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Procedia Engineering 00 (2013) 000–000
Procedia Engineering
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The Manufacturing Engineering Society International Conference, MESIC 2013

Upper Bound solutions of Ring Compression Test

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Abstract

In this paper a particularization of forging process is presented, studying the deformation of a ring specimen. Plastic forming is performed by means of the Upper Bound Theorem and the model of Triangular Rigid Blocks to calculate the minimal charge needed to deform the part. The establishing the part to deform an annular geometry is determined by the so-called Ring Compression Test under its canonical configuration, that allows assimilating to a plane strain process. A new perspective to calculate the neutral plane (radius at which the workpiece material flows in opposite directions) is proposed, a basic element in the solution of the problem.

Keywords: Upper Bound Theorem; Ring Compression Test; Triangular Rigid Blocks; Plane Strain; Neutral Radius

1. Introduction

Obtaining the most suitable solution for the calculation of the forces necessary to plastically deform a material requires a thorough knowledge of the most important factors in the process of plastic deformation. Factors among which are the properties of the material under deformation, friction conditions (friction factor and Tresca factor), and most especially, the geometry and material flow, both in terms its direction and speed.

The establishment of a specific model of study should pursue the achievement of a solution as accurate as possible, within a range of values acceptable enough, minimize the possibility of testing required to validate the procedure. Various analytical methods have been developed that Local Analysis of Stress, Slip Lines Field and Limit Analysis based all of them on particular treatment and involving different approximation range to the exact

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Selection and peer-review under responsibility of Universidad de Zaragoza, Dpto Ing Diseño y Fabricacion.

solution of the problem, allowing or not these treatments discriminate inclusion of the above factors [Rubio (2012), Ajiboye (2009), Bermudo et al (2011), Martin et al (2012), Bermudo et al (2012)].

Upper Bound Theorem (UBT), as a solution not only provides the minimum load value which ensures its deformation, but allows to discriminate the parameters that is possible to introduce in the equations optimizing process conditions.

The methods based on Limit Analysis (Upper and Lower Limits) offer, from an analytical perspective, solutions that limit the range in which this solution will be found exactly. In between these limits, the Upper it guarantees the required deformation will be possible by calculated solution.

Expression of UBT, formulated by Prager and Hodge [Prager and Hodge (1951)] (Eq.1) take into account the discontinuity surfaces between different Triangular Rigid Blocks (TRB) considered, among all kinematically admissible fields possible, the chosen is one that minimizes the following expression

$$J^* = \frac{2}{\sqrt{3}}\sigma_0 \int_V \sqrt{\frac{1}{2}\dot{\varepsilon}\dot{i}_j\,\dot{\varepsilon}_{ij}}\,dV + \int_{S\Gamma} \tau|\Delta v|\,ds - \int_{St} T_i v_i\,ds \tag{1}$$

Expression in which the external strain energy (*J**) will never exceed the value calculated from the above equation. The first term expresses the energy consumed due to the internal distortion of the deformation produced on the workpiece. The second term includes the energy produced by the shear forces existing in the discontinuity surfaces, including the contact area tool-workpiece. The third term of the equation provides the energy consumed by the potential external efforts tensile (or compression) that arise in the forming processes.

The aim of this work is the UBT application on cylindrical inner bore, thereby extending the use of this method in plastic forming cases in which, although the geometry of the part is not strictly under plane strain, are acceptable.. It develops therefore a universal model to obtain the load required to deform a ring (axial-symmetric part) subjected to forging, using the UBT [ASM Handbook (1996)].

When on a cylindrical ring-shaped plane, compression is done on their flat sides, keeping constant temperature conditions, the shape change depends on the magnitude of the applied direction compressive and conditions friction at the interfaces tool-workpiece. If the friction in the contact surfaces is zero, the ring is deformed in a similar manner to a solid disk, wherein each element flows radially outward at a speed proportional to their distance from the center of the piece.

When the friction in these contact surfaces is moderate, the outer diameter due to this effect, is lower than that generated in the case of zero friction. If friction exceeds a critical value, the friction resistance to the outward flow becomes so high that some of the material of the part flows into the ring. The measurements of the inner diameters of the rings compressed provide a particularly sensitive to the study of friction, since the inner diameter increases if the friction is low and decreases if the friction is high.

