understanding of fatigue crack growth using the crack tip opening displacement (CTOD). This parameter has not been totally exploited in the context of crack closure and fatigue propagation, and has a great potential. A numerical analysis was therefore developed for a wide range of constant amplitude tests in two aluminium alloys (6016-T4 and 6082-T6). Without contact of crack flanks there is a well defined relation between maximum CTOD and ΔK , which indicates that there is no influence of stress ratio, and validates the linear elastic fracture mechanics. A linear correlation was found between the CTOD and ΔK in log-log scales. The CTOD predicted with contact of crack flanks overlap the results without contact only when ΔK_{eff} is used, which validates the crack closure concept. A linear relation was found between da/dN and CTOD in log-log scales for the 6082-T6 aluminium alloy.

KEYWORDS: Fatigue crack propagation; non-linear crack tip parameters; crack tip opening displacement (CTOD),

1. INTRODUCTION

Engineering analysis of fatigue crack propagation is usually performed by relating the crack advance per unit cycle, da/dN , to the stress intensity factor range, ΔK . A power law relationship, named Paris law, is generally observed at intermediate values of ΔK . Paris and Erdogan [1] proved that da/dN versus ΔK for long

cracks in the small-scale yielding range retains the advantage of LEFM, namely an invariance relatively to the shape and size of cracked solids. However, several limitations were found in the use of da/dN - ΔK relations. First, the da/dN - ΔK curves are completely phenomenological and not derived from physics. The fit parameters have units for which no physical reasoning is given. Second, the da/dN - ΔK curves are only valid in

the small-scale yielding range. Additionally, da/dN depends on other parameters, like stress ratio or load history. Crack closure concept [2] was proposed to explain variations associated with mean stress, overloads, short cracks and specimen thickness. The T-stress concept was used to explain the effect of specimen geometry [3]. However, in our opinion, the emphasis on ΔK parameter must be replaced by a close look to non-linear crack tip parameters (NLP), because these effectively control fatigue crack growth rate (FCGR). Antunes *et al.* [4] used non-linear parameters to prove the validity of crack closure concept and to identify the best crack closure parameter. The non-linear crack tip parameters identified in literature were the range of cyclic plastic strain [5,6], the size of reversed plastic zone [7,8], the total plastic dissipation per cycle [9,10] and the crack opening displacement.

The crack opening displacement (COD) is a classical parameter in elastic-plastic fracture mechanics. Crack tip blunting under maximum load and re-sharpening of the crack-tip under minimum load is widely used to explain fatigue crack growth under cyclic loading [11]. It was also shown by various authors that there is a relationship between the amplitude of crack tip blunting over a full fatigue cycle and the crack growth rate. Pelloux [12], using microfractography, showed that the concept of COD allowed the prediction of fatigue striations spacing and therefore the crack growth rate. Nicholls [13] assumed a polynomial relation between crack growth rate and COD:

$$\frac{da}{dN} = b(\text{COD})^{1/p} \quad (1)$$

where b and p are constants. Tvergaard [14] and Pippan and Grosinger [15] indicated a linear relation between da/dN and COD for very ductile materials:

$$\frac{da}{dN} = c \times \text{COD} \quad (2)$$

being c a constant. Nicholls [13] proposed the following expression:

$$\text{COD} = \frac{\lambda K^2}{E \sigma_{ys}} \quad (3)$$

where E is Young's modulus, K is the stress intensity factor and σ_{ys} is the yield stress. The elastic crack profile is given by:

$$\text{COD}_{\text{elas}} = \pm \frac{4K}{E} \sqrt{\frac{d}{2\pi}} \quad (4)$$

being d the distance from crack tip. The positive and negative signals indicate the superior and inferior crack flanks, respectively. Note also that different measurements of crack opening displacement have been used in literature. In CT specimens an extensometer with blades is used to measure the opening of the specimen at the edge. Therefore, this parameter is usually called crack mouth opening displacement (CMOD). A similar approach is used in the M(T) specimen. A pin extensometer is placed at the center of the specimen, fixed in two small holes to avoid sliding [16]. The resulting force versus displacement curves are used to calculate the crack closure level. Digital Image Correlation (DIC) is a non-contact full-field measurement technique to measure the total (elastic plus plastic) strain on the surface of the specimen. The capability of DIC to measure total strain is unrivalled. DIC has found increasing application for the study of crack-tip strain fields and it has been possible to extract fracture mechanics information such as closure stresses [17], plastic zone sizes [18], crack opening displacements and effective stress intensity factors at the crack-tip [18]. The crack tip opening displacement (CTOD) can only be measured numerically or analytically. Matos and Nowell [19,20] studied the variation of the first node behind crack tip using numerical and analytical methods. They analysed a M(T) specimen made of Ti-6Al-4V titanium alloy, considering finite element elements with 5 or 10 μm near the crack tip. In numerical studies the CTOD is usually defined as the distance between two points found by intersecting the finite element model with two (+45° and -45°) lines originated from the crack tip.

The main objective here is to improve the understanding of fatigue crack growth using the CTOD. This parameter has not been adequately exploited in the context of crack closure and fatigue crack closure, and has a great potential. Specific objectives are the determination of CTOD versus ΔK curves for different materials, with and without contact of crack flanks, and the determination of da/dN versus CTOD curves for materials with known da/dN - ΔK_{eff} curves.

2. NUMERICAL MODEL

A Middle-Tension specimen was studied, having $W=60\text{mm}$ and a small thickness ($t=0.2\text{ mm}$) in order to obtain a plane stress state (Figure 1). A straight crack was modelled, with an initial size, a_0 , of 5 mm ($a_0/W=0.083$). The specimen is symmetric about three orthogonal planes and therefore only 1/8 was simulated considering proper boundary conditions. The material considered in this research were the 6016-T4 ($\sigma_{ys}=124\text{MPa}$) and 6082-T6 ($\sigma_{ys}=238\text{MPa}$) aluminium alloys. Since Plasticity Induced Crack Closure (PICC) is a plastic deformation based phenomenon, the hardening behavior of the material was carefully modelled. From the experimental data and curve fitting results, for

different constitutive models, it was determined that the mechanical behaviour of this alloy is better represented using an isotropic hardening model described by a Voce type equation combined with a non-linear kinematic hardening model described by a saturation law. An anisotropic yield criterion was considered.

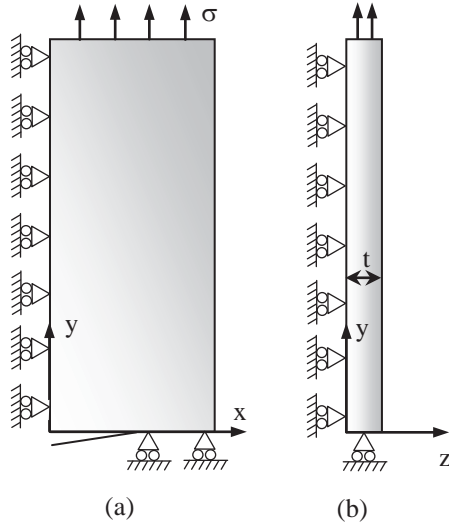


Figure 1. *M(T)*, specimen. (a) Frontal view. (b) Plane strain state. (c) Plane stress state.

The finite element model of the *M(T)* specimen had a total number of 6639 linear isoparametric elements and 13586 nodes. The finite element mesh was refined near the crack tip, having $8 \times 8 \mu\text{m}^2$ elements there. Only one layer of elements was considered along the thickness. Crack propagation was simulated by successive debonding of nodes at minimum load. Each crack increment corresponded to one finite element and two load cycles were applied between increments. In each cycle, the crack propagated uniformly over the thickness by releasing both current crack front nodes. The total crack propagation was $\Delta a = 1.28 \text{ mm}$, which corresponds to 160 propagations, each with $8 \mu\text{m}$. A wide range of load cases was considered for a deep understanding of the mechanisms behind PICC.

