Bulk Density of Chopped Alfalfa Hay

C. L. DAY AND H. H. PANDA

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INTRODUCTION

Bulk densities* of agricultural products are of interest to agricultural engineers and others concerned with storing and drying farm crops. Such information is not only essential for designing storage structures, but, regarding hay, is also extremely important in the design of forced air drying systems. Previous investigations (3) have shown that the resistance of hay to air flow increases approximately as the third power of its bulk density. Thus, if the bulk density is doubled, the resistance to air flow is increased by eight times.

Values for bulk density of chopped hay are given in various textbooks and handbooks. Most references ignore the fact that the bulk density varies with the moisture content of the hay and the depth of storage. Wooley (7) lists the bulk density of "cut" hay as 10 pounds per cubic foot. Midwest Farm Handbook (4) lists a range of values from 5.5 to 7 pounds per cubic foot for chopped alfalfa hay. Newbauer and Walker (6) list values ranging from 6 to 12 with 10 pounds per cubic foot as an average value. Barre and Sammet (1) list values ranging from 8 to 10 pounds per cubic foot.

Zerfoss (8), in an article published in 1947, pointed out that the bulk density of hay is affected by many factors. He listed the following as being most important: stage of maturity at the time of cutting, moisture content at the time of storage, kind of hay, depth of storage, method of harvesting and handling before storage and method of handling in mow storage. He listed some densities which he had determined, as well as additional hay density data from Michigan State University.

Davis and Baker (2) attempted to evaluate two of the factors listed by Zerfoss. They made observations on the effect of initial moisture and storage depth on the final density attained by hay when it is dry. Their results show that the final density of hay increases with an increase in either the initial moisture or the depth of storage.

Although the values given by Zerfoss and by Davis and Baker include the effect of depth of storage, the values are averages for the entire hay mass. Apparently no attempt was made to determine the bulk density at various levels (or depths) of storage.

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* The term "bulk density" is defined as the weight of bulk stored material per unit volume. Bulk densities are expressed herein in pounds per cubic foot.

HYPOTHESES AND ASSUMPTIONS

A systematic study of bulk density of hay in a full size mow, unless limited to few tests, was considered to be too expensive and time-consuming. Therefore, an experiment was conducted in the laboratory using a small container as a model of a mow.

The bulk density of a layer of hay was believed to depend primarily upon the depth of storage or the superimposed load over it, moisture content of the layer, density of water, and kind of hay. The length of cut of pieces of hay was believed to affect the bulk density to some extent in a small container. It was further assumed that the impact of a falling load over the layer may affect the rate of change, but not the final bulk density. Of the geometrical properties of the container, such as cross-sectional area, perimeter, average horizontal dimensions, area of contact with hay and least horizontal dimension, the last one, if any, was considered to have significant influence. Though the kind of hay was recognized to be a factor that may influence the bulk density, this study was limited to freshly chopped alfalfa hay.

The bulk density (S_a) of a layer of hay can be considered to be the sum of two components: the initial bulk density S_{ai} and the increase in bulk density ΔS_a over the initial bulk density due to the superimposed load over the layer, i.e.,

$$S_a = S_{ai} + \Delta S_a. \hspace{3cm} \text{Equation 1}$$

The initial bulk density of hay, visualized as the density of a layer of hay one foot thick at the top of the stack, is due only to its own weight (independent of superimposed load). For a given kind of hay, the initial bulk density depends upon moisture content M, length of cut L_c, specific weight of water S_w, and least horizontal dimension of container L_h. These factors may be expressed in terms of the basic dimensions, force and length. L_c and L_h have dimensions of length (L), S_w has dimensions of FL^{-3}, and moisture content is dimensionless.

Since two independent dimensions were involved, the Buckingham Pi theorem (5) indicates a relationship between three dimensionless groups or Pi terms. One appropriate form for the expression is

$$\frac{S_{ai}}{S_w} = F \left(M, \frac{L_c}{L_h} \right). \hspace{3cm} \text{Equation 2}$$

The first Pi term is the ratio of initial bulk density of hay to specific weight of water; the second is the moisture content of hay; the last one is the ratio of the mean length of cut of pieces of hay to the least dimension of container (for

brevity called length ratio), and is designated as r. As S_w is a constant for all practical purposes, determination of the nature of the function F in Equation 2 reduces to an evaluation of the effects of moisture content and length ratio on bulk density of hay.

In addition to the factors mentioned above, ΔS_a depends on the vertical distance Z of the layer from the top of the hay mass. The relationship between ΔS_a and other factors can be written as

$$\frac{\Delta S_a}{S_w} = \theta \ (M, r, \frac{Z}{L_h}). \qquad \text{Equation 3}$$

This means that the evaluation of the function θ involves determination of the combined effect of moisture content, length ratio, and depth of hay over the layer under consideration on the change in bulk density.

EXPERIMENTATION AND RESULTS

Determination of bulk density of hay was divided into the following experiments:

A. Laboratory experiments to evaluate the functions F and θ.

B. Field tests to compare the results of the laboratory tests.

A. Laboratory experiments. Instead of running two separate experiments to evaluate the functions F and θ, only one experiment was planned to evaluate the function θ, which in turn will furnish necessary data to establish the nature of the function F. For such problems, it is customary to vary one of the Pi terms on the right hand side of Equation 3, while holding the others constant, and to determine the effect on the dependent Pi term. In the laboratory tests, this procedure was not feasible since it was not practical to vary the last Pi term. A different approach was therefore taken in this case. Sand, instead of hay, was used to apply weight to the hay and the load-density relationship was converted to a depth-density relationship which was finally used to evaluate the function θ.

Apparatus. A plywood container, reinforced with aluminum angles at the corners, was used to test the effects of moisture content and load on the bulk density of hay. The container was 12 inches square and 16 inches deep inside. A wooden board (top board), measuring approximately 11¾ inches square and ¾ inch thick, was used on the top of the hay in order to distribute the load evenly throughout the cross section. Two metallic containers, 10 inches in diameter and 13½ inches deep, each capable of holding approximately 60 pounds of dry sand were used to apply weight to the hay. A funnel, 10 inches in diameter, with a device to close the bottom opening temporarily was used to store an arbitrary unit weight of sand before it was released into the sand containers. The arrangement is shown in Figure 1.

