# The dynamics of user channels in massive MIMO systems

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***Abstract*—The SNR associated with the different users in large-scale MIMO systems depends on the magnitude of the eigenvalues of the channel power matrix $HH^{\mathcal{H}}$. While it is known that the spread between the best and worst channels is reduced when the number of antennas $N$ at the BS grows, there is little known about how these channels change due to the user mobility. Do all parallel channels change at the same rate, or conversely is their dynamic behavior different for the best and worst channels? We evaluate the interplay between the number of BS antennas $N$ and the number of (single-antenna) users $K$ in MIMO systems, and investigate the effect of letting $N$ grow on the dynamics of the best and worst channels in this multiuser set-up.**

## I. Introduction

In conventional MIMO systems the channel is described as a random matrix $\mathbf{H}$ whose size is determined by the number of transmit and receive antennas. This technique has been incorporated in current wireless communication standards such as LTE or Wi-Fi as a means of increasing the system capacity.

In order to boost the performance of cellular networks to a much higher level, a new variation of MIMO systems is being considered [1]: in massive MIMO systems, a cellular base station (BS) with $N$ antennas serves $K$ single-antenna user terminals over the same time-frequency interval. If the number of BS antennas is much larger than the number of users, the spectral efficiency is dramatically increased.

Just like in conventional MIMO systems, the SNR per parallel channel depends on the magnitude of the eigenvalues of the matrix $\mathbf{W} \triangleq \mathbf{H}\mathbf{H}^{\mathcal{H}}$; for this reason, the distribution of these eigenvalues is a well-studied subject [2, 3].

However, wireless communication systems are in general non-static and hence the stochastic process associated with $\mathbf{H}$ exhibits a variation along different dimensions due to mobility of users or objects in the propagation environment.

The random process associated with the largest eigenvalue of $\mathbf{W}$ was recently studied in [4], and a similar result can be obtained for the smallest eigenvalue. Since the expressions for the joint cdfs obtained in [4] are analytically tractable, they can be evaluated even for large numbers of BS antennas. Here, we use this result to study whether the best and worst eigen-channels have a similar behavior in non-static environments[1].

## II. System Model

Let us consider a $N \times K$ MIMO system with $N > K$, where $\mathbf{H} \in \mathbb{C}^{K\times N}$ represents the Rayleigh fading channel matrix with i.i.d. entries $\sim \mathcal{CN}\left(0, \sigma^2\right)$. Then, $\mathbf{W} \in \mathbb{C}^{K\times K}$ follows a complex central Wishart distribution, i.e. $\mathbf{W} \sim \mathcal{CW}\left(t, \sigma^2\mathbf{I}_K, \mathbf{0}_K\right)$, where $\mathbf{I}_K$ and $\mathbf{0}_K$ are the identity and the null $K \times K$ matrices, respectively.

We consider two realizations of the random process $\mathbf{W}$ at two different instants, i.e. $\mathbf{W}(t) \triangleq \mathbf{W}_1$ and $\mathbf{W}(t+\tau) \triangleq \mathbf{W}_2$. The diagonal matrices formed by the ordered eigenvalues of $\mathbf{W}_1$ and $\mathbf{W}_2$ are then given by $\mathbf{\Lambda} \triangleq \text{diag}\{\lambda_1, ..., \lambda_K\}$ and $\mathbf{\Phi} \triangleq \text{diag}\{\varphi_1, ..., \varphi_K\}$, where $\lambda_n$ and $\varphi_n$ represent the $n^{th}$ eigenvalue of the $\mathbf{W}_1$ and $\mathbf{W}_2$ matrices, respectively.

The correlation between the underlying Gaussian processes $\mathbf{H}(t) \triangleq \mathbf{H}_1$ and $\mathbf{H}(t+\tau) \triangleq \mathbf{H}_2$ corresponding to the two realizations of the channel matrix can be modelled as

$$\mathbf{H}_2 = \rho\mathbf{H}_1 + \sqrt{1-\rho^2}\mathbf{\Xi}, \quad (1)$$

where $\rho$ is the correlation coefficient between the $\{i, j\}$ entries of $\mathbf{H}_1$ and $\mathbf{H}_2$, and $\mathbf{\Xi}$ is an auxiliary $K \times N$ matrix with i.i.d entries $\sim \mathcal{CN}\left(0, \sigma^2\right)$, which is independent of $\mathbf{H}_1$.

