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AGRICULTURAL EXPERIMENT STATION

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# GROWTH AND DEVELOPMENT

*With Special Reference to Domestic Animals*

XXXIX. Relation Between Monetary Profit and Energetic Efficiency of Milk Production with Special Reference to the Influence of Live Weight Thereon.

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## FOREWORD

The special investigation on growth and development is a cooperative enterprise in which the departments of Animal Husbandry, Dairy Husbandry, Agricultural Chemistry, and Poultry Husbandry have each contributed a substantial part. The parts for the investigation in the beginning were inaugurated by a committee including A. C. Ragsdale, E. A. Trowbridge, H. L. Kempster, A. G. Hogan, F. B. Mumford, Samuel Brody served as Chairman of this committee and has been chiefly responsible for the execution of the plans, interpretation of results and the preparation of the publications resulting from this enterprise.

The investigation has been made possible through a grant by the Herman Frasch Foundation, now represented by Dr. F. J. Sievers.

F. B. MUMFORD

*Director Agricultural Experiment Station*

## ACKNOWLEDGMENTS

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## ABSTRACT

Gross energetic efficiency is not dependent physiologically on body weight of cow; but monetary profit is. Monetary profit per cow, increases with increasing body weight; monetary profit per unit body weight decreases with increasing body weight; monetary profit per unit milk (FCM) production or per unit feed consumption tends to be independent of body weight and is almost directly proportional to energetic efficiency. While profit for the entire dairy enterprise (not necessarily per cow or per unit live weight) is an index of the commercial value of a cow under given conditions, energetic efficiency is an index of physiological value. Milk production by itself is a poor index of either commercial or physiological value and should be replaced by gross energetic efficiency in the estimate of which body weight as well as milk production enter as factors. The above statements are analyzed and illustrated graphically in detail. Alignment charts (nomographs) are presented for quick estimation of energetic efficiency and various kinds of profit (per cow, per unit body weight, per unit milk).

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### I. INTRODUCTION

The preceding bulletins (Missouri Research Bulletins 222 and 238) were concerned with energetic *efficiency* of milk production; that is, with the *ratio* of energy output as milk to energy expended as digestible feed. The present bulletin is concerned with monetary *profit* of milk production; that is, with the *difference* between money realized for milk produced and money expended for feed consumed.

Profits from milk production are dependent not only on energetic efficiency of milk production but also on other factors such as milk and feed prices and costs of: labor, management, housing, taxes, etc. The differential between milk price and feed cost is obviously the most important factor. No matter how efficient a cow may be energetically, no matter how much milk she may produce, she will not yield a profit if price of milk is below cost of feed required to produce milk. Indeed, if price of milk is below cost of feed then the more milk produced the greater the loss. Monetary profit is therefore by no means identical with energetic efficiency, and the dairyman naturally wishes to be informed concerning the relation between monetary profit and energetic efficiency. The purpose of this bulletin is to supply some of this desired information with special reference to rapid methods for computing monetary profit and to the influence of live weight and of energetic efficiency of animal on the monetary profit of milk production.

### II. DEFINITIONS OF PROFIT FROM MILK PRODUCTION AND THE INFLUENCE OF LIVE WEIGHT AND ENERGETIC EFFICIENCY ON PROFIT.

The first question needing clarification is: what does one mean by profit from milk production?

1. Is it profit per cow?
2. Is it profit per unit live weight of cow?
3. Is it profit per unit milk produced?
4. Is it profit per unit feed consumed?

This is Paper 110 in the Herman Frasch Foundation Series.

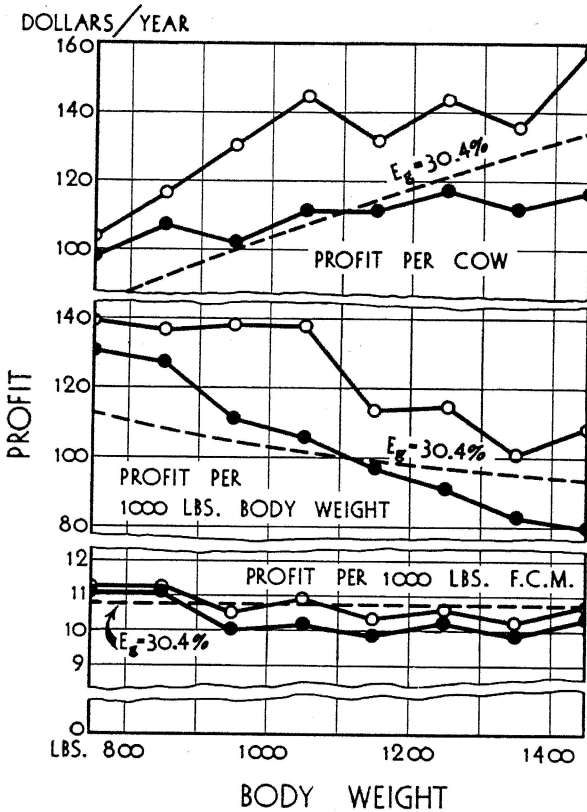


Fig. 1.—Influence of live weight on profit per cow (top segment), per 1000 pounds body weight (middle segment), and per 1000 pounds milk (FCM) production (bottom segment). The light broken curves indicate the effect when gross energetic efficiency is assumed to be the same, namely, 30.4% for all live weights. The continuous curves represent the observed data listed in the appendix. Profit is defined by the difference between monetary return for milk at \$2.00 per 100 pounds FCM and monetary cost of feed at \$1.50 per 100 pounds TDN, ignoring all other expenses.

Figure 1 illustrates the influence of live weight of cows on: profit per cow (top segment); profit per 1000 pounds live weight (middle segment); profit per 1000 pounds FCM production (bottom segment). The broken curves indicate the effects of live weight on the assumption that gross energetic efficiency is the same (30.4%) for small and large cows. The curves connecting the data points, represent data listed in the appendix of this bulletin.

It is instructive to discuss the answers to each of these three questions in some detail. To simplify the discussion, we shall leave out from computations the overhead cost of management, housing, labor,

etc., and define "profit" by the difference between monetary return for milk produced and monetary cost of feed consumed. In constructing Fig. 1 we assumed that the price of milk is \$2.00 per 100 pounds FCM\*, and of feed \$1.50 per 100 pounds TDN.

**1. Profit Per Cow, or "Productive Capacity":** (See top segment in Fig. 1.) It is well known that other conditions being the same (especially fatness), large animals produce more milk than small. Under farm conditions, as illustrated by dairy cattle herd improvement association records, there is a difference of about 250 pounds of FCM per year for every 100 pounds live weight. Under official-test conditions, there is a difference of about 500 pounds FCM per year for every 100 pounds live-weight. Since a large cow tends to produce more milk than a small, then if gross energetic efficiency and profit per pound of milk is the same, the large cow producing more milk than the small, will also bring in the greater total monetary return per cow. If, therefore, the aim is to secure the largest production *per cow*, and if other conditions are the same, then large cows should generally be chosen.

**2. Profit per Unit Live Weight, or "Productive Intensity":** (See middle segment in Fig. 1) While a large cows tends to produce more milk than a small, a small cow tends to produce more *in proportion to her body weight* than a large. If, therefore, the aim is to secure the largest production and largest profit per unit weight of cow, small animals should be generally chosen. This statement does not imply that smaller animals are more efficient energetically than large; because, as indicated in the preceding bulletin (Res. Bull. 238), maintenance feed needs per unit live weight of animal increase with decreasing size of animal in such manner that energetic efficiency (i. e., ratio of energy in milk to energy in feed) tends to be independent of live weight. True that the small cow produces more FCM per unit of body weight than the large, but she also consumes more feed per unit of body weight; with the net result that energetic efficiency remains the same.

**3. Profit per Unit Milk Produced, or "Productive Efficiency":** (See bottom segment in Fig. 1.) If overhead costs of management, housing, labor, etc. are left out of consideration, and profit is defined by the difference between monetary return for milk produced and monetary costs of feed consumed, then profit per unit milk produced tends to be the same for large and small cows for the same reason that energetic efficiency tends to be the same. This is illustrated in the lower segment of Fig. 1 and in Fig. 3.

\*FCM represents milk corrected to 4% fat according to the methods of Gaines. TDN represents total digestible nutrients.

**4. Profit per Unit Feed Consumed:** As indicated in section 6f, profit per unit feed consumed, like profit per unit milk produced, tends to be independent of body weight and almost proportional to energetic efficiency.

**5. Influence of Gross Energetic Efficiency on Profit:** Figure 2 shows that profit per cow and profit per 1000-pounds live weight of cow increase more rapidly than gross energetic efficiency. The greater the efficiency level, the greater the profit per efficiency increment. Profit approaches infinity as energetic efficiency approaches 100%. (It will be noted later that a 50% gross energetic efficiency with respect to TDN consumed is perhaps near the biologic maximum. The most TDN that can be recovered in milk is about 50%.)

It is interesting to note from Fig. 2 that for the assumed milk and feed prices, gross energetic efficiency must exceed 14% in order to make any profit at all; also that *per cow*, large cows bring in a larger profit than small; but *per unit live weight* small cows bring in a larger profit than large cows. This is evident from the fact, previously explained, that at a given energetic efficiency, a large cow produces more milk than a small and therefore more profit; but *per unit body weight*, a small cow tends to produce more milk than a large even if the energetic efficiency is the same in both.

Figure 2 shows that unlike *per cow* or *per unit body weight*, profit per unit milk (FCM) is represented by a single curve, i. e., it is a function of efficiency alone, and tends to be independent of live weight for the same reason that efficiency tends to be independent of live weight. The ratio of profit per unit milk to energetic efficiency, is then, nearly the same for all live weights.\* The independence of profit per unit FCM and body weight is shown in a direct manner in Fig. 3, and in the bottom segment of Fig. 1.

\*Ratios of profit to energetic efficiency of cows represented in Table 2, Research Bulletin 222 is practically independent of live weight as shown below:

Body Weight, lbs.	700	800	900	1000	1100	1200	1300	1400
Ratio profit per unit FCM to energetic efficiency	0.344	0.341	0.328	0.347	0.338	0.349	0.332	0.351

In the above example FCM is in terms of 1000 pounds per year.

The ratio of profit per unit milk,  $\frac{P}{FCM}$ , to energetic efficiency,  $E_g$ , is obtained from equation (4) in the Appendix. Dividing (4) by  $E_g$  gives,

$$\frac{P/FCM}{E_g} = \frac{c_1}{E_g} - \frac{c_2k}{E_g^2}$$

Assuming a milk price of \$2.00 per 100 pounds FCM and a feed cost of \$1.50 per 100 pounds TDN eliminates variations due to market conditions. The equation is multiplied by 365 and  $\frac{1000}{365}$  converting to profit units of dollars per year per 1000 pounds FCM per year. The resulting equation is

$$\frac{P/FCM}{E_g} = \frac{20}{E_g} - \frac{281}{E_g^2}$$

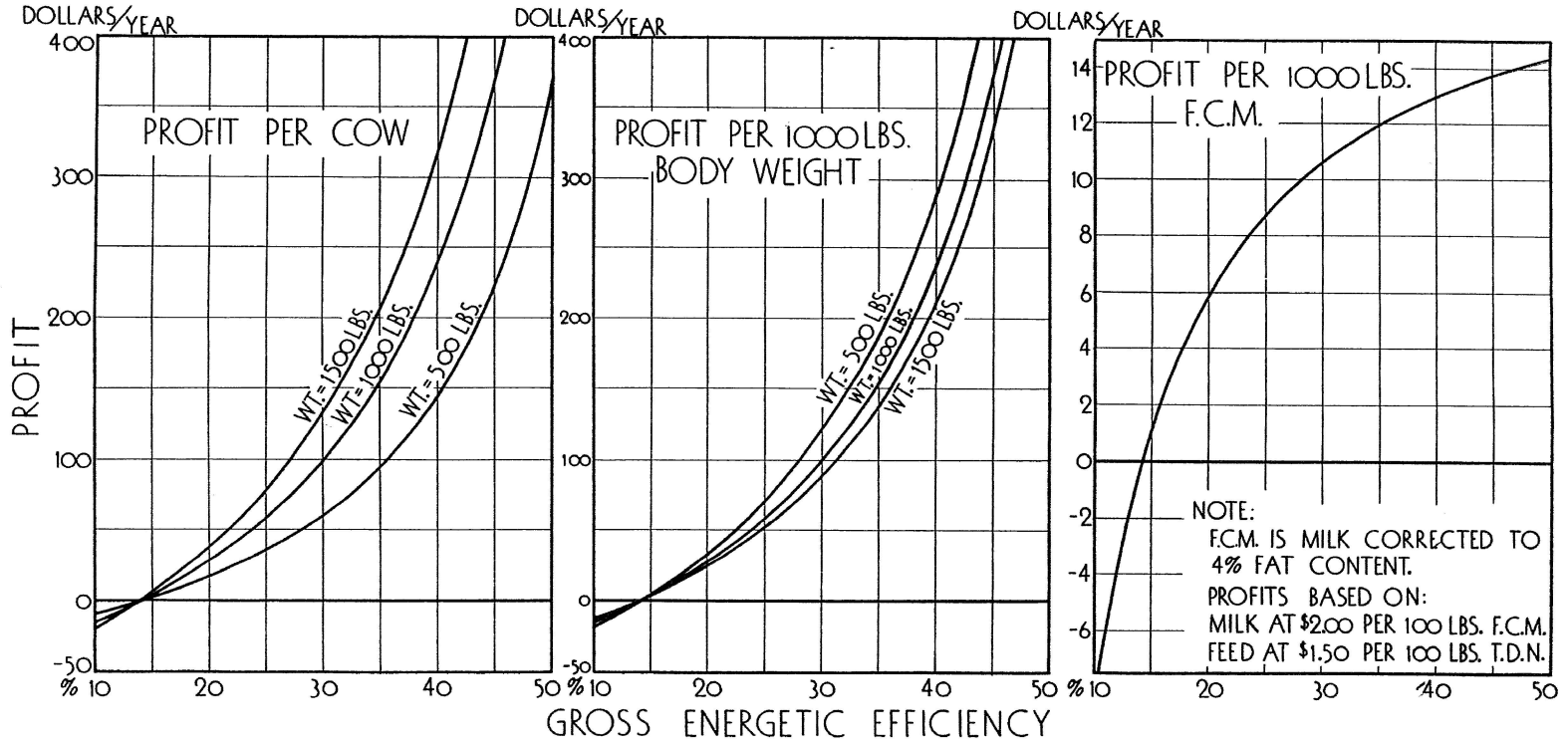


Fig. 2.—Influence of gross energetic efficiency on profit per cow, per unit live weight, and per unit milk production. These curves were computed from equations (13), (15), and (17) of the Appendix.

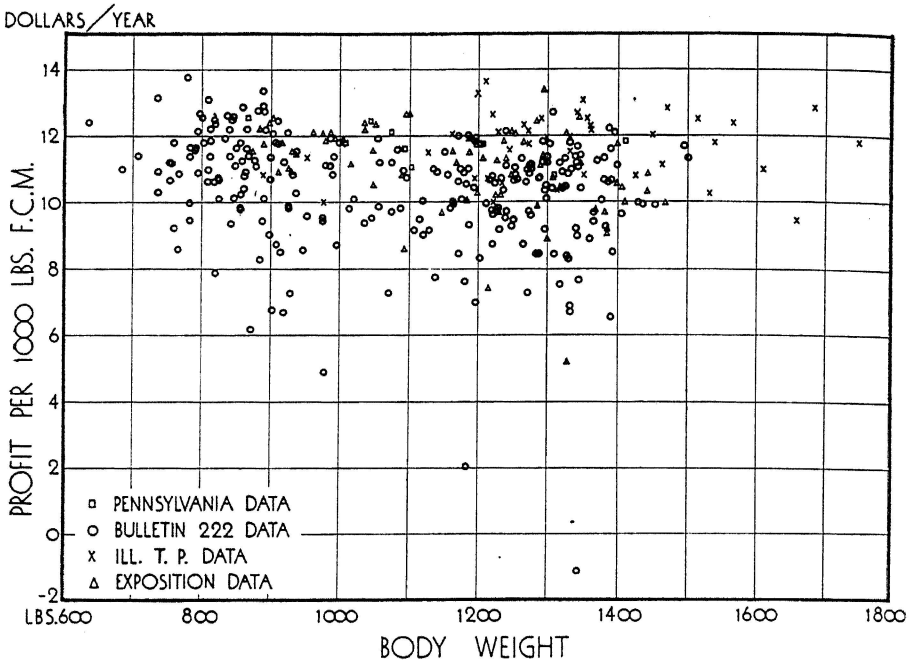


Fig. 3.—Profit per unit milk (1000 pounds FCM) is nearly the same for large and small cows; plotted from data listed in the appendix. It is assumed that milk sells at \$2.00 per 100 pounds FCM and feed costs \$1.50 per 100 pounds TDN.

