Reasoning in Interval Temporal Logics - New Frontiers

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- A linguistic issue. Logical formalisms have always featured in the study of natural languages; they arise as suitable frameworks for modeling progressive tenses and expressing language constructions involving both time points and periods.
- An Artificial Intelligence/Computer Science issue. Temporal languages and logics have sprung up from expert systems, planning systems, theories of actions and change, natural language analysis and processing, formal verification systems, among others.

Representing Time: some Questions

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  - Discrete or dense?
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- Should time (representation) be:
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- If we choose to represent time as made of intervals, instead of points, then:
  - Should intervals include their end-points or not?
  - Can they be unbounded?
  - Are point-intervals (i.e. with coinciding endpoints) admissible or not?
  - How are points and intervals related?

Temporal logics: Points

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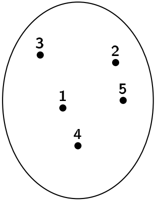
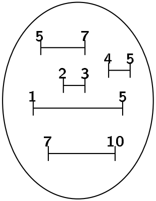


set of worlds
primitive temporal entity
time points/instants

accessibility relations
$\longrightarrow$: next
$\longrightarrow^{\star}$: finally

Temporal Logics: Intervals

- worlds are intervals (time period — pairs of points)



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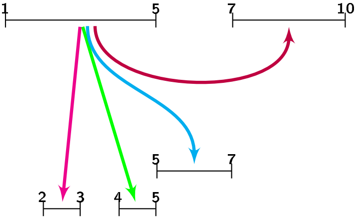
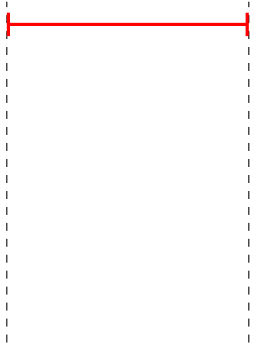


accessibility relations
all binary relations between pairs of intervals

Allen's relations: Algebra and Logic

There are 13 different binary relations between intervals:



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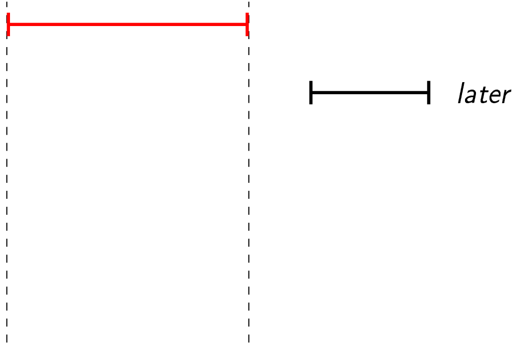

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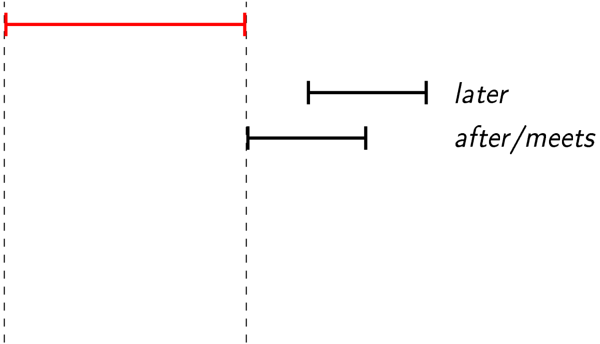

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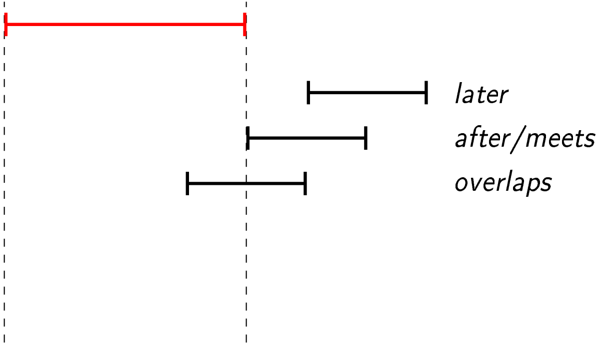

