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Geometric Numerical Integration of Gradient Systems**

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We construct and analyse numerical methods that preserve a Lyapunov function of a dynamical system. Therefore we focus on gradient-like Ordinary Differential Equations:

$$\frac{dy}{dt} = L(y)\,\nabla V(y)$$

where L is a negative-definite matrix and V is a scalar function, which fulfils the conditions to be a Lyapunov function, chiefly that it decreases over time. Gradient systems are models of dissipative physical systems, and their trajectories converge towards an asymptotically stable equilibrium. There are few published numerical methods that approximate the original system while preserving the downwards path of the Lyapunov function. We consider discrete-gradient methods [**?**], defined as:

$$\frac{y_{k+1} - y_k}{h} = \widetilde{L}(y_k, y_{k+1})\,\overline{\nabla} V(y_k, y_{k+1})$$

where $\widetilde{L}(y,y) = L(y)$ and $\overline{\nabla}$ is a *discrete gradient*. We choose a particular discrete gradient, namely the coordinate increment, and show that, under mild assumptions, discrete gradient methods are designed that can be computed explicitly. An alternative method, based upon projection, is also described, and the relative merits and shortcomings of each algorithm are brought to light. Numerical results are presented for some simple test equations, supporting the validity of the proposal. For particular examples, the experiments show that discrete gradient methods preserve the Lyapunov function, whereas the Euler rule fails to do so, since periodic solutions appear, and, besides, numerical accuracy of the discrete gradient method is also favourable.

Referencias

[1] Robert I. McLachlan, G.R.W. Quispel and Nicolas Robidoux. Geometric Integration using discrete gradients. **21**, 1998.

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