**6. Influence of Live Weight of Cow on Profit:** The preceding discussion made it clear that if other conditions remain the same, profit per cow increases with increasing size; profit per 1000 pounds live weight of cow decreases with increasing size; profit per 1000 pounds milk (FCM) production is practically independent of size.

In the above discussion "profit" was defined by the difference between money received for milk and money expended for feed. Expenses other than cost of feed were ignored. This section lists some of the other expenses in relation to the influence of size of cow on profit.

a. *Cost of milking:* Cost of milking per cow is nearly the same for small and large as shown in Fig. 4; since a large cow produces, on the average, more milk than a small, the cost of milking, *per unit milk*, is less for a large than small.

b. *Cost of managing and housing:* Housing and managing costs per cow are nearly the same for large and small; since a large cow produces, on the average, more milk than a small, the housing and managing cost *per unit milk* produced is less for a large than small.



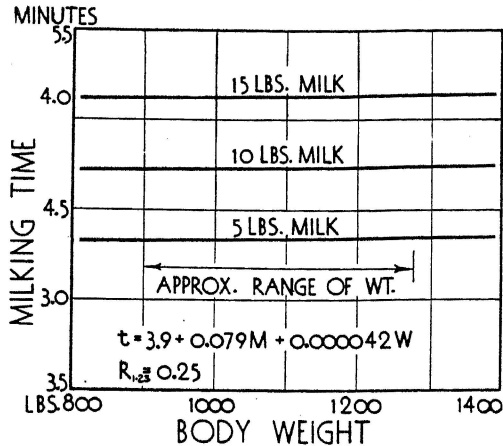


Fig. 4.—Time required to milk cows of different body weights at 5, 10, and 15-pound milking levels. The equation constants on the chart ( $t$ , time taken to milk;  $M$ , production per milking;  $W$ , body weight) indicate that the time required to milk a cow is practically independent of live weight. This chart represents 1289 timed milkings done by milking machine. (From unpublished data by A. C. Ragsdale).

c. *Taxes*: Taxes are paid on a per cow and not size basis; since a large cow produces, on the average, more milk than a small, the tax *per unit milk* is less for a large than small.

d. *Investment*: The market price per cow is in some cases nearly the same for a large and small; when such is the case, then since a large cow produces, on the average, more milk than a small, the investment cost *per unit milk* is less for a large than small. This is not an important item. More important is the investment on buildings and equipment which is roughly the same for large and small cows on a *per cow* basis, and therefore greater for a small cow on a *per unit milk* basis.

e. *Market demands*: Where milk is sold by volume regardless of fat percentage, as is the case of many market milks, then, if the energetic efficiencies are the same, the profit per unit milk will be greatest when the milk has least fat. Since large cows tend to produce a greater volume of milk of lower fat content than small, the milk profits *per cow* and *per unit milk* are likely to be larger for large cows.

When milk is sold for its solids-not-fat (as for fat-poor cheese), cows producing relatively large quantities of relatively fat-poor milk (usually large cows) may be more profitable *per unit milk* and *per cow* than cows producing relatively small quantities of relatively fat-rich milk (usually small cows). The opposite may be true when milk is

sold for its fat (as for butter making). It is generally known that while milk from large Holsteins tends to be more economical for cheese making, milk from small Jerseys tends to be more economical for butter making. One reason for the more economic production of butter from small cow's milk, was pointed out by Whetham and Hammond. Milk from small cows has larger fat globules, which therefore separate out more easily from the milk; with the result that: (1) milk from smaller cows has a better cream line in market milk; (2) fat churns more completely, so that under home churning conditions a pound of milk fat from small cows yields more butter than from large cows.

"It requires, at one extreme with Friesian milk, 1.160 lbs of fat to make 1 pound of butter, and at the other extreme with Jersey milk only 1.011 pounds of fat to make 1 pound of butter."\* Does this mean that the utilization of the butter fat for butter making is about 25% greater for Jersey than Friesian milk? ( $1.260 - 1.011 = .249$  or about 25%).

f. *Limitation of market for milk or supply of feed*: If the dairyman has to produce a *given, constant, amount of milk*, should he produce it from large or small cows? Would large or small cows producing the given amount of milk bring the larger profit? If the gross energetic efficiency of milk production is the same, and if overhead expenses (labor, management, housing, taxes, environment) are left out of consideration, then the profit for the given amount of milk will be the same when the milk is produced by large or small cows as indicated by the bottom segment in Fig. 1†, and also by Figs. 2 and 3.

If the dairyman has a *given limited amount of feed* for milk production, should he use it for feeding large or small cows? Would

\*J. Dairy Res. 6, 320, 1935. Incidentally, the following figures cited from Whetham and Hammond on the amount of fat required to make a pound of butter from different cows will appear incredible to American dairymen who usually make one pound of butter from 0.80 pounds of butterfat.

Breed	Friesian (Holstein)	Red Poll	Short-horn	Blue Albion	South Devon	Ayrshire	Devon	Guernsey	Kerry	Dexter	Jersey
"Weight of fat in the milk required to make 1 lb. of butter".....	1.260	1.213	1.213	1.209	1.167	1.148	1.124	1.102	1.090	1.061	1.011

†The profit in this case is simply the difference between FCM produced times price,  $C_1$ , per pound FCM, and TDN consumed times price,  $C_2$ , per pound TDN; or in equation form

$$\text{Profit} = C_1\text{FCM} - C_2\text{TDN}$$

The dollar efficiency may be expressed:

$$\text{Dollar efficiency} = \frac{C_1\text{FCM}}{C_2\text{TDN}} = \frac{k_1\text{FCM}}{k_2\text{TDN}} = C \times \text{energetic efficiency, where } C_1 \text{ and } C_2 \text{ are the}$$

factors for converting FCM and TDN to dollar units and  $k_1$  and  $k_2$  are the factors for converting FCM and TDN to energy units. In accordance with the assumptions of this case TDN is the only *variable* in the formulas for dollar efficiency and profit. Then the smaller the TDN, and hence the greater the efficiency, the greater is the profit. In this case the most efficient is the most profitable, regardless of size or number of cows.

large or small cows produce more milk from the given amount of feed? The answer to this question is identical with that to the preceding question where milk production was the constant factor. If the gross energetic efficiency of milk production is the same in large and small cows, and if overhead expenses (labor, management, housing, taxes, etc.) are left out of consideration, then the return for the given feed will be the same when fed to large or to small cows, as indicated by the bottom segment in Fig. 1, and also by Figs. 2 and 3.

If, however, overhead expenses (labor, management, housing, taxes, investment, etc.) are deducted from the milk return, and if the gross energetic efficiency of milk production is the same in large and small cows, then profit per unit milk (FCM) from large cows is likely to be greater than from small; because, as previously pointed out, costs of labor, management, bookkeeping, housing, investment, taxes, etc. are likely to be less *per unit milk* produced by large than by small cows. This is illustrated by the following example:

Each of two herds is producing 1000 pounds of milk per day. One herd consists of 900-pound cows the other of 1400-pound cows. The energetic efficiency of milk production is the same in both cases, namely, 30%.

	900 lb. Cows	1400 lb. Cows
No. of cows required to produce 1000 lbs. FCM per day at an energetic efficiency of 30%-----	42	30
TDN required-----	625 lbs./day	625 lbs./day
Approximate time for milking-----	13.6 hrs.	10.0 hrs.
Housing, records, taxes, etc.-----	\$42 x (x = 10c per cow per day)	\$30 x
Cost for 900 lb. Herd =		
feed	\$9.37	feed \$9.37
labor	2.72	labor \$2.00
recds. etc.	4.20	recds. 3.00
	16.29	14.37
	Return from milk = \$20 per day.	

Profit on the small cows = \$3.71 per day; on the large cows = \$5.63 per day. It cost 13% more to produce the 1000 pounds FCM by the small than large cows. The profit was computed on the assumption that milk sells for \$2.00 per 100 pounds FCM; feed costs \$1.50 per 100 pounds TDN; labor, 20c per hour; housing, records, taxes, etc. 10c per day.

g. *Tendency for selective breeding for small cows of high efficiency and large cows of low efficiency:* The above discussion indicates that if large and small cows produce milk with the same energetic efficiency, and if allowance is made for overhead expenses, large cows return the larger net profit per unit milk and consequently larger profit on the enterprise than small, because cost—energy expenditure—of labor, housing, managing, recording, taxes, etc. is less per unit of milk

from large cows than small. This conclusion is true but is complicated by present dairy practice described below.

It was explained in Research Bulletins 222 and 238 that there is no mechanical relation between body size and energetic efficiency; however, larger cows tend to be less efficient energetically than small as inferred from the following situation: Dairymen judge cows by their milk production level and not energetic efficiency. Thus 900 and 1100 pound cows are judged to be equally good if they produce the same amounts of milk (FCM), in spite of the fact that the 900-pound cow is more efficient if she produces as much milk as the 1100-pound cow. The smaller the cow the greater is her struggle for survival in the herd; the smaller the cow the more she must compensate for her small size by high energetic efficiency. In other words, our present system of judging cows by milk production level regardless of size tends to associate in selection high energetic efficiency with small size. It is entirely possible that the relatively superior overhead economies of large cows are on the average compensated by the tendency (for the reason just explained) of small cows to produce milk with higher energetic efficiency; so that *on the average* large and small cows may be equally profitable. This discussion leads to the conclusion that if energetic efficiency and other conditions are the same for small and large cows, large cows produce greater profit per unit FCM than small because the overhead expense (cost of milking, housing, taxes, etc.) per unit milk will be less for large cows. However, energetic efficiency and other conditions are not the same for large and small cows. Large cows tend to produce milk at lower efficiencies on account of selective breeding and for other reasons explained in Research Bulletins 222 and 238. Market and technical conditions are likewise influential. Thus as previously noted butter is produced more economically from small cows and milk from small cows gives a better cream line for market milk. For these and other reasons, applications of the generalization that large cows tend to return a larger net profit per unit milk than small, must be qualified by special considerations.

### III. METHODS FOR ESTIMATING ENERGETIC EFFICIENCY AND MONETARY PROFIT

This section presents a series of alignment or nomograph charts for estimating energetic efficiency and monetary profit on milk production.

The first four charts have similar designs, and may for convenience be described together. The known variables are live weight and milk (FCM) production; the desired unknown variables are respectively: (1) gross energetic efficiency of milk production; (2) profit per cow;

(3) profit per 1000 pounds live weight; (4) profit per 1000 pounds milk (FCM) production. The charts are composed of three axes so spaced and graduated that a straight line drawn through the points representing live weight and FCM production on the live weight and FCM axes, intersects the third, energetic efficiency or profit axes.

It is only necessary to describe the technique of using one of these charts, such as the gross energetic efficiency chart; the use of the other charts will then be clear without further explanation.

1. **Gross Energetic Efficiency:** Gross energetic efficiency is the ratio of milk energy produced to digestible feed energy consumed

TABLE 1.—TO EQUALIZE ENERGY IN MILK, CONVERT MILK TO GIVEN FAT PERCENTAGE TO "4 PER CENT MILK" BY MEANS OF TABLE 1.\*

A Per Cent Fat in Milk	B Factor for Con- verting to 4% Milk	A Per Cent Fat in Milk	B Factor for Con- verting to 4% Milk
2.5	0.775	5.0	1.150
2.6	0.790	5.1	1.165
2.7	0.805	5.2	1.180
2.8	0.820	5.3	1.195
2.9	0.835	5.4	1.210
3.0	0.850	5.5	1.225
3.1	0.865	5.6	1.240
3.2	0.880	5.7	1.255
3.3	0.895	5.8	1.270
3.4	0.910	5.9	1.285
3.5	0.925	6.0	1.300
3.6	0.940	6.1	1.315
3.7	0.955	6.2	1.330
3.8	0.970	6.3	1.345
3.9	0.985	6.4	1.360
4.0	1.000	6.5	1.375
4.1	1.015	6.6	1.390
4.2	1.030	6.7	1.405
4.3	1.045	6.8	1.420
4.4	1.060	6.9	1.435
4.5	1.075	7.0	1.450
4.6	1.090	7.1	1.465
4.7	1.105	7.2	1.480
4.8	1.120	7.3	1.495
4.9	1.135	7.4	1.510

Column A gives fat percentages, column B corresponding conversion factors, which when multiplied by pounds of milk produced, will convert the given milk to 4% milk. Thus if a cow produced 10,000 pounds of 3% milk multiply 10,000 by 0.850 and get the answer 8500 pounds of 4% milk. In other words 10,000 pounds of 3% milk contains the same amount of energy as 8500 pounds of 4% milk.

during production. Details of the method for measuring gross energetic efficiency are described in Missouri Station Bulletin 351. To determine the energetic efficiency of a cow, the first step is to convert the milk of the given fat per cent to 4% milk (FCM) by the method indicated in Table 1\* or Fig. 5\*. The exact procedure of using Table 1 and Fig. 5 are described in the respective legends.

\*Table 1 and Fig. 6 with the accompanying discussions, are taken from Missouri Station Bulletin 351. Fig. 5 is taken from Missouri Research Bulletin 222.

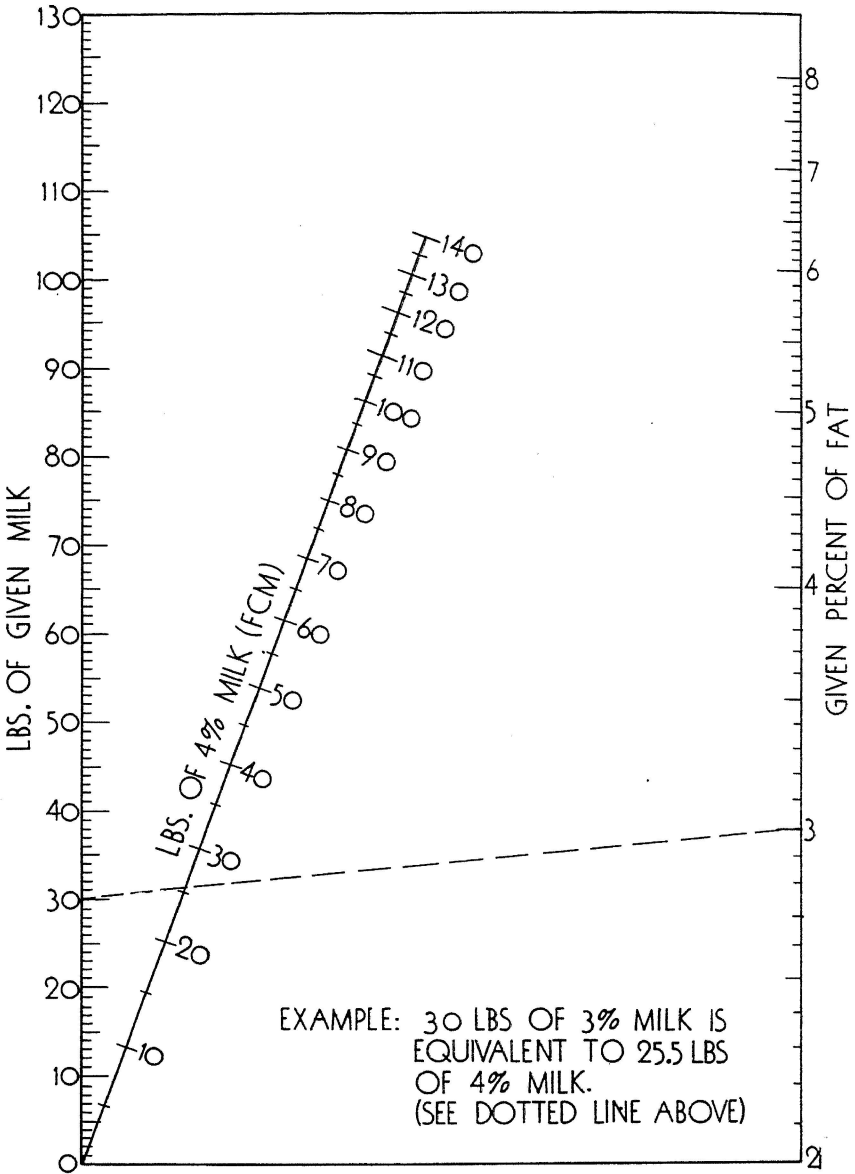


Fig. 5.—Nomograph for converting pounds of milk containing any per cent of fat to milk containing 4 per cent of fat. Thus to convert 30 pounds 3% milk to pounds 4% milk, stretch a string between 30 on the left scale and 3 on the right scale and read the answer (25.5 pounds 4% milk) on the middle scale. This nomograph was constructed from Gaines' well known formula, as was also the conversion table in Missouri Station Bulletin 351.