The opening load, necessary for the determination of the closure level, was obtained considering the contact status of the first node behind the current crack tip, and the contact forces at minimum load along crack flank [21]. In this second approach, the stress intensity factor needed to eliminate each of the contact forces at minimum load is calculated, and the opening load is considered to be the linear summation of all the values. This method involves all the nodes in contact behind crack tip. The CTOD was assumed to be the vertical displacement of the node behind crack tip. The numerical simulations were performed with the Three-Dimensional Elasto-plastic Finite Element program (DD3IMP). This software was originally developed to model deep drawing [22], and was adapted to study PICC due to its great competence in the modelling of

plastic deformation. Further details of the numerical procedure may be found in previous publications of the authors [21,23].

3. RESULTS

3.1. Typical curves

Figure 2a presents a typical plot of CTOD versus load, obtained for $\Delta a = 1.272 \text{ mm}$. The remote stress is calculated from the load, F , by dividing the area of cross section: $\sigma = F/A$, being $A = 30 \times 0.1 = 3 \text{ mm}^2$. The crack closes at minimum load (A) and only opens when the load reaches point B, which is the crack opening load. After opening the CTOD increases linearly, but after point C there is some deviation from linearity which indicates plastic deformation. The extrapolation of the linear regime to the maximum load, as is represented, shows that the major part of the deformation is elastic. The decrease of load from point D produces a linear decrease of CTOD. The rate of variation of CTOD in regions DE and BC is similar. After point E, reversed plastic deformation starts and the crack closes again at point F. It is also interesting to note that the crack opening and crack closure levels are slightly different, which is explained by the plastic blunting during loading.

Figure 2b also plots the CTOD versus load, but without contact of crack flanks. There are negative values of CTOD, which indicate that the material is overlapping. Naturally this is only possible in the numerical analysis and has no physical sense. The range of CTOD is therefore significantly higher than with contact. The slope of the elastic regime is however the same with and without contact, since it only depends on the elastic properties of the material, and distance from the crack tip, where the CTOD is measured, as can be seen in equation 4.

3.2. Effect of load parameters

Figure 3a presents the maximum CTOD versus ΔK for different load cases. Without contact of crack flanks there is a well defined relation, which indicates that there is no influence of stress ratio. In other words, without crack closure, fatigue crack propagation only depends on ΔK . This curve is called the master curve. Besides, the existence of a well defined relation between the elastic parameter, ΔK , and a crack tip parameter that is supposed to control da/dN , validates the linear elastic fracture mechanics (LEFM). The results of CTOD versus ΔK with contact show a great scatter. Additionally the values are significantly below the master curve, i.e., the CTOD was supposed to be higher. However, when the same results are plotted versus ΔK_{eff} , the points overlap the master curve. This indicates that the crack closure concept is valid and able

to explain the effect of the contact of crack flanks on crack tip parameters and therefore on da/dN . Figure 3b plots CTOD versus ΔK (or ΔK_{eff}) in log-log scales, and a well defined linear variation can be seen. The slight deviation of the data obtained with contact is explained by the difficulties in defining a crack closure level that effectively quantifies the effect of contact on da/dN . The contact of crack flanks is used here to quantify the crack closure level, and seems to be quite effective.

