Public debt frontiers: The Greek case*

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Abstract

Using a DGE model where the government is fully characterized, we compute the steady state relationship between the public debt/output ratio and the size of the government, measured as the total public expenditures/output ratio. We find the existence of a negative relationship between public debt long-run sustainable limit and government size. Calibration of the model for the Greek economy reveals that, for the period just before the current recession, i.e. 2002-2006, the steady state debt to GDP ratio (the long-run sustainability level) was 236.5%, whereas the observed figure for the same period was around 100%. Nevertheless, the crisis starting in 2007 provokes fast growth in the total public expenditures to output ratio, driving the Greek economy to the long-run unsustainable-debt region. We conclude that an original fiscal indiscipline did not cause the debt crisis and we have to look for alternative causes such as a credit crunch and/or gambling for redemption. We find that a gambling for redemption attitude towards the recent crisis triggered the Greek public financial disaster by crossing the debt frontier.

JEL Classification: H5; H6.

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1 Introduction

One of the many derivations caused by the ongoing international financial crisis has focused the attention of economists and policy makers on the sovereign debt crisis, which is hitting some countries of the Euro Area with particular intensity. The severity of the debt crisis starting in 2008 and its disastrous potential consequences

have fueled a debate where different measures have been proposed to prevent a similar crisis in the future and to provide stability to the European currency union. Those proposals can be summarized in two categories: economic reforms to provide long-term stability and zero budget deficits, and automatic economic and political sanctions for deficit sinners, to provide short-term stability. These proposals made by European leaders have been endorsed by the European Commission led by Barroso and the European Council led by Van Rompuy, and seem to be the most fundamental part of a comprehensive solution package to solve the European crisis.

Other proposals such as the creation of a Eurobond or the increased involvement of the European Central Bank acting as a lender of last resort have also been discussed. However, such economic policies have important caveats: they would imply huge transfers of income from the North to the South, are legally dubious according to the Treaty and the statutes of the ECB and consequentially, these proposals have encountered enormous resistance from several Member States regarding their implementation.

A reverse-engineering of the proposed solutions for the European debt crisis shows that the origins of this crisis can be found in i) A crisis of imbalances, caused by the weak competitiveness of peripheral Europe, and ii) A fiscal crisis, due to either direct fiscal indiscipline in the cases of Portugal and Greece, and irresponsible financial policies that triggered excessive fiscal guarantees, as in the cases of Ireland and Spain.

The question we want to study in this paper is whether the current European fiscal crisis is a consequence of past fiscal indiscipline (as implied by the austerity policies), or a bad response from political leaders pursuing generalized fiscal stimulus as providing excessive fiscal guarantees. In particular, we want to explore the Gambling for Redemption theory of Conesa and Kehoe (2011), where a fixed and exogenous probability of fiscal revenues recovery invites for a gambling not even precluded by the possibility of a bailout. While at times this gambling works, sometimes the bet is lost triggering all aspects of an economic disaster.

We test the Gambling for Redemption theory using a diagram where two key ratios of fiscal data of a given country are plotted together with a line obtained from the collection of steady states from a general equilibrium model. To provide a clearer idea of the diagram we use, consider the budget equation of a government that cannot resort to money printing:

$$T_t + (B_{t+1} - B_t) = G_t + rB_t$$

Fiscal revenues plus newly issued debt must finance current spending plus the service of existing debt at a given exogenous rate. The steady state equation is:

$$T_{ss} - G_{ss} = rB_{ss}$$

stating that an economy has to generate enough primary fiscal surpluses to finance the service of its debts to be sustainable long-term. If we divide the above equation by the steady state GDP, Y_{ss}, we obtain:

$$\frac{B_{ss}}{Y_{ss}} = \frac{\tau}{r} - \frac{1}{r}\frac{G_{ss}}{Y_{ss}}$$

where τ is just the average steady state tax rate T_{ss}/Y_{ss}. This equation, provides a mechanical relation between the two key ratios that we use to test the Gambling for Redemption hypothesis. However, the above equation is too simple. It does not take into consideration the various forms of taxation and expenditures that a government can use. The non-linearities that arise from government interventions in the economy are an important part of our test, and therefore our goal is to model a rich public sector that captures those non-linearities.

To this end we construct a DGE model where the role of the government affects a large variety of fiscal policies on both sides of the government budget restriction: revenues and expenditures.

In our model, total government spending is divided into several variables: public consumption of goods and services; public investment in physical capital; a public wage bill; transfer payments to households; and interest payments of public debt. As we will show in this paper, the amount of total debt is not independent from the spending policies, as different shares of total government spending have different effects on fiscal income: for example, spending in social transfers does not improve productivity of private factors, whereas increasing public investment does. Therefore, the amount of sustainable debt varies across policies. On the other hand, public revenues are raised by taxation and new debt issuance. We consider the existence of five taxes: consumption tax, labor income tax, capital income tax, social security tax and a corporate tax. Additionally, we include the fiscal funding of the social security system of the economy as a pay-as-you-go system.

As debt is modelled as if bond markets were infinitely liquid, the term structure of the debt is irrelevant in our model. Any maturing bond can always be rolled over at the given rate in the steady state. This paper attempts to quantify the maximum amount of debt that a government can sustain by assuming that lenders always lend. We do not allow for self-fulfilling crises. These crises arise when lenders think that a government will not repay its debt. If lenders think a government will not repay, they do not lend. If a government cannot roll over the portion of its debt becoming due within a period, it may choose to default even though it would not default if the lenders do lend. This is the idea in Cole and Kehoe (1996, 2000). The maximum level of debt that can be sustained if lenders do not lend is much lower than the maximum that can be sustained if they do lend. Consesa and Kehoe (2011) show that governments with low debt can choose to run this debt up to levels where they risk crises if their country is unlucky enough to be in a recession period after period.

This is the idea of Gambling for Redemption.

With this assumption of a perfect rolling over of debt, we compute the maximum level of sustainable debt, and provide a picture of the frontier dividing the sustainable region from the unsustainable region for any given level of public expenditure to GDP ratio. We find the existence of a steady state negative relationship between public debt/GDP ratio long-run sustainable limit and government size (measured as the total public expenditures/output ratio).

We have chosen Greece for our study because it was the first country under the currency union to lose its triple A rating on government bonds, and the country has faced strong pressure to consolidate the budget. We carefully calibrate the model to reach the conclusion that Greece was well inside the sustainable debt to GDP ratio when the crisis hit. Then, the government decided not to respond with an immediate reduction in government spending. On the contrary, government spending smoothly kept increasing. The government consumption to GDP ratio increased as a consequence, rapidly driving the economy to the unsustainable region. In the meantime, the recovery didn't happen. We conclude that a Gambling for Redemption attitude rather than fiscal indiscipline is behind the Greek debt crisis drama.

The structure of the rest of the paper is as follows: Section 2 presents the model, Section 3 discusses the calibration exercise, the main results from the calibrated model to the Greek economy are shown in Section 4, and finally, Section 5 concludes.

2   The model

We develop a general equilibrium model where the government affects private decisions in a number of ways. We consider the role of taxes, public consumption of goods and services, public investment in public capital, public labor markets and public debt.

We first describe the behavior of the government, then the firms, and finally the households. The government displays a high degree of disaggregation in both expenditures and fiscal income sides. On the expenditure side, we distinguish four components: public consumption of goods and services; public investment in capital; public wage bill; and transfers. On the fiscal income side, we consider four income taxes (consumption tax, labor income tax, capital income tax and corporate tax) plus revenues from the social security tax.

Firms are represented by a CES production function nested within a standard Cobb-Douglas. The production of the final output requires four factors: labor services and capital, both private and public. Finally, consumers are modeled in a standard way, but including public goods in the utility function and splitting worked hours between the private and the public labor sectors.

2.1 The Government

First, we describe the instruments at the government's disposal with the elements present in the government budget constraint:

$$G_t + R_t^B B_t + \Delta D_t = T_t + R_t^D D_t + CBT_t + \Delta B_t \tag{1}$$

Equation (1) says that all cash outlays (including transfer payments to households) - for non-interest total government spending (G_t), interest payments of total government debt (R_t^B times B_t), and new purchases of financial assets (ΔD_t) - must be funded by some combination of tax receipts (T_t), interest earnings on government assets (R_t^D times D_t), transfers from the central bank (CBT_t), and new debt issuance (ΔB_t).