Knowing live weight and the amount of 4% milk produced, the energetic efficiency of milk production is then estimated from Fig. 6.

Figure 6 has three scales. The left scale shows production in pounds of 4% milk per year and per day; the right scale shows live-weight; and the center scale gives percentage efficiency. To find the efficiency of the 700-pound cow producing (in terms of 4% milk) 8500 pounds per year (an average of 23.3 pounds per day) place a straight edge (or stretch a string) across the chart between points 8500 (or 23.3) on the left (milk) scale and 700 on the right (body weight) scale, as shown by line (1). Line (1) cuts the center (efficiency) scale at  $32\frac{1}{2}$ , which is the percentage efficiency with which the 700 pound cow produced milk. To find the efficiency of the 1550-pound cow producing 9300 pounds per year (an average of 25.5 pounds per day), place the straight edge across points 9300 (or 25.5) on the left scale and 1550 on the right scale as shown by line (2). The center scale shows that the efficiency is 25 per cent.

It will be observed that the percentage scale in Fig. 6 is divided into three zones: 15 to 25 per cent zone, containing inferior producers; 25 to 35 per cent zone, containing good producers; 35 to 40 per cent zone, containing superior producers. The energetic efficiency of milk production is on the average about 30 per cent for good producing cows, the exact value varying with the milk yield.

It is instructive to recall in this connection the following conclusions listed in the preceding bulletins (Research Bulletins 222 and 238) that: energetic efficiency of milk production is independent of body weight (if other conditions are the same); energetic efficiency (with respect to TDN consumed) of good experiment station cows is of the order of 30%; the energetic efficiencies of cows exhibited at the St. Louis "World's Fair" were 34.3% for Holsteins, 33.6% for Jerseys, 29.6% for Shorthorns, 28% for Brown-Swiss; a group of superior Holstein cows at the Dixon, Illinois, Testing Plant produced milk at an efficiency of 34.3%; the highest gross energetic efficiency encountered in our studies (estimated with the nomograph, Fig. 6) was 47.5% for the 700-pound Jersey champion Stonehurst Patrician's Lily who produced 25,946 pounds FCM for the year.

While the 1700-pound 1936 Holstein champion, Carnation Ormsby Butter King "Daisy," produced 36,476 pounds FCM, that is, 40.6%  
 $(= \frac{36,476 - 25,946}{25,946} \times 100)$  more FCM than the 700-pound Jersey champion, yet the gross energetic efficiency of the Holstein was estimated to be only 43.5% because the Holstein weighed 143%  $(= \frac{1700 - 700}{700} \times 100)$  more than the Jersey, so that the Holstein expended

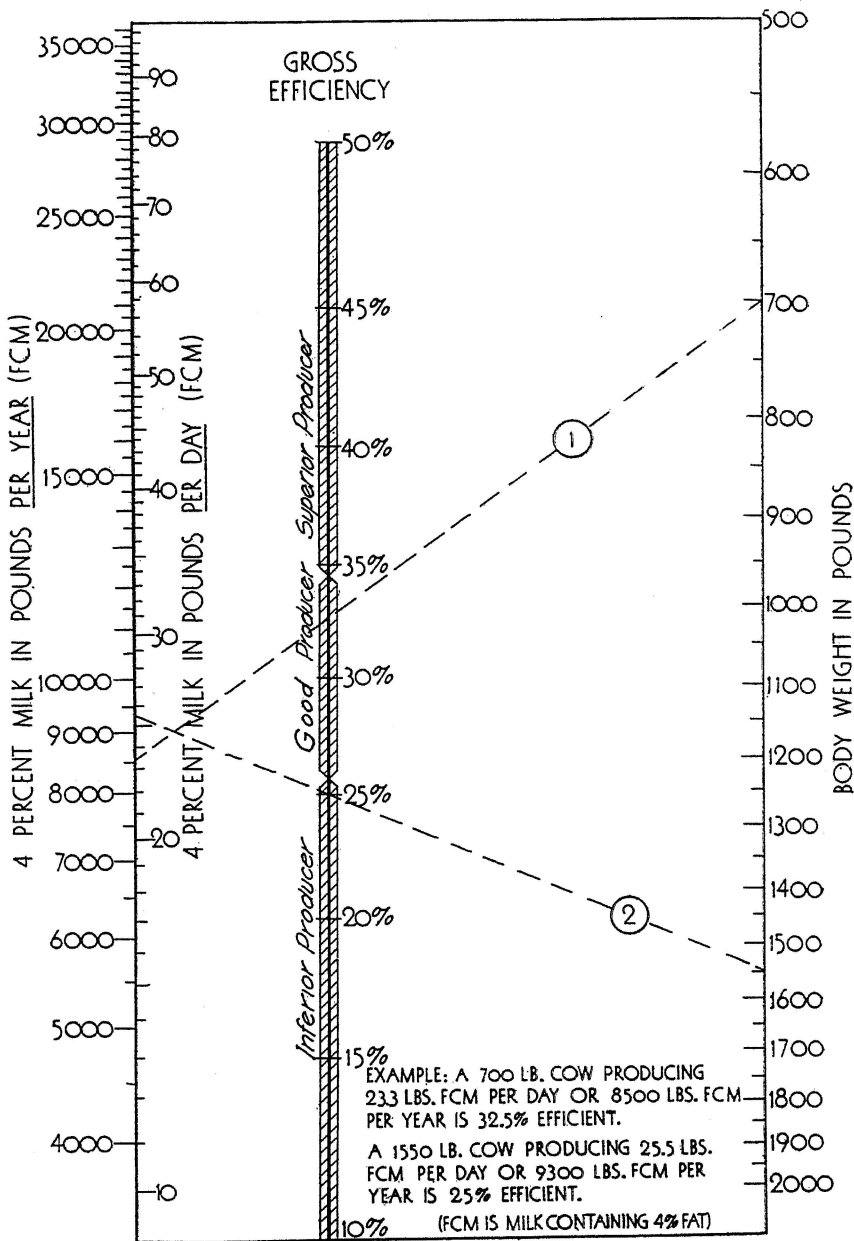


Fig. 6.—Nomograph for estimating gross energetic efficiency with which cows produce milk. First, the pounds of the given milk must be converted to pounds "FCM" (Gaines) that is, to milk containing 4% fat. The efficiency of milk production is then read from this chart. Thus if it is desired to find the efficiency of a 700-pound cow producing 8500 pounds yearly or on the average 23.3 pounds daily from FCM (4% milk), place a straight edge between 23.3 on the left (or milk) scale, and 700 on the right (or body-weight) scale, and read the answer 32.5 on the center (or efficiency) scale.



perhaps 90% more energy for maintenance than the smaller Jersey. The contrast between estimated production and efficiencies of the very small Jersey and the very large Holstein illustrates the fact that milk production level is by itself not a good measure of dairy value; that live-weight must be related with milk production in such manner as to give energetic efficiency in order to have a good estimate of a cow's dairy value. Fig. 6 makes it simple to estimate energetic efficiency, and progressive breeders of dairy cattle should select breeding animals primarily on the basis of the efficiency of individual cows by the aid of Fig. 6 rather than by milk production level. (See Missouri Station Bulletin 351.)

**2. Profit per Cow at a Given Milk and Feed Price:** As in the case of energetic efficiency, the first step is to convert milk of the given fat percentage to FCM by the method indicated in Table 1 or Fig. 5. Profit per cow is then estimated from Fig. 7 in exactly the same manner as was energetic efficiency from Fig. 6. Fig. 7 was prepared on the basis of the partition equation  $TDN=0.305 FCM+0.053 M^{0.73}+2.1\Delta M$  described in Missouri Research Bulletins 222 and 238, and on the assumption that the price of FCM is \$2.00 per 100 pounds and of TDN is \$.150 per 100 pounds.

It is again instructive to compare the 1700-pound Holstein champion Daisy with the 700-pound Jersey champion Lily. From Fig. 7 the line for Daisy connecting 1700 on the live weight scale, and 36,500, on the milk scale passes through 500 on the profit scale; that is, if Daisy's milk sold at \$2.00 per 100 pounds FCM, and if feed cost \$1.50 per 100 pounds TDN, the milk return above feed cost would be \$500 for the year. The line for Lily connecting 700 on the live-weight scale and 25,946 on the milk scale, gives for Lily a return of \$367 for the year above the assumed feed cost.

The fact that 1700-pound Daisy made a larger monetary return for the year than 700-pound Lily does not, as previously explained, mean that Daisy produced milk with a higher energetic efficiency than Lily. In fact the reverse is true: the energetic efficiency of Daisy is only 43.5%, as compared to that of Lily which is 47.5%. While the profit per cow was less for Lily than for Daisy, the energetic efficiency was less for Daisy than Lily. Since the profit *per pound* of milk is nearly the same in large and small cows therefore the cow which produces more milk pounds naturally makes the greater profit. This conclusion is obvious, and serves as another illustration of the fact that energetic efficiency and profitableness *per cow* may differ depending on live weight, as illustrated in Figs. 1 and 2.

**3. Profit per Unit Live Weight of Cow at Given Milk and Feed Prices:** The procedure for estimating profit per unit live

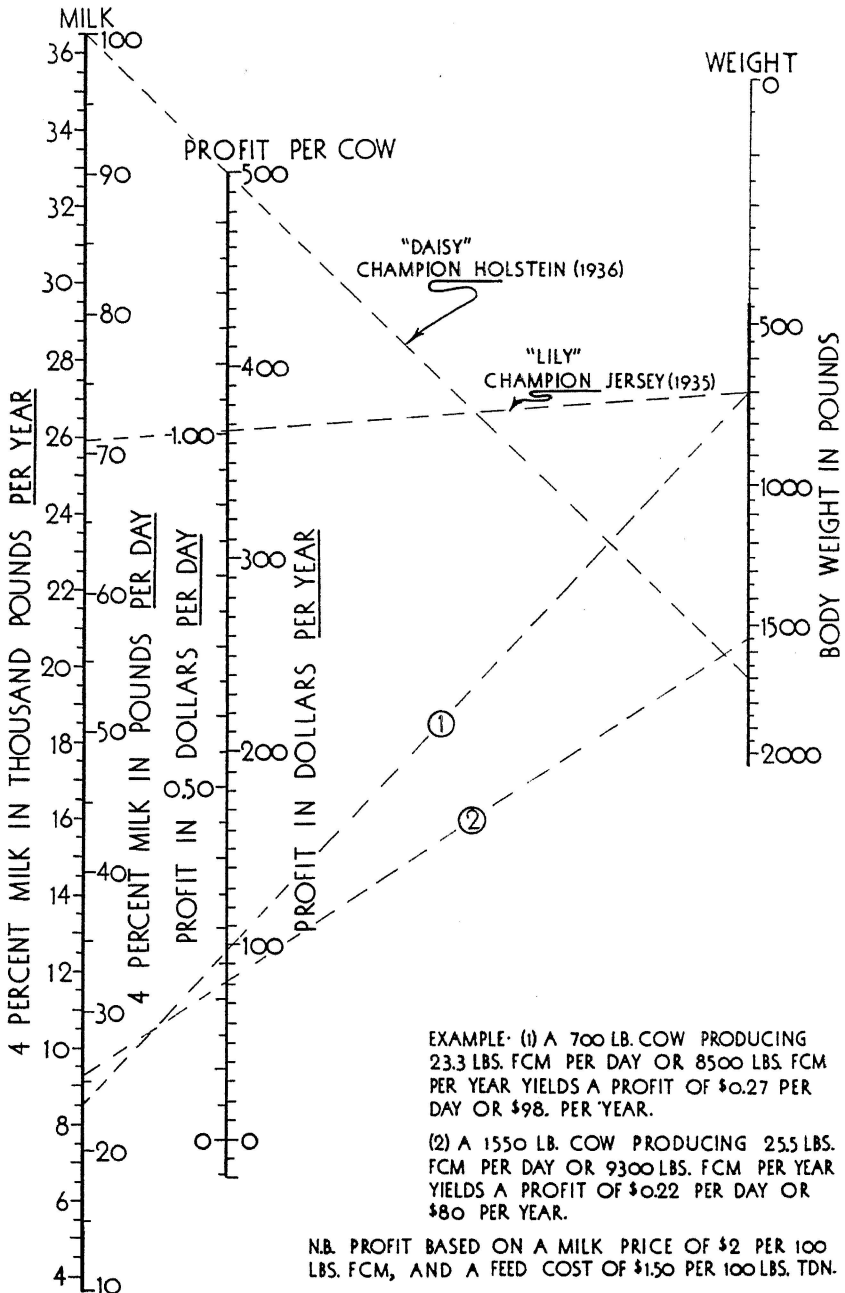


Fig. 7.—Nomograph for estimating profit per cow from body weight and milk production (FCM) assuming that price of milk is \$2.00 per 100 pounds FCM and of feed \$1.50 per 100 pounds TDN. Broken lines (1) and (2) indicate the solutions of examples (1) and (2) on the chart. The profits for Daisy and Lily, discussed in the text, are also indicated by broken lines, as labelled. This nomograph is derived from equation (13) of the Appendix.

weight from Fig. 8 is exactly the same as for estimating profit per cow from Fig. 7.

The profit line for the 1700-pound Holstein champion, Daisy, and for the 700-pound champion, Lily, are also shown in Fig. 8. From Fig. 7, the profit *per cow* is \$500 per year for Daisy, and \$367 for Lily; but from Fig. 8, the profit *per 1000 pounds live weight* is only \$292 for Daisy and \$522 for Lily. The profit *per cow* is  $\frac{500-367}{367} \times 100 = 36\%$  greater for Daisy than for Lily, but profit *per 1000 pounds live weight* is  $\frac{522-292}{292} \times 100 = 79\%$  greater for Lily than for Daisy. Which of the two ways of estimating profit is the better for indicating dairy qualities of a cow, profit per cow, or profit per unit live weight? Is Lily or Daisy the better dairy animal? We believe that neither profit per cow nor profit per unit live weight is a good measure of dairy quality, but energetic efficiency as estimated from Fig. 6; or profit per unit milk production which tends to be proportional to energetic efficiency.

**4. Profit per Unit Milk (FCM) Production at Given Milk and Feed Prices:** Figure 9 presents a nomograph for estimating profit per 1000 pounds FCM. Here again we compare the relative performances of 1700-pound Daisy with 700-pound Lily. It is estimated on a *per cow* basis that Daisy returned 36% greater profit than Lily; on a *per 1000-pound body weight* basis, Lily returned 79% greater profit than Daisy; on a *per 1000-pound milk (FCM)* basis, Lily returned  $\frac{1415-1365}{1365} \times 100 = 4\%$  greater profit than Daisy. Lily's gross en-

ergetic efficiency is  $\frac{47.5-43.5}{43.5} \times 100 = 9\%$  higher than Daisy's. It is

therefore obvious that profit per unit milk production is a better index of a cow's gross energetic efficiency than either profit per cow or profit per 1000 pounds live weight. To summarize: on a *per cow* basis Daisy brought in a 36% greater profit than Lily; on a *per 1000-pound live weight* basis, Lily brought a 79% greater profit than Daisy; on a *per 1000-pound milk (FCM)* basis, Lily brought in a 4% greater profit than Daisy; the gross energetic efficiency of Lily is 9% greater than Daisy's. All figures are estimates made on the basis of the nomograph presented in Figs. 6 to 9. Is large Daisy or small Lily the better *dairy* cow? Is large Daisy or small Lily the better *commercial* cow? Under what conditions is one *commercially* superior to the other? Under what conditions is one *genetically* or *biologically* superior to the other?

