

# On the Use of F-transform on the Reduction of Concept Lattices

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**Abstract.** In this paper, we show that F-transform can be used to reduce relational databases. Subsequently, we show that the respective concept lattice is reduced significantly as well. Moreover, we present a clarifying example of the procedure.

**Keywords:** F-transform, Fuzzy Sets, Fuzzy Concept Analysis, Knowledge Reduction.

## 1 Introduction

Fuzzy Formal Concept Analysis deals with the processing of imprecise knowledge in information systems [2, 3]. In this theory, the information of a relational databases is represented in terms of a complete lattice where its elements are called concepts. However, despite the information represented by the concept lattice is valuable, the complexity and the size (which increases exponentially with respect to the size of the relational database) makes the use of this theory impractical in many applied tasks. For this reason, recent approaches have dealt with Knowledge Reduction in relational databases to simplify the formal concept analysis of them [1].

On the other hand, F-transforms [4] is a theoretical tool that has shown its effectiveness on representing the information of signals (like temporal series, images, etc.) to a vector of few components. This paper applies F-transforms (based on residuated lattice) to Knowledge Reduction. Specifically, we begin by showing that objects (or attributes) in a relational database can be grouped in a new set of objects (or attributes). Then, we transfer the information of the original database to another where objects are given by the grouping previously mentioned. The transfer of information is given by F-transforms and therefore, there are two possible new relational databases.

This paper has the following structure. In Section 2 we recall briefly the theories of fuzzy property-oriented concept lattices and F-transforms. Then, in Section 3 we describe the reduction of relational tables by means of F-Transforms. Moreover, we illustrate the consequences of the reduction in concept lattices with an example. Finally, in Section 4 we present conclusions and future work.

## 2 Preliminaries

### 2.1 F-transforms on residuated lattices

In this section, we briefly recall the basic definitions and the main principles of F-transforms based on operations of a residuated lattice [4]. Let  $(L, \leq, \&, \rightarrow)$  be a residuated lattice. A fuzzy partition of a finite set  $\mathcal{U}$  is a set of  $L$ -fuzzy sets on  $\mathcal{U}$   $\mathcal{A}_1, \dots, \mathcal{A}_n$  fulfilling the covering property namely, for all  $x \in \mathcal{U}$  there exists  $k \in \{1, \dots, n\}$  such that  $\mathcal{A}_k(x) > 0$ . The membership functions  $\mathcal{A}_k(x)$ ,  $k = 1, \dots, n$  are called the *basic functions*.

**Definition 1.** Let  $f: \mathcal{U} \rightarrow L$  be a function and  $\mathcal{A}_1, \dots, \mathcal{A}_n$ , with  $n \leq |\mathcal{U}|$ , be basic functions which form a fuzzy partition of  $\mathcal{U}$ . We say that the  $n$ -tuple of real numbers  $\mathbf{F}_n^\uparrow[f] = [F_1^\uparrow, \dots, F_n^\uparrow]$  is the (direct)  $F^\uparrow$ -transform of  $f$  w.r.t.  $\mathcal{A}_1, \dots, \mathcal{A}_n$  if

$$F_k^\uparrow = \bigvee_{x \in \mathcal{U}} (\mathcal{A}_k(x) \& f(x)). \quad (1)$$

Moreover, we say that the  $n$ -tuple of real numbers  $\mathbf{F}_n^\downarrow[f] = [F_1^\downarrow, \dots, F_n^\downarrow]$  is the (direct)  $F^\downarrow$ -transform of  $f$  w.r.t.  $\mathcal{A}_1, \dots, \mathcal{A}_n$  if

$$F_k^\downarrow = \bigwedge_{x \in \mathcal{U}} (\mathcal{A}_k(x) \rightarrow f(x)). \quad (2)$$

The elements  $F_1^\uparrow, \dots, F_n^\uparrow$  and  $F_1^\downarrow, \dots, F_n^\downarrow$  are called *components* of the  $F^\uparrow$ -transform and  $F^\downarrow$ -transform, respectively.

The following lemma ([4]) shows that the components of the  $F^\uparrow$ -transform ( $F^\downarrow$ -transform) are lower mean values (upper mean values) of an original function which give least (greatest) element to certain sets.