A shaker (Figure 2) with a 1.75-inch vertical stroke and a frequency of 26 cycles per minute was fabricated, for the purpose of vibrating the hay container just after filling.
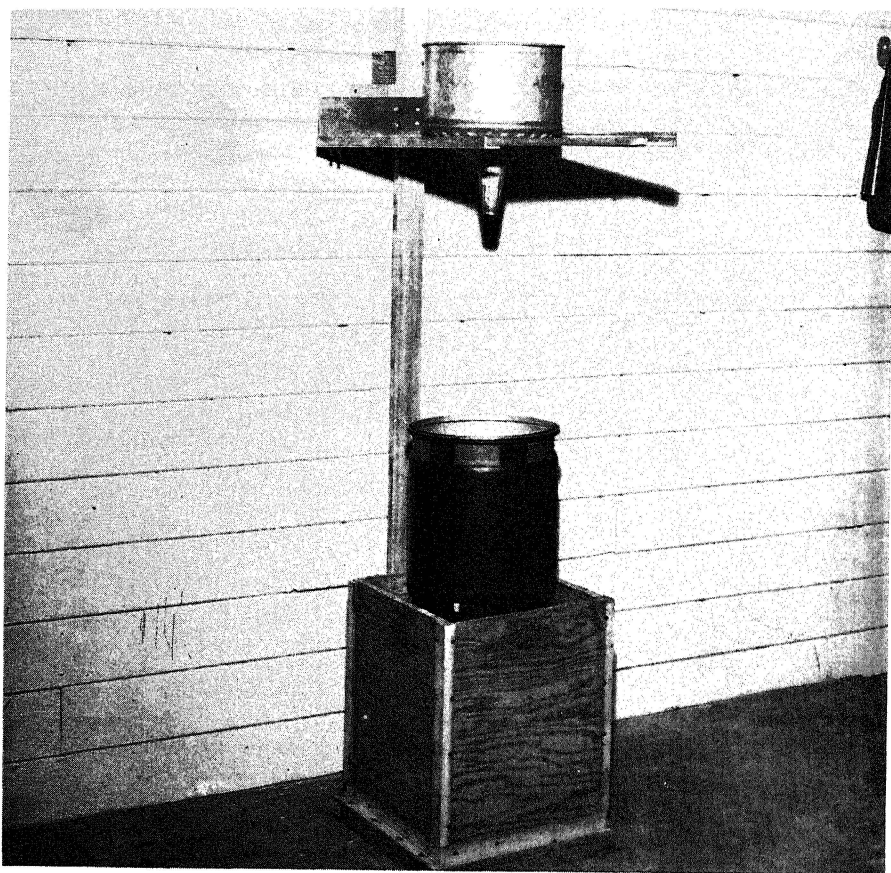
Figure 1. Device Used To Load Hay With Sand

Procedure. Two sets of tests, that vary in details, were planned to determine:
1. The effect of moisture content and depth, and
2. The effect of length of cut on the bulk density of hay.
Hence, the procedure adopted in conducting laboratory experiments is described below under two separate headings.

1. Effect of moisture content and depth. A quantity of freshly cut, machine-chopped, alfalfa hay was brought to the laboratory. After mixing the hay

thoroughly, a sample was taken at random and placed in the oven for determination of moisture content. The plywood container was completely filled with hay and vibrated for two minutes. The container was rotated four times by 90 degrees during each period of vibration to avoid unequal settlement. After vibrating, only one cubic foot of hay was retained within the container for subjecting to load. The weight of the one cubic foot of hay remaining inside the container was determined.

The hay was loaded progressively with a measured weight of sand dropped from the funnel placed 30 inches above the top of the hay. The loads were 0 to 30 pounds with 3-pound increments, 30 to 72 pounds with 6-pound increments, and 72 to 90 pounds with 9-pound increments.

Sand was released from the funnel and fell into the sand container placed on
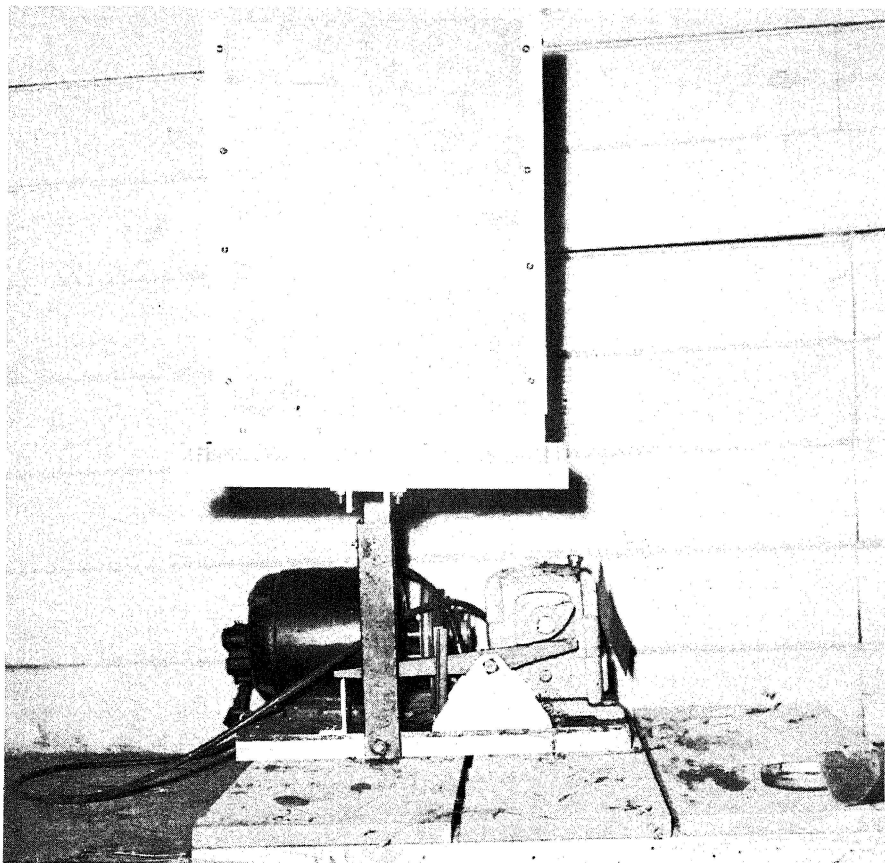

Figure 2. Shaker and Plywood Container

the top board. The weight of the top board and the sand container was included in the load. The depth of hay was measured at each increment of loading from a reference point on the container. In some instances, a given load was allowed to remain on the hay for a period of up to 12 hours. No appreciable settling was found to occur after the first few minutes in this small container.