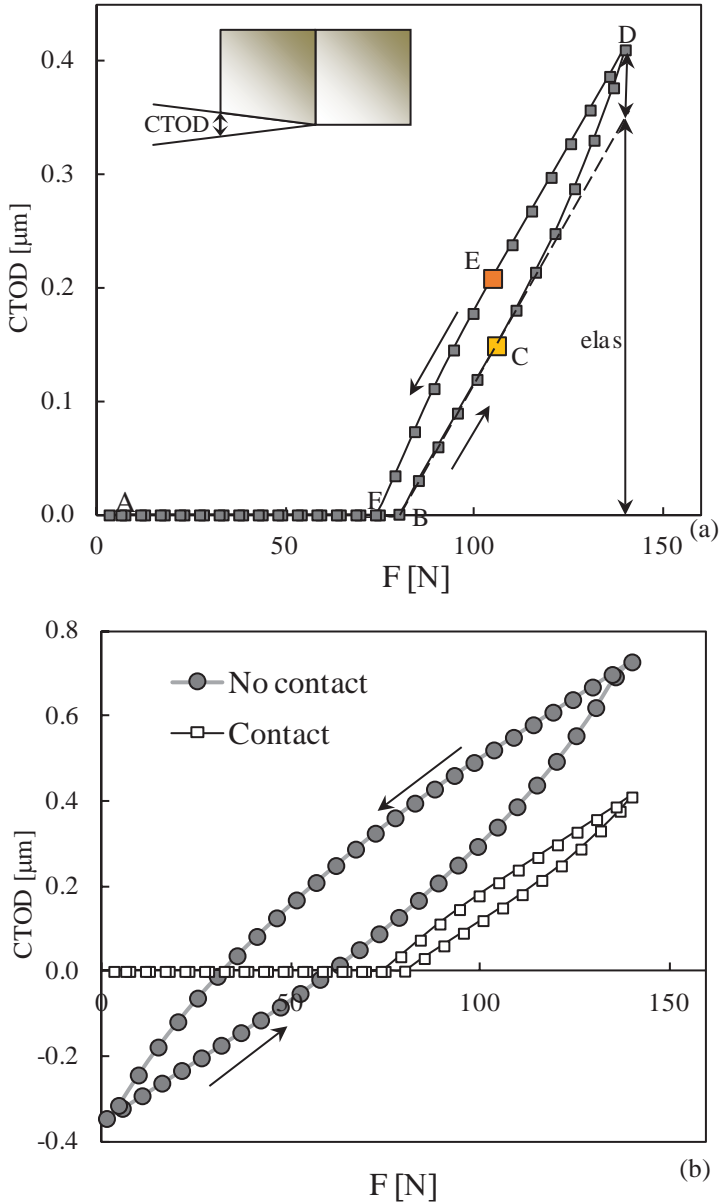


Figure 2. Typical CTOD versus load plot. (a) With contact of crack flanks. (b) Without contact. (plane stress, $\Delta a=1.272$ mm).

3.3. Effect of material

Figure 4 compares the master curves (free of the influence of crack closure) obtained for two aluminium alloys. The yield stress is 124 MPa for the 6016-T4 AA,

and 238 MPa for the 6082-T6 AA. The CTOD values obtained for the 6082-T6 are of the same order of those for the 6016-T4, which was not expected considering the yield stresses. The elastic CTOD is exactly the same, therefore the difference is due to the plastic deformation. Anyway, a well defined between CTOD and ΔK still exists for the 6082-T6 AA, linear in log-log scales, which validates the LEFM once again.