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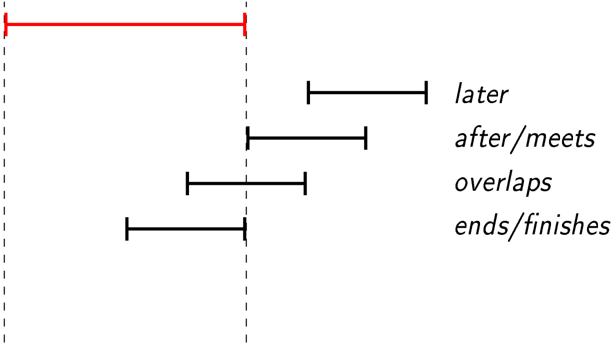

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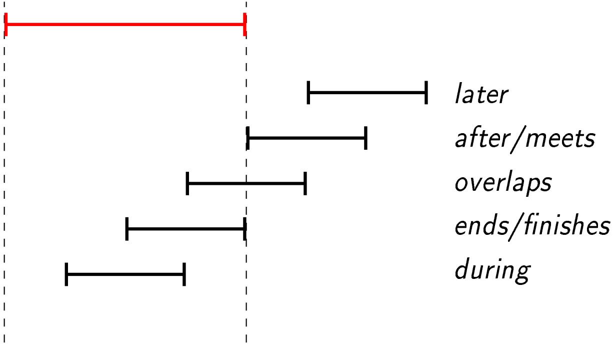

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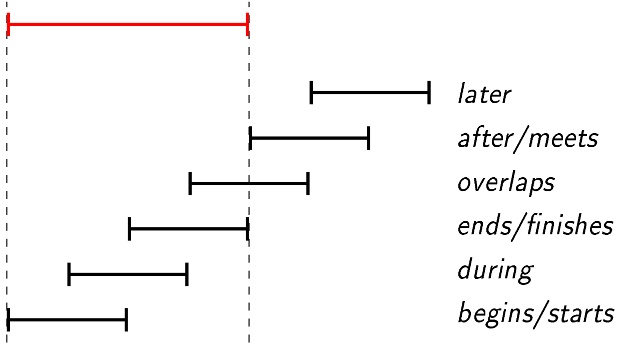

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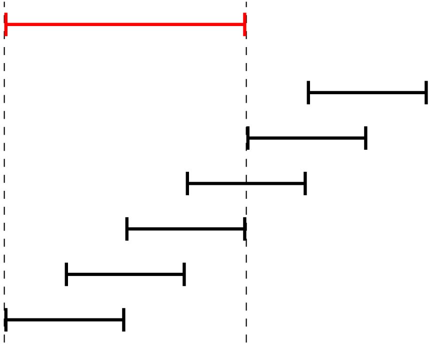

later	$\langle L \rangle$
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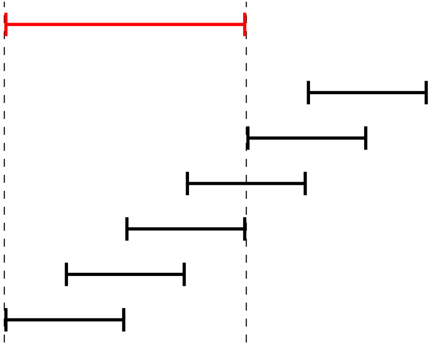

later	$\langle L \rangle$	$\langle \overline{L} \rangle$
after/meets	$\langle A \rangle$	$\langle \overline{A} \rangle$
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together with their inverses.

Setting a Language: Halpern-Shoham's Modal Logic of Time Intervals

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Halpern and Shoham's logic modal logic of time intervals HS:

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \psi \mid \langle \mathsf{B} \rangle\varphi \mid \langle \mathsf{E} \rangle\varphi \mid \langle \overline{\mathsf{B}} \rangle\varphi \mid \langle \overline{\mathsf{E}} \rangle\varphi \mid \langle \mathsf{A} \rangle\varphi \mid \langle \overline{\mathsf{A}} \rangle\varphi.$$

Interpreted on Interval models

$$\mathbf{M} = \langle \mathbb{I}(\mathbb{D}), V \rangle,$$

where $V : \mathcal{AP} \mapsto 2^{\mathbb{I}(\mathbb{D})}$ is the valuation function.