Five series of tests were conducted with hay at 11, 22, 30, 48, and 66 percent moisture.

Bulk densities under different loads were calculated from the volume of hay at the time of corresponding measurements and initial bulk density. Results are shown in Figure 3.

The depth-density relationship was derived from each series of tests from load-density curves, assuming uniform density within each successive layer of one-foot depth. Results are shown in Figures 4 and 5. The sample calculations for derivation of depth-density relationship from load-density relationship are given in the Appendix.
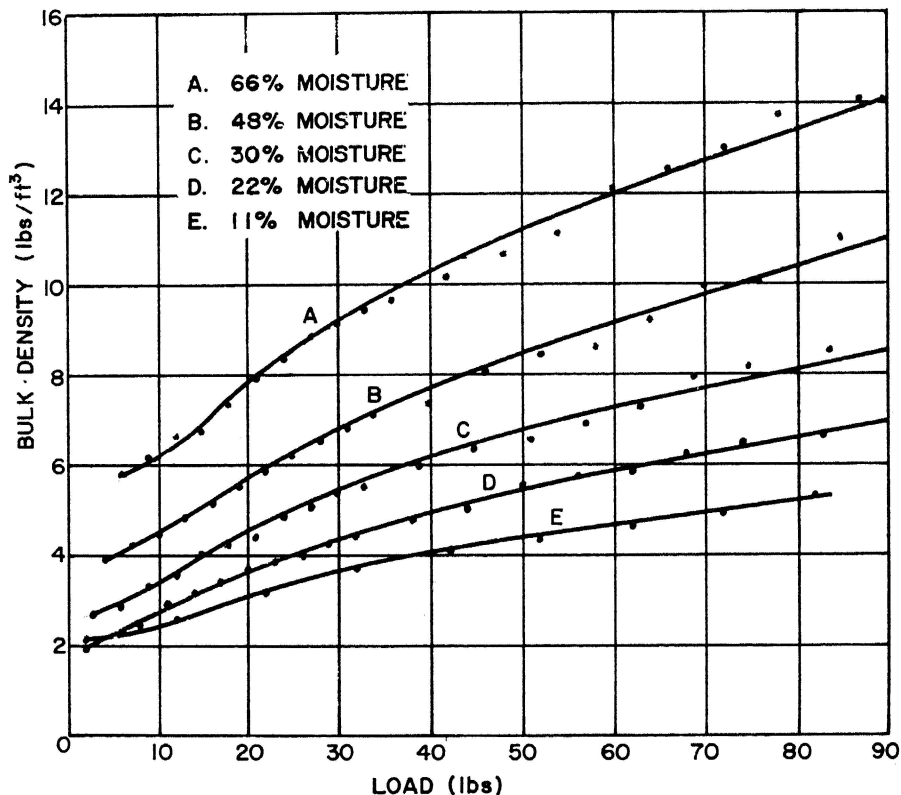

Figure 3. Relationship Between Bulk Density and Load For A Length Ratio of 0.34 and Moisture Contents Ranging From 11 to 66 Percent.

2. Effects of length of cut. The effects of length of cut were studied in four series of tests. The first three series of tests were conducted for two-, four-, and six-inch lengths of cut in the plywood container and the fourth series was conducted in a different container, 22 inches in diameter, made by cutting a 55-gallon metal barrel in half. Hay was chopped manually to desired lengths for the first three series of tests. Machine-chopped hay was used for the fourth series.
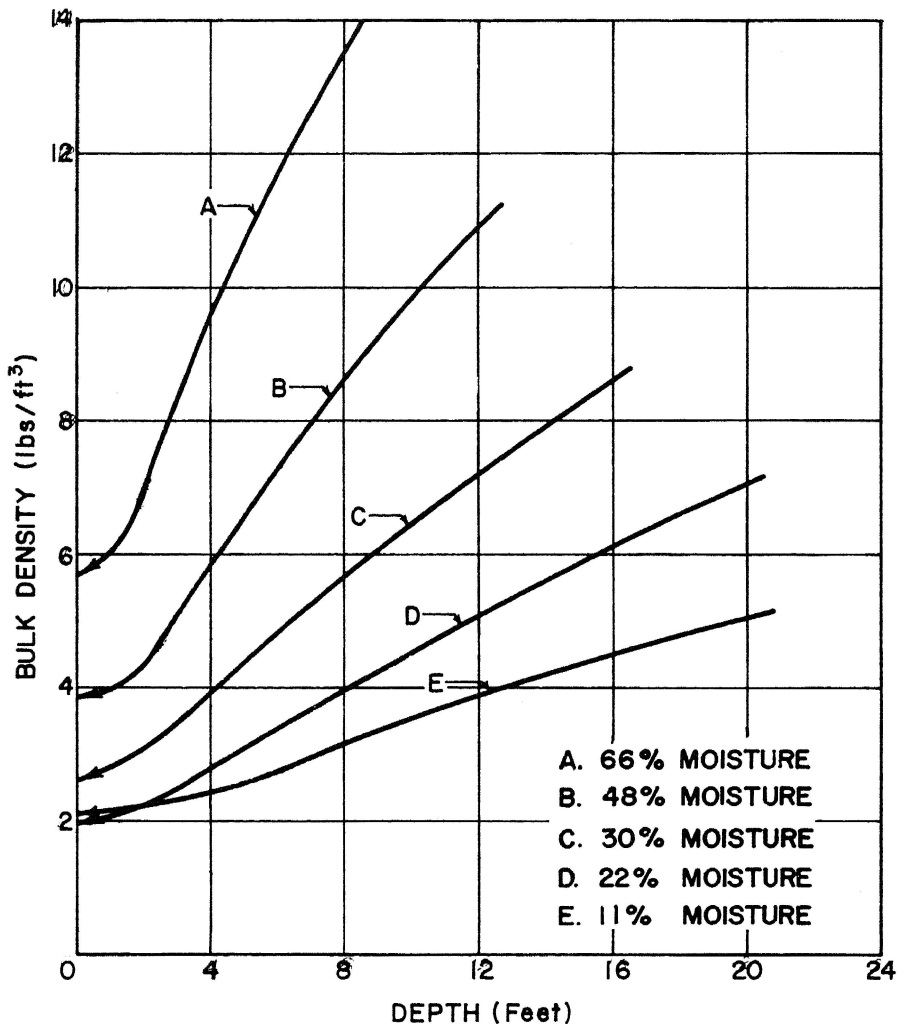

