Selfo: A class of self-organizing connection games

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Abstract

Selfo is defined as a class of abstract strategy board games subscribed to the category of connection games. Its name derives from the phenomenon of self-organization (i.e. the increase in a system's organization without external guidance), since during the game the sets of pieces might flow in a coordinated way as they step on the board. Despite its very simple definition ("group all your pieces by moving in turns to adjacent cells") complex self-organization processes takes place under concrete circumstances (a balanced distribution of pieces and similar levels of expertise in the players), and are the result of abrupt and deep changes in the tactics. Since a big number of variants have been found to meet the conditions for self-organization, the particular values given to the traditional parameters that define a game (i.e., board tiling, size and initial position, or number of pieces and players) are not so relevant. The *Selfo* class of connection games is defined, instead, by the interrelations among parameters in order to favor self-organization.

1. Introduction

Since the invention of *Go*, traditional connection games have been widely played and studied. In the last decades this genre of games have proliferated, and now they constitute a significant contribution to strategy games. An ambitious compilation has been published by C. Browne [2005].

Some connection games are well-known and have become popular as board games: *Lightning*, the first connection game by several decades [Polczynski 2001]; *Hex*, devised by the mathematician and Nobel laureate John F. Nash, and whose publication [Gardner 1957] raised the connection game genre; *Y*, from which *Hex* is a special case, was proposed in the early 1950s by C. Shannon, the father of the Communication Theory; or *Twixt*, a game that has been marketed by six different companies since 1961.

Far from pure connection games, Browne classifies under the category "convergent connective goal" those connection games whose winning condition implies amalgamating a set of pieces into a single connected group. A number of games have been proposed under this convention, like *Lines of Action* (invented by C. Soucie [Sackson 1969]), or *Groups* (proposed in 1998 by R. Hutnik [Browne 2005]).

The *Selfo* class of games defined in this report subscribes to the convergent family, since the ultimate goal is to knit together the pieces. Despite their apparent similarity, *Lines of Action* and *Groups* do not belong to this class; some rules are added to the definition, and they have been designed for fast games (usually under 10 turns in *Groups*). On the contrary, the fun of playing *Selfo* will be more in going through balanced positions (like in *Tetris*); with a tempo that switches frequently among evenly matched players.

2. Definition of *Selfo*

This section describes the main rules that apply to a game in this class.

After setting the board's grid and size, move length, and the initial board position, each player is assigned a set of pieces, and a random order of play. Players move their pieces in turns, and after the first player's turn, a swap option is given to the rest of the player on their first move. The game ends when a set of pieces gets arranged into a single connected group, or when all players decide a draw (e.g. if some pieces get isolated).

2.1. Board

Selfo does not impose any restriction on the particular board topology. This aspect of the game is indeed critical, since the adjacency graph (based on a triangular, square, hexagonal, or even irregular grid) strongly influences the particular dynamics of the game. But, as said before, self-organized dynamics does emerge on any particular grid if the connectivity is balanced with other parameters. For simplicity, in this report we will constraint ourselves to board surfaces with hexagonal tessellations, where board points will be cell interiors.

With respect to board size, in principle *Selfo* can be played on a theoretically infinite board, where pieces are not confined, and can wander around without limits. This option reduces dramatically the possibilities of self-organized play, which easily turns into a race where any initial advantage cannot be neutralized by the opponents.

Limiting the number of cells introduces a major difference in the development of the game. Boundary effects clearly favor the use of tactics for isolating competing pieces over densely occupied finite boards. This balancing mechanism strongly favors the self-organization during the game, or, said in another way, it expands the range of the parameter space where self-organization takes place.

2.2. Initial board position

Some constraints apply to the initial position: the minimal number of moves necessary for a set of pieces to reach the winning condition (keeping the opponents' pieces on their initial cells) must be high (proportional to the number of pieces) and similar for all sets. Also, subsets of pieces cannot be isolated by competing chains of pieces, neither in the initial position, nor after the first moves (i.e. there must be a chance for any piece to avoid isolation).

All the pieces are placed on the board before starting the game (players cannot place pieces, as in *Go*). This can be done according to a fixed arrangement, or by an algorithm that, respecting the previous constraints, randomly assigns pieces to empty cells. Section 3 develops some examples of fixed initial positions, and algorithms for randomly sorting the pieces.

