

# Workshop on numerical approximations of PDEs. Honoring the 60th birthday of Frédéric Hecht. Málaga, April 20-22, 2015

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*"Reduction of complexity and variational data assimilation"*

Reduced basis methods belong to a class of approaches of *\emph{model reduction}* for the approximation of the solution of mathematical models involved in many fields of research or decision making and in data assimilation. These approaches allow to tackle, in --- close to --- real time, problems requiring, a priori, a large number of computations by formalizing two steps : one known as "offline stage" that is a preparation step and is quite costly and an "online stage" that is used on demand and is very cheap.

The strategy uses the fact that the solutions we are interested in belong to a family, a manifold, parametrized by input coefficients, shapes or stochastic data, that has a small complexity. The complexity is measured in terms of a quantity like the "Kolmogorov width" that, when it is small, formalizes the fact that some small dimensional vectorial spaces allow to provide a good approximation of the elements on the manifold.

We shall make a review of the fundamental background and state some results proving that such a dimension is small for a large class of problems of interest, then use this fact to propose approximation strategies in various cases depending on the knowledge we have of the solution we want to approximate : either explicit through values at points, or through outputs evaluated from the solution, or implicit through the Partial Differential Equation it satisfies. We shall also present a strategy available when a mixed of the above informations is available allowing to propose new efficient approaches in data assimilation and data mining.

The theory on the numerical analysis (a priori and a posteriori) of these approaches will also be presented together with results on numerical simulations.

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