The Ring Compression Test is, therefore, a compression test which incorporates a friction measurement. Be possible to measure the dimensions of the ring and calculating the value of both friction and the load required to deform the workpiece [Avitzur (1968), Altan (1972)].

The analysis of the deformation of the ring is contemplated for a perfect rigid-plastic material. Based on these assumptions (material and type of friction), the equations of plasticity have provided solutions to various geometries of rings within a complete range of values in the adhesion (Tresca) friction factor (*m*) between 0 to 1. The ring Thickness is usually expressed in relation to the inner and outer diameters. Under maximum friction conditions, the largest usable height rings obtained with such dimensions satisfying the relation $D_o{:}D_i{:}h$ *6:3:1* see Fig.1, where D_o is the outer radius of the ring, the inner radius D_i and the height of the ring h. For normal lubrication, it can use a canonical geometric relationship *6:3:2* [Petersen (1998)] which may yield sufficient accuracy for most applications.

Avitzur [Avitzur and Van Tyne (1982)] have been discussed analysis limit techniques, the resulting mechanical compression between die on the ring-shaped workpieces in plastic deformation. From these studies it can be estimated that the progressive increase of the shear stress at the interface tool-part [Camacho et al (2006), Johnson and Mellor (1983), Rubio et al (2008)] may be assumed to be constant during the deformation process. In any way, it is comparable to real processes, suggesting a constant value of Tresca factor *m*.

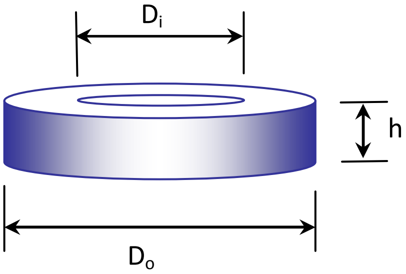


Fig.1. Ring geometric parameters.

2. Methodology

Initial analysis was performed on the corresponding canonical geometric relationship to moderate friction conditions (*6:3:2*). The ring will have an outer diameter of 9 mm, an inner 4.5 mm and a height of 3 mm, with consideration of perfect rigid-plastic material. The canonical account of this relationship, widely tested, allows the assumption of plane strain condition, it still remains a clear axial-symmetric geometry, the relationship between the length of the development of the piece compared to the dimensions of the section is sufficiently high to affirm the validity of this condition. Therefore, each section of the ring is deformed (at a high degree of approximation) within its own plane.

Friction range considered for an adhesion friction factor (Tresca) encompass values from *0* to *1*. Whereas previous studies of UBT on forging processes [Martin (2009)] [Martin et al (2007), Martin et al (2012-a), Martin et al (2012-b)], we analyze one quarter of the workpiece section (ring), to establish a double symmetry condition, which simplifies the problem and pose the boundary conditions by this double condition in the application of TRB model within the UBT.

Assuming a quarter of the overall ring, develops a model of TRB comprises two modules separated by the so-called neutral radius (*Rn*) see Fig.2, from which the material flows in the same and opposite directions. Each of these two modules is composed of 3 TRB. Neutral radius position with respect to the outer radius has been established by Avitzur [Avitzur and Van Tyne (1982)] empirically to axial-symmetric case, for configurations where $R_i < R_n < R_0$ (where R_i = inner radius R_n = neutral radius, R_0 = outer radius) and with different ring geometric relationships to those in this study.

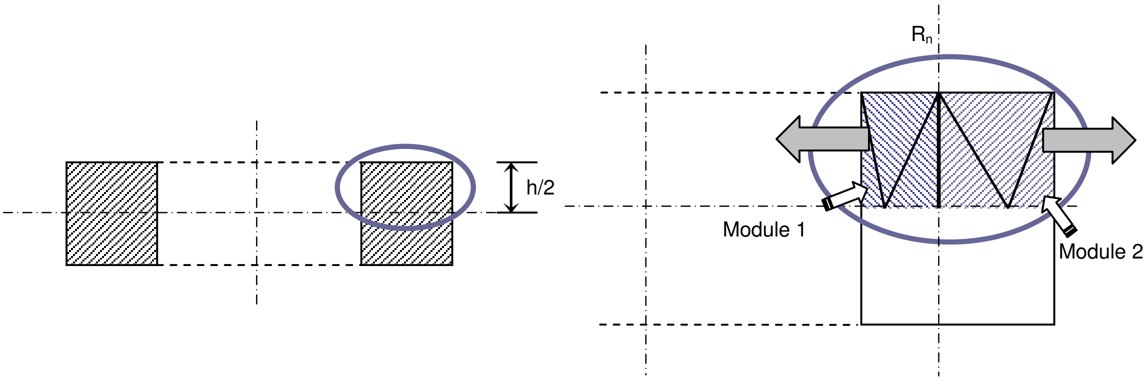


Fig.2. Fourth part under analysis and Modules composition.