For Eurozone countries, transfers from the central bank are zero, and direct purchases of government bonds are precluded by the Treaty (i.e. $CBT_t = 0$). If we denote by B_t the net position of the government, we can also set financial purchases to zero (i.e. $D_t = 0$).

2.1.1 Government spending

Non-interest total government spending is defined as:

$$G_t = C_{g,t} + (1 + \tau_t^{ss})W_{g,t}L_{g,t} + I_{g,t} + Z_t \tag{2}$$

where $C_{g,t}$ is public consumption of goods and services, $I_{g,t}$ is public investment, $W_{g,t}L_{g,t}$ is the wage bill for public employees and Z_t are transfer payments to households, such as welfare, social security or unemployment benefit payments.

We assume that a certain level of public capital is necessary in the aggregate production function. Public investments accrue into the public structures stock. We assume the following accumulation process for the public capital:

$$K_{g,t} = (1 - \delta_{K_g})K_{g,t-1} + I_{g,t} \tag{3}$$

which is analogous to the private capital accumulation process.

2.1.2 Government decision rules

We need to specify the government decision rules. These decision rules imply the election of i) a certain level of public spending and ii) its distribution among the different components.

The level of government spending in the long run, given a certain amount of fiscal revenues, depends on the target level for the public deficit and public debt. While

the Maastricht Treaty[1] establishes limits together with sanctions for deficit and debt sinners, these limits have only been respected to enter into the monetary union, but never after that date. Therefore, we do not consider the Maastricht criteria to be binding for these two variables.

The distribution among the different components of public spending is as follows.

$$C_{g,t} = \theta_1 G_t$$

$$I_{g,t} = \theta_2 G_t$$

$$(1 + \tau_t^{ss})W_{g,t}L_{g,t} = \theta_3 G_t$$

$$Z_t = \theta_4 G_t$$

where $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 1$. McGrattan et al. (1997) assume that public spending on goods and services is a stochastic process around a constant proportion of total output. We follow the same framework for the components of total government spending but in a deterministic environment.

2.1.3 Public labor market

The public labor market is modeled following the work of Fernández de Córdoba, Pérez and Torres (2012). The purpose of the mechanism described in this section is to distort the labor market to prevent wages equalization between the private and the public sector. An analysis of the public labor market among OECD countries show that the public wage bill is a source of major differences among these countries. Our analysis shows that government interventions in the wage setting of public wages can have a significant effect not only on the wage bill, but also in the growth path of the economy affecting the income shares of private inputs, having therefore a long-term effect on the debt frontier.

We have chosen a mechanism where the government has preferences over the number of public workers and their pay. To provide an objective function for the government, we follow a standard text-book approach (for example see Oswald et al.,

[1]The Treaty on European Union was signed on 7 February 1992 by the members of the European Community in Maastricht, Netherlands. The Treaty led to the creation of the euro, and stablished a set of rules imposing control over inflation, public debt and the public deficit, exchange rate stability and the convergence of interest rates. With regard to public finances it imposed an annual limit of 3% in the ratio of government deficit to GDP, and a 60% of gross government debt prior to the entry in the European Monetary System

1984[2]) and pose an objective function for the government as the solution of a game between a public sector union that cares about the wages of public-sector employees, $W_{g,t}$ and a government that cares about the level of public employment, $L_{g,t}$ given its budget constraint. Thus, the government wants to maximize the following objective function subject to a budget constraint:

$$\max \left[\omega W_{g,t}^{\theta} + (1 - \omega) L_{g,t}^{\theta} \right]^{1/\theta} \tag{4}$$

where ω is the weight given to wages and θ is a negative parameter indicating the curvature of the trade-off between the elements present in the objective function of the government. If ω is close to zero, then the main goal of the government is to maximize public employment (benevolent government preference), whereas if ω is close to one, the main goal of the government is to maximize public wages (public sector union's preferred option).

Note that expression (4) encompasses the different approaches found in the literature. On the one hand, it takes into account the fact that public employment and wages are determined in an environment different to the private sector. The government itself can increase the number of public employees or can increase public wages subject to the budgetary constraint. On the other hand, it takes into account the fact that trade unions are more important in the public labor sector than in the private sector (see for instance Blanchflower, 1996).

As defined previously, the government wage bill is defined as:

$$\theta_3 G_t = (1 + \tau_t^{ss}) W_{g,t} L_{g,t} \tag{5}$$

Maximizing the government objective function subject to the government budget constraint is to find critical values for the auxiliary Lagrangian function:

$$\pounds_g (\bullet) = \max \left[\omega W_{g,t}^{\theta} + (1 - \omega) L_{g,t}^{\theta} \right]^{1/\theta} + \xi \left(\theta_3 G_t - (1 + \tau_t^{ss}) W_{g,t} L_{g,t} \right)$$

That provides, upon differentiation, the first order necessary conditions:

$$\frac{\partial \pounds_g (\bullet)}{\partial W_{g,t}} = \left[\omega W_{g,t}^{\theta} + (1 - \omega) L_{g,t}^{\theta} \right]^{1/\theta - 1} \omega W_{g,t}^{\theta - 1} - \xi (1 + \tau_t^{ss}) L_{g,t} = 0$$

$$\frac{\partial \pounds_g (\bullet)}{\partial L_{g,t}} = \left[\omega W_{g,t}^{\theta} + (1 - \omega) L_{g,t}^{\theta} \right]^{1/\theta - 1} (1 - \omega) L_{g,t}^{\theta - 1} - \xi (1 + \tau_t^{ss}) W_{g,t} = 0$$

Dividing orderly:

$$\omega W_{g,t}^{\theta} = (1 - \omega) L_{g,t}^{\theta} \tag{6}$$

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[2]On related grounds Ardagna (2007) and Forni and Giordano (2003) consider the wage bill of the government, employment and wages, separately as arguments of the objective function of the government or the public sector union.

Combining this expression with equation (5) we obtain that public wages and employment are equal to:

$$W_{g,t} = \left(\frac{\omega}{1-\omega}\right)^{-1/2\theta} \left[\frac{\theta_3 G_t}{(1+\tau_t^{ss})}\right]^{1/2} \tag{7}$$

$$L_{g,t} = \left(\frac{\omega}{1-\omega}\right)^{1/2\theta} \left[\frac{\theta_3 G_t}{(1+\tau_t^{ss})}\right]^{1/2}, \text{ if } W_{g,t} > W_{p,t} \tag{8}$$

This distribution of the public resources depends on government preferences. However, private and public sectors are competing for the same labour input and as a consequence there is a relationship between public sector and private sector wages inducing a wage premium. The wage premium is implicit in equation (8) and it is part of the solution of the governments problem. This wage premium ensures the government that it's demand for labor will be satisfied. This relationship will become clearer once we present the household's problem.

2.1.4 Tax revenues

The government obtains resources from the economy by taxing consumption and income from labor, capital and profits, whose effective average tax rates are denoted by $\tau_t^c, \tau_t^l, \tau_t^k, \tau_t^\pi$, respectively. Additionally, we consider a pay-as-you-go social security system and thus we include the social security tax, τ_t^{ss}. The government budget in each period is given by,

$$\begin{aligned} T_t &= \tau_t^c C_{p,t} + \tau_t^l(W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau_t^k(R_t - \delta_{K_p})K_{p,t-1} \\ &\quad + \tau_t^k R_t^B B_t + \tau_t^{ss}(W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau_t^\pi \Pi_t \end{aligned}$$

where $C_{p,t}$ is private consumption, $W_{p,t}$ is private sector wages, $L_{p,t}$ is private labor, R_t is the rental rate of private capital, δ_{K_p} is the depreciation rate of private capital, $K_{p,t}$ is private capital stock, and Π_t are profits to be defined later.

2.1.5 The government identity

As we previously argued the government budget constraint can be written as:

$$G_t + (1 + R_t^B)B_t = T_t + B_{t+1}$$

With the meaning that non financial spending, plus servicing the existing government debt must be financed through taxes plus new debt.