In order to avoid first-move advantage, a swap option is offered to each player. Every player can (only on their first turn) either make a move, meaning that they keep the set, or swap sets with any other player. This determines the final assignment of sets of pieces to the players.

2.3. Density

Defined only for finite boards, the density of a *Selfo* game is the relation, as a percentage, between the overall number of pieces and the number of cells of the board (i.e. a ratio of empty vs. non-empty cells).

The density has to be high enough to provide a strong interaction among the sets of pieces. But a too dense game will raise the probability of a deadlocked game. Densities in the range 35-40% have demonstrated to favor self-organized play.

2.4. Winning condition

The winning condition of any *Selfo* game is to form a single group connecting all the pieces of a player. The connectivity in this group will be assumed to be that of the adjacency graph of the board, i.e. two pieces are connected if they are in adjacent cells.

For example, on a square board where pieces could only move orthogonally, the winning condition would be to form a single orthogonally connected group.

Players can also resign. Resigning must be announced on a player's turn, and the effect is like the player passing on the following turns: the pieces do not move any more, they stay on the current cells, remaining as non-empty cells for the rest of the game.

2.5. Number of players

Selfo can be played by two or more players that are assigned the same number of pieces. Since the density of pieces has been defined as a very influencing parameter, and must be kept, the size of the sets of pieces will be the total number of pieces divided by the number of players. This size must be bigger than one in order to make groups, but less than four pieces per player is not recommended.

2.6. Move length

This parameter is defined as the maximum number of moves through empty adjacent cells that a player can perform with a single piece in a turn. The length of the move has a lower limit of zero (meaning that the player can pass the turn on). This is the parameter that influences the depth of the game more significantly, since the collection of possible movements increases exponentially with higher values of the maximum length allowed. For this reason the class will be divided into subclasses according to the maximum move length: *Selfo*-1 being the simplest subclass, where any piece can perform single moves by stepping onto one empty adjacent cell (and players can pass on turns), and *Selfo*-*n* being the class where a piece can make a sequence of up to *n* single moves in a turn.

2.7. What a *Selfo* game is not

Possible refinements and extensions of the class can be considered, but in order to keep the simplicity of its definition a number of restrictions should always be met:

- all pieces move according to the same rules,
- players cannot influence the initial position,
- the winning condition should not be altered.

With respect to the first condition, having pieces with different behaviors would introduce a significant complexity in the definition. In such a case it would be necessary to specify a distinctive shape for each type of piece, and their role in group formation.

The second condition is also important, since it affects the simplicity of the strategy: moving pieces on a densely populated board is one thing, and placing pieces on an empty board according to some rules is another. To reach a balanced initial position would mean that the players match also in their skills to select an advantageous constellation of cells.

The proposed winning condition is a standard one (see the Introduction section), and any deviation from this simple goal will increase the solutions, giving less chance for a balanced play.

2.8. Summarized rule set

All the above definition can be condensed on the following rules for the simplest variant of the class, and two players:

Box I. Rule set for *Selfo*-1.

- A board with a given topology and size, and a number of pieces are chosen,
- all black and white pieces are distributed on the board with a proper algorithm,
- colors are randomly assigned to players,
- starting with blacks, players move in turns, and a swap option is given to whites on the first movement,
- in each turn, a player can either pass on the turn, or move one piece to an adjacent empty cell,
- the player that first arranges all her/his pieces in a single connected group wins the game,
- players can decide a draw if the game lasts for too long or some pieces get isolated.

Or even more concisely:

Starting with an initial board position, players take turns moving a piece of their color an adjacent empty cell. The first player to connect all of their pieces into a single group wins.

3. Initial board position

Given that the expertise of the players match (or has been balanced somehow), the main point for a *Selfo* game to develop self-organized dynamics is to start with a board position where the sets of pieces are distributed in a way that do not favor the grouping of one nor the other sets. Some initializations are proposed based on regular and irregular distributions of pieces.

3.1. Regular positions

Figure 1 shows a number of fixed initial positions of hex hex boards of size 6 (for clarity, hexagonal cells are painted as circles), for two players (first row) and three players (last row).