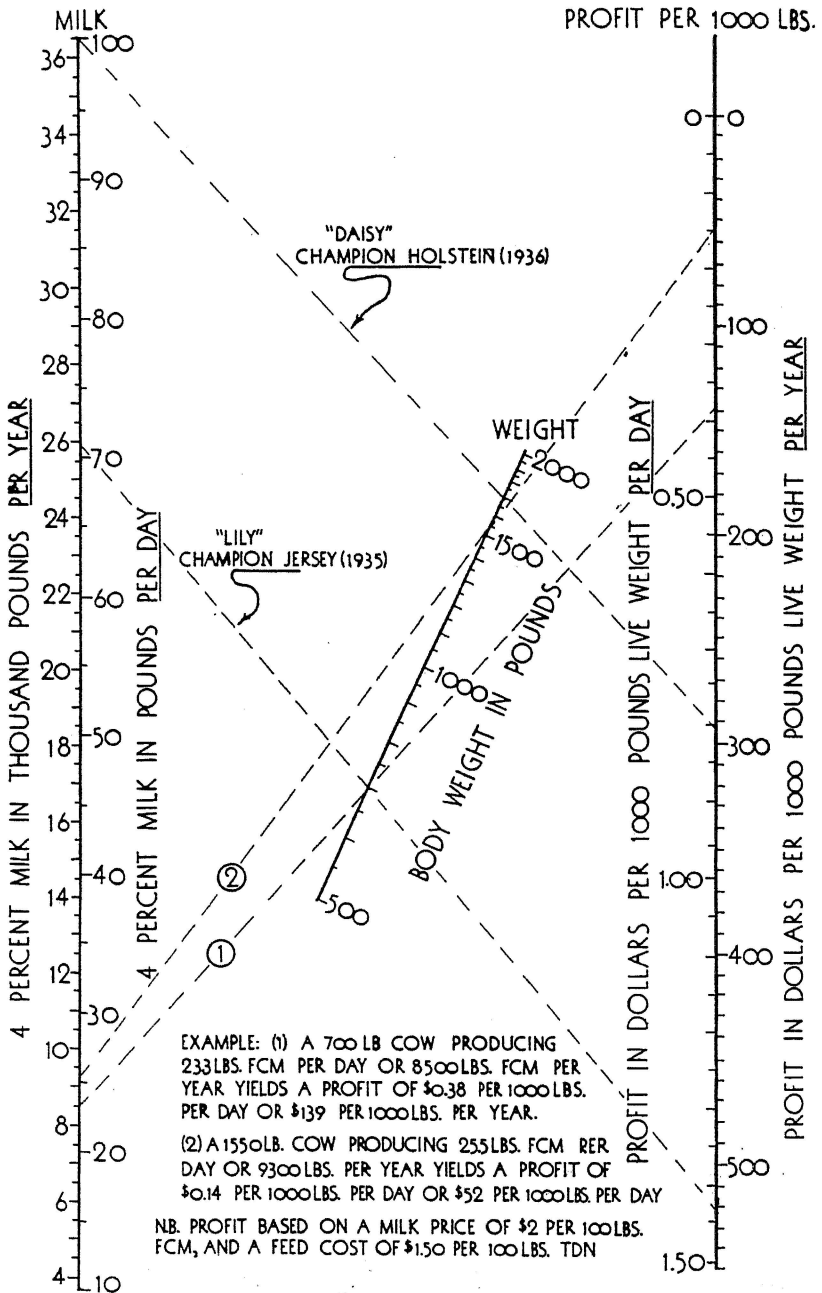


Fig. 8.—Nomograph for estimating profit per 1000-pounds live weight. See text and legend for Fig. 7 for method of reading chart. This chart is derived from equation (15) of the Appendix.

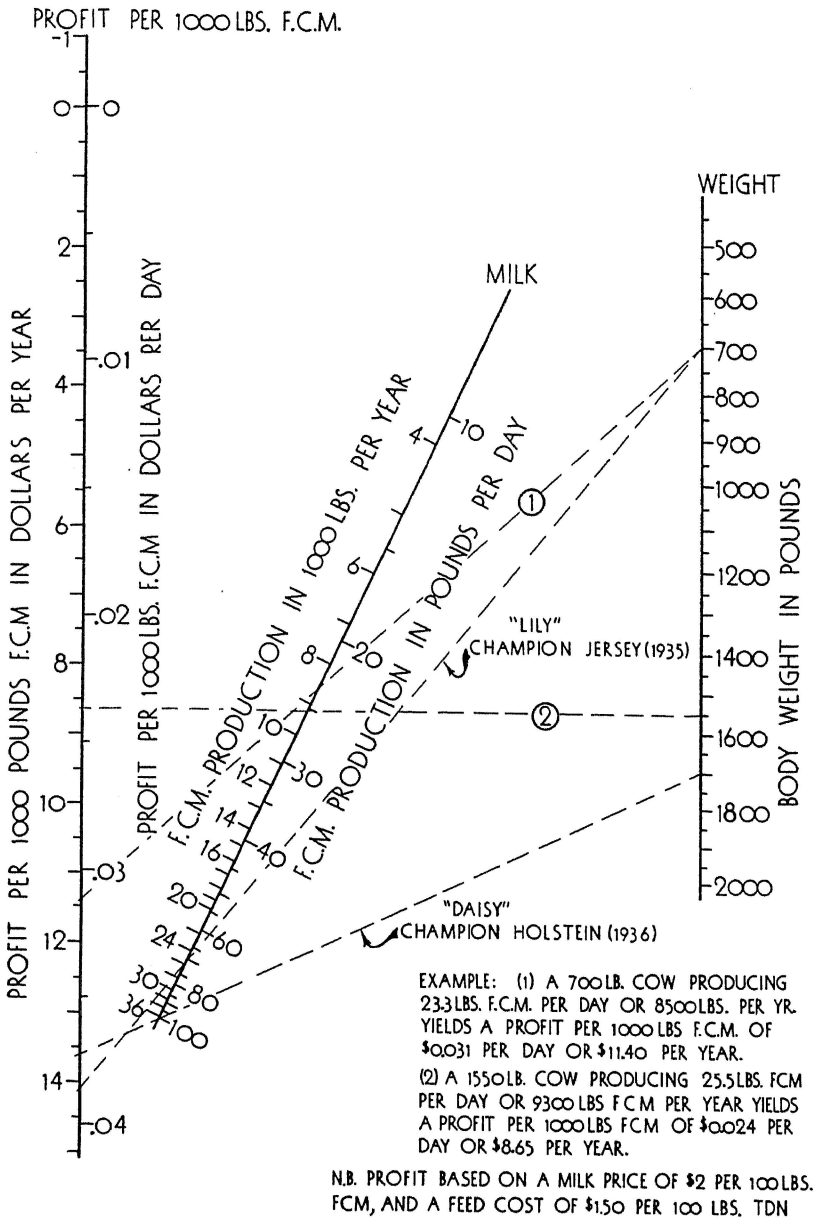
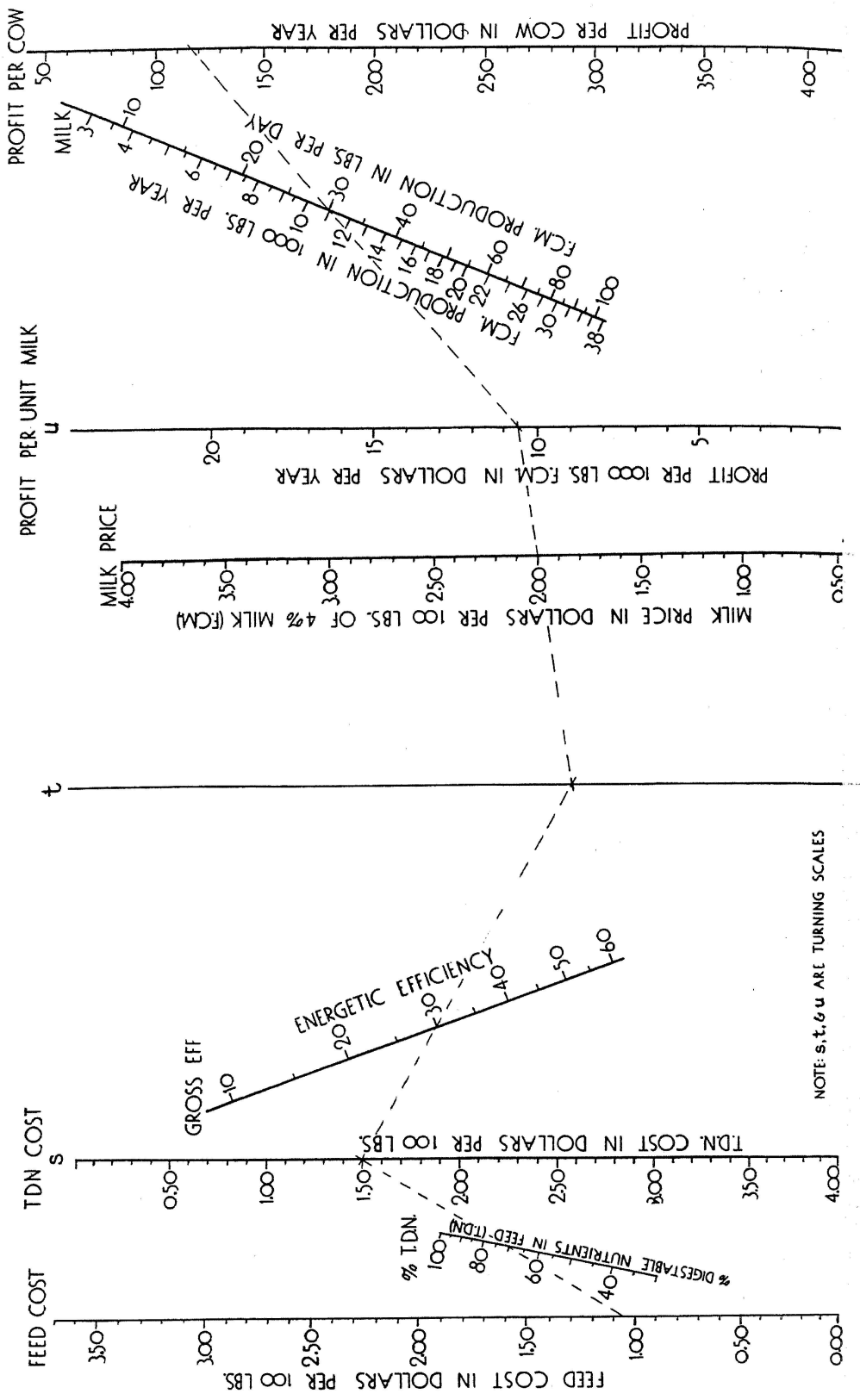


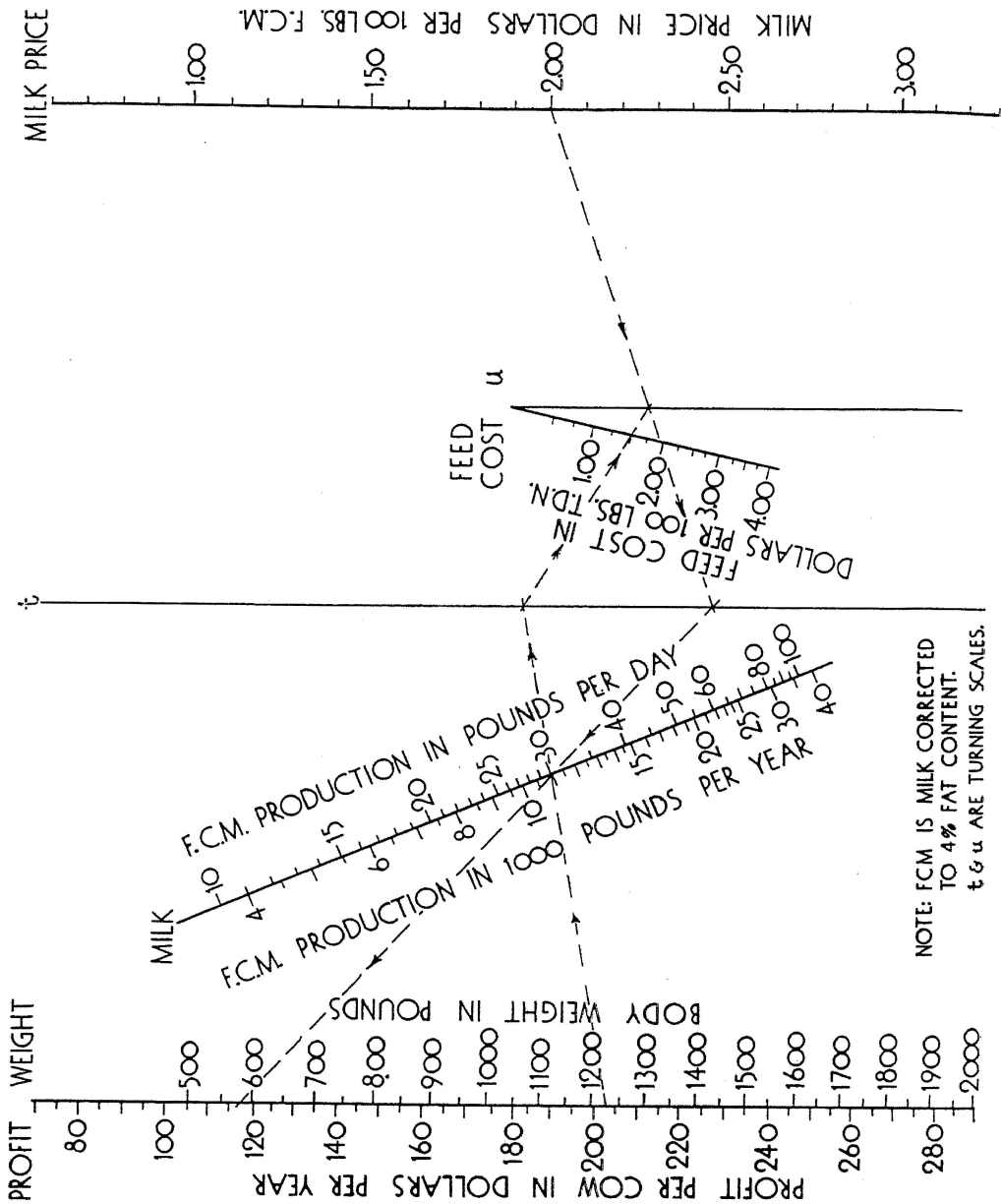
Fig. 9.—Nomograph for estimating profit per 1000-pound milk (FCM). As in Figs. 7 and 8, solutions of two examples are indicated, and a comparison is shown between the profits of the Holstein and Jersey champions. This nomograph is derived from equation (17) of the Appendix.



NOTE: s, t, & u ARE TURNING SCALES

**EXAMPLE:** Feeding a ration, costing \$1.05 per 100 lbs. and yielding 70% T. D. N., to a cow producing 30 lbs. F. C. M. per day at a gross efficiency of 30% and selling her milk at \$2.00 per 100 lbs. F. C. M., returns a profit of \$117 per year or \$10.50 per 1000 lbs. F. C. M.

Fig. 10.—Nomograph for estimating profit per 1000 pounds FCM and per cow from milk (FCM) and feed prices, and from energetic efficiency. The broken line indicates the solution of the example on the chart. Begin on the left side with feed cost (\$1.05 per 100 pounds feed); connect with percentage TDN in feed (70% TDN); extrapolate to the TDN cost axis which gives cost per unit TDN (\$1.50 per 100 pounds TDN). Connect \$1.50 on the TDN cost axis with gross energetic efficiency (30%); extrapolate to line t, connect the point on line t with milk price (assumed to be \$2.00 per 100 pounds FCM) and extrapolate to the u line which gives profit per unit milk (\$10.50 profit per 1000 pounds FCM). Connect point on u line with milk (FCM) line (cow assumed to produce 30 pounds FCM per day) and extrapolate to profit line, indicating a profit of \$117 per year, the answer. The derivation of this nomograph is given in the Appendix, equations 4 - 7.



