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Reference point approaches in Stochastic Multiobjective Programming

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Mariano Luque
Universidad de Málaga.
Campus de Excelencia Internacional Andalucía Tech

Introduction

Stochastic Multiobjective Programming Problem (SMOP)

$$\underset{\mathbf{x}\in X}{Max} \left(\tilde{z}_1(\mathbf{x},\tilde{\mathbf{c}}), \tilde{z}_2(\mathbf{x},\tilde{\mathbf{c}}), \cdots, \tilde{z}_k(\mathbf{x},\tilde{\mathbf{c}})\right)$$

$\mathbf{x} \in X \subset \mathrm{R}^{\mathrm{n}}$ is the deterministic feasible set

$\tilde{\mathbf{c}} \in E \subset \mathrm{R}^{\mathrm{s}}$ is a continuous random vector

$\tilde{z}_i(\mathbf{x},\tilde{\mathbf{c}})$ $i = 1, 2, \ldots, k$ are the stochastic objective functions.

Introduction

Most widely used criteria for the stochastic - deterministic transformation

- Minimum risk (Stancu-Minasian, 1984):

$$\begin{aligned} Max \quad & \left(P(\tilde{z}_1(\mathbf{x},\tilde{c}) \geq u_1), \cdots, P(\tilde{z}_k(\mathbf{x},\tilde{c}) \geq u_k)\right) \\ \text{s.t.} \quad & \mathbf{x} \in X \end{aligned}$$

$\mathbf{u}-$efficient solution

- Gen. solution with probability β (Kataoka, 1963):

$$\begin{aligned} Max \quad & \left(u_1, u_2, \cdots, u_k\right) \\ \text{s.t.} \quad & \mathbf{x} \in X \\ & P(\tilde{z}_i(\mathbf{x},\tilde{c}) \geq u_i) \geq \beta_i \quad i = 1, \cdots, k \end{aligned}$$

$\boldsymbol{\beta}-$efficient solution

Interactive methods clue for this approach

- Synchronous Approach in Interactive Multiobjective Optimization (Miettinen and Mäkelä, 2006)
- Interactive multiobjective fuzzy random linear programming: Maximization of possibility and probability (Katagiri, Sakawa, Kato and Nishizaki, 2008)
- INTEREST method (Muñoz, Luque and Ruiz, 2010)

Interactive methods clue for this approach

Synchronous Approach in Interactive Multiobjective Optimization (Miettinen and Mäkelä, 2006)

- Linear and nonlinear problems (only multiobjective but not stochastic).
- Interactive Method is based on the Reference Point approach.
- At each iteration, the DM must provide some aspirations levels.
- Using the same preference information (aspiration levels), four solutions are generated by considering four different achievement scalarizing functions.

Interactive methods clue for this approach

Interactive multiobjective fuzzy random linear programming: Maximization of possibility and probability (Katagiri, Sakawa, Kato and Nishizaki, 2008)

- Linear problems with fuzzy random variables in coefficients.
- Interactive Method is based on the Reference Point approach.
- Initially, the DM must provide some reference levels for the objective functions. After, the DM provides some reference probabilities for these reference values, which he or she can change at each iteration.
- Reference probabilities are regarded as reference point for the achievement scalarizing function, while the reference levels are considered as hard constraints.

Interactive methods clue for this approach

INTEREST method
(Muñoz, Luque and Ruiz, 2010)

- Linear and nonlinear problems.
- Parameters are continuous random variables with known distribution
- Interactive Method is based on the Reference Point approach.
- At each iteration, the DM must provide, for each objective, a reference (desired) level and a minimum probability he is willing to assume.
- The desired reference levels are regarded as the reference point for the achievement scalarizing function, while all the desired reference probabilities are considered as hard constraints.

A Synchronous Reference Point scheme for SMOP

- Linear and nonlinear problems.
- Parameters are continuous random variables with known distribution
- Interactive Method is based on the Reference Point approach.
- At each iteration, the DM can provide, for each objective, a reference (desired) level and a reference (desired) probability.
- Given this information, three different scalarized optimization problems are used in order to generate three solutions, taking into account the nature of SMOP problems.

A Synchronous Reference Point scheme for SMOP

SOLUTION (Case 1):

All the desired probabilities $\bar{\boldsymbol{\beta}} = (\bar{\beta}_1, \dots, \bar{\beta}_k)$ must be verified (as hard conditions) while the vector with the reference levels $\bar{\mathbf{u}} = (\bar{u}_1, \dots, \bar{u}_k)$ is regarded as the reference point.