Formal semantics of HS

$\langle B \rangle$: $\mathbf{M}, [d_0, d_1] \Vdash \langle B \rangle \phi$ iff there exists d_2 such that $d_0 \leq d_2 < d_1$ and $\mathbf{M}, [d_0, d_2] \Vdash \phi$.

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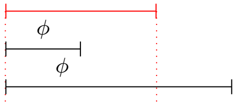

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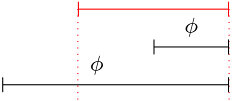

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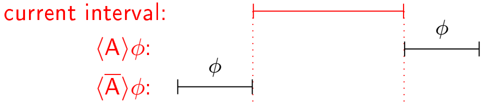

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- The dividing instant dilemma ("if the light is on and it is turned off, what is its state at the instant between the two events?")

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- Temporal databases: offer a logical framework as a basis of a conceptual design.

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$$[G](A \rightarrow (\langle\overline{\mathrm{O}}\rangle B \wedge [\overline{\mathrm{D}}]\neg C))$$

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- If satisfiability is decidable, then, for example, one can build a plan, or deduce the consequences of a set of assumptions, or answer a temporal query...

Computational Properties of Satisfiability

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- Ontology: point intervals are admitted or not?
- Ontology: is the class of models finite, discrete, dense, based on the reals, based on $\mathbb{N}, \mathbb{Z}, \ldots$?
- Expressive power: which are the allowed modalities?
- Semantical choices: do we admit all intervals built on a linear order?
- Syntactical choices: do we admit propositionally complete formulas?
- Syntactical choices: do we admit every combination of existential and universal modalities?

The Satisfiability Zoo

HS

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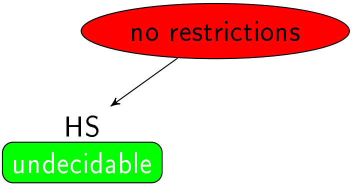

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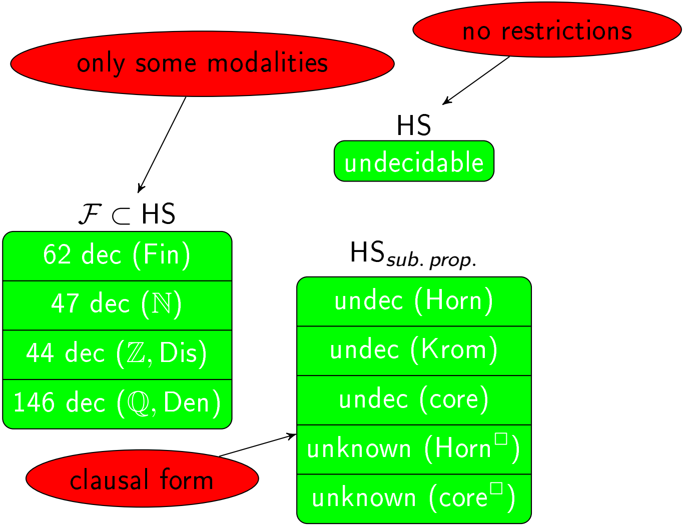

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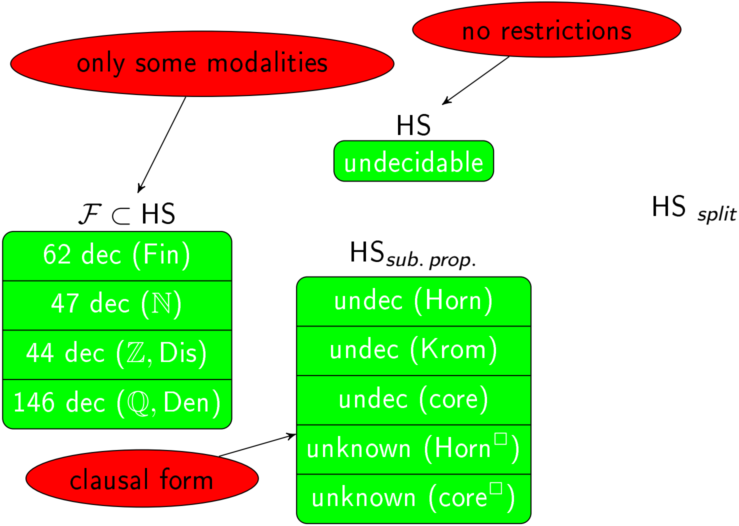