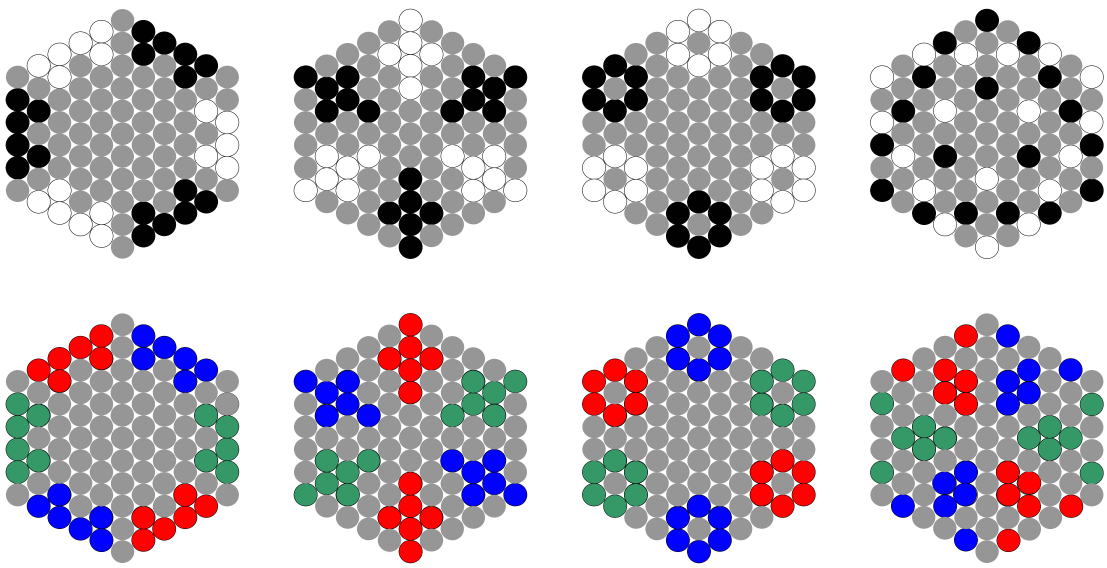

Figure 1. Regular initial positions for a hex hex board of size 6. Colors identify the different sets.

Similar positions can be obtained for a higher number of players by re-coloring the pieces in a way that keeps the symmetry. All these board positions contain 36 pieces (≈40% density).

A number of regular positions for two players (can be extended to more players) derive from defining a pattern of pieces on one of the six triangles that form the hex hex board (see Figure 2-left), and copying it after reversing colors and rotating the pattern to fit the neighboring triangles. Figure 2 shows an example of initial position after reproducing the pattern on the left. A variation would be to define the pattern on two or three adjacent triangles, and copying and inverting colors three or two times, respectively.

This method is simple in its definition, and ensures a good distribution of the pieces on a hex hex board, whatever its size.