Figure 4.   Relationship Between Bulk Density and Depth For A Length Ratio of 0.34 and Moisture Contents Ranging From 11 to 66 Percent.

In order to determine the length of cut of machine-chopped hay, the lengths of individual hay pieces were measured by means of a steel rule. A large number of fine particles and detached leaves were found in the sample. There was concern that the inclusion of these particles, which would outnumber the intact pieces, would bring down the mean length to too low a value to be representative. Moreover, the length of these particles was not easy to determine. Therefore, these were excluded from the sample. There were 137 pieces of intact stems ranging from ½ inch to 8 inches in length. The mean length was found to be 4.08 inches.
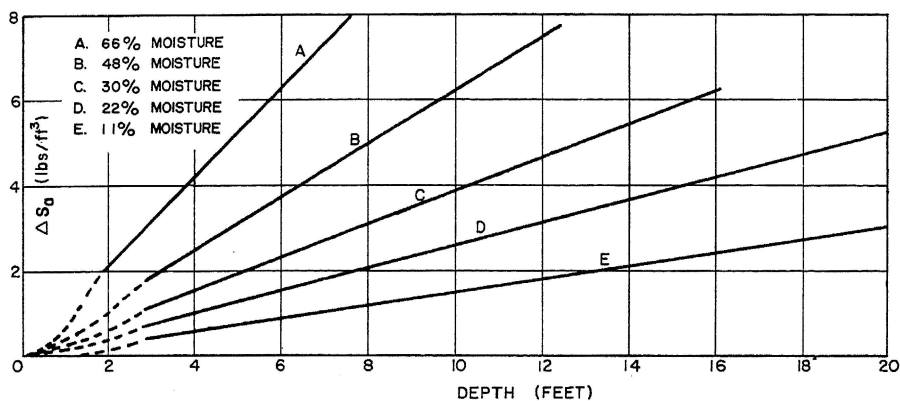

Figure 5. Relationship Between Increase in Bulk Density and Depth
At Moisture Contents Ranging From 11 to 66 Percent.

The moisture content of the hand-chopped hay was 11 percent and that of the machine-chopped hay was 14 percent. As the moisture content of hay and the method of cutting used in the fourth series were different from the others, the results of this series cannot be compared directly with results of the other three series. However, Figure 6 shows that the results of the machine-cut series do not differ greatly from the others.

The procedure for placing the hay in the containers, shaking, and weighing was essentially the same as previously described.

Loads of 0 to 80 pounds were applied progressively in 10-pound increments for these tests. The moisture content was determined from a 200-gram sample of hay. Calculation of bulk density and derivation of depth-density relationship were made in the same manner as previously described.

The effects of length of cut are shown in Figures 6 through 8.

B. *Field tests to compare the results of the laboratory tests.* A bin (Figure 9) measuring 10 feet x 8 feet x 10 feet, with a false floor at the bottom, constructed inside a building was used to compare the results of the laboratory tests to actual conditions in a full size mow. The bin was provided with a translucent
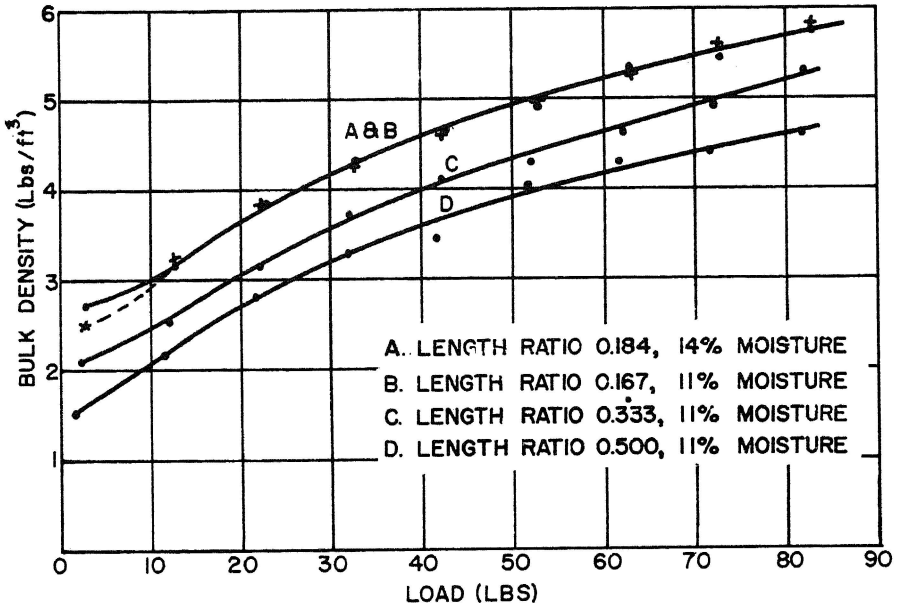
Figure 6.   Relationship Between Bulk Density and
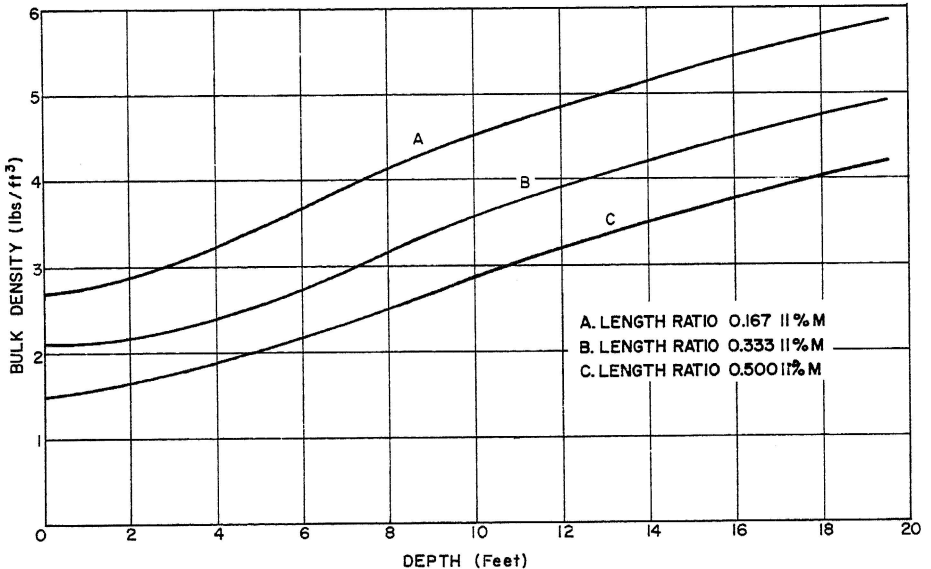Load For Different Length Ratios.