NOTE: FCM IS MILK CORRECTED TO 4% FAT CONTENT.  
t & u ARE TURNING SCALES.



**EXAMPLE:** A 1225 lb. cow producing 30 lbs. FCM per day is fed a ration costing \$1.50 per 100 lbs. TDN, selling her milk at \$2.00 per 100 lbs. F.C.M., returns a profit of \$117 per year.

Fig. 11.—Another method for estimating profit per cow. The solution of the example on the chart is indicated by the directed broken lines. See text for details. See Appendix equations 11 and 18-21 for the derivation of the chart.

5. **Profits per Cow at Variable Milk and Feed Prices:** Figures 7 to 9 were prepared on the assumption that the price of milk was \$2.00 per 100 pounds FCM, and of feed, \$1.50 per 100 pounds TDN. Fig. 10 on the other hand indicates profit per cow and profit per 1000 pounds milk (FCM) for any price of feed and milk. The broken line in Fig. 10 indicates the solution of the example given at the bottom of the chart. The example gives cost of feed (\$1.05 per 100 pounds); percentage TDN in feed (70%); energetic efficiency of the cow (30%); price of milk (\$2.00 per 100 pounds FCM); milk production (30 pounds FCM per day). What is the profit per year (profit in the sense of difference between income for milk and expense for feed)? The broken line in Fig. 10 indicates the method of solving this problem. Connect \$1.05 (price of feed) on feed-cost axis with 70 (% TDN in feed) on % TDN axis and extrapolate to the TDN cost axis. Connect \$1.50 on the TDN-cost axis with 30%, on the energetic efficiency axis and extrapolate to line t; connect the point on t with \$2.00, on the milk-price axis and extrapolate to line u where it intersects \$10.50, which is profit per 1000 pounds FCM. Connect point \$10.50 with 30 on the milk production axis, and extrapolate to the profit per cow axis where it intersects at \$117, which is the profit per cow per year.

Figure 11 presents another method of estimating profit per cow per year. The directed broken curves in Fig. 11 indicate the method of solving the example given on the chart. Connect 1225 (weight of cow) on the body weight axis with 30 (FCM yield) on the the FCM-yield axis and extrapolate to line t. Connect this point with 1.50 (cost per 100 pounds TDN) on the feed cost axis and extrapolate to line u. Connect 2.00 (price per 100 pounds FCM) on the milk-price axis with the point on line u intersected by the line extrapolated from the feed cost axis, and continue to line t. Connect this point on line t with 30 (FCM production of cow) on the FCM production line, and extrapolate to the profit axis, where it cuts point 117, profit in dollars per year, which is the answer to the problem given on the chart.

#### IV. SUMMARY AND CONCLUSIONS

Two preceding bulletins (Missouri Research Bulletins 222 and 238) analyzed milk production data from the standpoint of energetic efficiency with special reference to the influence of live weight thereon; the present bulletin is an extension of the analysis to include monetary profit.

The present bulletin on monetary profit analyzes the following questions; Should profit be computed per cow, per unit live weight (such as per 1000-pounds live weight), per unit milk produced (such as per 1000 pounds FCM), or per unit feed consumed? What is the influence of live weight on each of these varieties of profit? What is

the influence of gross energetic efficiency on each of these varieties of profit? What is the influence of overhead expenses and related factors on each of these varieties of profit (such factors as: cost of milking, housing, managing, taxes, investment, market demands, limitation of market for milk, limitation of supply of feed, tendency for selective breeding for small cows of high efficiency and large cows of low efficiency)?

The analysis led to the following conclusions: 1. Profit on a per cow basis increases with increasing size of cow. 2. Profit on a per body weight basis (as per 1000 pounds live weight) decreases with increasing size. 3. Profit on a per unit milk basis (as per 1000 pounds FCM) tends to be independent of size and almost directly proportional to gross energetic efficiency of milk production. 4. Profit on a per unit feed consumption basis tends to be independent of size and almost directly proportional to gross energetic efficiency.

If gross energetic efficiency is the same, large cows are more profitable than small because large cows produce more milk, and because there is less expense in producing a unit milk from a large than a small cow. (Milking, housing, taxes, etc. are nearly the same for a large as for a small cow, and therefore less per unit milk for the large cow than the small.) But as matter of fact gross energetic efficiency under present dairy practice tends to be higher in small cows than in large not for physiological reasons, but because of a tendency for selective breeding of small cows of high energetic efficiency and large cows of small efficiency as previously explained.

Gross energetic efficiency of milk production, that is ratio of energy in milk produced to energy in digestible nutrients consumed, is unquestionably the only proper index of physiologic dairy value. Using this index it was shown that the production of 25,946 pounds FCM by the 700-pound Jersey champion Lily is a considerably superior performance to the production of 36,476 pounds FCM by the 1700-pound Holstein champion Daisy.

Alignment charts (nomographs) are presented for estimating gross energetic efficiency and each of the several kinds of profit (per cow, per unit body weight, per unit milk production). However, "profit" was defined as return for milk above cost of feed. The costs of milking, housing, taxes, etc. were not considered in the preparation of the nomographs.

The important lesson of this bulletin is that milk production by itself is a poor index of either commercial or physiologic value of a cow. Gross energetic efficiency, which takes account of both milk production and live weight, is the best measure of physiological dairy value; profit for the entire dairy enterprise (not necessarily per cow or per unit body weight, or per unit milk) is the best measure of commercial value.

## V. APPENDIX

## 1. Algebraic Solution of the Profit Equations and Their Application to Alignment Charts

The following list of symbols will be used throughout:

- $P$  = profit per cow in dollars per day  
 $FCM$  = milk corrected to 4% fat content in pounds per day  
 $TDN$  = total digestible nutrient of feed in pounds per day  
 $M$  = body weight of cow in pounds  
 $E_g$  = gross energetic efficiency in per cent  
 $E_m$  = monetary efficiency in per cent  
 $k$  = proportionality constant (ratio of energy in 1 pound of FCM to energy in 1 pound of TDN)  
 $c_1$  = milk price per pound FCM ( $N_1 = 100 c_1$ )  
 $c_2$  = feed cost per pound TDN ( $N_2 = 100 c_2$ )

In this treatment only milk and feed are considered. The simplest statement of profit is:

$$P = c_1 (FCM) - c_2 (TDN). \quad (1)$$

by definition

$$E_g = k \frac{FCM}{TDN} \%, \quad (2a)$$

or

$$TDN = k \frac{FCM}{E_g}. \quad (2b)$$

Substituting (2b) in (1) we get

$$P = c_1 (FCM) - c_2 k \frac{FCM}{E_g}, \quad (3a)$$

or

$$P = FCM \left\{ c_1 - \frac{c_2 k}{E_g} \right\}. \quad (3b)$$

Dividing both sides of (3a) or (3b) by FCM gives the profit per unit milk, which may be written

$$\frac{P}{FCM} = c_1 - \frac{c_2 k}{E_g}. \quad (4)$$

Letting  $\frac{P}{FCM} = Y$ ,  $c_1 = Z$ , and  $\frac{c_2 k}{E_g} = X$ , equation (4) may be written

$$Y = Z - X \quad \text{or} \quad X + Y = Z \quad (5)$$

which is the general equation for the parallel line alignment chart. (see Appendix 2, section A)

In the above relation  $X$  and  $Y$  are complex functions which may be solved by alignment charts by making further substitutions. Letting

$P = A$  and  $FCM = B$ , the relation is

$$Y = \frac{P}{FCM} = \frac{A}{B}, \quad \text{or} \quad B \cdot Y = A. \quad (6)$$

Equation (6) is the general equation of a "Z" chart which may be laid out with Y on one of the parallel axes. (see Appendix 2, section B)

By substituting  $c_2k = D$  and  $E_g = E$ , X becomes:

$$X = \frac{c_2k}{E_g} = \frac{D}{E} \quad \text{or} \quad E \cdot X = D. \quad (7)$$

Equation (7) is also the general form of a "Z" chart in which X is on one of the parallel axes.

Equations (5), (6), and (7), representing three alignment charts, are combined into a single diagram (Fig. 10) as indicated in Fig. 12.

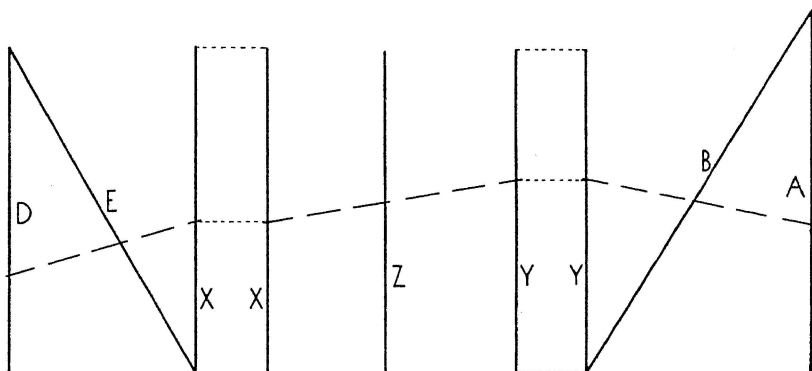


Fig. 12.—Schematic diagram indicating the component nomograms of Fig. 10. Axes connected by dotted lines are identical in the complete chart.

The scales are so arranged that axes connected by dotted lines are identical and coincident in the final chart.

In case the feed cost is in terms of pounds of *feed* then the cost of TDN will be  $c_2 = \frac{\text{cost of feed}}{\% \text{ TDN in feed}} \times 100$ . For convenience in computing this cost another "Z" chart is included in Fig. 10 such that the axis D ( $= c_2k$ ) is computed from cost of feed and % TDN.

Summarizing, equation (4) is stated in terms of general functions to illustrate how the general equations of the alignment charts (given in the following section) are applied to a specific problem in the form of Fig. 10.

To study the effect of body weight on the profit it is necessary to refer to the partition equation discussed previously (Bulletins 222 and 238),

$$\text{TDN} = .305 \text{ FCM} + .053 \text{ M}^{0.73}.$$

Substituting this equation in (1):

$$P = c_1 \text{ FCM} - c_2 (.305 \text{ FCM}) + .053 \text{ M}^{0.73}, \quad (8)$$

collecting terms

$$P = (c_1 - .305 c_2) \text{ FCM} - .053 c_2 \text{ M}^{0.73}. \quad (9)$$

By dividing both sides of equation (9) by the body weight the relation is the profit per unit weight:

$$\frac{P}{M} = (c_1 - .305 c_2) \frac{FCM}{M} = .053 c_2 \frac{M^{0.73}}{M}. \quad (10)$$

To find the profit per unit milk, divide both sides of equation (9) by FCM giving

$$\frac{P}{FCM} = (c_1 - .305 c_2) - \frac{.053 c_2 M^{0.73}}{FCM}. \quad (11)$$

To show the effect of weight on profit, other things remaining the same, assume the prices fixed at \$1.50 per 100 pounds TDN and \$2.00 per 100 pounds FCM. Substituting these prices in equation (9) the profit per day becomes

$$P = (.0200 - .305 \times .0150) FCM - .053 \times .0150 M^{0.73}. \quad (12)$$

Multiplying by 365 gives the profit in dollars per year:

$$P_y = 5.63 FCM - 0.290 M^{0.73}. \quad (13)$$

Similarly equation (10) becomes,

$$\frac{P_y}{M} = 5.63 \frac{FCM}{M} - 0.290 M^{-.27}. \quad (14)$$

Multiplying by 1000 gives the profit per 1000 pounds live weight in dollars per year,

$$\text{Profit per 1000 pounds wt.} = P_m = 5630 \frac{FCM}{M} - 290 M^{-.27}. \quad (15)$$

Also, equation (11) becomes,

$$\frac{P_y}{FCM} = 5.63 - 0.290 \frac{M^{0.73}}{FCM}. \quad (16)$$

Multiplying by  $\frac{1000}{365} = 2.74$  gives the profit per 1000 pounds annual milk production in dollars per year,

$$\text{Profit per 1000 pounds FCM} = P_F = 15.42 - .795 \frac{M^{0.73}}{FCM}. \quad (17)$$

The conversion of equations (13), (15), and (17) into forms corresponding to general equations of alignment charts may be accomplished in single steps. In the case of (13), let  $P_y = X$ ,  $5.63 FCM = Z$ , and  $.290 M^{0.73} = Y$ , then equation (13) becomes,

$$X = Z - Y \quad \text{or} \quad X + Y = Z$$

which is the equation for a parallel line chart. In equation (15) let

$-P_m = X$ ,  $5630 FCM = Y$ ,  $\frac{1}{M} = Z_1$  and  $290 M^{-.27} = Z_2$ . Equation

(15) becomes,

$$-X = Y \cdot Z_1 - Z_2$$

or by rearranging terms and changing signs

$$X + Y \cdot Z_1 = Z_2$$

which is the general equation of the straight and curved line chart. (see Appendix 2, section C.)

Equation (17) may be rearranged to read,

$$- (P_F - 15.42) = -.795 \frac{M^{0.73}}{FCM}$$

Letting  $-(P_F - 15.42) = Y$ ,  $FCM = X$ , and  $.795 M^{0.73} = Z$ , equation (17) becomes,

$$- Y = - \frac{Z}{X} \quad \text{or} \quad X \cdot Y = Z$$

which is the equation for a "Z" type chart.

Thus, by referring to the partition equation and assuming prices fixed, it is possible to devise simple, three axis charts which show the relative profitableness of different weights and productions. Although it is realized that the actual values may vary widely, due to market conditions, nevertheless, the most profitable cow under one set of prices will be the most profitable under any other, where the milk prices are based on FCM equivalents.

To devise a chart to estimate the profitableness of a cow which will include prices as variables as well as body weight and milk production, it is necessary to refer back to equation (11). Letting  $P = X$ ,  $FCM = Y$ , and  $[c_1 - .305 c_2 - .053 c_2 \frac{M^{0.73}}{FCM}] = Z$  equation (11) becomes,

$$\frac{X}{Y} = Z \quad \text{or} \quad X = Y Z \quad (18)$$

which may be solved by a "Z" type chart. It is obvious, though, that the function, Z, is very complex, requiring several steps to complete. The value of Z may be rearranged, becoming,

$$Z = c_1 - c_2 (.305 + .053 \frac{M^{0.73}}{FCM}). \quad (19)$$

$$\text{or } Z = c_1 - c_2 D.$$

Letting  $.053 M^{0.73} = A$  and  $FCM = B$  the terms inside the parenthesis, denoted by D, may be solved by a "Z" type chart. The relation now reduces to

$$Z = c_1 - c_2 D \quad \text{or} \quad Z = c_1 - E \quad (20)$$

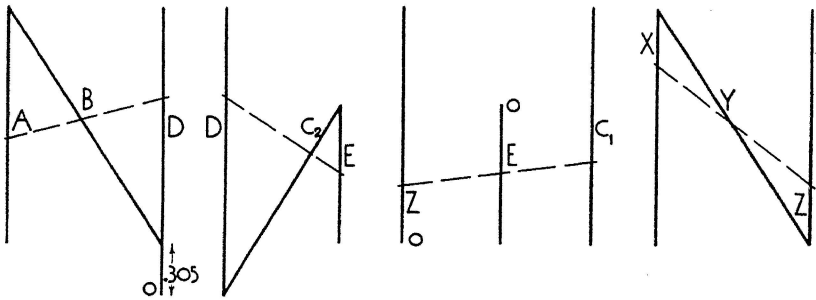
where E may also be solved by a "Z" type chart, since  $E = c_2 D$ .

Equation (20) is recognized as the general form of the parallel line chart when it is rearranged to read

$$Z + E = c_1. \quad (21)$$

Having thus determined the value of Z by a series of three "Z" type charts and one parallel line chart, the ultimate solution follows from

equation (18). The combination of these charts into a single diagram (Fig. 11), although straightforward in principle, is rather complex. The diagram below indicates this combination.