The neutral radius position in this study is based on the assumption of volume conservation in plastic deformation processes. It starts from the initial section of the fourth part, of dimensions 2.25 mm x 1.5 mm, reducing its height *h* in increments of 0.2 mm to reach final height 0.5 mm (1.5, 1.3, 1.1, 0.9, 0.7, 0.5) (Fig.3).

Determining the geometric R_n by (Eq.2):

$$h\cdot\pi\cdot(R_n^{2}-R_i^{2})=h\cdot\pi\cdot(R_o^{2}-R_n^{2})\Rightarrow R_n=\sqrt{\frac{(R_o^{2}+R_i^{2})}{2}} \tag{2}$$

As the volume of the workpiece analyzed: $Vi = \pi \bullet (R_0^2-R_i^2) \bullet h = 572.55\ mm^3$, and the initial contact area: $A_i = 190.85\ mm^2$ (initial total interface tool-die contact area).

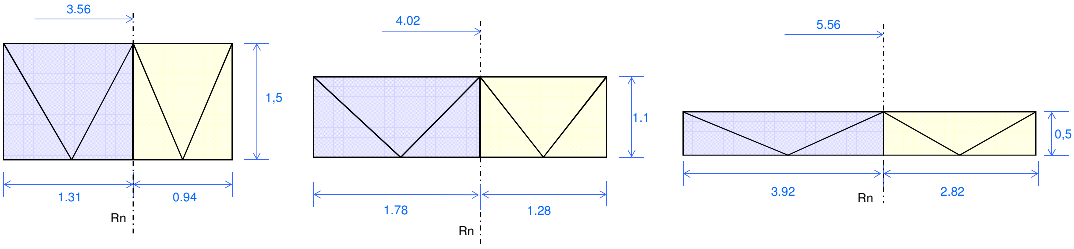


Fig.3. R_n values versus different workpiece heights.

Establishing as a comparison neutral radius values calculated from Avitzur equation (Eq. 3) [Avitzur and Van Tyne (1982)]:

$$\frac{R_n}{R_o}=\frac{2\cdot\sqrt{3}\cdot m\cdot\left(\frac{R_o}{h}\right)}{\left(\frac{R_o}{R_i}\right)^{2}-1}\left[\sqrt{1+\frac{\left(1+\left(\frac{R_i}{R_o}\right)\cdot\left[\left(\frac{R_o}{R_i}\right)^{2}-1\right]\right)}{2\cdot\sqrt{3}\cdot m\cdot\left(\frac{R_o}{h}\right)}}-1\right] \tag{3}$$

Once the different radii neutral have been estimated, the UBT by TRB method requires determine velocities of each of the blocks that make up the modules in the geometry of the section of the workpiece. In particular, are the velocities of the inner and outer end blocks (blocks 3 and 6, Fig. 4) to provide the greater values in both directions. The velocities of each one of the end blocks of each module, calculated from their hodographs (Fig. 4), used to verify the end at which there is a greater flow of the material, allowing include a greater number of modules in the case large ring linking the input and output velocities of material. This addition of modules achieves a better adaptation to the geometry of the examined ring.

The velocities of the material flow at the ends of the ring fourth section (V_o ring outward velocity; V_i velocity into the ring) will be determined by the ratio $V_1 \bullet b / h$ [Martin et al (2009)], where *b* is the section width (the total width of the two modules), *h* the height of the section, and V_1 the initial value of the descent rate of the die, taken with unit value (Fig. 5). The values of the velocities indicated for the different heights are compiled in Table 2.