Figure 7.   Relationship Between Bulk Density and
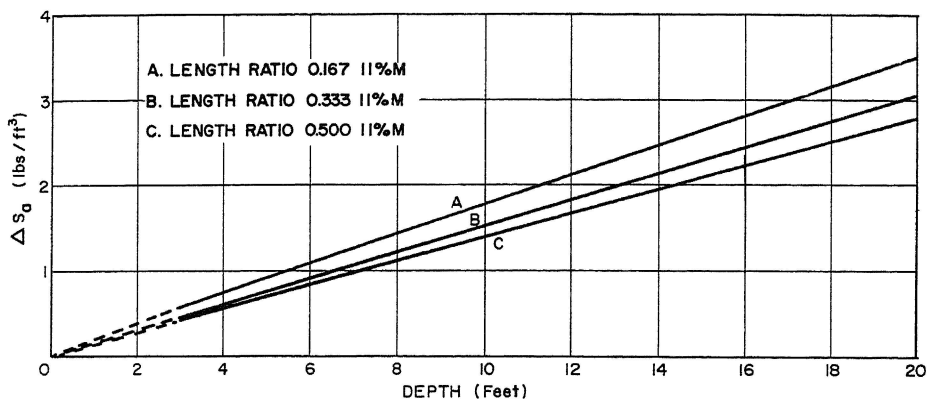Depth for Different Length Ratios.

Figure 8. Relationship Between Increase in Bulk Density
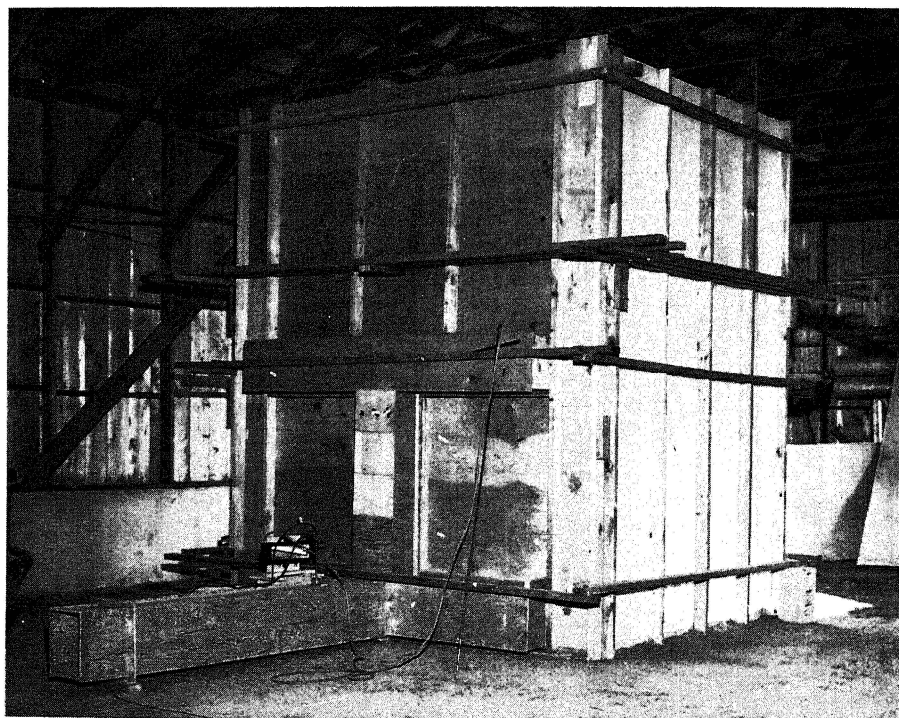and Depth for Different Length Ratios.

Figure 9. A View of the Bin Used in the Study.

vertical strip two feet wide at the center of each of the 10-foot walls. It was filled five times during 1962, 63, and 64. Each filling consisted of several loads of hay approximately 1500 pounds per load. The weight of each load was determined by a large platform scale with 20-tons capacity and 5-pounds least division. The hay was raised to the top of the bin by means of an elevator and was distributed evenly in the bin. At no time was the hay walked upon or otherwise packed. An iron rod (marker), 8 feet long having red reflectors at the ends, was placed on the surface after each load of hay was distributed in the bin. The reflectors were oriented so as to be visible from the outside through the translucent strips. The moisture content of each load was determined by collecting three samples of hay from each load at the time of filling, and drying the samples in the oven.