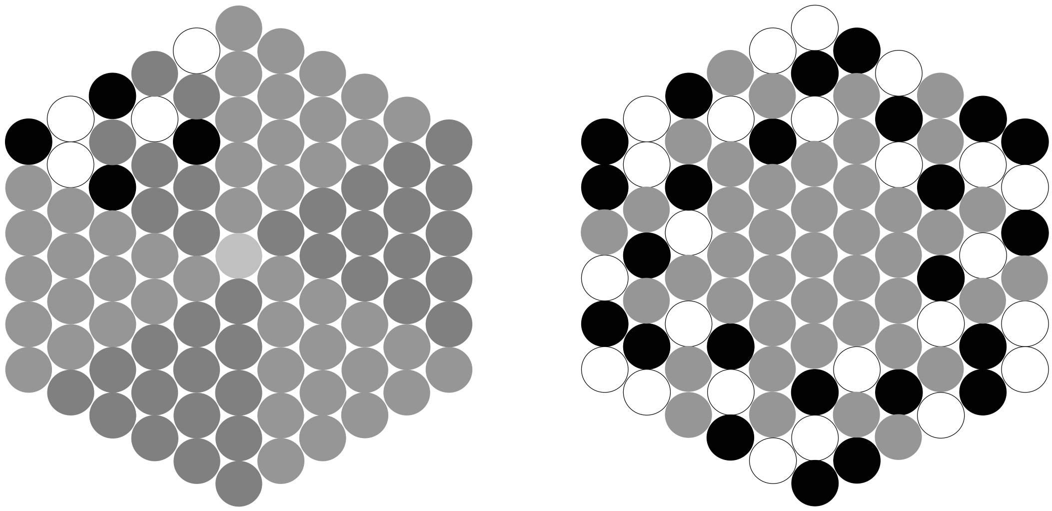

Figure 2. Shown in two differentiable gray tones are the six triangles that, arranged around the central cell, form a hex hex board; and a pattern of black and white pieces (left). The resulting position of 36 pieces (right).

3.2. Irregular positions

Irregular initial positions of the board can be obtained by different algorithms. The main restriction is that the sets of pieces are balanced (see section 2.2). Two main strategies are proposed to determine an initial position: perturbing a regular position, and selecting random cells for each piece after indexing the board.

The first option starts with a regular distribution, and all pieces are randomly numbered. Starting from the first piece, a random number from 1 to 6 is selected that indicates a direction according to the scheme in Figure 3, the piece then steps onto the corresponding cell if it exists and is empty. This procedure applies to all the pieces on the board, and can be easily implemented on a physical board with a standard die. The result is a random rearranging of the sets of pieces that generally keeps a good distribution of pieces. Figure 4 shows an example starting with one of the positions proposed in the previous section.

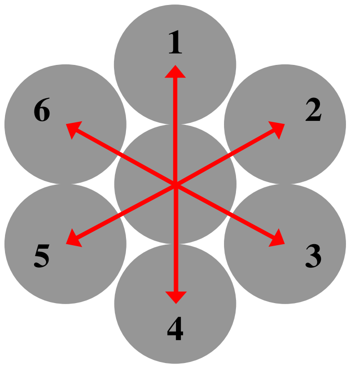

Figure 3. The six possible directions of movement from a given cell.

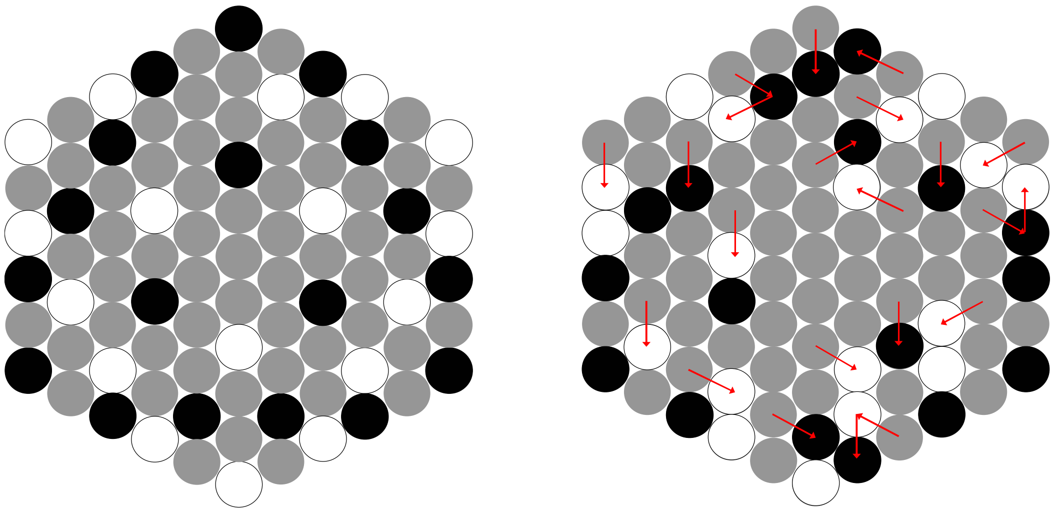

Figure 4. An irregular position derived from a regular one.

The second strategy places the pieces following a spiral course that starts from the central cell and goes clockwise. In this case a random number is selected, and a piece of the first player is placed on the corresponding cell; counting from the central cell. A piece of the next player is placed after moving a random number of steps forward from the last piece. The process goes on until the last piece of the last player is placed on the board. This procedure can also be implemented with a die, renumbering the six faces accordingly (for example, on a size 6 hex hex board, numbers from 1 to six would be reinterpreted as {1, 2, 2, 3, 3, 4} for a 36% mean density). In case some of the last pieces cannot be placed on the board, the whole procedure should be repeated.