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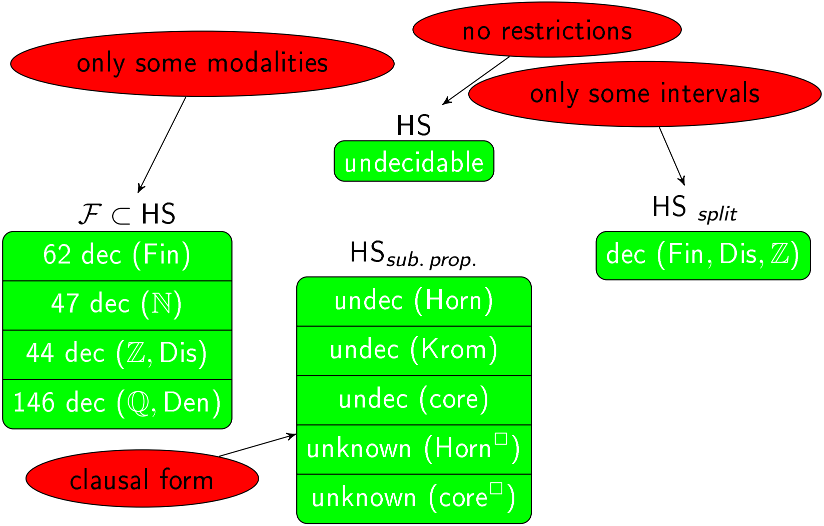

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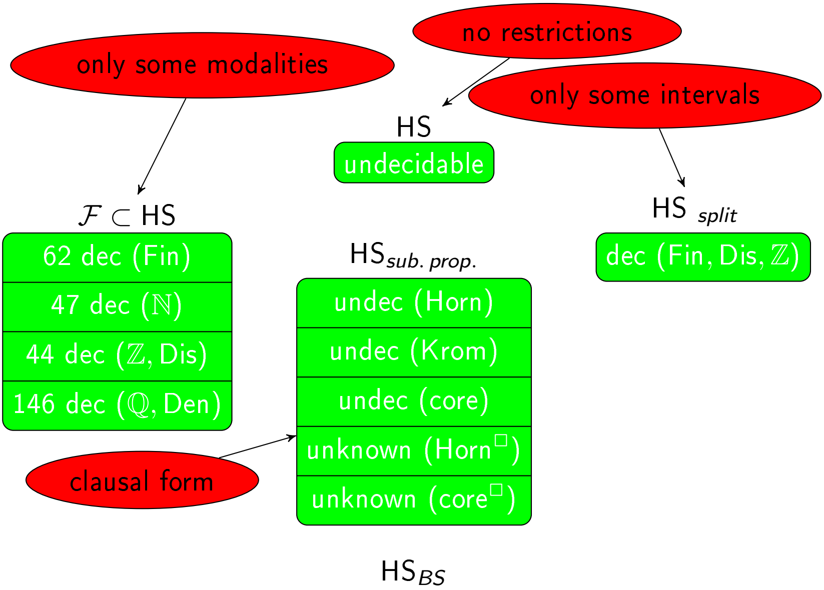