The thickness of successive layers of hay was determined, immediately after the filling operation was over, by measuring the vertical distances between the reflectors with a steel tape. The average thickness of the layers was calculated

TABLE I

ORIGINAL DATA FOR HAY IMMEDIATELY AFTER FILLING THE BIN
AT THE AGRICULTURAL ENGINEERING FARM ON THE DATES INDICATED

Date	Load Number	Weight of hay (lbs.)	Moisture Content	Thickness of layer (inches)	Position of Marker From Top of Hay (inches)
July 25, 1962	1	1390	25.0	23.0	130.0
	2	1195	21.5	29.0	97.0
	3	1560	22.0	26.75	68.0
	4	1075	21.0	41.25	41.25
					0.0
July 19, 1963	1	1230	25.2	18.0	124.0
	2	1500	25.1	35.5	106.0
	3	1545	26.1*	35.5	70.5
	4	1085	25.6	35.0	35.0
					0.0
September 6, 1963	1	1785	41.69	25.5	103.5
	2	2050	39.31	42.0	78.0
	3	1860	43.26	36.0	36.0
					0.0
June 2, 1964	1	1535	57.2	17.0	98.25
	2	1625	56.8	15.75	81.25
	3	2175	56.2	24.0	65.50
	4	1280	55.4	17.25	41.50
	5	1400	56.7	24.25	24.25
					0.0
July 15, 1964	1	1528	31.6	21.0	138.0
	2	1500	24.8	37.0	117.0
	3	1355	27.6	32.0	80.0
	4	1370	30.1	48.0	48.0
					0.0

*The moisture content of third load ranged from 31.5 to 22%.

from the two readings obtained for each layer from each end of the markers. The weights and mean values of thickness of layers for hay placed in storage on five different dates are given in Table I.

Once the bin was filled, the drying fan was started and was then operated continuously until the hay was dry. The positions of markers were also obtained on different dates for the fourth filling (out of five) in order to determine the rate of settlement. The settlement for different dates from June 2, 1964 to June 25, 1964, along with the moisture content of the top layer of hay, is shown in Figure 10.
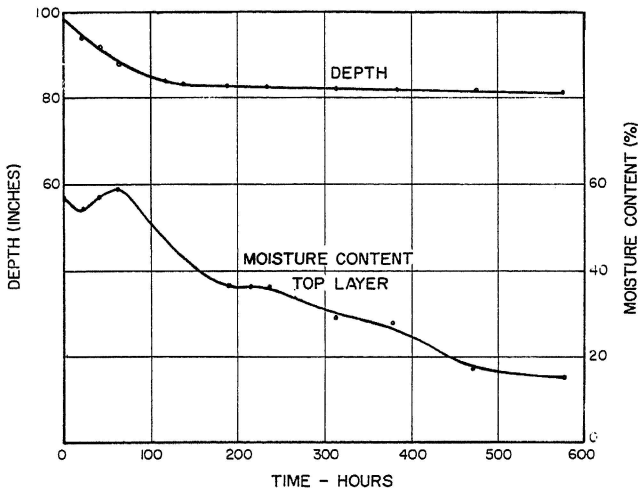
Figure 10. Variation in Depth and Moisture Content
of the Top Layer During Drying

DEVELOPMENT OF EQUATIONS

The data relating initial bulk density of hay to the moisture content and length ratio as obtained in the laboratory tests are presented in Table II. A plot of initial bulk density (S_{ai}) with a 0.34 length ratio versus moisture content M (M being expressed as a decimal), results in a straight line on semilog coordinates (Figure 11). The relation between S_{ai} and M can be expressed as

$$S_{ai} = 1.4e^{2.138M}, \text{ when } r = 0.34 \qquad \text{Equation 4}$$

Figure 12 shows a plot of S_{ai} versus length ratio r for a moisture content of 11 percent on semilog coordinates. The relationship between S_{ai} and r is linear on semilog paper, r being plotted along the linear scale, and the mathematical expression for it is

$$S_{ai} = 3.69 \times e^{-1.77r}, \text{ when } M = 0.11 \qquad \text{Equation 5}$$
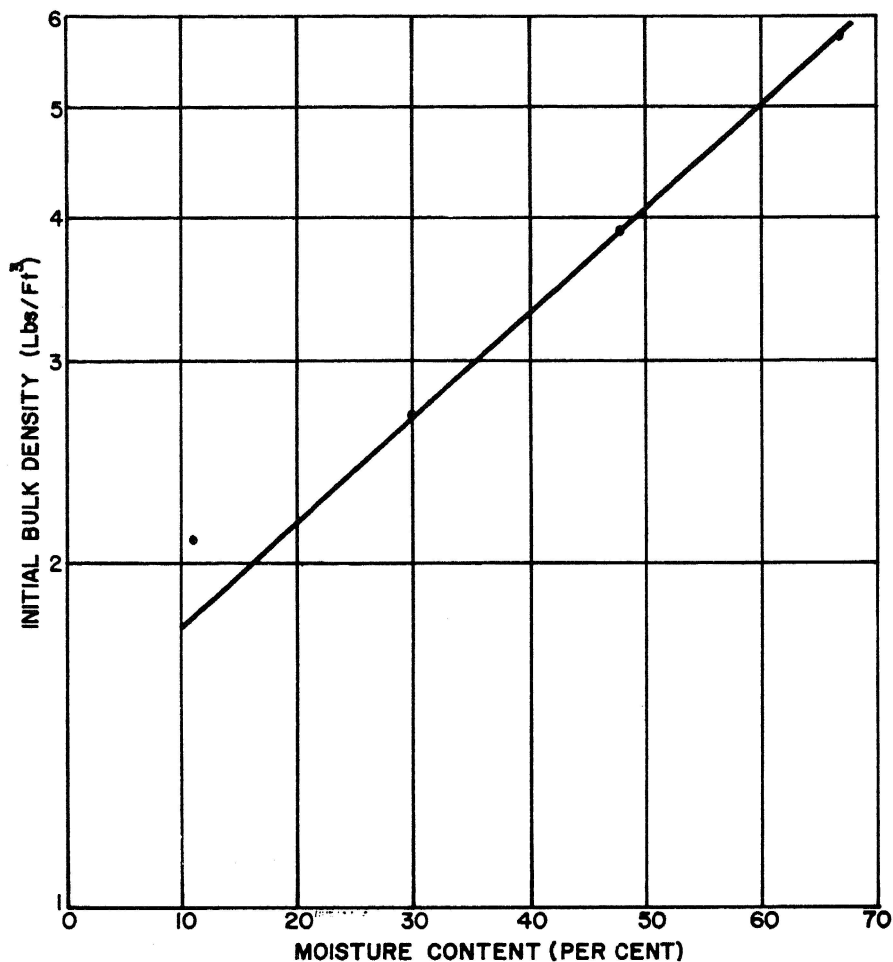
Figure 11.   Relationship Between Initial Bulk Density
and Moisture Content.