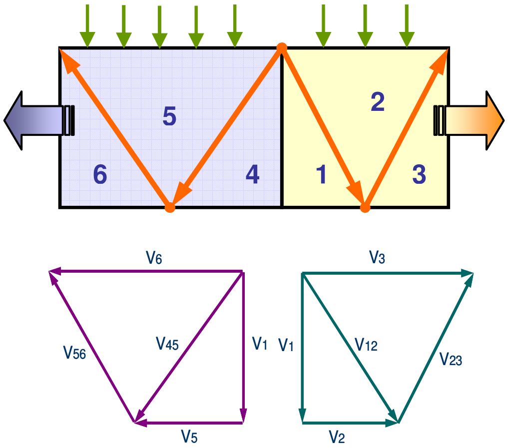
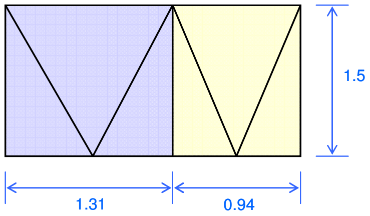


Fig. 4. Material flow directions of the two modules and hodographs.



$$V_i = V_1 \cdot \frac{b}{h} = 1 \cdot \frac{1{,}31}{1{,}5} = 0.871$$

$$V_o = V_1 \cdot \frac{b}{h} = 1 \cdot \frac{0{,}94}{1{,}5} = 0.628$$

Fig. 5. Inner and outer velocities of each module example.

Finally, from *p/2k* expression of Equation 1 is obtained with the dimensionless ratio to determine the pressure and taking into consideration the area on which it acts, the load required to achieve the expected deformation:

$$\frac{p}{2k} = \frac{h}{b} + \frac{b \cdot (1+m)}{4h} \tag{4}$$

From this relationship, we obtain the applied pressure p, which follows, after determining the area of the tool-workpiece interface, the load required to ensure the deformation of the piece (being k shear stress to the workpiece material in Eq 4).

3. Results

Keeping canonical dimensions defined at the beginning of this work, the constancy of volume and applying equation 3 for cases of variable m sets the neutral radius from a purely geometrical consideration. The results obtained are shown in Table 1 and are expressed graphically in Fig. 6.

Can be seen as the use of Equation 3 gives a neutral radius values (R_n) are lower than those obtained from geometrical considerations without taking into account the existing friction. In the latter case the radius will be larger, since the material flow is not "brake" for friction.

Table 1. Neutral radius values (R_n)

R_o (mm)	Geometric (mm)	Avitzur m=0.1 (mm)	Avitzur m=0.3 (mm)	Avitzur m=1 (mm)
4.50	3.56	1.64	2.27	2.85
4.83	3.77	1.74	2.40	3.00
5.25	4.02	1.85	2.54	3.16
5.81	4.36	1.93	2.66	3.34
6.59	4.82	1.93	2.70	3.46
7.79	5.56	1.50	2.24	3.14

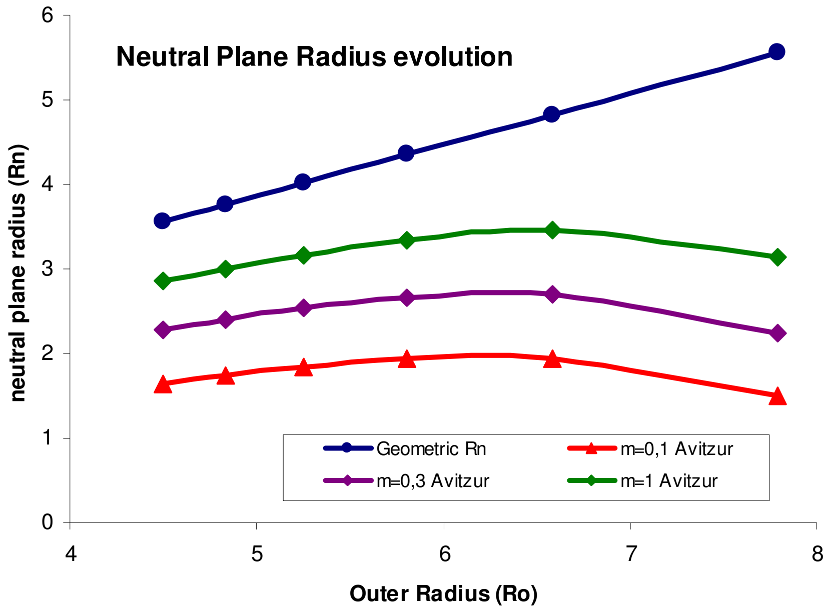


Fig. 6. Neutral Plane Radius evolution (R_n).