- Scalarized problem

$$\min_{\mathbf{x} \in X(\bar{\boldsymbol{\beta}}, \mathbf{u})} \max_{i=1,\dots,k} \left\{ \mu_i \left(\bar{u}_i - u_i \right) \right\}$$

- The same problem

$$\min_{\mathbf{x}, \mathbf{u}} \max_{i=1,\dots,k} \left\{ \mu_i \left(\bar{u}_i - u_i \right) \right\}$$

$$\text{s.t.:} \quad P\left(\tilde{z}_i(\mathbf{x}, \tilde{\mathrm{c}}) \geq u_i \right) \geq \bar{\beta}_i \quad i = 1, \dots, k$$

$$\mathbf{x} \in X$$

A Synchronous Reference Point scheme for SMOP

SOLUTION (Case 2):

We will assume that all reference levels $\bar{\mathbf{u}} = (\bar{u}_1, \ldots, \bar{u}_k)$ must be satisfied (as hard conditions) while the vector with the probabilities $\bar{\boldsymbol{\beta}} = (\bar{\beta}_1, \ldots, \bar{\beta}_k)$ is regarded now as the reference point.

- Scalarized problem

$$\min_{\mathbf{x} \in X(\boldsymbol{\beta}, \bar{\mathbf{u}})} \max_{j=1,\ldots,k} \left\{ \omega_j \left(\bar{\beta}_j - \beta_j \right) \right\}$$

$$\omega_j = 1 \quad j = 1, \ldots, k$$

- The same problem

$$\min_{\mathbf{x}, \boldsymbol{\beta}} \max_{j=1,\ldots,k} \left\{ \omega_j \left(\bar{\beta}_j - \beta_j \right) \right\}$$

$$\text{s.t.:} \quad P\left(\tilde{z}_i(\mathbf{x}, \tilde{c}) \geq \bar{u}_i \right) \geq \beta_i \qquad i = 1, \ldots, k$$

$$\mathbf{x} \in X$$

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SOLUTION (Case 3):

Both the probabilities $\bar{\boldsymbol{\beta}} = (\bar{\beta}_1, \ldots, \bar{\beta}_k)$ and the reference levels $\bar{\mathbf{u}} = (\bar{u}_1, \ldots, \bar{u}_k)$ are considered as reference points simultaneously.

- Scalarized problem (γ_1, γ_2 > 0)

$$\min_{\mathbf{x} \in X(\boldsymbol{\beta}, \mathbf{u})} \gamma_1 \max_{i=1,\ldots,k} \left\{ \mu_i \left(\bar{u}_i - u_i \right) \right\} + \gamma_2 \max_{j=1,\ldots,k} \left\{ \omega_j \left(\bar{\beta}_j - \beta_j \right) \right\}$$

- The same problem (γ_1, γ_2 > 0)

$$\min_{\mathbf{x}, \mathbf{u}, \boldsymbol{\beta}} \quad \gamma_1 \max_{i=1,\ldots,k} \left\{ \mu_i \left(\bar{u}_i - u_i \right) \right\} + \gamma_2 \max_{j=1,\ldots,k} \left\{ \omega_j \left(\bar{\beta}_j - \beta_j \right) \right\}$$

$$\text{s.t.:} \quad P\left(\tilde{z}_i(\mathbf{x}, \tilde{c}) \geq u_i \right) \geq \beta_i \qquad i = 1, \ldots, k$$

$$\mathbf{x} \in X$$

A Synchronous Reference Point scheme for SMOP

Preference information provided by the DM can be relaxed

- If only reference levels $\mathbf{\bar{u}} = (\bar{u}_1, \dots, \bar{u}_k)$ for the stochastic functions are provided, we use Solution 2, setting $\bar{\beta}_j = 1 \;\; \forall j = 1, \dots, k$
- If only reference probabilities $\boldsymbol{\bar{\beta}} = (\bar{\beta}_1, \dots, \bar{\beta}_k)$ are provided, we use Solution 1, setting $\bar{u}_i = z_i^U \;\; \forall i = 1, \dots, k$

A Synchronous Reference Point scheme for SMOP

Efficiency results

- If $\mu_i > 0 \ \forall i = 1,\ldots,k$, the Solution 1 is $\bar{\boldsymbol{\beta}}-$weakly efficient solution of SMOP (efficient if it is unique)

- If $\omega_j > 0 \quad \forall j = 1,\ldots,k$, the Solution 2 is $\bar{\mathbf{u}}-$weakly efficient solution of SMOP (efficient if it is unique)

- If $\mu_i, \omega_i > 0, \quad \forall i = 1,\ldots,k$, the Solution 3 is $\boldsymbol{\beta}^*-$weakly efficient solution and $\mathbf{u}^*-$weakly efficient of SMOP (efficient if it is unique), being $\left(\mathbf{x}^*, \mathbf{u}^*, \boldsymbol{\beta}^*\right)$ the optimal solution.

Conclusions

- The interactive reference point approaches can be very useful in SMOP, similar to deterministic MOP.
- Preference information from the DM (reference levels and probabilities) can be reflected in different ways by considering different scalarized problems and taking into account the stochastic nature of these problems.
- Even, it is possible to relax this preference information, asking for the DM only reference levels or only probabilities.
- Scalarized problems generate (weakly) efficient solutions.