The Equations 4 and 5 can be combined as

$$S_{ai} \propto e^{-1.77r} \, e^{2.138M},$$

when both r and M vary or

$$S_{ai} = C \, e^{2.138M - 1.77r} \qquad \text{Equation 6}$$

The values of the constant C in Equation 6 for moisture contents of 11 percent
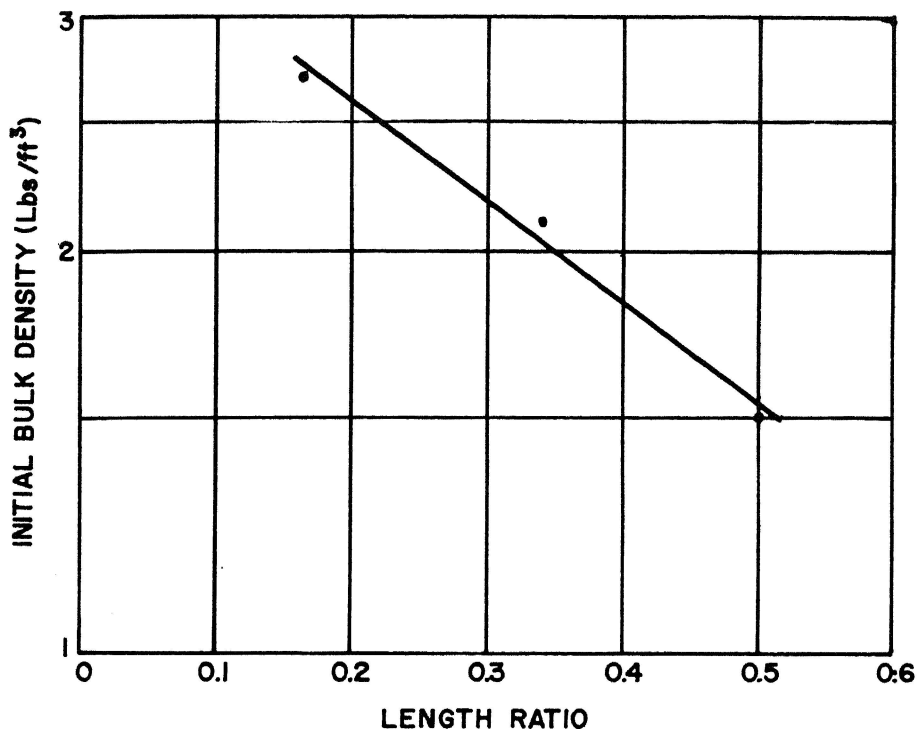
Figure 12. Relationship Between Initial Bulk Density
and Length Ratio.

TABLE II

BULK DENSITY OF FIRST ONE FOOT (INITIAL BULK DENSITY) OF HAY
FOR DIFFERENT MOISTURE CONTENTS AND LENGTH RATIOS

Moisture Content (Percent)	Length Ratio	Initial Bulk Density (lbs/ft³)
66	0.34	5. 8
48	0.34	3. 9
30	0.34	2. 7
22	0.34	2. 0
14	0.182	2. 5
11	0.34	2. 1
11	0.5	1. 5
11	0.167	2. 7

TABLE III

VALUE OF THE CONSTANT C IN THE EQUATION 6, FOR HAY
WITH 11% TO 66% MOISTURE CONTENT AND 0.5
TO 0.167 LENGTH RATIOS

Moisture Content M (Percent)	Length Ratio (r)	Initial Bulk Density (S) $(lbs/ft^3)^{ai}$	Value of C
66	0.34	5.8	2.58
48	0.34	3.9	2.60
30	0.34	2.7	2.60
22	0.34	2.0	2.28
11	0.34	2.1	3.03
11	0.5	1.5	2.83
11	0.167	2.7	2.87
11	0.182	2.5	2.73

Mean Value of C = 2.69

to 66 percent and length ratios of 0.167 to 0.5 are given in Table III. The mean value of C is 2.69 and the dispersion of the values is negligible. Therefore, Equation 6 can be taken as valid for moisture contents of 11 percent to 66 percent and length ratios of 0.167 to 0.5. Substituting the mean value of C in Equation 6 we get

$$S_{ai} = 2.69 \; e^{\; 2.138M - 1.77r} \qquad \text{Equation 7}$$

An examination of Equation 6 reveals that the constant C has dimensions of FL^{-3}, i.e. the same dimensions as density. In fact, the constant is the value of bulk density of hay at 0 percent moisture content and 0 length ratio or the bulk density when $M = 0.832r$. It further appears that when M is equal to 1, S_{ai} is not equal to the density of water (62.3 pounds per cubic foot) but equal to 22.9 pounds per cubic foot, but this does not invalidate the equation as the equation is not the mathematical model for a physical system where free water is present. It represents a system where water remains as an integral part of hay and hay seldom contains more than 80 percent water. Therefore, the condition that when M approaches one, S_{ai} should approach the density of water, is considered to be a desirable but not an essential characteristic.

A preliminary examination of the relation between ΔS_a, the increase in bulk density over initial bulk density caused by load or depth of storage and moisture content M, shows that a simple function of M such as $F_1(M) = a_1 M^{b_1}$, $F_2(M) = a_2 e^{b_2M}$, or $F_3(M) = a_3 + b_3 M$ does not satisfy some of the conditions. For example, one essential condition is $M = 0$, $\Delta S_a \neq 0$ and a desirable (but not essential) condition is $M \gg 1, \Delta S_a \gg 0$. In addition, the primary requirement of the function is to conform to the data.