Putting together all the elements defined above, the government budget constraint can be written as:

$$C_{g,t} + (1 + \tau_t^{ss})W_{g,t}L_{g,t} + I_{g,t} + Z_t + (1 + R_t^B)B_t$$
$$= \tau_t^c C_{p,t} + \tau_t^l(W_{p,t}L_{p,t} + W_{g,t}L_{g,t})$$
$$+\tau_t^k(R_t - \delta_{K_p})K_{p,t-1} + \tau_t^{ss}(W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau_t^\pi \Pi_t + B_{t+1} \qquad (9)$$

or, collecting uses and resources:

$$C_{g,t} + W_{g,t}L_{g,t} + I_{g,t} + Z_t + (1 + R_t^B)B_t$$
$$= \tau_t^c C_{p,t} + \tau_t^l(W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau_t^k(R_t - \delta_{K_p})K_{p,t-1}$$
$$+\tau_t^{ss}W_{p,t}L_{p,t} + \tau_t^\pi \Pi_t + B_{t+1} \qquad (10)$$

2.2   Firms

The problem of the firm is to find optimal values for the utilization of labor and capital given the presence of public inputs. The representative firm operates a CES production function nested within a standard Cobb-Douglas production function, and thus this technology exhibits a constant return to private factors. The production of final output, Y, requires labor services, L and capital, K, both private and public. Goods and factors markets are assumed to be perfectly competitive. The firm rents capital and hires labor to maximize period profits, taking factor prices and public labor and capital as given. The technology is given by:

$$Y_t = A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta]^{\frac{(1-\alpha_p-\alpha_g)}{\eta}} \qquad (11)$$

where Y_t is aggregate output, A_t is a measure of total-factor productivity, α_p and α_g are private and public capital share of output respectively, μ measures the weight of public employment relative to private employment and $\sigma = 1/(1 - \eta)$ is a measure of the elasticity of substitution between public and private labor inputs.

If we assume final output to be the unit of account, profits are defined as:

$$\Pi_t = A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta]^{\frac{(1-\alpha_p-\alpha_g)}{\eta}} - (1 + \tau_t^{ss})W_{p,t}L_{p,t} - R_t K_{p,t-1} \qquad (12)$$

Under the assumptions that private workers are paid their marginal productivity, we get:

$$(1 + \tau_t^{ss})W_{p,t} = \mu(1 - \alpha_p - \alpha_g)A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta]^{\frac{(1-\alpha_p-\alpha_g-\eta)}{\eta}} L_{p,t}^{\eta-1}$$

$$R_t = \alpha_p A_t K_{p,t-1}^{\alpha_p-1} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta]^{\frac{(1-\alpha_p-\alpha_g)}{\eta}}$$

From the above equations, it is found that private factor incomes are:

$$
\begin{aligned}
(1 + \tau_t^{ss})W_{p,t}L_{p,t} &= \mu(1 - \alpha_p - \alpha_g)A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g}[\mu L_{p,t}^{\eta} + (1 - \mu)L_{g,t}^{\eta}]^{\frac{(1-\alpha_p-\alpha_g-\eta)}{\eta}} L_{p,t}^{\eta} \\
&= \frac{\mu(1 - \alpha_p - \alpha_g)L_{p,t}^{\eta}}{\mu L_{p,t}^{\eta} + (1 - \mu)L_{g,t}^{\eta}}Y_t
\end{aligned}
$$

$$
R_t K_{p,t-1} = \alpha_p Y_t \tag{13}
$$

The aggregate production function has four productive factors. However, the two public factors have no market price. The government does not usually charge a price that covers the full cost of the services provided with the contribution of public factors. This implies that those rents generated by public factors are not assigned to public factors. As public factors are paid by the government, there is a positive profit, Π_t, which turns out to be[3]:

$$
\Pi_t = Y_t - R_t K_{p,t-1} - (1 + \tau_t^{ss})W_{p,t}L_{p,t} > 0
$$

We assume that profits are paid out to households given that they are the owners of the firm.

2.3 Households

In our model economy, the decisions made by consumers are represented by a stand-in consumer with a period utility where consumption can be decomposed into two components:

$$
U(C_t, L_t) = U(C_{p,t}, C_{g,t}, L_t) \tag{14}
$$

where $C_{p,t}$ is private consumption and $C_{g,t}$ is consumption of the same private good provided by the government to the consumer. We assume that households obtain utility from the public spending in good and services. In particular, we assume that:

$$
C_t = C_{p,t} + \pi C_{g,t} \quad \text{with } \pi \in (0, 1] \tag{15}
$$

Households' preferences are given by the following instantaneous utility function:

$$
U(C_t, N_t\overline{H} - L_t) = \gamma \log C_t + (1 - \gamma) \log(N_t\overline{H} - L_t) \tag{16}
$$

Leisure is $N_t\overline{H} - L_t$, where $\overline{H}$ is total time endowment and it is calculated as the number of effective hours in the week times the number of weeks in a year times population in the age of taking labour-leisure decisions, N_t, minus the aggregated

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[3]See appendix A.2

number of hours worked in a year, L_t. The parameter γ $(0 < \gamma < 1)$ is the fraction of private consumption on total private income. Households consume final goods and supply labour to the private and the public sectors,

$$L_t = L_{p,t} + L_{g,t} \tag{17}$$

where L_t is the aggregate level of employment, $L_{p,t}$ is private employment and $L_{g,t}$ is public employment. Public employment is chosen by the government and thus it is exogenously given to the households. At an aggregate level, the household can only choose the supply of private labour, $L_{p,t} = L_t - L_{g,t}$. Recall that public employment demand is fully covered by the household, provided that $W_{g,t} > W_{p,t}$.

The budget constraint faced by the stand-in consumer is:

$$(1 + \tau_t^c)C_{p,t} + K_{p,t} - K_{p,t-1} + B_{t+1} - B_t \tag{18}$$

$$= (1 - \tau_t^l)[W_{p,t}L_{p,t} + W_{g,t}L_{g,t}] + (1 - \tau_t^k)(R_t - \delta)K_{p,t-1} \tag{19}$$

$$+ (1 - \tau_t^k)R_t^B B_t + Z_t + (1 - \tau_t^\pi)\Pi_t \tag{20}$$

where $K_{p,t}$ is private capital stock, $W_{p,t}$ is private compensation per employee, $W_{g,t}$ is public compensation per employee, R_t is the rental rate of capital, δ_{K_p} is the capital depreciation rate which is modelled as tax deductible and Π_t denotes profits from firms, as defined previously. The budget constraint states that consumption and investment, in physical capital and government bonds, cannot exceed the sum of labour and capital rental incomes and profits net of taxes .

Capital holdings evolve according to:

$$K_{p,t} = (1 - \delta_{Kp})K_{p,t-1} + I_{p,t} \tag{21}$$

where $I_{p,t}$ is household's gross investment.

The problem faced by the stand-in consumer is to maximize the value of her lifetime utility given by:

$$Max_{\{C_t, L_t\}_t^\infty} \sum_{t=0}^{\infty} \beta^t \left[\gamma \log(C_{p,t} + \pi C_{g,t}) + (1 - \gamma) \log(N_t \overline{H} - L_{p,t} - L_{g,t}) \right] \tag{22}$$

subject to the budget constraint, where (K_{p0}, K_{g0}, B_0) and the paths of public employment and taxes are given, and where $\beta \in (0, 1)$, is the consumer's discount factor. The Lagrangian auxiliary function is:

$$
\begin{aligned}
\pounds(\bullet) &= \sum_{t=0}^{\infty} \beta^t [\gamma \log(C_{p,t} + \pi C_{g,t}) + (1 - \gamma) \log(N_t \overline{H} - L_{p,t} - L_{g,t}) + ... \\
&\quad \lambda_t \{ (1 - \tau_t^l)[W_{p,t}L_{p,t} + W_{g,t}L_{g,t}] + (1 - \tau_t^k)(R_t - \delta)K_{p,t-1} \\
&\quad + (1 - \tau_t^k)R_t^B B_t + Z_t + (1 - \tau_t^\pi)\Pi_t - ... \\
&\quad ((1 + \tau_t^c)C_{p,t} + K_{p,t} - K_{p,t-1} + B_{t+1} - B_t) \}]
\end{aligned}
$$

The first order conditions for the consumer maximization problem are:

$$\frac{\partial \pounds}{\partial C_{p,t}} = \gamma \frac{1}{C_{p,t} + \pi C_{g,t}} - \lambda_t(1 + \tau_t^c) = 0 \tag{23}$$

$$\frac{\partial \pounds}{\partial L_{p,t}} = -(1 - \gamma)\frac{1}{N_t\overline{H} - L_{p,t} - L_{g,t}} + \lambda_t(1 - \tau_t^l)W_{p,t} = 0 \tag{24}$$

$$\frac{\partial \pounds}{\partial K_{p,t}} = \beta^{t+1}\left[\lambda_{t+1}\left(1 + (1 - \tau_{t+1}^k)(R_{t+1} - \delta_{Kp})\right)\right] - \lambda_t\beta^t = 0 \tag{25}$$

$$\frac{\partial \pounds}{\partial B_t} = \beta^{t-1}\lambda_{t-1} - \beta^t\lambda_t(1 + (1 - \tau_t^k)R_t^B) = 0 \tag{26}$$

Plus the budget constraint and two transversality conditions stating that the today-value of long distant future values of assets are zero.