**Lemma 1.** Let  $f: \mathcal{U} \rightarrow L$  be a function and  $\mathcal{A}_1, \dots, \mathcal{A}_n$ , with  $n \leq |\mathcal{U}|$ , be basic functions which form a fuzzy partition of  $\mathcal{U}$ . Then the  $k$ -th component of the  $F^\uparrow$ -transform is the least element of the set

$$S_k = \{a \in L \mid \mathcal{A}_k(x) \leq (f(x) \rightarrow a) \text{ for all } x \in \mathcal{U}\}$$

and the  $k$ -th component of the  $F^\downarrow$ -transform is the greatest element of the set

$$T_k = \{a \in L \mid \mathcal{A}_k(x) \leq (a \rightarrow f(x)) \text{ for all } x \in \mathcal{U}\}$$

where  $k = 1, \dots, n$ .

### 2.2 Fuzzy property-oriented concept lattices

In this section we recall briefly a simplification of property-oriented concept lattices introduced in [2, 3]. So, because of the lack of space, here we restrict to residuated lattices instead of adjoint triples. The notion of fuzzy property-oriented context is defined below.

**Definition 2.** Let  $(L, \leq, \&, \rightarrow)$  be a residuated lattice. A context is a tuple  $(A, B, R)$  such that  $A$  and  $B$  are non-empty sets (usually interpreted as attributes and objects, respectively),  $R$  is an  $L$ -fuzzy relation  $R: A \times B \rightarrow L$ .

From now on, we fix a context  $(A, B, R)$ . The mappings  $\uparrow^\pi: L^B \rightarrow L^A$  and  $\downarrow^\pi: L^A \rightarrow L^B$  are defined, for  $g \in L^B$  and  $f \in L^A$  as,  $g^{\uparrow^\pi}$  and  $f^{\downarrow^\pi}$ , where

$$g^{\uparrow^\pi}(a) = \bigvee_{b \in B} R(a, b) \& g(b)$$

$$f^{\downarrow^\pi}(b) = \bigwedge_{a \in A} R(a, b) \rightarrow f(a)$$

It is not difficult to prove that  $(\uparrow^\pi, \downarrow^\pi)$  forms an isotone Galois connection (also known as adjunction) and, therefore,  $\uparrow^\pi \downarrow^\pi: L^B \rightarrow L^B$  is a closure operator and  $\downarrow^\pi \uparrow^\pi: L^A \rightarrow L^A$  is an interior operator. A concept is a pair of mappings  $\langle g, f \rangle$ , with  $g \in L^B, f \in L^A$ , such that  $g^{\uparrow^\pi} = f$  and  $f^{\downarrow^\pi} = g$ , which will be called *fuzzy property-oriented concept*. In that case,  $g$  is called the *extent* and  $f$ , the *intent* of the concept. The set of all these concepts will be denoted as  $\mathcal{F}_{\Pi N}$ .

**Definition 3.** The associated fuzzy property-oriented concept lattice to the context  $(A, B, R)$  is defined as the set

$$\mathcal{F}_{\Pi N} = \{\langle g, f \rangle \in L^B \times L^A \mid g^{\uparrow^\pi} = f \text{ and } f^{\downarrow^\pi} = g\}$$

in which the ordering is defined by  $\langle g_1, f_1 \rangle \preceq \langle g_2, f_2 \rangle$  iff  $g_1 \preceq_2 g_2$  (or equivalently  $f_1 \preceq_1 f_2$ ).

### 3 Reducing the size of Relational Tables.

Throughout this section we consider a frame  $(A, B, R)$  and a residuated lattice  $(L, \leq, \&, \rightarrow)$ . The idea underlying in the reduct is the creation of two smaller relational tables  $R^\uparrow$  and  $R^\downarrow$  that keep as much information from  $R$  as possible. In order to reduce the size of the table is needed to reduce either the number of attributes or the number of objects. In this paper we focus on objects. In this way, we define a new set of objects  $\bar{B}$  that can be considered as a set of fuzzy sets that group objects according to certain attributes in  $A$ . For instance, consider a relational table where objects are people and attributes are physical features of them. Then, we could group people according to their high and then, to define the following set of “new” objects  $\{B_1 = \text{VerySmall}, B_2 = \text{QuiteSmall}, B_3 = \text{Medium}, B_4 = \text{QuiteTall}, B_5 = \text{VeryTall}\}$ . To conclude the reduction, we only need to define the relations  $R^\uparrow$  and  $R^\downarrow$  between the new set of objects and the original set of attributes. For such a task we consider direct F-transforms. In this framework, each basic function from the chosen fuzzy partition determines a new object and the value assigned to it by the direct F-transform determines the value of the relation.