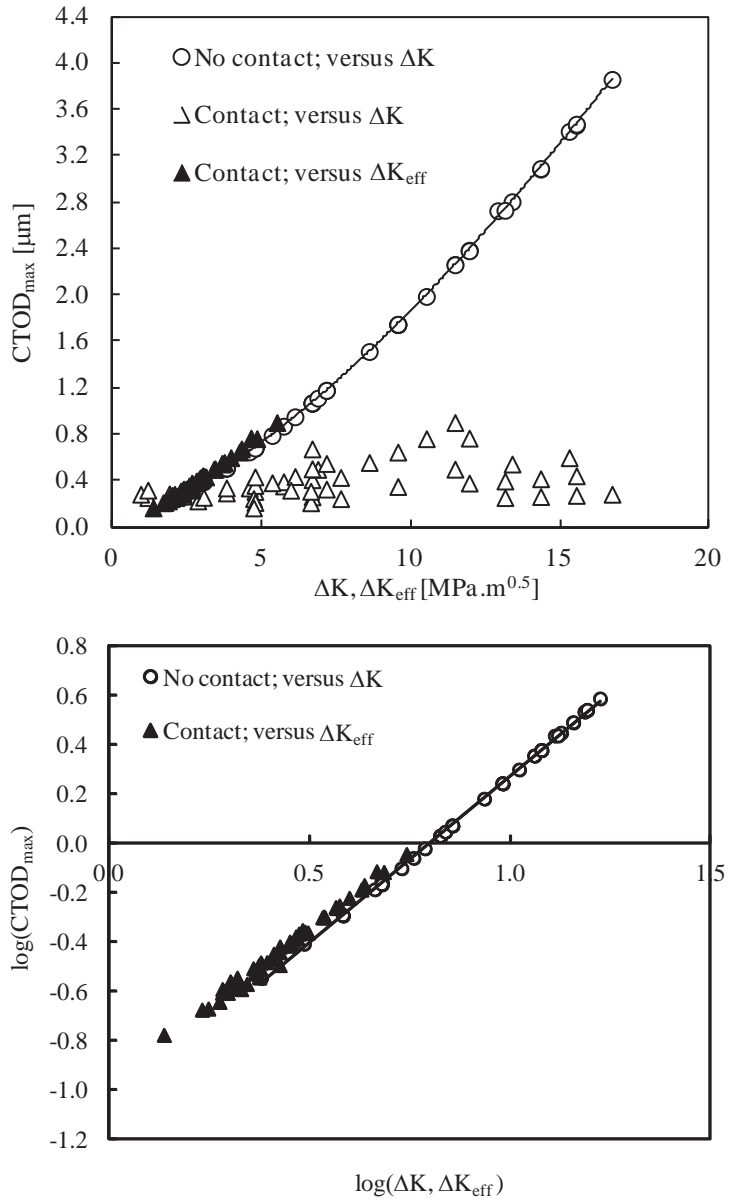


Figure 3. CTOD versus ΔK or ΔK_{eff} . (a) Linear scales. (b) Log-Log representation. (6016-T4 aluminium alloy)

For the 6082-T6, a model of da/dN versus ΔK_{eff} was found in literature [16]:

$$\frac{da}{dN} = C(\Delta K_{eff})^m \quad (5)$$

being $m=3.389$, $C=1.229 \times 10^{-7}$, $[da/dN]=\text{mm/cycle}$ and $[\Delta K_{\text{eff}}]=\text{MPa.m}^{0.5}$. These constants are valid within the range $2.5 < \Delta K_{\text{eff}} < 12 \text{ MPa.m}^{0.5}$ [16]. The relation between CTOD and ΔK_{eff} was used to link da/dN with CTOD. Figure 5 shows this relation in log-log scales. A linear variation is evident, which indicates that the CTOD is a viable alternative to ΔK . In fact, it is a crack tip parameter, which quantifies things where they really matter, i.e., where the crack propagation happens. The following relation was found:

$$\frac{da}{dN} = 5.291 \times 10^{-4} (\text{CTOD}_{\text{max}})^{1.185} \quad (6)$$

This expression is valid for the 6082-T6 aluminium alloy, for plane stress, $[da/dN]=\text{mm/cycle}$ and $[\text{CTOD}]=\mu\text{m}$. The CTOD was measured 8 μm behind crack tip.