THE COMBINED CHART BECOMES:

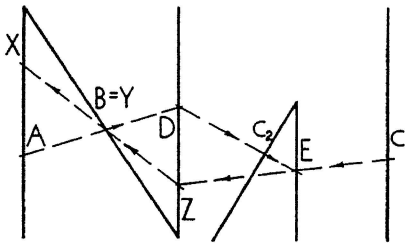


Fig. 13.—Schematic diagram indicating the component nomograms of Fig. 11. The bottom half shows the combination as used for Fig. 11.

The letters refer to the functions as assumed in the discussion above. Since the values of D, E, and Z are of no importance to the reader the axes upon which they appear become turning lines and no scale is plotted.

**2. Geometric Proof of Alignment Chart Equations**

To justify the use of the various types of alignment charts it is necessary to prove, by the methods of plane geometry, that these charts give the solutions to general equations. The proofs are included here merely for the convenience of the reader in checking the work or in preparing similar charts.

*A. Parallel Line Chart*

Figure 14 illustrates the general case of the parallel line chart consisting of three parallel axes (AX, BY, and CZ) cut by a transversal (UV). The variables x, y, and z are measured, as indicated, from A, B, and C. Drawing DV and EW parallel to AB forms two similar triangles giving the following relation:

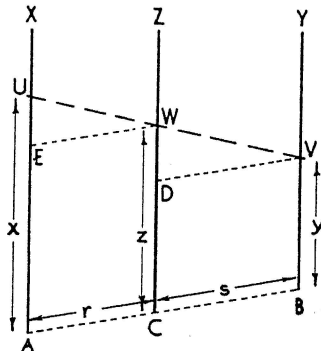


Fig. 14.



$$UE : WD = EW : DV \tag{22}$$

let  $EW = r$  and  $DV = s$   
 then substituting equivalents

$$(x - z) : (z - y) = r : s \tag{23}$$

or by solving the proportion and separating variables

$$xs + yr = z(r + s). \tag{24}$$

Dividing by  $r \cdot s$ ,

$$\frac{x}{r} + \frac{y}{s} = z \frac{r + s}{rs} \tag{25}$$

If  $X, Y,$  and  $Z$  represent functions of three variables and these functions are plotted on the axes such that

$$X = \frac{x}{r}; \quad Y = \frac{y}{s}; \quad \text{and } Z = z \frac{r + s}{rs},$$

then  $X + Y = Z$  is solved by this chart. (26)

**B. "N" or "Z" Type Chart**

Figure 15 shows the "N" or "Z" type chart in which the axes  $AX$  and  $BY$  are parallel with the third axis  $AZ$  at any angle. These three axes are cut by a transversal at  $U, V,$  and  $W$ . The variables  $x, y,$  and  $z$  are measured from the intersections of the axes ( $A$  and  $B$ ), as indicated. By similar triangles:

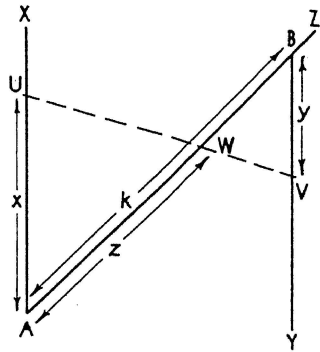


Fig. 15.

$$AU : BV = AW : BW. \tag{27}$$

Let  $AB = k$ , then by substituting identities:

$$x : y = z : (k - z) \tag{28}$$

solving

$$x(k - z) = yz \tag{29}$$

or

$$x = y \cdot \frac{z}{k - z} \tag{30}$$

If  $X, Y,$  and  $Z$  represent functions of three variables and these functions are plotted on the axes such that

$$x = \theta_1 X \quad \text{and} \quad y = \theta_2 Y$$

where  $\theta_1$  and  $\theta_2$  are proportionality constants, which add to the convenience in plotting  $X$  and  $Y$  (i. e., they may be plotted to any scale).

By substitution

$$\Theta_1 X = \Theta_2 Y \frac{z}{k - z} \tag{31}$$

or 
$$X = Y \frac{\Theta_2 z}{\Theta_1 (k - z)}.$$

Letting 
$$Z = \frac{\Theta_2 z}{\Theta_1 (k - z)} \quad \text{or} \quad z = \frac{k \Theta_1 Z}{\Theta_1 Z + \Theta_2} \tag{32}$$

then 
$$X = Y \cdot Z. \tag{33}$$

This, then, is the simplified general equation of the "Z" type chart.

C. *Straight and Curved Line Chart*

Figure 16 illustrates the straight and curved line chart in which axes AX and BY are parallel straight lines and CZ is any curved intermediate axis. These axes are cut by the transversal UV. Referring to the figure: DW is parallel to AX and BY. EW and FV are parallel to AB. Let AU = x, BV = y, AB = k, EW = AD = s, and DW = r. By similar triangles

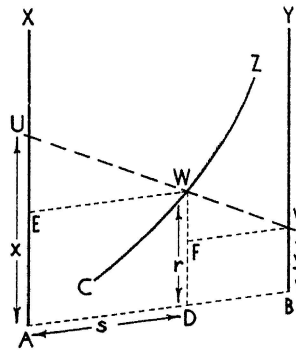


Fig. 16.

$$UE : WF = EW : FV. \tag{34}$$

or by substituting the identities above,

$$(x - r) : (r - y) = s : (k - s). \tag{35}$$

Solving,

$$(k - s)(x - r) = s(r - y) \tag{36}$$

or 
$$x(k - s) + ys = rs + r(k - s) \tag{37}$$

and 
$$x(k - s) + ys = kr. \tag{38}$$

Dividing by (k - s),

$$x + y \frac{s}{k - s} = \frac{kr}{k - s}. \tag{39}$$

Then if  $x = \Theta_1 X$ ,  $y = \Theta_2 Y$ ,  $\frac{s}{k - s} = \frac{\Theta_1}{\Theta_2} Z_1$ , and  $\frac{kr}{k - s} = \Theta_1 Z_2$

( $\Theta_1$  and  $\Theta_2$  are proportionality constants depending upon the scales used) and substituting identities,

$$\Theta_1 X + \Theta_2 Y \frac{\Theta_1}{\Theta_2} Z_1 = \Theta_1 Z_2 \tag{40}$$

or 
$$X + YZ_1 = Z_2. \tag{41}$$

Thus, this type chart solves an equation involving functions of X and Y and two functions of Z ( $Z_1$  and  $Z_2$ ).

These three proofs then lead to the solutions of any equation involving three variables where the functions of these three variables may be written in the forms:

- (1)  $X + Y = Z$
- (2)  $X = YZ$
- (3)  $X + YZ_1 = Z_2$ .

### 3. Slide-Rule for Estimating Efficiency and Profitableness of Milk Production

Instead of using page nomographs for estimating efficiency and profit of milk production, it is possible to make such estimates from a more compact slide rule as illustrated in Fig. 17a.

A brief outline will serve to illustrate the method of designing the slide-rule scales. The equation giving energetic efficiency of milk production is:

$$E = \frac{18.74}{0.0535M^{0.73} \left( 0.305 + \frac{\text{FCM}}{\text{FCM}} \right)} \quad (\text{see page 22 Res. Bul. 222})$$

Taking the reciprocal

$$\frac{1}{E} = \frac{.305}{18.74} + \frac{0.0535M^{0.73}}{18.74 \text{ FCM}}$$

or

$$\left( \frac{1}{E} - .305 \right) = \frac{0.0535M^{0.73}}{18.74 \text{ FCM}}$$

By taking logarithms of both sides the linear relation is obtained,

$$\log \left( \frac{1}{E} - .01628 \right) = \log 0.0535M^{0.73} - \log 18.74 \text{ FCM}.$$

This may be written

$$\Theta_3 = \Theta_1 - \Theta_2$$

by substituting term for term as indicated. In this form the equation yields to solution by the circular rule.

Values of  $\Theta_1$  were computed for each value of M desired on the nomogram; similarly  $\Theta_2$  and  $\Theta_3$  values were computed for FCM and E respectively. In the particular lay-out shown, a unit of logarithm equals  $259^\circ$ . For convenience in plotting the scales the initial value of  $\Theta$  was taken as zero by subtracting it from succeeding values. The scales were plotted with arbitrary origins for weight and FCM. However, the efficiency scale must be placed so that the value of 10% is displaced  $97^\circ$  clockwise from the value of 4000 pounds FCM per year. With the efficiency scale in such a position the index pointer on the top card (see

efficiency slide of Fig. 17a) will be diametrically opposite 1000 pounds body weight.

Table 2 gives the angular displacement in degrees for scale values as plotted in Fig. 17.

Knowing the energetic efficiency of milk production, and feed and milk prices, the profit per unit milk may be gotten indirectly by a second circular slide rule, which may be printed on the reverse side of the first one. This slide rule is shown in Fig. 17b, and was prepared as follows:

$$\text{Energetic Efficiency} = E = 18.74 \frac{\text{FCM}}{\text{TDN}}$$

$$\text{Cost of feed} = c_2 \text{ TDN}$$

$$\text{Return on milk} = c_1 \text{ FCM.}$$

If the milk return equals the feed cost then

$$c_2 \text{ TDN} = c_1 \text{ FCM,}$$

or 
$$\frac{c_2}{c_1} = \frac{\text{FCM}}{\text{TDN}}$$

But

$$\frac{\text{FCM}}{\text{TDN}} = \frac{E}{18.74}$$

therefore,

$$\frac{c_2}{c_1} = \frac{E}{18.74}.$$

Taking logarithms of both sides,

$$\log c_2 - \log c_1 = \log \frac{E}{18.74}.$$

Let

$$\theta_1 - \theta_2 = \theta_3$$

be the corresponding slide-rule equation and proceed as before.

Suppose the question to be, "If I feed a cow a ration costing  $N_2$  dollars per 100 pounds of TDN and she produces milk with an energetic efficiency of  $E\%$ , what price must I get per 100 pounds FCM for her milk to pay for the feed?" The equation and the slide-rule answer this question. Thus, the difference between the price you are getting for milk and the price indicated by the slide-rule is the profit per 100 pounds FCM (from the standpoint of feed and milk only).

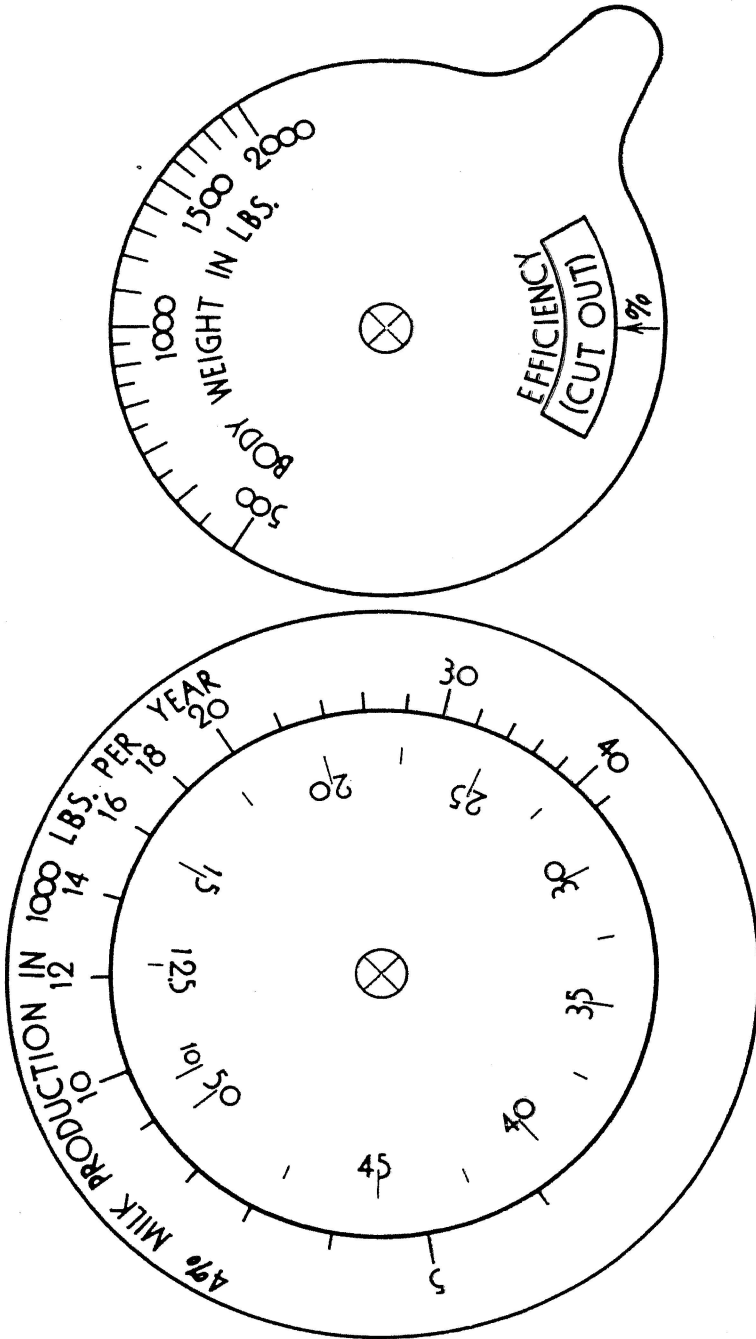
Table 3 gives the angular displacement in degrees for scale values as plotted in Fig. 17b.

TABLE 2.—ANGULAR DISPLACEMENTS IN DEGREES FOR SCALE VALUES

Body Weight in Pounds M	Angle in degrees $\Theta_1$	4% Milk production in 1000 lbs./year FCM	Angle in degrees $\Theta_2$	Gross efficiency E %	Angle in degrees $\Theta_3$
500	0	4	0	10	0
550	9	5	25	12.5	31
600	16	6	45	15	57
650	22	7	63	17.5	81
700	28	8	78	20	103
750	34	9	91	22.5	122
800	39	10	103	25	142
850	44	12	124	27.5	160
900	48	14	141	30	179
950	53	16	156	32.5	197
1000	57	18	169	35	216
1100	65	20	181	37.5	235
1200	73	22	192	40	255
1300	79	24	202	42.5	275
1400	85	26	210	45	298
1500	91	28	219	47.5	322
1600	96	30	227	50	351
1700	100	32	234		
1800	105	34	241		
1900	110	36	247		
2000	114	38	253		
		40	259		
		42	265		

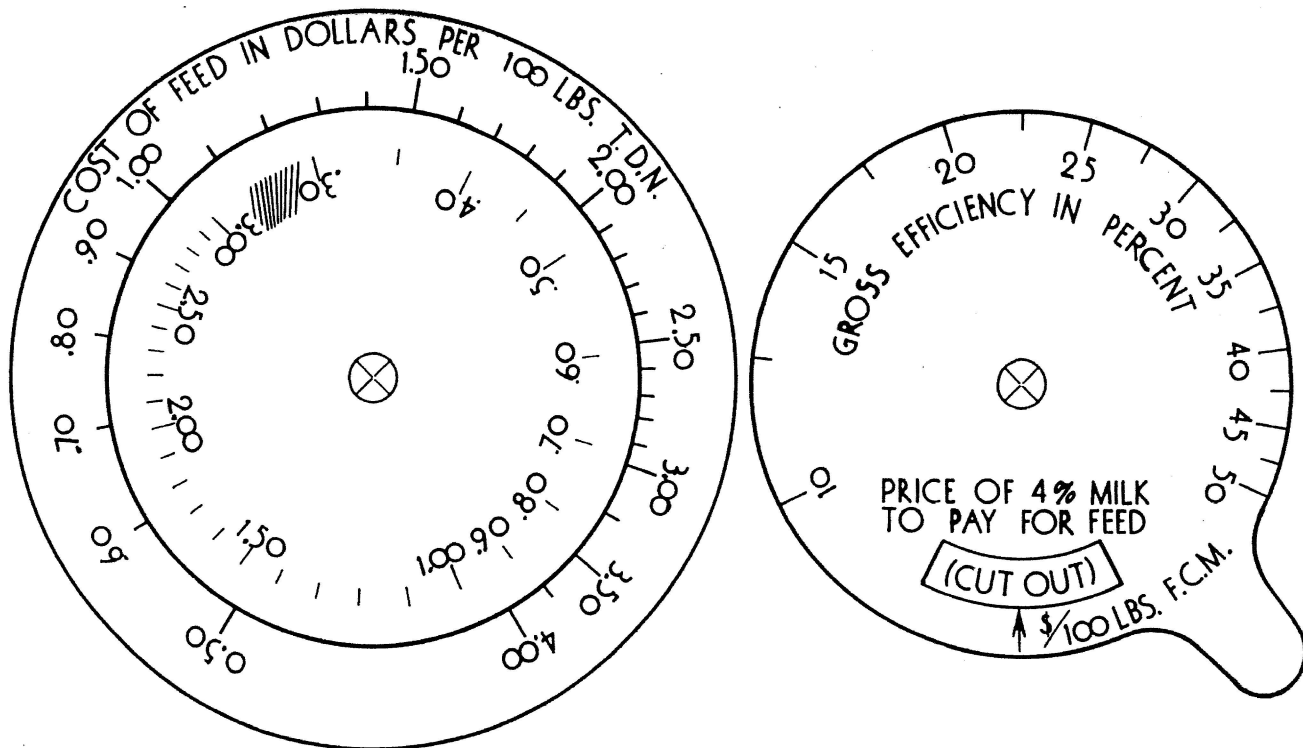
TABLE 3.—ANGULAR DISPLACEMENT IN DEGREES FOR SCALE VALUES

Prices in dollars per 100 lbs. $N_1$ or $N_2$	Angle in degrees $\Theta_1$	Gross efficiency %	Angle in degrees $\Theta_3$
0.30	-73	10	0
0.35	-51	12.5	32
0.40	-32	15	58
0.45	-15	17.5	80
0.50	0	20	99
0.60	26	22.5	116
0.70	48	25	131
0.80	68	27.5	145
0.90	84	30	157
1.00	99	32.5	169
1.10	113	35	180
1.20	126	37.5	189
1.30	137	40	198
1.40	148	42.5	207
1.50	158	45	215
1.60	167	47.5	223
1.70	176	50	231
1.80	184		
1.90	191		
2.00	199		
2.10	206		
2.20	212		
2.30	219		
2.40	225		
2.50	231		
2.60	236		
2.70	242		
2.80	247		
2.90	252		
3.00	257		
3.25	268		
3.50	279		
3.75	289		
4.00	298		



A — SLIDE RULE FOR ESTIMATING EFFICIENCY OF MILK PRODUCTION

Fig. 17A.