In [4], the joint cdf $F_{\lambda_1,\varphi_1}(\lambda, \varphi) \triangleq \Pr\{\lambda_1 < \lambda, \varphi_1 < \varphi\}$ and the joint cdf $F_{\lambda_K,\varphi_K}(\lambda, \varphi) \triangleq \Pr\{\lambda_K < \lambda, \varphi_K < \varphi\}$ that characterize the random processes $\lambda_1(t)$ and $\lambda_K(t)$ were derived. Now, we use these results to study the dynamic behavior of these parallel channels in terms of $N$ and $K$.

## III. Performance metric

The joint distributions characterized in [4] incorporate the dynamics of the CW random process through the correlation coefficient $\rho$ of the underlying Gaussian channel matrix, according to (1). However, the relation between $\rho$ and the correlation coefficient $\rho_i$ of each one of the ordered eigenvalues of the CW matrix is not fully understood. Analytical results for this correlation coefficient are hard to obtain, as they require a two-fold numerical integration over the joint distribution of the eigenvalue of interest [4, 6, 7].

Observing the influence of $\rho$ in the joint distributions, we see that if $\rho = 1$ then the two samples of the random process are identical, i.e. $F_{\lambda,\phi}(\gamma, \gamma)|_{\rho=1} = F_\lambda(\gamma)$. On the contrary if $\rho = 0$ then the two samples are independent and hence $F_{\lambda,\phi}(\gamma, \gamma)|_{\rho=0} = F_\lambda(\gamma, \gamma)^2$. Thus, when the joint distributions are evaluated in $\gamma$, the bivariate cdf takes values in the range $[F_\lambda(\gamma)^2, F_\lambda(\gamma)]$.

Using some adequate normalization factors, we define the *outage correlation coefficient* (OCC) $\rho_o(\gamma, \rho)$ as

$$\rho_o(\gamma, \rho) \triangleq \frac{F_{\lambda,\phi}(\gamma, \gamma)|_\rho - F_\lambda(\gamma)^2}{F_\lambda(\gamma)\left(1 - F_\lambda(\gamma)\right)}. \quad (2)$$

The OCC provides a similar information than the bivariate cdf, while having some practical advantages that are easy to show: It has dimensions of correlation coefficient, i.e.

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[1] We must note that the results here derived correspond to a Gaussian channel matrix with i.i.d. entries. Even though this assumption does not hold when the number of transmit antennas is very large, it is usually considered as a reference case and in some scenarios it is a reasonably good approximation for the massive linear array case [5].

$\rho_o(\gamma,\rho) \in [0,1] \quad \forall\gamma,\rho$; it has the same zeros as $\rho$, i.e. $\rho_o(\gamma,0)=0 \quad \forall\gamma$, and if $\rho=1$, then $\rho_o(\gamma,1)=1, \forall\gamma$.

## IV. Numerical results and discussion

We are interested in understanding how the dynamics of MIMO parallel channels are affected by $N$ and $K$. In Fig. 1, we represent the outage correlation coefficient $\rho_o(\gamma,T)$ as a function the product $T=f_d\cdot\tau$ for different numbers of BS antennas $N$ and considering $K=2$, where $f_d$ is the Dopler frequency. This case is very simple, as it considers only two channels; however, it will prove to be very insightful to study the impact of using more BS antennas in the dynamics of MIMO parallel channels. We assume a value of $\gamma$ that yields a outage probability of $10^{-2}$, and a correlation profile according to Clarke's model, i.e. $\rho=J_0(2\pi f_d\tau)$.