The study of the modules allows establishing, from the corresponding hodographs in each case (Fig. 4), the material flow velocities, both the outside (from the outer radius) as to the interior of the workpiece (from the inner radius), being able to appreciate the behavior of the ring and the declining value of the inner radius at a faster rate than the increase in the outer radius.

The Tresca friction conditions are simple to incorporate into Equation 4, showing for a range of $m = 0$ to $m = 1$ the results in the Fig. 7.

Table 2. Neutral radius velocities

Height (h)	Inward velocity V_i	Outward velocity V_o
1.5	0.871	0.628
1.3	1.160	0.836
1.1	1.621	1.168
0.9	2.421	1.745
0.7	4.002	2.885
0.5	7.845	5.654

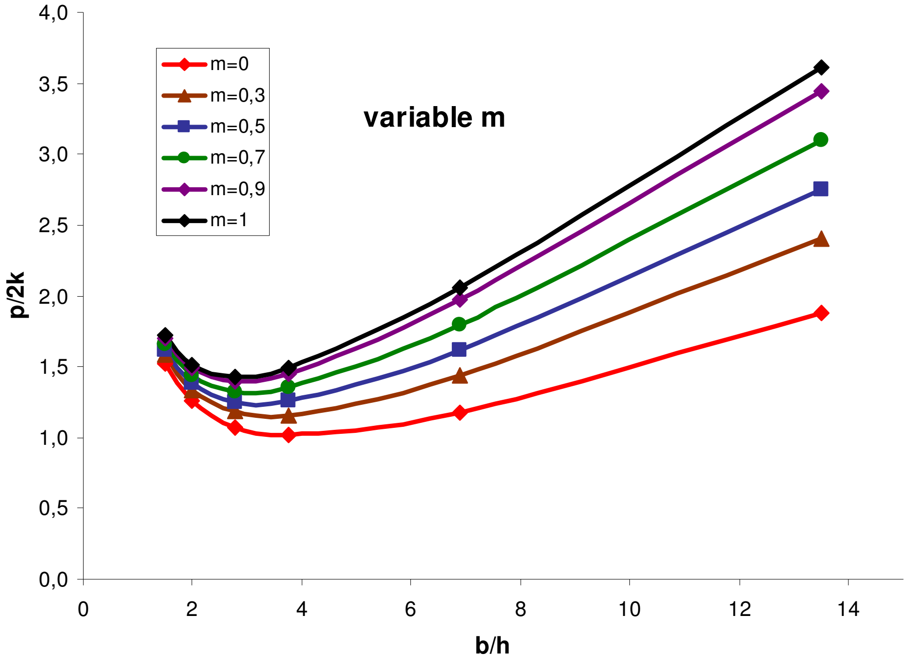


Fig. 7. *p/2k* evolution versus *m* values

Fig. 7 can observe that the results have a similar behavior in the UBT by TRB application versus the curves of typical *p/2k* dimensionless ratio provided in applications of the same method on other plastic forming processes.

These curves provide a minimum with which you can achieve the best adaptation of the number of modules before the limit value is optimal.

4. Conclusions

Higher values of velocities workpiece inner section versus outward velocities, as reflected in Table 2, confirm the results obtained experimentally by Ring Compression Test, meaning the effect of friction that prevents the material's natural flow outwards and causes a withdrawal of the material towards the inner central hole, thereby reducing the radius of the latter. For this reason, the Ring Compression Test is considered a suitable test for determining the influence of the friction forging process.

According to the results obtained show an evolution of the dimensionless ratio similar to those generated *p/2k* other plastic deformation processes, whether they are stationary (drawing) as no-stationary (forging). *p/2k* evolution curves appear with minimum application of load coincidental with the geometrical configuration which provides minimal distortion of the material.