This formulation implies that the wage-setting process in the private sector is totally different to that of the public sector. Whereas in the private sector wages are determined in terms of their marginal products, in the public sector a given amount from the government's budget constraint is distributed between public wages and public employment.

Note that the above expressions imply that the consumer can only choose the supply of private labour, given that public labour is determined inelastically by the government at a wage that includes a positive premium that guarantees that all public labor demand is covered by the consumer at any market $W_{p,t}$.

Walras's Law is satisfied at all times[4]. From equations (25) and (26) we obtain a non arbitrage steady state condition

$$R = \delta_{K_p} + R^B$$

The real return to capital has to equate the depreciation rate due to the use of physical capital plus the real return of the competing bond, including any risk premium.

3   Calibration

In this section we calibrate the model for the Greek economy. We select this economy as our case study given that it represents a benchmark for studying the causes of a debt crisis, as it was the first country under the currency union to lose its triple A rating on government bonds. Figure 1 plots the evolution of the public debt/GDP ratio (left scale) and total public expenditure/GDP ratio (right scale) for the period 2002-2011. Both ratios remain almost constant for the period 2002-2006. As these figures are central in our analysis, we choose the average values of the macroeconomic

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[4]See Appendix A for a proof.

variables for the Greek economy for this period as the steady state for our model economy.

[Insert here Figure 1]

The steady state of the model economy is computed as follows: Some parameter values are computed from ratios taken from the national accounts, other parameters are taken from the set of equilibrium conditions while the technological parameters of the nested CES (production function are estimated using standard econometric techniques. The parameter $\phi = G_t/Y_t$, takes values from the interval $[0, \widehat{\phi}]$. For any given vector that defines the fiscal policy $(\tau^k, \tau^l, \tau^c, \tau^{ss}, \tau^\pi, \theta_1, \theta_2, \theta_3, \theta_4)$, we compute the steady state values for prices and quantities that satisfy the set of first order conditions and the market clearing equations described above, for each value $\phi = G_t/Y_t$.

First, the parameters of the model are calibrated to replicate some salient features of the Greek economy. Total output at the calibration point (years 2002-2006), is set to $Y_{SS} = 100$. From OECD statistics we obtain the ratio of total government expenditures ($G_{SS}/Y_{SS} = 0.4504$), total (both public and private) investment ($I_{SS}/Y_{SS} = 0.2236$), and total consumption ($C_{SS}/Y_{SS} = 0.7764$), as well as the fraction of total labour force actually employed ($L_{SS}/H_{SS} = 0.5750$). Notice that public and private consumption plus total investment is total GDP. The reason for this is that our measure of G_t adds to public consumption all transfers to the consumer such as public education, public health, transfers for the unemployed and the public wage bill. The public wage bill of Greece represents about 31% of total government expenditure. In particular, this parameter is set to $\theta_3 = 0.3280$. Public investment was about 15% of total investment, which yields a value of $\theta_2 = 0.0745$, while total public consumption was about 19% of GDP. The value taken from National Accounts yield a value for $\theta_1 = 0.1923$. These figures yield a value for $\theta_4 = 1 - \theta_1 - \theta_2 - \theta_3 = 0.4052$ for total transfers to consumers. The values for depreciation rates are calculated from Greek National Accounts, where gross capital consumption values are provided. Accordingly we set a value for $\delta_{K_p} = 0.06$ and $\delta_{K_g} = 0.04$, we compute steady state values for capital as $K_{p,SS} = I_{p,SS}/\delta_{K_p} = 316.7667$ and $K_{g,SS} = I_{g,SS}/\delta_{K_g} = 83.8500$.

The values for effective average tax rates $(\tau^c, \tau^l, \tau^k, \tau^{ss}, \tau^\pi)$ are taken from Boscá et al. (2012), who use the methodology developed by Mendoza et al. (1994). The real return of the Greek bond at the calibration period was $R^B = 0.045$. Equation (26), provides a steady state relationship between R^B and β, given a value for $\tau_k = 0.1640$. The value we obtain is $\beta = 0.9673$. Once we have this value, equation (25) together with the value of δ_{K_p} delivers $R = 0.1005$. From equation (13) and the value for public capital we obtain a value for $\alpha_p = 0.3184$.

Equate the marginal products of public and private capital to get:

13

$$R_{pt} = \alpha_p A_t K_{p,t-1}^{\alpha_p-1} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^{\eta} + (1-\mu) L_{g,t}^{\eta}]^{\frac{(1-\alpha_p-\alpha_g)}{\eta}}$$

$$R_{gt} = \alpha_g A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g-1} [\mu L_{p,t}^{\eta} + (1-\mu) L_{g,t}^{\eta}]^{\frac{(1-\alpha_p-\alpha_g)}{\eta}}$$

$$\frac{R_{pt}}{R_{gt}} = \frac{\alpha_p}{\alpha_g} \frac{K_{g,t-1}}{K_{p,t-1}}$$

If we assume that the real return to public capital is equal to the real return to private capital, we can compute a value for $\alpha_g = \alpha_p(K_g/K_p) = 0.0843$. With the data from the OECD data series on public sector labor and wages for Greece, we obtain the ratio of public labor to private labor in 2002-2006, $L_{g,SS}/L_{p,SS} = 0.2399$, while the wage premium for the same year was, $W_{g,SS}/W_{p,SS} = 1.400$.

Comparing Greece's labour market with Europe (figures 2 and 3), we observe the same trend of a decrease in the participation of public labor in total employment, with an increase in the wage premium. The dissimilarity that might have fiscal consequences relates to the level of the wage premium. While in Europe in 2008 the average wage premium was about 1.3, the same magnitude for Greece was 1.4. That is 7.69% higher in Greece. The weight of public employment relative to private employment and the elasticity of substitution between public and private labor inputs are estimated econometrically. From the production function, we can obtain the ratio of public wages to private wages as[5]:

$$\frac{W_{g,t}}{W_{p,t}} = \frac{1-\mu}{\mu} \frac{L_{g,t}^{\eta-1}}{L_{p,t}^{\eta-1}} \tag{27}$$

From this expression we get, taking logs

$$\log\left(\frac{W_{g,t}}{W_{p,t}}\right) = \log\left(\frac{1-\mu}{\mu}\right) + (\eta-1)\log\left(\frac{L_{g,t}}{L_{p,t}}\right) \tag{28}$$

and estimate by OLS[6]. From the estimation[7] for Greece we obtain the values for $\mu = 0.6008$ and $\eta = 0.4326$. Results from the estimation are represented in the lower panel of Figures 2 and 3. When we estimate the coefficients of equation (28) we find values of η and μ that imply that a wage premium is being paid by the government to public workers.

---

[5]See appendix A.2 for a derivation of equation (27)

[6]The estimation procedure is explained in Fernández-de-Córdoba, Pérez and Torres (2012). An alternative to the estimation is to fix $W_p = 1$, and set $W_g = wage\ premium * Wp$, and then obtain the value for μ, and fix η to a reasonable value.

[7]The OLS estimation for Greece produces a $R^2 = 0.7468$, the F statistic is $F = 138.5908$, and a p value for the model $p = 0.0000$, while the same values for Europe are $R^2 = 0.9199$, $F = 424.6571$, and $p = 0$.

We set a total labor endowment of $H = 100$, and from the OECD labor statistics we have $L = 57.500$ for the year 2002. This number, plus the public to private labor ratio yield the corresponding values for L_p, and L_g. The production function gets fully calibrated computing A as a residual:

$$A = \frac{Y}{K_p^{\alpha_p} K_g^{\alpha_g} [\mu L_p^{\eta} + (1-\mu)L_g^{\eta}]^{\frac{(1-\alpha_p-\alpha_g)}{\eta}}} = 1.4733$$

Fix $\theta = -1$, in the government public sector objective function, and compute the value for ω as

$$\omega = \frac{1}{1 + \left(\frac{W_g}{L_g}\right)^{\theta}} = 0.0765$$

Finally, we compute γ as:

$$\gamma = \frac{C_p + \pi C_g}{C_p + \pi C_g + (H - L_p - L_g)W_p \frac{1-\tau_l}{1+\tau_k}} = 0.8437$$

We collect the parameter values in two tables. The first table (Table 1) contains values taken from National Accounts, average effective tax rates and depreciation rates. Table 2 shows the set of parameters that we calibrate using the equilibrium conditions from the model.