B—SLIDE RULE FOR ESTIMATING MINIMUM MILK PRICE

Fig. 17B.

#### 4. Individual Records on Which this Bulletin and Missouri Research Bulletin 238 Are Based

##### I. LOUISIANA PURCHASE "WORLD'S FAIR" DATA

Average Live wt. lbs.	Feed TDN lbs./day	Milk FCM lbs./day	Live wt. gain lbs./day	Gross Energetic Effic.		Profit per year, dollars*		
				Not corrected for wt. gain	Corrected for wt. gain	per cow	per 1000 lbs. live wt.	per 1000 lbs. FCM
<b>Brown Swiss, 5 cows</b>								
1181	23.97	36.14	.550	28.3	29.0	132	112	10.05
1248	28.31	46.46	.617	30.8	31.6	184	147	10.85
1367	29.16	42.58	.417	27.4	27.8	151	110	9.73
1384	27.23	37.34	1.225	25.7	27.1	124	90	9.07
1467	30.24	44.93	.567	27.8	28.5	162	110	9.92
<b>Holsteins, 15 cows</b>								
1167	25.76	43.72	.967	31.7	33.3	178	153	11.15
1225	25.51	43.54	.700	32.0	33.1	178	145	11.20
1236	26.53	48.82	1.392	34.5	36.8	211	171	11.84
1239	26.54	45.85	.750	32.4	33.5	189	153	11.32
1247	27.51	52.36	.567	35.7	36.6	232	186	12.38
1252	26.59	50.11	.842	35.3	36.7	220	176	12.03
1264	27.01	49.46	.725	34.3	35.4	213	169	11.81
1283	26.19	51.78	.717	37.1	38.3	235	183	12.41
1286	26.04	45.50	.975	32.8	34.3	190	148	11.42
1292	27.55	62.45	.450	42.5	43.3	305	236	13.37
1327	26.77	48.65	.967	34.1	35.6	208	157	11.75
1343	27.14	55.15	.767	38.1	39.4	254	189	12.63
1344	26.99	45.73	.308	31.8	32.2	186	138	11.15
1398	26.24	47.59	.992	34.0	35.6	204	146	11.73
1438	27.08	41.50	1.225	28.7	30.4	155	108	10.22
<b>Jerseys, 25 cows</b>								
782	23.82	41.98	.458	33.0	33.8	176	225	11.48
786	22.58	42.80	.758	31.4	37.0	189	240	12.08
868	23.92	42.26	.842	33.1	34.6	178	205	11.51
883	25.14	47.72	.633	34.2	36.7	211	239	12.11
884	24.06	43.28	.358	33.7	34.3	184	217	11.67
899	26.09	50.77	.683	36.5	37.6	228	254	12.30
902	25.46	50.46	.425	37.2	37.9	229	254	12.44
914	25.66	42.30	.753	30.9	32.0	168	184	10.90
914	26.04	47.48	.292	34.2	34.6	204	223	11.78
927	22.98	38.35	.483	31.3	32.1	154	166	11.01
928	24.29	39.88	.658	30.8	31.8	158	170	10.88
930	23.30	41.34	.425	33.2	34.0	174	187	11.53
936	23.32	41.07	.558	33.0	34.0	172	184	11.48
961	24.63	46.61	.742	35.5	36.8	205	213	12.08
974	24.49	46.18	.542	35.3	36.3	203	208	12.05
976	26.46	48.62	.525	34.4	35.3	210	215	11.84
984	25.35	47.79	.617	35.3	36.4	210	213	12.05
990	25.56	47.35	.675	34.7	35.8	206	208	11.89
1002	25.80	47.56	.608	34.6	35.5	206	206	11.86
1036	26.54	50.61	.583	35.7	36.7	224	216	12.14
1036	27.47	54.16	.642	37.0	38.0	245	236	12.38
1046	27.28	39.94	.708	29.6	30.6	153	146	10.49
1047	25.86	45.78	.533	33.2	34.0	193	184	11.53
1050	25.95	50.57	.642	36.5	37.6	227	216	12.30
1090	27.27	44.85	.850	30.8	32.0	178	163	10.88
<b>Shorthorns, 28 cows</b>								
1014	19.90	33.62	.775	31.7	33.2	136	134	11.12
1093	16.10	21.07	1.167	24.5	26.8	66	60	8.55
1094	18.86	38.53	.625	38.4	39.8	178	163	12.66
1100	21.28	43.40	1.158	38.2	40.9	200	182	12.66
1146	22.46	32.38	1.067	27.0	28.6	113	99	9.59
1172	19.57	34.79	1.092	33.3	35.7	147	125	11.56
1184	20.17	37.53	.725	34.9	36.4	164	139	11.94
1186	20.06	35.36	.942	33.0	35.0	148	125	11.48
1196	21.67	33.46	1.050	28.9	30.7	126	105	10.27
1206	19.15	32.98	1.067	32.3	34.6	136	113	11.29
1214	20.14	24.05	1.525	22.4	24.6	65	54	7.42
1214	20.63	31.36	.992	28.5	30.2	116	96	10.14
1223	17.64	26.94	1.100	28.6	28.8	100	82	10.16
1226	22.16	32.49	1.058	27.5	29.1	116	95	9.78
1230	21.45	32.58	.950	28.5	30.0	120	98	10.14

\*Assuming that price of feed is \$1.50 per 100 lbs. TDN; price of milk is \$2.00 per 100 lbs. FCM.



Average Live wt. lbs.	Feed TDN lbs./day	Milk FCM lbs./day	Livewt. gain lbs./day	Gross Energetic Effic.		Profit per year, dollars		
				Not corrected for wt. gain	Corrected for wt. gain	per cow	per 1000 lbs. live wt.	pers 1000 lbs. FCM
M 1286	18.81	27.92	.667	27.8	29.0	101	79	9.89
1289	19.86	33.71	.383	31.8	32.6	137	106	11.15
1297	23.44	31.55	.917	25.2	26.5	102	79	8.85
1306	20.48	33.05	1.058	30.2	32.2	129	99	10.71
1314	20.89	32.76	.817	29.4	30.8	125	95	10.44
1326	18.70	35.40	.342	35.5	36.3	156	118	12.08
1329	20.86	21.11	1.950	19.0	21.4	40	30	5.18
1380	23.91	34.72	1.192	27.2	28.9	122	88	9.67
1386	20.85	28.97	.825	26.0	27.3	97	70	9.20
1398	25.62	40.60	.642	29.7	30.6	156	112	10.55
1405	22.85	35.88	.850	29.4	30.8	137	98	10.44
1408	20.58	30.69	.667	27.9	29.1	111	79	9.94
1442	21.28	34.79	1.158	30.6	32.8	137	95	10.82

## II. PENNSYLVANIA (FORBES) HOLSTEINS, 10 cows

867	16.02	32.53	-.056	38.0	37.8	149	172	12.55
1008	16.55	30.56	.036	34.7	34.7	132	131	11.84
1042	19.99	39.71	.259	37.3	38.0	181	174	12.49
1073	18.87	36.08	.205	35.8	36.4	160	149	12.14
1092	19.76	35.51	-.056	33.7	33.5	151	138	11.64
1102	20.61	34.59	.652	31.5	33.0	140	127	11.10
1198	21.55	39.35	.580	34.3	35.7	169	141	11.75
1207	19.14	34.94	.154	34.3	34.6	150	124	11.75
1308	20.66	33.62	.542	30.5	31.7	132	101	10.77
1408	24.39	45.13	-.165	34.7	34.3	195	138	11.83

## III. DIXON, LLINOIS, TESTING PLANT HOLSTEINS, 42 COWS

889	26.60	43.55	.370	30.7	30.8	172	194	10.82
952	25.12	43.68	.192	32.6	32.7	181	190	11.34
976	25.86	38.68	-.110	28.0	28.0	141	144	10.00
1086	23.15	41.31	.520	33.4	33.7	175	161	11.62
1126	26.44	47.02	.358	33.3	33.5	198	176	11.53
1161	27.71	52.47	.233	35.5	35.6	231	199	12.05
1194	26.41	42.69	-.356	30.3	30.2	167	140	10.71
1198	29.92	67.22	.096	42.1	42.2	327	273	13.32
1210	28.65	67.82	.110	44.4	44.4	338	279	13.64
1211	26.61	43.01	.000	30.3	30.3	168	139	10.71
1220	29.99	61.43	.000	38.4	38.4	284	233	12.66
1221	27.32	41.02	.559	28.1	28.3	150	123	10.03
1229	27.98	53.54	-.301	35.9	35.7	237	193	12.14
1233	29.44	47.02	-.301	33.2	33.3	182	148	10.60
1245	28.13	50.16	-.068	33.4	33.4	212	170	11.59
1247	27.08	44.70	.206	30.9	31.0	178	143	10.90
1265	28.31	55.31	.274	36.6	36.7	249	197	12.33
1271	28.95	55.59	.247	36.0	36.1	247	194	12.16
1272	28.75	52.51	-.333	34.2	34.1	226	178	11.78
1291	26.94	53.84	-.329	37.5	37.3	246	191	12.52
1297	27.26	50.86	.301	35.0	35.1	222	171	11.95
1312	28.18	43.90	-.274	29.2	29.1	166	127	10.36
1330	28.46	53.25	.000	35.1	35.1	233	175	12.00
1331	25.85	45.64	.315	33.1	33.2	192	144	11.53
1340	26.10	53.39	-1.041	38.4	37.8	247	184	12.68
1347	27.76	59.93	.109	40.5	40.5	286	212	13.07
1350	25.48	41.87	.247	30.8	30.9	166	123	10.85
1354	27.11	54.25	.055	37.5	37.5	248	183	12.52
1358	29.22	56.81	-.192	36.4	36.4	255	188	12.30
1360	29.50	56.85	.041	36.1	36.1	253	186	12.19
1425	26.19	42.64	.137	30.5	30.6	168	118	10.79
1448	30.17	56.66	-.269	35.2	35.1	248	171	12.00
1464	27.65	46.91	.167	31.8	31.9	191	130	11.15
1469	30.68	64.59	-.110	39.3	39.3	303	206	12.85
1514	28.62	57.28	-.055	37.5	37.5	261	172	12.49
1531	23.78	36.42	.548	28.7	28.9	136	89	10.22
1538	28.97	53.02	.055	34.3	34.3	228	148	11.78
1566	28.73	56.29	.205	36.7	36.8	254	162	12.36
1609	30.34	50.28	.208	31.1	31.1	201	125	10.96
1660	27.99	39.95	.192	26.8	26.8	138	83	9.45
1683	29.41	61.35	.137	39.1	39.2	287	171	12.82
1748	31.94	58.39	.274	34.3	34.4	251	144	11.78

## IV. EXPERIMENT STATION DATA

Average Live wt. lbs.	Feed TDN lbs./day	Milk FCM lbs./day	Live wt. gain lbs./day	Gross Energetic Effic.		Profit per year, dollars		
				Not corrected for wt. gain	Corrected for wt. gain	per cow	per 1000 lbs. live wt.	per 1000 lbs. FCM
<b>Haecker's Guernseys, 16</b>								
765	12.62	16.55	.196	24.6	25.4	52	68	8.60
766	13.85	22.66	.050	30.7	30.9	90	117	10.88
782	12.50	21.86	.102	32.8	33.4	91	116	11.40
790	13.81	24.73	.008	27.6	27.6	84	98	9.84
801	12.39	22.67	-.133	34.3	33.5	98	122	11.84
840	17.19	29.83	.029	32.5	33.7	124	148	11.40
843	13.51	26.95	-.240	37.4	36.0	123	146	12.49
855	15.94	23.43	.008	27.6	27.6	84	98	9.84
864	16.95	30.38	-.109	33.6	33.1	129	149	11.64
887	13.98	29.68	-.488	39.8	37.0	140	158	12.93
909	17.97	32.79	-.082	34.2	33.9	141	155	11.78
925	15.08	22.27	-.060	27.7	27.4	80	86	9.84
947	15.29	20.13	-.095	24.7	24.4	63	67	8.58
996	15.78	21.07	.103	25.0	25.4	67	67	8.71
998	20.53	37.62	-.191	34.3	33.7	162	162	11.81
1019	17.37	26.28	.143	28.4	28.9	97	95	10.11
<b>Haecker's Holsteins, 27</b>								
781	15.08	22.49	.077	27.9	28.2	82	105	10.00
817	14.14	22.69	.095	30.1	30.5	88	108	10.63
844	14.60	29.64	-.429	38.0	35.8	136	161	12.58
872	15.97	17.27	.165	20.3	20.7	39	45	6.19
877	17.35	29.35	-.089	31.7	31.4	119	136	11.10
880	16.86	35.17	.050	39.1	39.4	164	186	12.77
887	13.42	30.15	.278	42.1	40.3	147	166	13.37
888	14.80	28.55	-.230	36.1	35.0	127	143	12.19
890	18.34	37.83	-.319	38.7	37.3	176	198	12.74
906	17.54	32.25	-.071	35.5	35.2	139	153	11.81
919	16.22	27.79	-.214	32.1	31.2	114	124	11.23
923	16.47	31.43	-.286	35.8	34.5	139	151	12.11
975	16.37	23.51	.191	26.9	27.6	82	84	9.56
976	18.81	26.65	.223	26.6	27.2	92	94	9.45
978	19.15	32.29	-.196	31.6	30.9	131	134	11.12
985	19.88	36.99	-.120	34.9	34.4	161	163	11.92
1086	18.75	27.69	.143	27.7	28.1	99	91	9.81
1095	18.44	29.83	-.042	30.2	30.2	117	107	10.74
1106	15.13	20.88	.326	25.9	26.7	70	63	9.18
1114	19.74	28.25	.161	26.8	27.3	98	88	9.51
1117	18.65	28.30	.113	28.4	28.8	104	93	10.05
1128	18.40	25.41	.238	25.9	26.6	85	75	9.18
1137	20.23	24.71	.326	22.9	23.4	70	62	7.75
1273	18.18	30.57	-.137	31.5	31.0	124	97	11.12
1292	19.66	36.09	.117	34.4	34.8	156	121	11.84
1298	20.07	30.41	.192	28.4	29.0	112	86	10.80
1315	18.64	31.84	.321	32.0	33.2	130	99	11.18
<b>Haecker's Jerseys, 55</b>								
634	14.45	28.47	-.137	36.9	36.2	129	203	12.41
683	11.10	18.65	-.053	31.5	31.2	75	110	11.01
706	14.66	25.48	-.286	32.6	31.3	106	150	11.40
734	12.51	27.48	-.010	41.2	41.4	132	180	13.15
735	12.57	20.79	-.034	31.0	31.2	83	113	10.93
735	14.64	22.60	.238	28.9	30.0	85	116	10.30
752	12.84	21.78	-.060	31.8	31.5	89	118	11.21
752	14.21	22.85	.006	30.1	30.1	89	118	10.66
754	13.78	23.49	-.027	31.9	31.8	96	127	11.21
758	15.14	27.59	-.114	34.1	33.6	119	158	11.81
760	11.79	16.34	.008	26.0	26.0	55	72	9.23
778	13.09	31.60	-.893	45.2	39.5	159	204	13.78
782	15.93	28.85	.137	33.9	34.6	123	157	11.67
783	12.49	19.35	-.080	29.0	28.6	67	86	9.48
789	13.45	24.09	.202	33.6	34.7	102	129	11.59
792	14.50	27.77	-.398	35.9	33.9	123	155	12.14
793	14.27	23.63	.115	31.0	31.6	94	119	10.90
796	12.54	25.51	-.092	38.1	37.5	118	148	12.68
800	14.10	28.56	-.095	38.0	37.4	131	164	12.58
809	14.17	30.90	-.414	40.9	38.5	148	183	13.12
809	15.31	25.41	.048	31.1	31.3	102	129	11.59
811	13.75	24.06	-.333	32.8	31.2	100	123	11.40
811	15.29	29.39	-.184	36.0	35.1	131	162	12.22