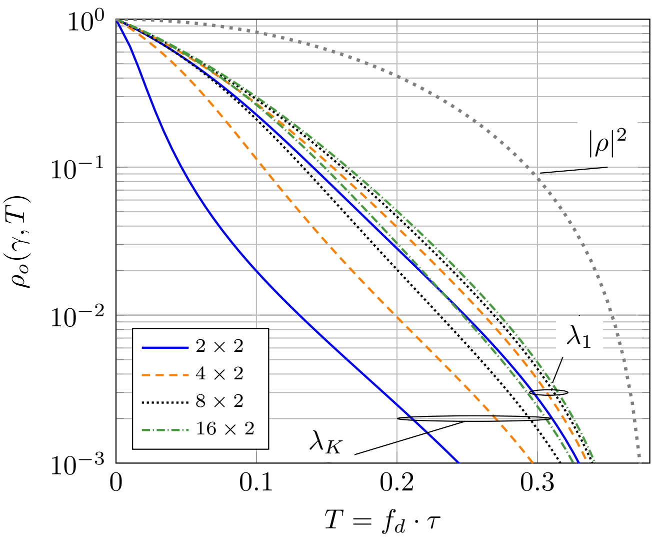


Fig. 1. OCC vs $f_d\cdot\tau$ for different numbers of BS antennas and $K=2$.

We see how the OCC of the best eigenchannel is barely affected by using more BS antennas; in fact, the value of $T$ that achieves a OCC $\rho_o=0.1$ is in the range $\approx[0.14-0.16]$ for the investigated configurations, which corresponds to $|\rho|^2\approx 0.5$. Conversely, we observe how the dynamic behavior of the worst eigenchannel is dramatically affected by the number of BS antennas. In this case, the value $\rho_o=0.1$ is attained for a wider set of values of $T$, i.e. $T\approx[0.05-0.15]$. Hence, this indicates that the worst channel decorrelates faster as $N$ is reduced. Indeed, the best eigenchannel takes longer to decorrelate as $N$ grows, but this difference is comparatively smaller.

Interestingly, the worst channel rapidly tends to exhibit a similar dynamic behavior than the best eigenchannel as $N/K$ is increased. In fact, we observe how the best channel in the $2\times2$ case and the worst channel in the $8\times2$ case have similar OCC. When 16 BS antennas are used, the gap between the best and worst channels is small, and the assumption that both channels present a similar dynamic behavior is reasonable.

Fig. 2 shows the OCC when considering $K=4$, and the same set of parameters as in the previous figure. Now, we observe that the dynamics of the best channel are even more stable, as the OCC is approximately constant with $N$. On the other hand, we see how increasing the number of BS antennas and users to 4 causes the worst channel to have a much faster rate of change. As the number of BS antennas is increased, we observe again how the worst channel tends to become more stable. We have observed that when the number of BS antennas and the number of users is similar, the worst channel has a much faster variation than the best channel. While the dynamics of the latter are barely affected by using more BS antennas, we notice that the worst channel tends to have a more stable behavior as $N$ is increased. One of the conclusions extracted in [5] stated that for $N\sim10\cdot K$, the spread between the best and worst channels is reduced and a stable performance can be ensured even in non favorable propagation conditions. Here, our results suggest that this performance can be also sustained in time with a similar behavior for all $K$ users, i.e. user channel variation in massive MIMO systems seems similar for users with the same mobility.

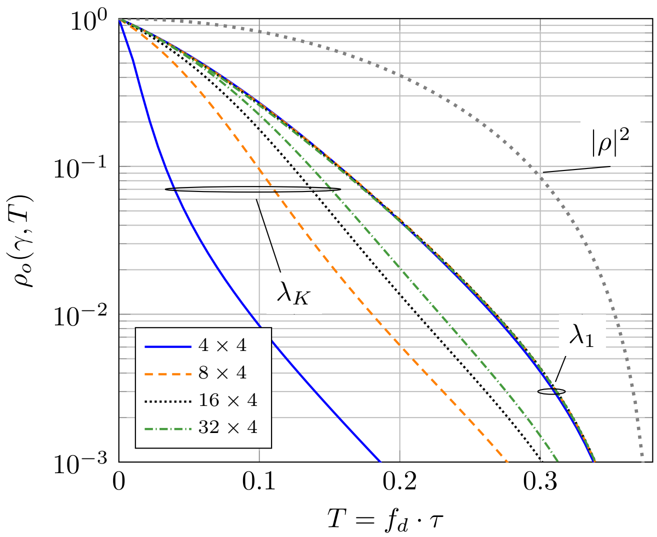


Fig. 2. OCC vs $f_d\cdot\tau$ for different numbers of BS antennas and $K=4$.

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