[Insert here Figure 2]
[Insert here Figure 3]

Table 1: The Greek economy calibrated parameters

Parameter	Definition	Value
Y	Total GDP	100
G/Y	Ratio total public spending/output	0.4504
I/Y	Ratio total investment/output	0.2336
C/Y	Ratio total consumption/output	0.7764
θ_1	Ratio public consumption/total government spending	0.1923
θ_2	Ratio public investment/total government spending	0.0745
τ^l	Labor income tax rate	0.4100
τ^k	Capital income tax rate	0.1640
τ^{ss}	Social security contribution	0.4406
τ^{π}	Profit tax rate	0.2500
τ^c	Consumption tax rate	0.1480
π	Private/public consumption rate of substitution	1
δ_{Kp}	Private capital depreciation rate	0.06
δ_{Kg}	Public capital depreciation rate	0.04

Table 2: The Greek economy model-calibrated parameters

Parameter	Definition	Value
R^B	Real return of a Greek bond	0.0405
β	Discount factor	0.9673
R	Real return to capital	0.1005
α_p	Private capital income share	0.3184
α_g	Public capital technical parameter	0.0843
η	Public-Private employment elasticity of substitution	0.4326
μ	Private employment weight	0.6008
θ	Public wages/employment elasticity of substitution	-1.0000
ω	Public wages weight	0.0765
θ_3	Ratio wage bill/total government spending	0.3280
θ_4	Ratio transfers/total government spending	0.4052
A	FTP	1.4733
γ	Consumption preferences	0.8437

4 The debt frontier

Given the calibrated parameters and the key macroeconomic ratios for the period 2002-2006 for the Greek economy, we compute the steady state of the model economy. A key result from the model is the existence of a negative relationship between public debt long-run sustainable limit and government size measured as the total government spending to GDP ratio, given a particular menu of taxes, i.e., a particular level of fiscal revenues and given an interest rate on public debt. A larger government size, given a constant level of public revenues, corresponds to a lower long-run sustainable level of public debt. The intuition behind this result is simple. In our model, public debt is modelled as if bond markets were infinitely liquid and thus, any maturing bond can always be rolled over at the given rate in the steady state. In this context, the long term sustainable amount of debt depends on both public revenues and expenditures and on the public bond interest rate. The sustainable debt limit is increasing in public revenues and decreasing in public expenditure and bond interest rate. A negative shock to output will reduce both the public income/output ratio and the public expenditure/output ratio, driving the economy toward the long-run unsustainable debt area on one hand, and reducing the long-run sustainable amount of debt on the other hand.

Our main result is better explained with Figure 4. The decreasing relationship between Public Expenditure to GDP ratio G_t/Y_t, and total debt to GDP ratio, B_t/Y_t in our notation, separates the space into two disjoint sub-spaces. Above the curve we have all pairs where given the ratio G_t/Y_t, the amount of endogenous fiscal

revenues are not enough to cover the services of total debt. Below the curve, we have all data pairs where fiscal revenues suffice to cover the given G_t/Y_t ratio and services the outstanding debt.

[Insert here Figure 4]

Two points are highlighted in the graph (blue circles). Both correspond to the observed ratio $G_{SS}/Y_{SS} = 0.4504$ for Greece. The upper point is on the Debt to GDP curve, showing that the maximum level of sustainable debt as a percentage of GDP that Greece can afford given the structure of public expenditures is 236.5467%. The lower point shows the actual level of Debt to GDP ratio at the steady state. As the reader can check, it belongs to the sustainable set. From this point, any reduction of total expenditure improves the credit position of Greece in the international debt markets. The vertical line drawn at $G_t/Y_t = 0.5423$, shows the ratio of total expenditure to GDP that would force Greece to cancel all its outstanding debt. It is clear from the graph that this point is sufficiently far away from the actual steady state ratio previous to the crisis.

Figure 4 also plots the actual values of both ratios for the period 2002-2011. These ratios, for the period 2002-2006 remain almost constant at a value of total public spending/GDP of 45% and a public debt/GDP of around 100%. We also highlight the "Gambling for Redemption" period, for the years 2007, 2008 and 2009. Whereas the figures for 2007 are still well inside the long-run sustainable area, the figures for 2008 are clearly in the unsustainable area. Moreover, the corresponding figures for 2009 reflects an even worse situation, as the public expenditure to GDP ratio reaches a level for which no positive amount of public debt is sustainable. By that time, financial markets were clearly betting for a default.

From this picture, we conclude that the current financial crisis affecting Greece has to be explained by an approach not directly linked to the fundamentals of the economy, as a carefully calibrated standard neoclassical growth model shows. Prior to the crisis, the Greek economy was well inside the long-run sustainable debt area with a public budget carrying with it a constant level of public debt/GDP ratio. Nevertheless, the crisis rapidly deteriorated GDP and public revenues, driving the Greek economy to the long-run unsustainable area. One can argue that the initial value of public debt was too high (around 100% of GDP) and that a lower level of public debt would have increased the strength of the Greek economy to cope with the crisis and remain in the long-run sustainable area. However, looking to the evolution of the Greek economy from 2007, an initial lower level of public debt does not guarantee that it would have avoided the debt crisis, given the evolution of Public Expenditures to GDP.

From Figure 4 it is clear that small reductions in the Public Expenditure to GDP ratio induce large increases in the Debt to GDP ratio. The immediate implication is that reductions of expenditure above the expected decrease in GDP, together with an increase in fiscal revenues from increased taxation should be enough to guarantee the solvency of the Greek State. Conversely, increases in the public expenditures to GDP ratio deteriorates the credit position very rapidly. The data shows that the swing to the right in the expenditures to GDP ratio from 2006 to 2009 was too large.

5   Conclusions

This paper develops a DGE model in which the government is fully characterized in both income and spending sides. The model shows the existence of a negative relationship between public debt long-run sustainable limit and government size, given a particular menu of taxes. As the government size becomes larger, given a constant level of public revenues, the long-run sustainable level of public debt becomes lower. Therefore, the model can be used to quantify for a particular economy, the distance between the current level of public debt and the long-run sustainable level.

Calibration of the model for the Greek economy reveals that, for the period just before the current recession, the steady state public debt/GDP ratio (the long-run sustainability level) was 236.5%, whereas the figure in 2002-2006, the steady state reference, was around 100%. We find evidence that a Gambling for Redemption attitude towards the crisis, as in Conesa and Kehoe (2011) and Arellano, Conesa and Kehoe (2012) can explain quite well the path of the Greek economy from 2007. As Conesa and Kehoe (2011) point out, countries that are in deep recessions have the incentive to cut government spending very slowly and increase the public debt, gambling that a recovery in the economy will lead to larger fiscal revenues. This argument is consistent with the recent experience of Greece during the period 2007-2009. Nevertheless, the debt-sustainability problem emerges when the recession is prolonged. In this case, government revenues never recover and the gamble for redemption cannot be maintained indefinitely, forcing the default.

The consequence we extract from this paper is that the government gambled for redemption and lost the bet. Period by period for three consecutive years, the global economy deteriorated, fiscal revenues never recovered, and suddenly astronomical bond yields indicated that the game was over.

The historically observed frequency of the cycle can entice governments to gamble for redemption with the hope that the next expected expansion will dissolve past fiscal deficits. This implies that the Gambling for Redemption attitude towards a crisis can be the product of our past statistical knowledge of the cycle. It is reasonable, as we argue, and also optimal as Conesa and Kehoe demonstrate, to gamble for redemption when purely statistically based policies are put in place.

Once the economic policy that emerges from a Gambling for Redemption strategy is proved incorrect by reality, some structural adjustments have to be put in place.

The table in Appendix B shows that policies oriented to increase productivity, together with a fiscal package that includes increases in VAT, labor taxes and corporate taxes, plus a re-structuring of public expenditures increasing public investment, at the expense of transfers, can be effective to solve a debt crisis. The proposed combination of increasing by 10% the following vector of policy instruments $(\tau_k, \tau_l, \tau_\pi, \theta_3)$ would depress output by -3.04%, it would depress private consumption and public consumption by -2.73% and -3.03% respectively, and it would depress private investment by -4.03%, but it would rise the debt ceiling by 48.37%.