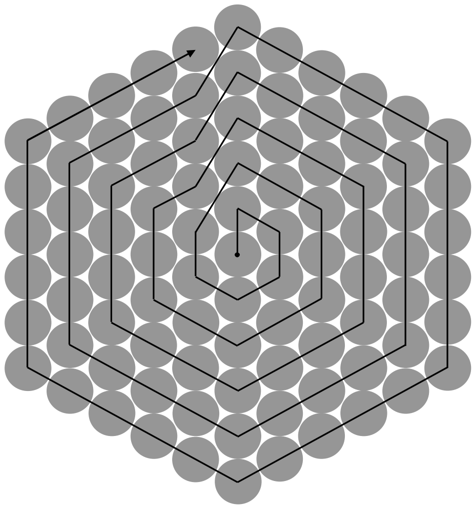

Figure 5. Trajectory followed on the hex hex board to place the pieces randomly.

The proposed methods give an initial position that warranties a balanced distribution of the pieces. But it could also happen that a piece gets isolated (or can be isolated with little effort) in this initial setting. In such a case, the procedure should be applied again.

4. Self-organized dynamics

As explained in previous sections, the development of the match in a *Selfo* game starts with a fixed board position. After the opening movement, and the consideration of the swap option by the rest of the players, pieces start to occupy strategic cells, and to form small groups. Each set of pieces will converge to a configuration that minimizes the overall distance inside a set, while obstructing the opponents' options to group their pieces first. Given the conditions of equilibrium in the initial distribution of the pieces and similar experience among the players, the tightly coupled position reached during the opening should develop into a phase of self-organization.

But, what does it mean to say that the sets of pieces self-organize during the game? Self-organization is a process widely studied in the field of Complex Systems. Models of cellular automata are good examples of self-organized behavior, where structure (order) emerges from disordered initial states, after the iterative updating of the cells values with a local rule (e.g. Wolfram's 1D cellular automata [Wolfram], and the 2D automata based on the Belousov-Zhabotinsky reaction [Dewney 1988]). Self-organization takes place similarly in *Selfo* games, once a player's set of pieces is distributed on the board, they start to move according to local rules (i.e. the strategy of the player) to try to find an ordered configuration. The fact that adversary pieces try to do the same, while preventing others from grouping their sets, allows a balance of forces that makes the pieces flow on the board in a self-organized fashion.

Self-organization occurs only over long lasting matches. It is a direct consequence of rapid changes in the tactics (alternating offensive and defensive movements), forced by the opponents' recent actions, and it is characterized by global long-range displacements of the pieces on the board. This phenomenon can be measured in different ways. A simple indicator is the "mean number of turns since last movement" applied to a player's set of pieces. A value of this mean fluctuating around the size of the set of pieces reveals a mobilization of the whole set, On the other hand, values significantly higher than the number of pieces are representative of the set having found a stable configuration (pieces that do not change positions, while a small number of them wander around).

4.1 An example of self-organized play

Figure 6 shows successive positions of a hex hex board of size 5 during a *Selfo*-1 game, played by players of similar experience. The initial board position is a regular one that soon shows the formation of some groups derived from the first black piece's movement. After 28 turns, the two players compensate their movements by establishing two ladders aligned horizontally (position after 18 and 28 moves). But in deciding who will be first connecting both sides of the board, tension grows on the upper-right corner (position after 38 moves). At this point the initial strategy changes, when the whites break the ladder, allowing some of the black pieces to cross it (positions after 42 and 49 turns), and converge to new diagonal ladders (position after turn 57) that finally align vertically. This option finally forces a draw when both sets of pieces reach a configuration where some pieces would become isolated before any whole set can get arranged into a single group (final position after 77 turns).

This simple example shows how self-organization takes place: the sets of pieces tend to balance configurations, and continue in this direction until the tension fractures the patterns, pushing the sets towards unstable sequences of positions that rapidly generate new patterns.

