A DISCRETE-TIME RETRIAL QUEUEING SYSTEM WITH SERVICE UPGRADE

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KEYWORDS

Discrete-time model; Retrials, Markov chain; Sojourn time; Stochastic decomposition.

ABSTRACT

This paper considers a discrete-time retrial queueing system with displacements. The arriving customers can opt to go directly to the server obtaining immediately its service or to join the orbit. In the first case, the displaced customer goes directly to the orbit. The arrivals follow a geometrical law and the service times are general.

We study the Markov chain underlying the considered queueing system obtaining the generating function of the number of customers in the orbit and in the system as well as the stationary distribution of the time that a customer spends in the server. We derive the stochastic decomposition law and as an application we give bounds for the proximity between the steady-state distributions for our queueing system and its corresponding standard system.

INTRODUCTION

Queueing systems with retrial queues have been widely used to model many practical problems in telephone switching systems, telecommunication networks and computers competing to gain service from a central processing unit. For a detailed review of the main results and the literature on this topic the reader is referred to (Artalejo and Gómez-Corral 2008). In the context of continuous time systems discrete time Markov chains play an important role. Defined as embedded chains they provide us with techniques to deal with continuous time single servers or even networks when the system is observed only at specific time points, e.g., arrival or departure instances. For an introduction see Kleinrock's monograph, Volume I, Part III (Kleinrock 1975), and for the application of these methods in performance analysis of computer and communication systems (Kleinrock 1976).

Performance prediction in communication switching queues, job processing in computers, etc., are always influenced by customers behavior, and the provision of this additional information will be useful in upgrading the service. This mechanism is called a synchronized or triggered motion (e.g., Artalejo, 2000; Gelenbe and Label, 1998). For the inverse order discipline, we refer to (Pechinkin and Svischeva 2004; Pechinkin and Shorgin 2008) as well as (Cascone et al. 2011).

Let us note that in our work is considered, with a certain probability, a LCFS discipline, that is, a customer can obtain service immediately after his arrival displacing the customer that is in the server.

Works related to discrete-time systems with server interruptions with or without expulsions and vacations can be found, including those by (Fiems et al. 2002; 2004; Vinck and Bruneel 2006; Morozov et al. 2011) as well as (Atencia and Pechinkin 2013; Atencia et al. 2013a; 2013b; Atencia 2014).

In the early times of queueing theoretical applications in telecommunication systems networks of transmission lines were modeled using brute force decomposition approximations which are still in use and are considered as unavoidable in many situations. First steps to overcome the restriction to decomposition approximations were done around 1950 by coupling classical exponential queues in lines. These were used to model jobshop like production systems, transportation lines, and complex distribution lines with inventories.

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