Appendix A.1: Walras' Law

Take the budget constraint faced by the consumer:

$$
\begin{aligned}
&(1 + \tau_t^c)C_{p,t} + K_{p,t} - K_{p,t-1} + B_{t+1} - B_t \\
= \ &(1 - \tau_t^l)[W_{p,t}L_{p,t} + W_{g,t}L_{g,t}] + (1 - \tau_t^k)(R_t - \delta)K_{p,t-1} \\
&+ (1 - \tau_t^k)R_t^B B_t + Z_t + \Pi_t
\end{aligned}
$$

And substitute the value of

$$
Z_t = G_t - C_{g,t} - (1 + \tau_t^{ss})W_{g,t}L_{g,t} - I_{g,t}
$$

to obtain:

$$
\begin{aligned}
&(1 + \tau_t^c)C_{p,t} + I_{gt} + K_{p,t} - K_{p,t-1} + B_{t+1} - B_t \\
= \ &(1 - \tau_t^l)[W_{p,t}L_{p,t} + W_{g,t}L_{g,t}] + (1 - \tau_t^k)(R_t - \delta)K_{p,t-1} \\
&+ G_t + (1 - \tau_t^k)R_t^B B_t - C_{g,t} - (1 + \tau_t^{ss})W_{g,t}L_{g,t} + \Pi_t
\end{aligned}
$$

$$
\begin{aligned}
&C_{pt} + C_{g,t} + I_{gt} + I_{pt} + B_{t+1} - B_t \\
= \ &-\tau_t^c C_{p,t} + (1 - \tau_t^l)[W_{p,t}L_{p,t} + W_{g,t}L_{g,t}] + R_t K_{p,t-1} - \tau_t^k \left(R_t - \delta_{K_p}\right)K_{p,t-1} \\
&+ G_t + (1 - \tau_t^k)R_t^B B_t - (1 + \tau_t^{ss})W_{g,t}L_{g,t} + \Pi_t
\end{aligned}
$$

But, the government identity establishes the following relation:

$$
(1 + R_t^B)B_t - B_{t+1} = T_t - G_t
$$

Direct substitution yields

$$
\begin{aligned}
&C_{pt} + C_{g,t} + I_{gt} + I_{pt} - T_t \\
= \ &-\tau_t^c C_{p,t} + (1 - \tau_t^l)[W_{p,t}L_{p,t} + W_{g,t}L_{g,t}] + R_t K_{p,t-1} - \tau_t^k \left(R_t - \delta_{K_p}\right)K_{p,t-1} \\
&- \tau_t^k R_t^B B_t - (1 + \tau_t^{ss})W_{g,t}L_{g,t} + \Pi_t
\end{aligned}
$$

Government fiscal income is given by:

$$
\begin{aligned}
T_t = \ &\tau_t^c C_{p,t} + \tau_t^l(W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau_t^k(R_t - \delta_{K_p})K_{p,t-1} \\
&+ \tau_t^k R_t^B B_t + \tau_t^{ss}(W_{p,t}L_{p,t} + W_{g,t}L_{g,t})
\end{aligned}
$$

Substitution and elimination drives to:

$$C_{pt} + C_{g,t} + I_{gt} + I_{pt}$$
$$= W_{p,t}L_{p,t} + R_t K_{p,t-1} + \Pi_t + \tau_t^{ss} W_{p,t} L_{p,t}$$

From the definition of profits we find that,

$$\Pi_t = Y_t - (1 + \tau_t^{ss})W_{p,t}L_{p,t} - R_t K_{p,t}$$

Substitution yields:

$$C_{pt} + C_{g,t} + I_{gt} + I_{pt} = Y_t$$

Therefore, Walras' Law is satisfied at all times.

Appendix A.2: Positive profits

In a private economy where the government supply capital and labor with market pricing, the firm would have a profit function as:

$$\bar{\Pi}_t = Y_t - (1 + \tau_t^{ss})(W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) - R_t(K_{p,t-1} + K_{g,t-1})$$

Where

$$Y_t = A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^{\eta} + (1-\mu)L_{g,t}^{\eta}]^{\frac{(1-\alpha_p-\alpha_g)}{\eta}}$$

Under the assumptions that private factors are paid their marginal productivity, we get:

$$(1+\tau_t^{ss})W_{p,t} = \mu(1-\alpha_p-\alpha_g)A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g}[\mu L_{p,t}^{\eta}+(1-\mu)L_{g,t}^{\eta}]^{\frac{(1-\alpha_p-\alpha_g-\eta)}{\eta}} L_{p,t}^{\eta-1} \quad (29)$$

$$(1+\tau_t^{ss})W_{g,t} = (1-\mu)(1-\alpha_p-\alpha_g)A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g}[\mu L_{p,t}^{\eta}+(1-\mu)L_{g,t}^{\eta}]^{\frac{(1-\alpha_p-\alpha_g-\eta)}{\eta}} L_{g,t}^{\eta-1}$$
$$(30)$$

$$R_t = \alpha_p A_t K_{p,t-1}^{\alpha_p-1} K_{g,t-1}^{\alpha_g}[\mu L_{p,t}^{\eta} + (1-\mu)L_{g,t}^{\eta}]^{\frac{(1-\alpha_p-\alpha_g)}{\eta}}$$

$$R_{g,t} = \alpha_g A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g-1}[\mu L_{p,t}^{\eta} + (1-\mu)L_{g,t}^{\eta}]^{\frac{(1-\alpha_p-\alpha_g)}{\eta}}$$

From the above equations we can obtain all income shares as:

$$(1+\tau_t^{ss})W_{p,t}L_{p,t} = \mu(1-\alpha_p-\alpha_g)A_tK_{p,t-1}^{\alpha_p}K_{g,t-1}^{\alpha_g}[\mu L_{p,t}^{\eta}+(1-\mu)L_{g,t}^{\eta}]^{\frac{(1-\alpha_p-\alpha_g-\eta)}{\eta}}L_{p,t}^{\eta} \quad (31)$$

$$= \frac{\mu(1-\alpha_p-\alpha_g)L_{p,t}^{\eta}}{\mu L_{p,t}^{\eta}+(1-\mu)L_{g,t}^{\eta}}Y_t \quad (32)$$

$$(1+\tau_t^{ss})W_{g,t}L_{g,t} = (1-\mu)(1-\alpha_p-\alpha_g)A_tK_{p,t-1}^{\alpha_p}K_{g,t-1}^{\alpha_g}[\mu L_{p,t}^{\eta}+(1-\mu)L_{g,t}^{\eta}]^{\frac{(1-\alpha_p-\alpha_g-\eta)}{\eta}}L_{g,t}^{\eta}$$

$$= \frac{(1-\mu)(1-\alpha_p-\alpha_g)L_{g,t}^{\eta}}{\mu L_{p,t}^{\eta}+(1-\mu)L_{g,t}^{\eta}}Y_t$$

$$R_tK_{p,t-1} = \alpha_pY_t$$

and

$$R_{g,t}K_{g,t-1} = \alpha_gY_t$$

Profits are zero because of the homogeneity of the production function:

$$\bar{\Pi}_t = Y_t - \frac{\mu(1-\alpha_p-\alpha_g)L_{p,t}^{\eta}}{\mu L_{p,t}^{\eta}+(1-\mu)L_{g,t}^{\eta}}Y_t - \frac{(1-\mu)(1-\alpha_p-\alpha_g)L_{p,t}^{\eta}}{\mu L_{p,t}^{\eta}+(1-\mu)L_{g,t}^{\eta}}Y - \alpha_pY_t - \alpha_gY_t,$$

$$\bar{\Pi}_t = Y_t\left(1-(1-\alpha_p-\alpha_g)-\alpha_p-\alpha_g\right) = 0$$

If, on the contrary, the government pays public factor through taxes, then there are positive profits.

Division of equation (29) by (30) yields equation (27) of section 3.