Average Live wt. lbs.	Feed TDN lbs./day	Milk FCM lbs./day	Live wt. gain lbs./day	Gross Energetic Effic.		Profit per year, Dollars		
				Not corrected for wt. gain	Corrected for wt. gain	per cow	per 1000 lbs. live wt.	per 1000 lbs. FCM
817	12.74	25.65	-.364	37.7	35.6	117	143	12.49
817	13.73	27.02	-.060	36.9	36.5	122	149	12.38
820	13.96	22.53	.113	30.3	30.8	88	107	10.71
820	15.08	18.47	.196	23.0	23.6	52	63	7.84
824	16.60	26.67	-.036	30.1	30.0	104	126	10.68
833	16.21	30.04	-.161	34.7	34.0	131	157	11.95
836	13.96	28.39	-.143	38.1	37.3	131	157	12.63
838	13.56	26.21	-.097	36.2	35.7	117	140	12.22
843	14.94	21.07	-.143	26.4	27.0	72	85	9.37
845	13.61	23.15	-.010	31.9	31.8	94	111	11.12
849	16.85	30.14	-.225	33.5	32.6	128	151	11.64
854	14.50	21.09	.017	27.3	27.3	75	88	9.75
855	16.01	29.25	-.025	34.2	34.1	126	147	11.81
856	15.27	23.50	-.080	28.8	28.5	88	103	10.25
858	13.40	23.12	-.500	32.3	30.0	95	111	11.26
859	12.28	25.95	-.311	39.6	37.6	122	142	12.88
861	14.85	24.34	.101	30.7	31.2	96	111	10.79
863	14.96	24.65	.018	30.9	30.8	98	114	10.90
867	14.68	25.64	-.214	32.7	31.8	107	123	11.42
876	13.85	24.01	-.082	32.5	32.1	99	113	11.29
887	15.44	22.05	-.103	26.8	26.4	76	86	9.45
890	15.16	23.04	.071	28.5	28.8	85	96	10.11
898	14.78	23.77	-.131	30.1	29.6	93	104	10.71
902	16.24	30.87	-.036	35.6	35.5	136	151	12.08
910	18.65	36.95	.024	37.1	37.2	168	185	12.47
915	13.70	17.99	.036	24.6	24.7	56	61	8.52
935	17.63	31.33	-.074	33.3	33.0	132	141	11.53
936	18.62	28.77	-.113	28.9	28.6	108	115	10.27
989	15.91	26.06	-.250	30.7	31.8	103	104	10.82
1013	15.81	23.18	-.009	27.5	27.4	83	82	9.81
1046	14.28	20.43	.148	26.8	27.4	71	68	9.53
1071	13.05	15.41	.077	22.1	22.4	41	38	7.29
<b>Eckles' Holsteins, 2</b>								
1056	20.03	34.26	.000	32.1	32.1	140	133	11.21
1319	18.13	29.89	.019	30.9	29.9	119	90	10.90
<b>Eckles' Jerseys, 5</b>								
807	15.47	24.75	.041	30.0	30.1	96	119	10.63
824	14.91	22.75	.137	28.6	29.2	84	102	10.11
899	16.42	28.65	.044	32.7	32.9	119	132	11.37
902	9.22	10.46	.049	21.3	21.5	26	29	6.77
952	15.03	21.73	.000	27.1	27.1	76	80	9.59
<b>Savage's Guernseys, 2</b>								
846	16.05	24.42	.200	28.5	29.3	90	106	10.14
1072	19.27	32.74	.514	31.8	33.8	134	125	11.21
<b>Savage's Holsteins, 14</b>								
985	21.20	35.81	.695	31.7	34.0	145	147	11.10
990	21.21	36.87	.086	32.6	32.9	153	155	11.37
1035	20.94	29.71	.838	26.6	29.1	102	99	9.40
1053	22.86	42.53	.295	34.9	35.9	185	176	11.92
1054	20.15	29.90	.029	27.8	27.9	108	102	9.89
1073	20.84	30.39	.162	27.3	27.8	108	101	9.73
1090	22.71	37.69	.657	31.1	33.2	151	139	10.96
1150	21.69	38.51	.638	33.3	35.5	162	141	11.53
1175	21.38	32.36	.581	28.4	30.1	119	101	10.08
1179	22.70	37.48	.019	30.9	31.0	149	126	10.90
1184	24.36	40.55	.209	31.2	31.8	163	138	11.01
1239	23.83	40.15	.419	31.6	32.8	163	132	11.12
1253	23.36	38.24	.752	30.7	29.4	151	121	10.82
1341	22.13	30.12	.438	25.5	26.6	99	74	8.99
<b>Savage's Jerseys, 6</b>								
812	18.81	34.67	.009	34.5	34.6	150	185	11.86
860	17.05	26.59	.095	29.2	29.6	101	117	10.41
865	17.86	34.44	.248	36.1	35.1	154	178	12.25
909	17.57	23.42	.505	25.0	26.6	75	83	8.74
925	16.64	24.71	.571	27.8	30.0	89	96	9.89
931	19.73	32.31	.752	30.7	33.4	128	137	10.85

Average Live wt. lbs.	Feed TDN lbs./day	Milk FCM lbs./day	Live wt. gain bs./day	Gross Energetic Effic.		Profit per year, dollars		
				Not corrected for wt. gain	Corrected for wt. gain	per cow	per 1000 lbs. live wt.	per 1000 lbs. FCM
<b>Hill's Holsteins, 4</b>								
1184	11.25	9.40	.487	15.7	17.3	7	6	2.05
1317	17.20	20.70	.752	22.5	23.4	57	43	7.53
1341	16.20	11.41	.893	13.2	14.9	-5	-4	-1.29
1390	18.40	20.49	.307	20.9	21.6	49	35	6.55
<b>Hill's Jerseys, 5</b>								
885	13.95	17.89	.627	24.0	26.6	54	61	8.30
899	14.20	19.43	.548	25.6	27.9	64	71	9.04
920	15.20	17.16	.613	21.1	22.8	42	46	6.71
929	16.55	19.45	.410	22.0	23.2	51	55	7.26
978	13.70	13.60	.458	18.6	20.0	24	25	4.88
<b>Perkin's Holsteins, 4</b>								
1288	21.96	28.58	.336	24.4	25.2	88	68	8.47
1330	22.03	28.23	.781	24.0	26.0	85	64	8.30
1360	18.33	24.76	.432	25.3	26.7	80	59	8.90
1393	21.36	27.83	.371	24.4	25.4	86	62	8.49
<b>Harrison and Savage's Holsteins, 103</b>								
1081	19.38	34.49	.155	33.4	33.9	146	135	11.59
1120	16.25	22.23	.192	25.6	26.3	73	65	9.04
1134	19.59	32.82	.075	31.4	31.7	132	116	11.01
1138	18.86	31.31	.367	31.1	32.5	125	110	10.93
1154	19.95	32.81	-.150	30.8	30.3	130	113	10.85
1159	16.50	24.36	.425	27.7	29.3	87	75	9.75
1162	18.88	28.45	-.376	28.2	27.1	104	90	9.84
1163	18.94	28.13	.214	27.8	28.5	102	88	10.03
1170	19.19	30.70	.711	28.9	31.3	119	102	9.95
1170	23.07	43.19	.169	35.1	35.6	189	162	12.00
1172	17.45	22.70	.432	24.4	25.7	70	60	8.47
1173	20.49	34.30	-.316	31.4	30.4	138	118	11.01
1181	16.60	20.14	.335	22.7	23.7	56	47	7.62
1182	18.80	29.60	-.078	29.7	29.4	115	97	10.58
1184	22.20	41.90	.171	35.4	36.0	184	155	12.03
1187	18.41	25.93	.244	26.4	27.2	88	74	9.34
1192	18.60	29.31	.004	29.5	29.6	112	94	10.47
1195	21.96	41.04	.120	35.0	35.4	179	150	11.95
1195	22.28	41.01	-.026	34.5	34.4	177	148	11.84
1196	16.57	19.09	.120	21.6	21.9	49	41	6.99
1203	21.72	39.39	-.030	34.0	23.9	169	140	11.75
1203	18.14	23.31	.530	24.1	25.7	71	59	8.33
1210	18.18	27.19	.150	28.0	28.5	99	82	9.97
1215	19.64	31.55	.398	30.1	31.5	123	101	10.68
1217	19.30	28.38	.384	27.6	28.8	101	83	9.75
1219	20.92	34.04	-.184	30.5	29.9	134	110	10.79
1220	16.91	22.57	.756	25.0	27.6	72	59	8.77
1220	16.14	23.41	.004	27.2	27.2	82	67	9.64
1223	16.76	26.85	.188	30.0	30.8	139	104	11.37
1224	19.11	28.00	1.053	27.5	31.1	100	82	9.75
1227	17.88	26.39	-.489	27.7	26.1	95	77	9.84
1230	16.81	23.36	.249	26.1	26.9	78	63	9.21
1232	20.42	33.19	.087	30.5	30.7	130	106	10.74
1234	16.16	25.11	-.200	29.1	28.4	95	77	10.36
1238	18.28	26.35	-.278	27.0	26.2	92	74	9.56
1239	20.05	38.16	.074	35.7	36.0	169	136	12.14
1239	19.65	28.71	.192	27.4	28.0	102	82	9.73
1248	18.86	26.46	.466	26.3	27.8	90	72	9.66
1250	19.67	31.60	.188	30.1	30.7	123	98	10.32
1251	19.39	32.29	-.278	31.2	30.3	130	104	11.04
1251	19.03	27.10	.015	26.7	26.7	94	75	9.51
1254	19.53	31.55	-.188	30.3	29.7	123	98	10.68
1255	18.24	29.59	-.049	30.4	30.2	116	92	10.74
1261	20.86	36.03	-.155	32.4	32.9	149	118	11.34
1266	17.74	23.72	.282	25.1	25.9	76	60	8.77
1267	19.55	31.23	.455	29.9	31.5	121	96	10.60
1270	20.42	33.75	-.116	31.0	30.6	134	106	10.96
1271	18.67	27.25	.587	27.4	29.3	97	76	9.73
1272	17.39	20.53	.327	22.2	23.1	55	45	7.31
1272	18.57	30.46	.098	30.7	31.1	121	95	10.88
1272	20.83	34.91	.216	31.4	32.1	141	111	11.07
1275	20.13	29.22	.470	27.2	28.6	103	102	9.64

Average Live wt. lbs.	Feed TDN lbs./day	Milk FCM lbs./day	Live wt. gain lbs./day	Gross Energetic Effic.		Profit per year, dollars		
				Not corrected for wt. gain	Corrected for wt. gain	per cow	per 1000 lbs. live wt.	per 1000 lbs. FCM
1275	20.71	34.56	.443	31.3	32.8	139	109	11.01
1283	17.51	22.75	.244	24.3	25.1	70	55	8.47
1285	20.45	32.83	.229	30.1	30.8	128	100	10.68
2186	17.87	23.19	.105	24.3	24.6	71	55	8.44
1287	19.70	31.87	.169	30.3	30.9	125	97	10.74
1292	20.68	35.09	.045	31.8	32.0	143	111	11.18
1295	19.54	30.40	.263	29.2	30.0	115	89	10.36
1296	19.08	26.50	.131	26.0	26.4	89	69	9.21
1297	20.73	32.63	.470	29.5	31.0	125	96	10.49
1298	20.40	34.94	.613	32.1	34.3	143	110	11.21
1298	20.98	36.36	.065	32.5	32.7	150	116	11.37
1301	21.53	39.18	.090	34.1	34.4	168	129	11.75
1305	20.76	32.38	.384	29.2	30.4	123	94	10.41
1306	18.50	27.76	.511	28.1	29.9	101	77	12.71
1308	17.53	22.74	.756	24.3	26.8	70	54	9.44
1320	19.65	30.79	.131	29.4	29.8	117	89	10.41
1323	19.70	33.91	.176	32.3	32.9	140	106	11.32
1323	20.65	32.29	.214	29.3	30.0	123	93	10.44
1327	20.78	34.06	.128	30.7	31.1	135	102	10.85
1327	17.63	22.85	.150	24.3	24.7	70	53	8.41
1331	21.71	39.74	-.237	34.3	33.5	171	128	11.78
1332	17.34	19.84	-.011	21.4	21.4	50	38	6.90
1332	17.94	20.25	.688	21.2	23.1	50	38	6.71
1333	21.48	35.65	.261	31.1	31.9	143	107	10.99
1337	19.35	33.53	.351	32.5	33.8	139	104	11.37
1338	20.80	35.29	.004	31.8	31.8	144	108	11.18
1340	18.72	26.11	-.045	26.1	26.0	88	66	9.23
1340	18.02	32.76	.327	34.1	35.4	140	104	11.70
1341	20.00	33.45	.192	31.4	32.0	135	101	11.07
1344	18.09	21.97	.853	22.8	25.3	61	45	7.64
1344	18.77	32.87	-.331	32.8	31.6	137	102	11.42
1346	21.00	32.88	.343	29.4	30.4	125	93	10.41
1364	18.40	26.62	.421	27.1	28.5	94	69	9.67
1366	19.49	27.62	.518	26.6	28.2	95	70	9.42
1368	18.89	32.59	.053	32.3	32.5	134	98	10.26
1375	19.26	29.17	.004	28.4	28.4	107	78	10.05
1379	19.08	30.56	.147	30.0	29.5	119	86	10.66
1379	19.61	33.87	-.131	32.4	31.9	140	102	11.32
1382	18.63	22.85	.150	24.3	24.7	70	53	8.41
1384	20.91	33.34	.286	29.9	30.8	129	93	10.60
1386	21.31	41.03	.016	36.1	36.1	183	132	12.22
1388	21.82	38.77	.265	33.3	34.2	164	118	11.59
1389	19.86	31.70	.447	29.9	31.4	123	89	10.63
1394	21.31	40.14	-.690	35.3	33.0	176	126	12.08
1398	20.89	35.07	.271	31.5	32.4	142	102	11.10
1404	15.83	22.86	.637	27.1	29.6	80	57	9.62
1428	19.18	28.58	-.387	27.9	26.8	104	73	9.97
1436	19.22	28.55	.082	27.8	28.1	103	72	9.92
1453	18.45	27.41	.494	27.9	29.5	99	68	9.89
1494	21.47	38.71	-.327	33.8	32.7	165	110	11.67
1500	21.11	36.35	-.019	32.3	32.2	150	100	11.32