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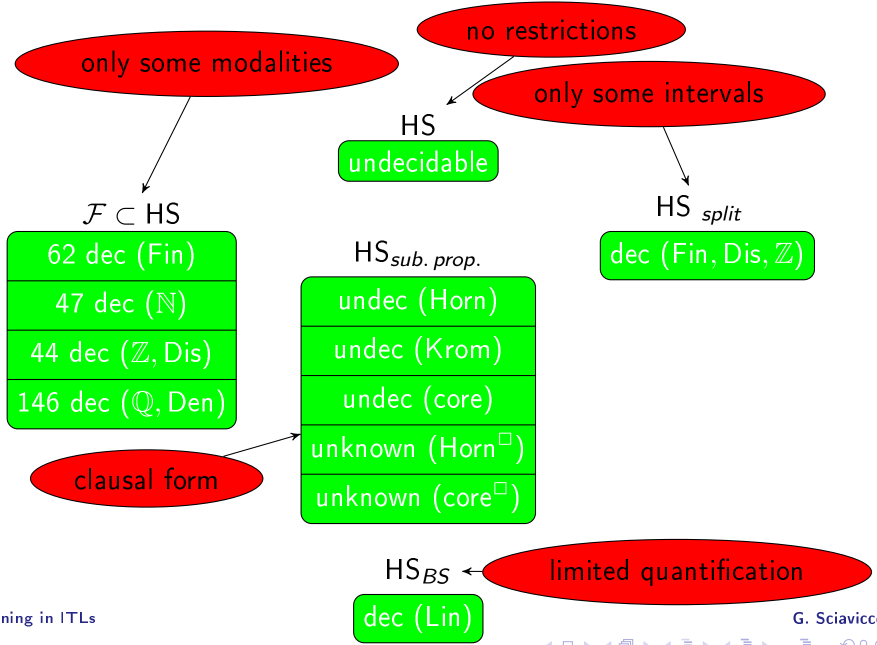
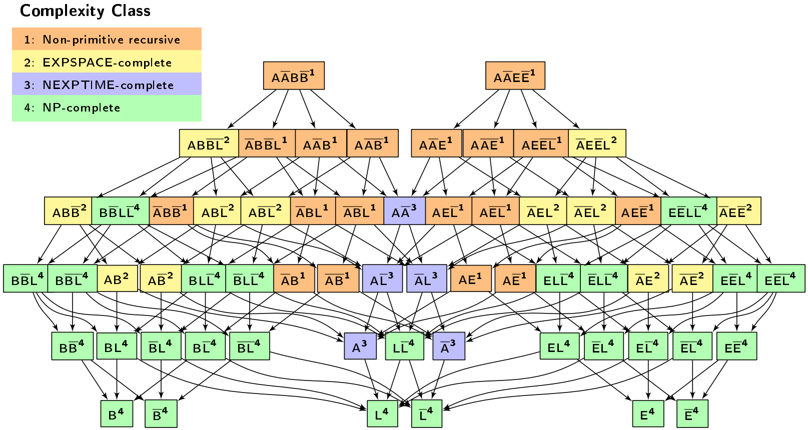


Limiting the modalities

Example: The complete picture (for finite orders)



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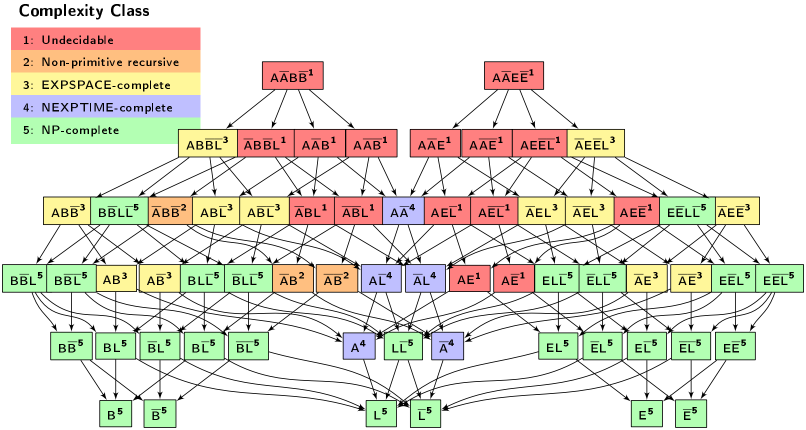

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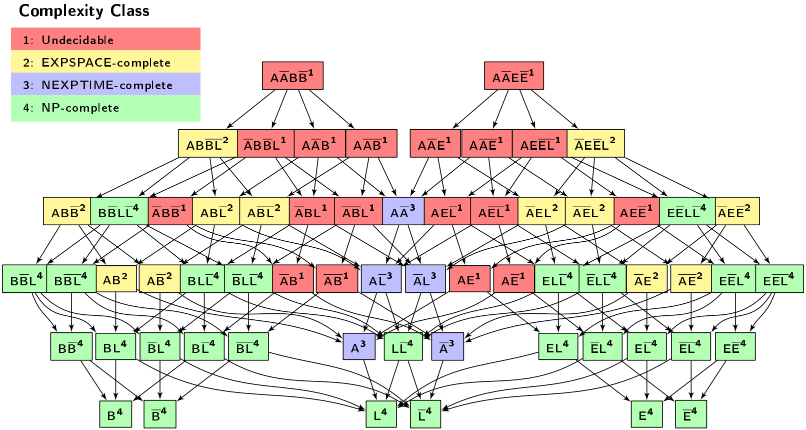