Appendix A.3: Equilibrium conditions and definition

The collection of the model's first order conditions, market clearing and resource constraints are:

$$\gamma \frac{1}{C_{p,t} + \pi C_{g,t}} - \lambda_t(1 + \tau_t^c) = 0 \tag{33}$$

$$-(1 - \gamma)\frac{1}{N_t\overline{H} - L_{p,t} - L_{g,t}} + \lambda_t(1 - \tau_t^l)W_{p,t} = 0 \tag{34}$$

$$\beta \left[\lambda_{t+1} \left(1 + (1 - \tau_{t+1}^k)(R_{t+1} - \delta_{Kp}) \right) \right] - \lambda_t = 0 \tag{35}$$

$$\lambda_{t-1} - \beta\lambda_t(1 + (1 - \tau_t^k)R_t^B) = 0 \tag{36}$$

$$L_t - L_{p,t} - L_{g,t} = 0 \tag{37}$$

$$Y_t - A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta]^{\frac{(1-\alpha_p-\alpha_g)}{\eta}} = 0 \tag{38}$$

$$R_t - \alpha_p A_t K_{p,t-1}^{\alpha_p-1} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta]^{\frac{(1-\alpha_p-\alpha_g)}{\eta}} = 0 \tag{39}$$

$$(1 + \tau_t^{ss})W_{p,t} -$$
$$\mu(1 - \alpha_p - \alpha_g)A_t K_{p,t-1}^{\alpha_p} K_{g,t-1}^{\alpha_g} [\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta]^{\frac{(1-\alpha_p-\alpha_g-\eta)}{\eta}} L_{p,t}^{\eta-1} = 0 \tag{40}$$

$$\Pi_t - \left[\alpha_g + \frac{(1 - \mu)(1 - \alpha_p - \alpha_g)L_{g,t}^\eta}{[\mu L_{p,t}^\eta + (1 - \mu)L_{g,t}^\eta]} \right] Y_t = 0 \tag{41}$$

$$K_{p,t} - ((1 - \delta_{Kp})K_{p,t-1} + I_{p,t}) = 0 \tag{42}$$

$$K_{g,t} - ((1 - \delta_{K_g})K_{g,t-1} + I_{g,t}) = 0 \tag{43}$$

$$G_t - (C_{g,t} + (1 + \tau_t^{ss})W_{g,t}L_{g,t} + I_{g,t} + Z_t) = 0 \tag{44}$$

$$C_{g,t} - \theta_1 G_t = 0 \tag{45}$$

$$I_{g,t} - \theta_2 G_t = 0 \tag{46}$$

$$(1 + \tau_t^{ss})W_{g,t}L_{g,t} - \theta_3 G_t = 0 \tag{47}$$

$$Z_t - \theta_4 G_t = 0 \tag{48}$$

$$W_{g,t} - \left(\frac{\omega}{1-\omega}\right)^{-1/2\theta} \left[\frac{\theta_3 G_t}{(1+\tau_t^{ss})}\right]^{1/2} = 0 \tag{49}$$

$$L_{g,t} - \left(\frac{\omega}{1-\omega}\right)^{1/2\theta} \left[\frac{\theta_3 G_t}{(1+\tau_t^{ss})}\right]^{1/2} = 0 \tag{50}$$

$$T_t - \left(\begin{array}{c} \tau_t^c C_{p,t} + \tau_t^l(W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau_t^k(R_t - \delta_{K_p})K_{p,t-1} \\ +\tau_t^{ss}(W_{p,t}L_{p,t} + W_{g,t}L_{g,t}) + \tau_t^k R_t^B B_t + \tau_t^\pi \Pi_t \end{array}\right) = 0 \tag{51}$$

$$G_t + (1 + R_t^B)B_t - (T_t + B_{t+1}) = 0 \tag{52}$$

This set of conditions fully characterizes a unique solution for any given policy vector. The complete set of equations of the model is completed with the budget constraint of the consumer and the following transversality conditions:

$$\lim_{t\to\infty} \beta^t \lambda_t K_{t+1} = 0$$

$$\lim_{t\to\infty} \frac{1}{(1+R)^t} B_t = 0$$

Definition of equilibrium: An equilibrium for this economy is a vector of prices (W_g, W_p, R), a vector of input quantities (L_g, L_p, K_g, K_p), and a vector of private consumption and investment (C_p, I_p) such that for a given fiscal policy summarized by a collection of taxes $(\tau_c, \tau_l, \tau_k, \tau_{ss}, \tau_\pi)$ and expenditure proportions $(\theta_1, \theta_2, \theta_3, \theta_4)$, induce a vector of public consumption, investment, transfers, and debt services (C_g, I_p, Z, RB), such that the optimization problems of the household, the firm, and the government are satisfied in a way that the resources constraints are satisfied and all markets clear.

Appendix B: Sensitivity Analysis

The results shown in the paper relate interest rates to the ratios G_t/Y_t and B_t/Y_t. We have seen during the crisis enormous variations in the yields that the Greek

bond had to pay to be attractive in the markets. In the calibration period 2002-2006, we observe a steady relation in the ratio $G/Y \simeq 0.45$, and $B/Y \simeq 100\%$. The implication is that when the yield of the bond increases by a factor of four, the expenditure made by the government in any other area has to decrease by a similar amount, and we know how extremely difficult this is. The result is that an enormous jump in the debt frontier has to take place.

In this appendix we analyze the sensitivity of the model to changes in some key parameters. Table B.1 shows the percentage change in the relevant variables given an increase of 10% in the parameters of the first row. Several interesting results emerge from this sensitivity analysis. Overall, this exercise shows the robustness of the model. As expected, a rise in Total Factor Productivity increases output, consumption and investment in the same amount. Additionally, the sustainability debt level increases by 16.5%, showing that public debt sustainability is also very sensible to productivity shocks.

An increase in taxes has a negative impact on all macroeconomic variables but on the sustainability debt level. From our model specification, a higher level of public revenues, given a particular government size, allows to cover a higher amount of debt services. The higher impact came from the labor income tax and consumption tax. Also note that the public debt interest rate is affected by the change in the capital income tax rate, increasing the cost of borrowing and partially compensating the positive effect on the creditworthiness of the Greek debt.

Also of interest is the reaction of our model economy to changes in total government spending composition. A rise in the proportion of public consumption (θ_1) does not affect output and investment, reducing private consumption and raising public consumption by the same amount. Nevertheless, this policy change reduces the long-run sustainability debt limit by around 2%. A rise in public investment (θ_2) has a positive impact on all macroeconomic variables, raising the long-run sustainability debt limit by 0.75%. The positive impact of a rise in public wage bill (θ_3) on output, consumption and investment is easily explained, as more public employment is added to the aggregate production function, in spite of a fall in private employment. At the same time, the residual parameter θ_4 is reduced by the same amount. Therefore public accounts remain unchanged while more factors are placed into the production function. The table also shows that a change in the composition of wages and public employment has mild effects on the economy. A reduction in civil servants' compensations increases the debt ceiling by just 1% at the cost of -0.67% decrease in output.

Table B.1: Sensitivity Analysis

$\Delta 10\%$	Y	C_p	C_g	I_p	I_g	B	L_p	L_g	R_B
A	**19.94**	**19.94**	**19.94**	**19.94**	**19.94**	**16.49**	**-1.79**	**9.51**	**0.00**
τ_k	-0.79	-0.47	-0.79	-1.80	-0.79	5.11	-0.06	-0.39	2.56
τ_l	**-2.87**	**-2.87**	**-2.87**	**-2.87**	**-2.87**	**21.33**	**-3.38**	**-1.44**	**0.00**
τ_c	-0.50	-0.50	-0.50	-0.50	-0.50	15.44	-0.60	-0.25	0.00
τ_π	**0.00**	**0.00**	**0.00**	**0.00**	**0.00**	**9.77**	**0.00**	**0.00**	**0.00**
δ_{K_p}	-3.38	-4.53	-3.38	0.28	-3.38	-7.08	0.85	-1.70	0.00
δ_{K_g}	**-1.66**	**-1.66**	**-1.66**	**-1.66**	**-1.66**	**-1.41**	**0.15**	**-0.83**	**0.00**
ω	-0.67	-0.67	-0.67	-0.67	-0.67	0.99	1.02	-5.34	0.00
θ_1	**0.00**	**-1.29**	**10.00**	**0.00**	**0.00**	**-2.08**	**0.00**	**0.00**	**0.00**
θ_2	1.89	1.33	1.89	1.89	12.08	0.75	0.06	0.94	0.00
θ_3	**0.62**	**0.62**	**0.62**	**0.62**	**0.62**	**12.41**	**-0.98**	**5.21**	**0.00**

We complete our sensitivity analysis with a variation of the yield. Figure 5 shows how the frontier moves inwards as a consequence of an increase of 25% in the yield of the Greek bond. Notice that the effective spreads of the Greek bond with respect to the German Bund were much larger. Since the very beginning of the negotiations of the details of the rescue package for Greece by April 2010, the spreads skyrocketed due to a number of reasons. One of those reasons is discussed in Chamley and Pinto (2011). They argue that the seniority of the new bonds issued to finance the rescue program would disincentivize other private investors from buying Greek bonds. However, we agree with Arellano, Conesa and Kehoe (2012) in saying that the rescue package was an effective mechanism to provide liquidity to the Greek State at a controlled yield.