The method of the Upper Bound Theorem by applying Triangular Rigid Blocks model presents a versatility that makes it suitable to be implemented in the calculation of minimum loads to be applied to parts to deform without necessarily presenting these typical geometric arrangements plane strain. Application of UBT by TRB model has traditionally been used in different plastic deformation processes stationary or non-stationary, but under plane strain consideration. The results of this study provide a new line of development of the method, without the need to impose this restriction, and therefore, even with some limitations in the form factor, applicable to parts with axial-symmetric configurations.

Acknowledgements

The authors thank University of Malaga – Andalucia Tech Campus of International Excellence for its economic contribution on this paper.

References

Ajiboye, J., 2009. Upper Bound for analysis of extrusion billets trough square circle and square / rectangular dies. Mechanical Journal of Science and Technology, 23, 461-474.

Altan, T., Lee, C.H., 1972. Influence of Flow Stress and friction Upon Metal Flow in Upset Forging of Rings and Cylinders. Journal of Engineering for Industry. Transactions of the ASME. 94 (3), 775-782.

ASM Handbook, 1996. Forming and Forging. ASM International. 14, Metals Park, Ohio, USA.Rubio, E.M., 2012. Energetic analysis of tube drawing Processes by the Upper Bound Method using theoretical work-hardening materials. Steel Research International, pp. 555-558.

Avitzur, B., 1968. Metal forming: Processes and analysis. Robert E. Krieger Publishing Company, Inc. Huntington, New York, USA.

Avitzur, B., Van Tyne, C.J., 1982. Forming Ring: An Approach Upper Bound. Part 1: Flow Pattern and Calculation of Power. Journal of Engineering for Industry. Transactions of the ASME. 104, 231-237.

Bermudo, C., Martin, F., Sevilla, L., 2011. Application of the Upper Bound Element Technique with Triangular Rigid Blocks in Indentation. Proc. Society of Manufacturing 4th International Conference, Cadiz, Spain,

Bermudo, C., Martin, F., Sevilla, L., 2012. Friction Influence on the Implementation of the Upper Bound Theorem in Indentation process. Annals of Daaam for 2012 & Proceedings of the 23rd International Symposium Daaam, 23 (1), 209-214.

Camacho, A.M., Marin, M., Sevilla, L., Domingo, R., 2006. Influence of strain hardening of forces and contact pressure distributions in forging processes. Journal of Achievements in Materials and Manufacturing Engineering, 15, 166-173.

Johnson, W., Mellor, P.B., 1983. Plasticity Engineering. Ellis Horwood. Chichester. England.

Martin, F., Sevilla, L., Bermudo, C., 2012. Analytical approach to the indentation process application of the Upper Bound Element Technique. Materials Science Forum, 713, 13-18.

Prager, W., Hodge, P.G., 1951. Theory of Perfectly Plastic Solids. Chapman & Hall, Ltd., London.

Martin, F., Sevilla, L., Rubio, E., Sebastian, M.A., 2007. Basis for application Upper Limit Theorem in forging processes on modular geometric configurations. Proc. of MESIC 2007: 2nd Manufacturing Engineering Society International Conference. Madrid, Spain.

Martin, F., 2009. Development, integration and optimization at the study of forging Processes by the Upper Bound Theorem Triangular Rigid Blocks trough model. PhD. Thesis. University of Malaga. Malaga. Spain.

Martin, F., Sevilla, L., Sebastian, M.A., 2009. Implementation of technological and geometrical parameters in forging Processes by means of the Upper Bound Element Technique. American Institute of Physics Conference Proceedings, 1181 (1), 455-463.

Martin, F., Camacho, A.M., Domingo, R., Sevilla, L., 2012. Modular Procedure to Improve the application of the Upper Bound Theorem in forging. Materials and Manufacturing Processes. (DOI: 10.1080/10426914.2012.718478)

Martin, F., Sevilla, L., Camacho, A.M., Sanz, A., 2012. Adaptive model to apply the Upper-Bound Theorem in plain strain forging. Applied Mechanics and Materials, 217-219, 2113-2116

Petersen, S.B., Martins, P., 1998. An alternative ring-test geometry for the evaluation and friction under ordinary low pressure. Journal of Materials Processing Technology, 79, 14-24.

Rubio, E.M., Camacho, A.M., Marcos, M., Sebastian, M.A., 2008. Analysis of the energy vanished by friction in tube drawing Processes with a fixed conical inner plug by the Upper Bound Method. Materials and Manufacturing processes, 23 (7), 690-697.

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