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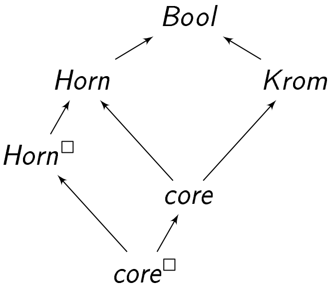
Clausal fragments of HS

Relative expressive power: all classes



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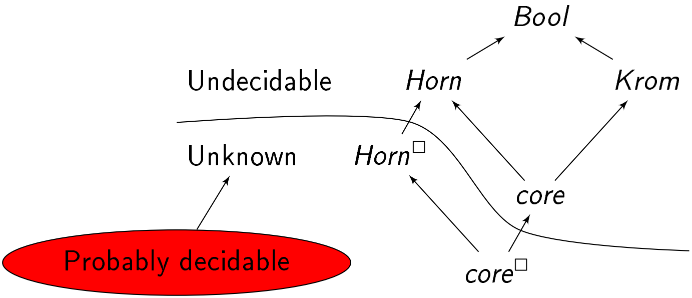

Mixing Fragments with Clausal fragments of HS

Relative expressive power: Fin

$$\begin{tikzcd}
 & & B\overline{B}L\overline{L} & \\
 & B\overline{B}L\overline{L}_{\mathrm{Horn}} \arrow[ur] & & B\overline{B}L\overline{L}_{\mathrm{Krom}} \arrow[ul] \\
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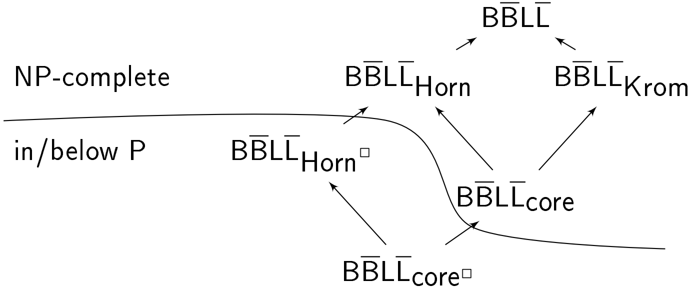
\end{tikzcd}$$

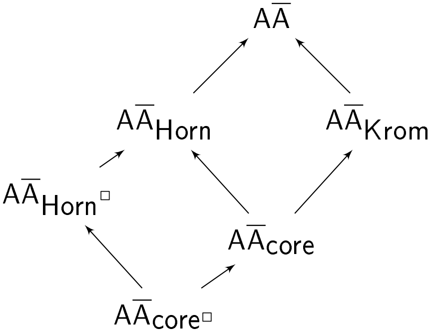
Mixing Fragments with Clausal fragments of HS

Relative expressive power: Fin



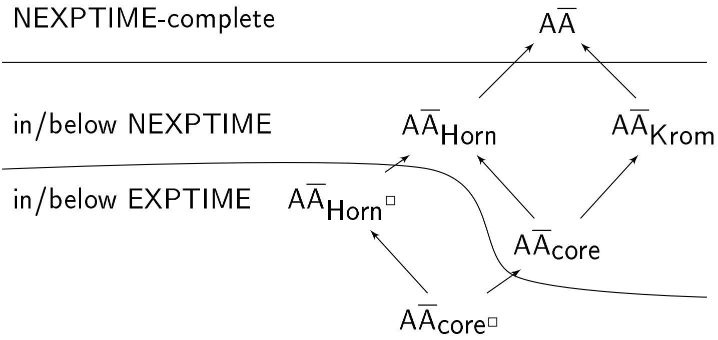
Mixing Fragments with Clausal fragments of HS (Cont.)

Relative expressive power: Fin



Mixing Fragments with Clausal fragments of HS (Cont.)

Relative expressive power: Fin



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