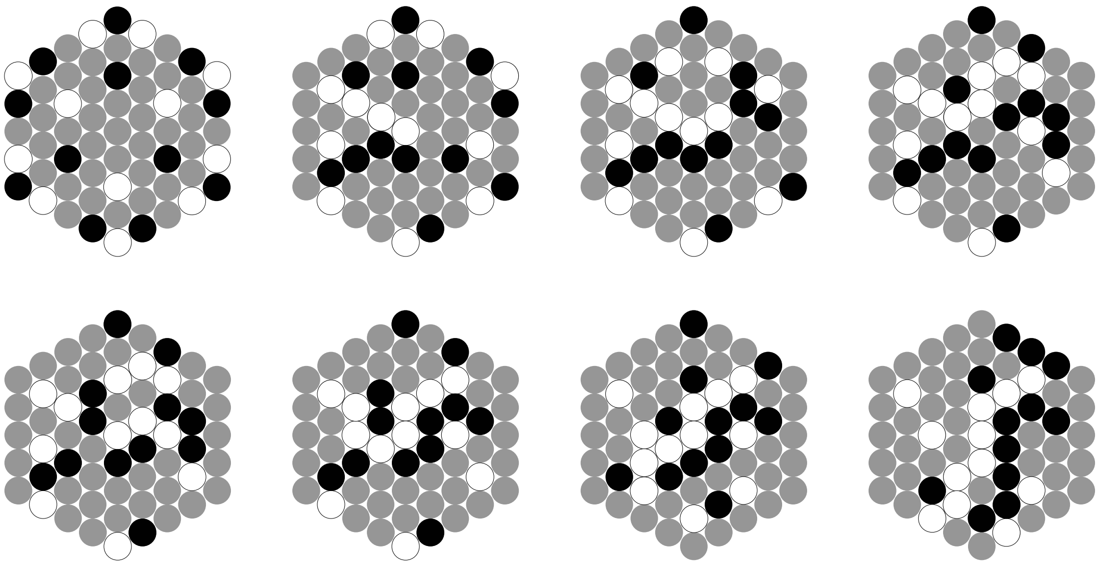

Figure 6. Evolution of the board position after 0, 18, 28, 38, 42, 49, 57 and 77 turns (from left to right and from top to bottom).

5. Discussion

We are living in a time (the Internet era) where less and less time can be dedicated to learning from reading, and mastering particular games. The success of new games is certainly influenced by this constraint. Traditional games, like backgammon or chess, could be categorized as complex to start playing, in the scope of the emerging family of online games and game consoles, operated by very simple rules and commands. Board strategy games will benefit much from the new Internet-based infrastructure, but simplicity will be a cutting factor for a game to become popular.

Taking this constraint as one of the main factors in game design, *Selfo* has been conceived in the spirit of E. de Bono's *L-game* [de Bono 1968]: simplest possible definition, a considerable depth, and indecisiveness of the game (playing all players perfectly, the winning condition would never be met). The result is a game where the only difficulty is in establishing the initial board position, but this handicap vanishes when the game is played on the computer, since the methods proposed in section 2.2 can be programmed. For the rest, a child starts playing *Selfo* correctly within minutes.

Interesting extensions of the *Selfo* class can be derived from assigning unequal forces to the players: variable number of pieces, or different lengths of move. In principle, move length looks like an adequate way to balance unpaired players with different degrees of expertise, but the fact is that a player with a slightly longer move, has in practice too much advantage. In general, the depth of a *Selfo* game is influenced by move length (more significantly), the number of pieces of a player, and the number of players.

Another variant might consider special pieces with a bigger (or unlimited) move length that help to block the winning strategy of an opponent. This proposal challenges the first condition imposed to the class in section 2.7, but it looks like the simplest variation when departing from a homogeneous set of pieces.

An important feature of *Selfo* is that it can be played with household stuff, by using the conventional chess (or draughts) board and pieces. For example, using the 16 pawns yields a 25% density, that works fine on an 8-neighbours-based topology. Initial board positions can be obtained from regular distributions or by iterating a pattern (4 times a square one, or 8 times a triangular one). Hexagonal boards can also be handcraft easily with a round cutting board.

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