To define the basic functions (and then also the set of new objects) let us consider firstly, a fuzzy partition of  $L$  given by fuzzy sets  $\{\mathcal{L}_k: k \in \{1, \dots, n\}\}$  and secondly, a subset of attributes  $\bar{A} \subseteq A$ . Then the fuzzy partition  $\bar{B} = \{B_{k\bar{a}}: k \in \{1, \dots, n\} \text{ and } \bar{a} \in \bar{A}\}$  of  $B$  is defined by

$$B_{k\bar{a}}(b) = \mathcal{L}_k(R(b, \bar{a})), \quad b \in B. \quad (3)$$

Note that the fuzzy partition  $\bar{B}$  groups original objects in fuzzy sets according to their relation with attributes in  $\bar{A}$ . Moreover, note the number of basic functions (i.e., the number of new objects) is  $k \cdot |\bar{A}|$ . So the size of the new set of objects depends on the number of attributes considered to define the partition. Once the fuzzy partition is fixed, we can define the following two  $L$ -fuzzy relational tables  $R^\uparrow$  and  $R^\downarrow$  between  $\bar{B} = \{B_{k\bar{a}}: k \in \{1, \dots, n\} \text{ and } \bar{a} \in \bar{A}\}$  and  $A$  as follows:

$$\begin{aligned} R^\uparrow: \bar{B} \times A &\rightarrow L \\ (B_{k\bar{a}}, a) &\mapsto \bigvee_{b \in B} B_{k\bar{a}}(b) \& R(a, b) \\ R^\downarrow: \bar{B} \times A &\rightarrow L \\ (B_{k\bar{a}}, a) &\mapsto \bigwedge_{b \in B} B_{k\bar{a}}(b) \rightarrow R(a, b) \end{aligned} \quad (4)$$

Note that original objects are used to define the values of the new ones. Finally, the reduction of the concept lattice given by the original frame  $(A, B, R)$  is the pair of concept lattices associated to the frames  $(A, \bar{B}, R^\uparrow)$  and  $(A, \bar{B}, R^\downarrow)$ . Below we show how the procedure works in a simple example.

	<i>HighPower</i>	<i>BigSpace</i>	<i>HighConsume</i>	<i>Expensive</i>	<i>Sport</i>	<i>Familiar</i>
$b_1$	1	0.2	1	0.8	1	0
$b_2$	1	1	0.8	1	0.6	1
$b_3$	0.6	0.8	0.4	0.6	0.2	0.6
$b_4$	0.8	0.6	0.6	0.6	0.6	0.6
$b_5$	0.6	0.4	0.2	0.6	0.2	0.2
$b_6$	0	0.2	0	0.2	0	0
$b_7$	0.8	0.2	0.8	0.8	0.8	0
$b_8$	1	1	1	1	0	1
$b_9$	0.6	1	0.4	0.6	0	1
$b_{10}$	0.6	1	0.6	0.6	0	1
$b_{11}$	0.6	0.6	0.4	0.4	0	0.6
$b_{12}$	0.2	0.4	0.4	0.2	0	0.2
$b_{13}$	0.8	0	0.8	1	0.8	0

**Fig. 1.** A car relational database.

*Example 1.* Let us consider the  $L$ -relational table in Figure 1 that relates types of cars (objects) with features (attributes). For the sake of simplicity, let  $L$  be the unit interval  $[0, 1]$  represented by finite set  $L = \{0, 0.2, 0.4, 0.6, 0.8, 1\}$ , and the adjoint pair considered for the reduction and the construction of the concept lattice is the one given by the Gödel connectives. Let us consider the partition  $\{\mathcal{L}_1, \mathcal{L}_2\}$  of  $L$  given by:

$x$	0	0.2	0.4	0.6	0.8	1
$\mathcal{L}_1(x)$	0	0.2	0.4	0.6	0.8	1

$x$	0	0.2	0.4	0.6	0.8	1
$\mathcal{L}_2(x)$	1	0.8	0.6	0.4	0.2	0

Now, for the sake of simplicity let us consider just the attribute *Familiar*  $\in A$  to make the reduct. Then, from Equation (3), we have that the partition of the set of objects  $B$  with respect to the attribute *Familiar* and the partition  $\{\mathcal{L}_1, \mathcal{L}_2\}$  of  $L$  is given by the following two fuzzy sets