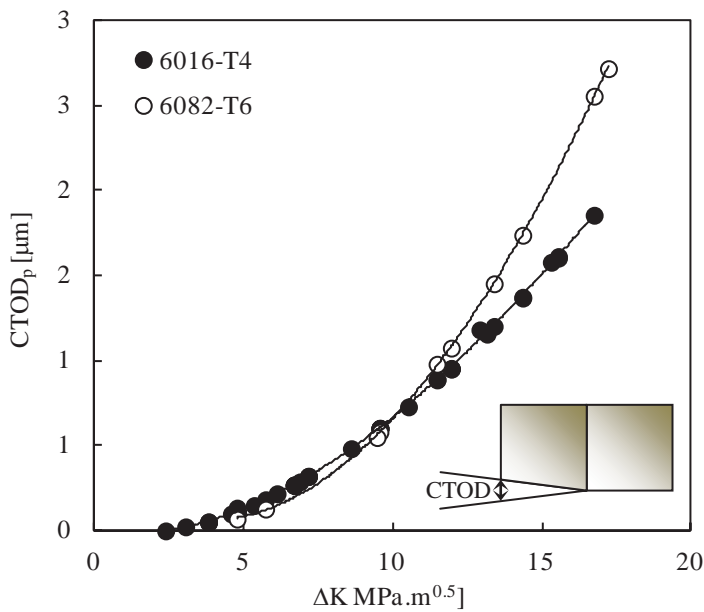


Figure 4. Effect of material on master curves.

3. CONCLUSIONS

A numerical study was developed to quantify the CTOD in a M(T) specimen made of aluminium alloy. Typical curves of CTOD versus load were obtained, showing the occurrence of crack closure, the crack opening level, an elastic regime and the plastic blunting. The elastic part of the CTOD is significantly higher than the plastic part for the 6016-T4 aluminium alloy.

Without contact of crack flanks there is a well defined relation between maximum CTOD and ΔK , which indicates that there is no influence of stress ratio, and validates the linear elastic fracture mechanics. With contact of crack flanks, the CTOD is below the values obtained without contact and show a great scatter.

However, when the same results are plotted versus ΔK_{eff} , the points overlap the results obtained without contact of crack flanks. This indicates that the crack closure concept is valid and able to explain the effect of the contact of crack flanks on crack tip parameters and therefore on da/dN . A linear correlation was found between the CTOD and ΔK in log-log scales.

The material, and particularly the yield stress, was found to have a significant influence on CTOD versus ΔK plots. Finally, a linear relation was found between da/dN and CTOD in log-log scales for the 6082-T6 aluminium alloy. This indicates that the CTOD is a viable alternative to ΔK in the analysis of fatigue crack propagation. Further work is however necessary to understand the effect of material, stress state and specimen geometry on the CTOD versus da/dN relation.

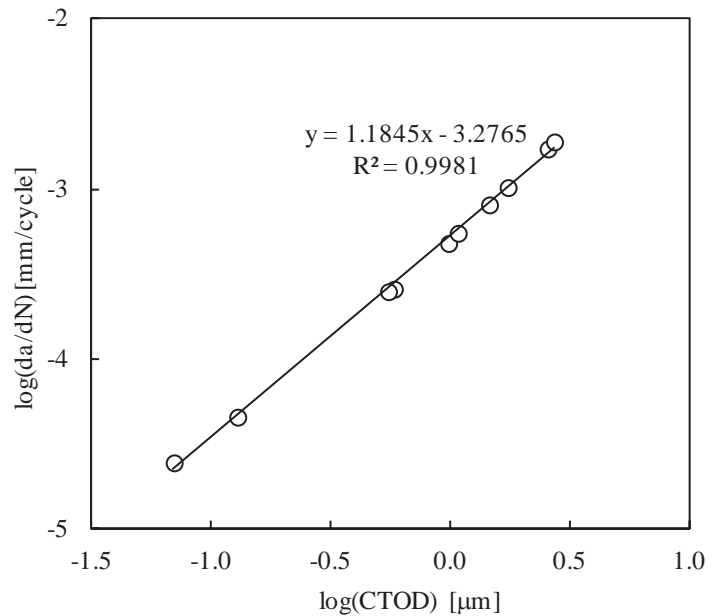


Figure 5. da/dN versus CTOD in log-log scales (6082-T6 AA).

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