[Insert here Figure 5]

Figure 5 shows that the fiscal ratios displayed by the Greek economy prior to the crisis were sustainable at the yield of 5%, that is, the real return of the rescue package bond was consistent with a long-term sustainability of the Greek State prior to the unfolding of events that drove Greece to the current crisis.

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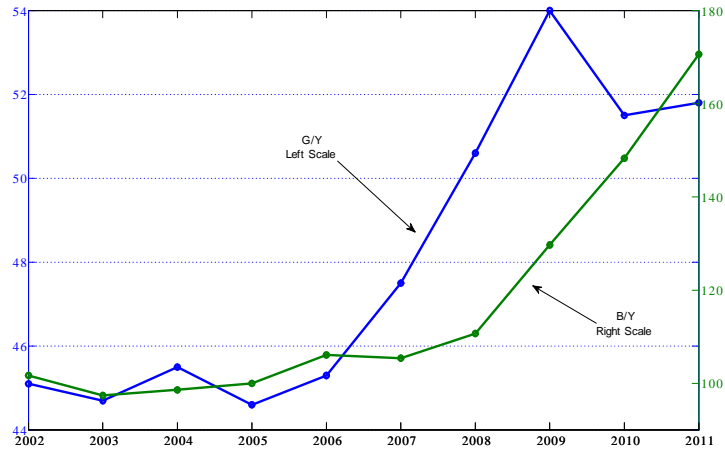
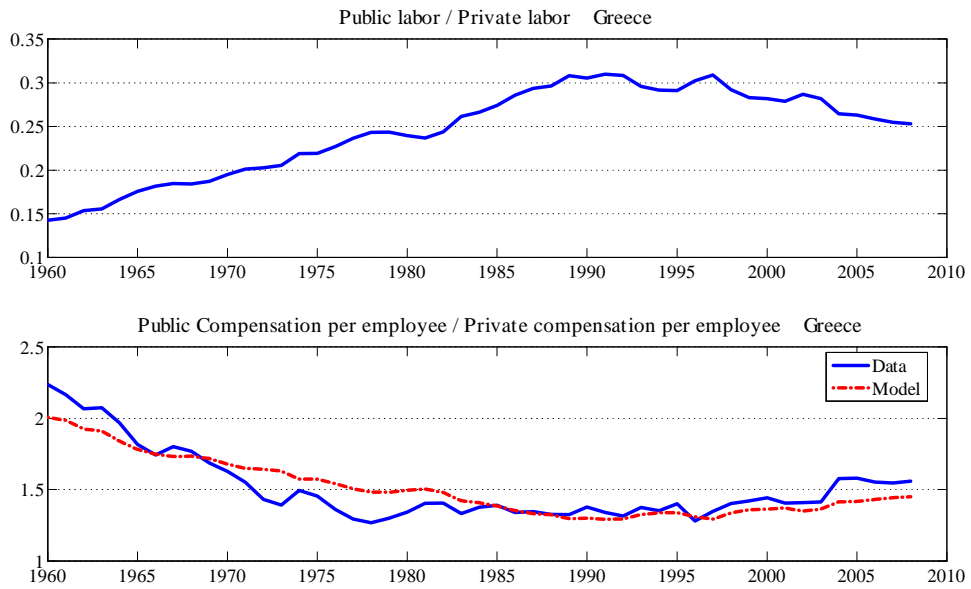Figure 1: Total public spending and public debt as percentage of GDP.
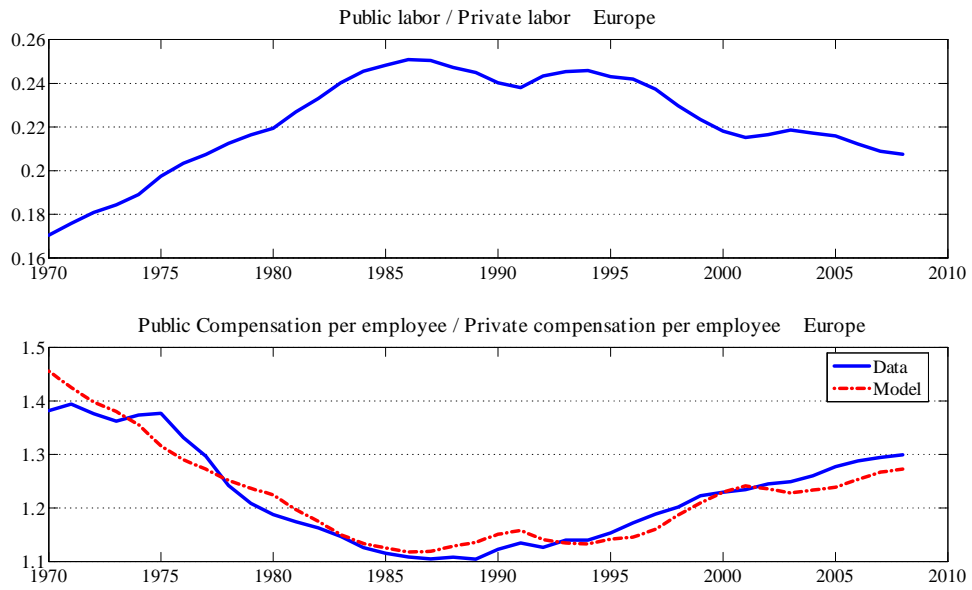

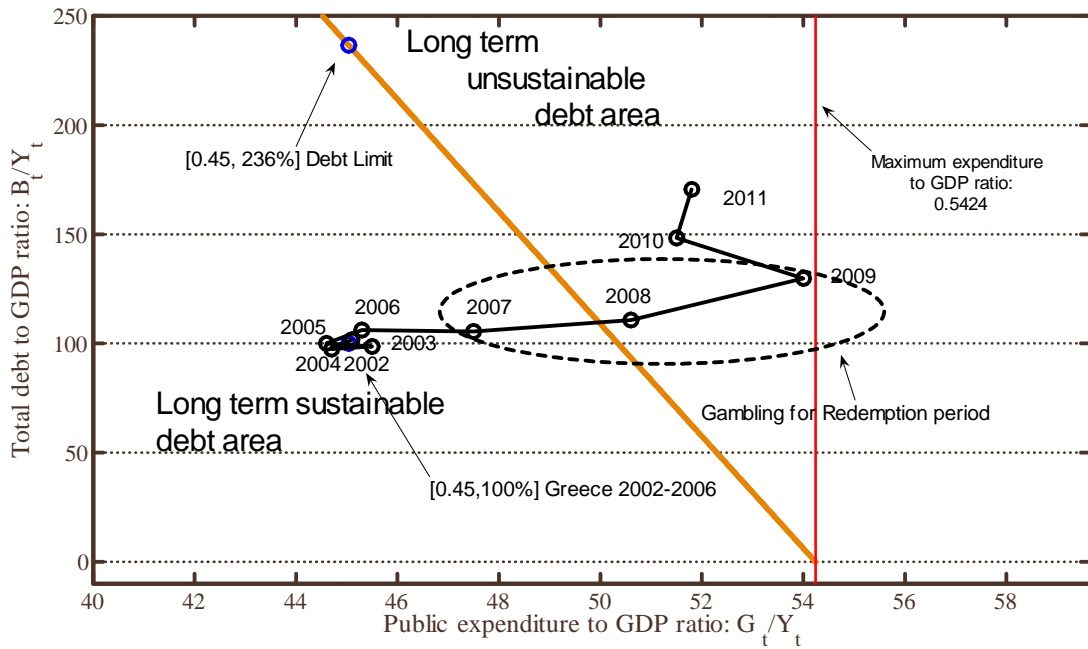Figure 2: Public/Private Labor in Greece

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Figure 3: Public/Private Labor in the Europe

Figure 4: The Greek Debt Frontier

Long term
unsustainable
debt area

r = 0.05

r = 0.07

r = 0.04

[0.45,100%] Greece 2002-2006

Long term sustainable
debt area

Total debt to GDP ratio: B_t/Y_t

Public expenditure to GDP ratio: G_t/Y_t