$b$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$	$b_9$	$b_{10}$	$b_{11}$	$b_{12}$	$b_{13}$
$B_{1\bar{a}}(b)$	0	1	0.6	0.6	0.2	0	0	1	1	1	0.6	0.2	0

$b$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$	$b_7$	$b_8$	$b_9$	$b_{10}$	$b_{11}$	$b_{12}$	$b_{13}$
$B_{2\bar{a}}(b)$	1	0	0.4	0.4	0.8	1	1	0	0	0	0.4	0.8	1

Note that partitions  $B_{1\bar{a}}$  and  $B_{2\bar{a}}$  above represent the fuzzy sets of cars that are familiar and non familiar, respectively. Thus, it has sense that the two new objects in the new tables are denoted by *FamCars* and *NonFamCars*. The new relational tables are given by F-transforms (4) as follows

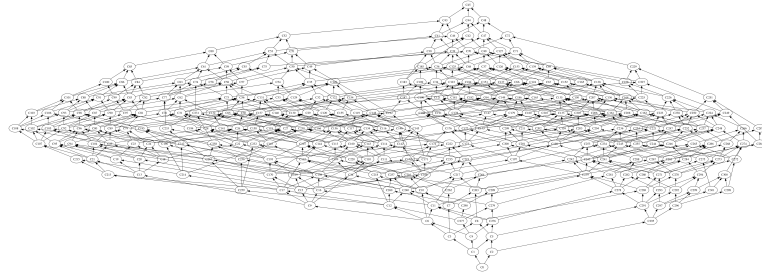
$R^\dagger$	<i>HighPower</i>	<i>BigSpace</i>	<i>HighConsume</i>	<i>Expensive</i>	<i>Sport</i>
<i>FamCars</i>	1	1	0.8	1	0.6
<i>NonFamCars</i>	1	0.4	1	1	1

$R^\downarrow$	<i>HighPower</i>	<i>BigSpace</i>	<i>HighConsume</i>	<i>Expensive</i>	<i>Sport</i>
<i>FamCars</i>	0.6	1	0.4	0.6	0
<i>NonFamCars</i>	0	0	0	0	0

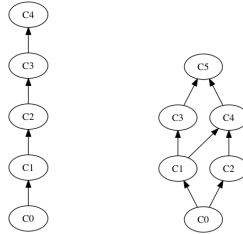
The tables above can be interpreted as follows. Tables  $R^\dagger$  and  $R^\downarrow$  represent the possibility and necessity, respectively, of a familiar car (in some degree) to have a certain attribute. So  $R^\dagger(\text{FamCars}, a)$  and  $R^\downarrow(\text{FamCars}, a)$  represent an upper and a lower bound, respectively, of the value  $R(b, a)$  for any familiar car  $b \in B$ , i.e.,  $R^\dagger(\text{FamCars}, a) \geq B_{1\bar{a}}(b) \& R(b, a)$  and  $R^\downarrow(\text{FamCars}, a) \leq B_{1\bar{a}}(b) \rightarrow R(b, a)$ .

It is interesting to mention that from the interpretability above, we can infer from the tables  $R^\dagger$  and  $R^\downarrow$  that a familiar car must have a big space because  $R^\dagger(\text{FamCars}, \text{BigSpace}) = R^\downarrow(\text{FamCars}, \text{BigSpace}) = 1$ . Moreover, the familiar cars are quite powerful and expensive as well as  $R^\downarrow(\text{FamCars}, \text{HighPower}) = R^\downarrow(\text{FamCars}, \text{Expensive}) = 0.6$ .

The concept lattice of the original relational table of Example 1 has 302 concepts and it is given by the following Hasse diagram



However, the concept lattices of the tables reduced by our procedure have only 5 and 6 concepts, respectively.



#### 4 Conclusion and Future Works

In this paper we have presented the reduction of relational tables aimed to keep as much information from the original table as possible. Our future work is to apply the reduct based on the ordinary F-transforms and measure, determine and/or bound the information which is on one hand lost by the reduction and on the other hand kept by the reduction.

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