None of the above functions satisfies these conditions. In order to develop a logical relation between ΔS_a and moisture content M, some other function must

be sought, which, in addition to fulfilling the above requirements, will conform to the data. By trial, a function of M of the form a $e^{\frac{b(M-C)^2}{1-M}}$, where a, b, C, are constants was found to be satisfactory for the purpose and $e^{\frac{(M-C)^2}{1-M}}$ was selected as Pi term instead of M itself. In this function, the value of C was also found using measured data to be 0.7 by trial and error. The function therefore appears hereafter in this publication as $e^{\frac{(M-0.7)^2}{1-M}}$. By a similar method, a function of r of the form e^r was selected as another Pi term instead of r itself. The two fundamental requirements of Pi terms are:

1. They must be dimensionless.
2. They must be independent.

Both of these conditions are satisfied by these functions. Now Equation 3 can be written as

$$\frac{\Delta S_a}{S_w} = \theta \left(e^{\frac{(M-0.7)^2}{1-M}} \cdot e^r, \frac{Z}{L_h} \right) \qquad \text{Equation 8}$$

or

$$\pi_1 = \theta \ (\pi_2, \ \pi_3, \ \pi_4)$$

where

$$\pi_1 = \frac{\Delta S_a}{S_w} = \frac{\Delta S_a}{62.3}$$

$$\pi_2 = e^{\frac{(M-0.7)^2}{1-M}}$$

$$\pi_3 = e^r$$

$$\pi_4 = \frac{Z}{L_h} = Z, \text{ as } L_h = 1 \text{ foot.}$$

The relations between π_1 and π_2 at $\pi_3 = 1.405$ (r = 0.34) for two values of π_4, $\pi_4 = 4$ and $\pi_4 = 8$, are shown in Figure 13. The relationships are mathematically expressed as

$$(\pi_1)_{4\ 3} = 0.06\ \pi_2^{-4.8}, \ \pi_4 = 4, \ \pi_3 = 1.405 \qquad \text{Equation 9a}$$

and

$$(\pi_1)_{4\ 3} = {}_{0.130}\ \pi_2^{-4.8}, \ \pi_4 = 8, \ \pi_3 = 1.405 \qquad \text{Equation 9b}$$

(The values of the Pi terms are calculated from Figure 5). Similarly, the relations between π_1 and π_4 at $\pi_3 = 1.405$ for two values of π_2 ($\pi_2 = 1.48$ cor-

Figure 13. $\dfrac{\Delta S_a}{S_w}$ Versus $e^{\dfrac{(M - .7)^2}{1 - M}}$ for 0.34 Length Ratio at Different Depths.

responding to 11 percent moisture, and $\pi_2 = 1.2575$ corresponding to 30 percent moisture) are presented in Figure 14, from which the Equations 9c and 9d are derived.

$$(\pi_1)_{\overline{2}\ \overline{3}} = 0.0024\ \pi_4,\ \pi_{\overline{2}} = 1.48,\ \pi_{\overline{3}} = 1.405 \qquad \text{Equation 9c}$$

$$(\pi_1)_{\overline{2}\ \overline{3}} = 0.006\ \pi_4,\ \pi_{\overline{2}} = 1.2575,\ \pi_{\overline{3}} = 1.405 \qquad \text{Equation 9d}$$
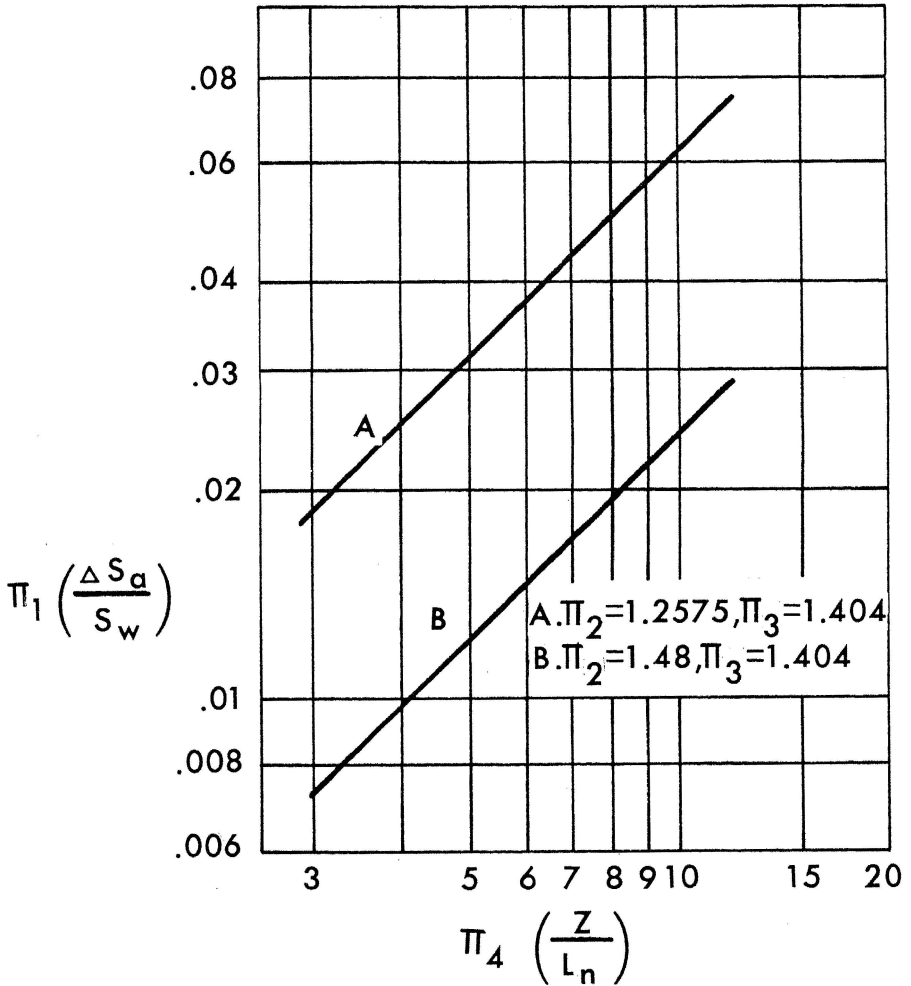
Figure 14. $\dfrac{\Delta S_a}{S_w}$ Versus $\dfrac{Z}{L_h}$ for 0.34 Length Ratio With Different Moisture Content.

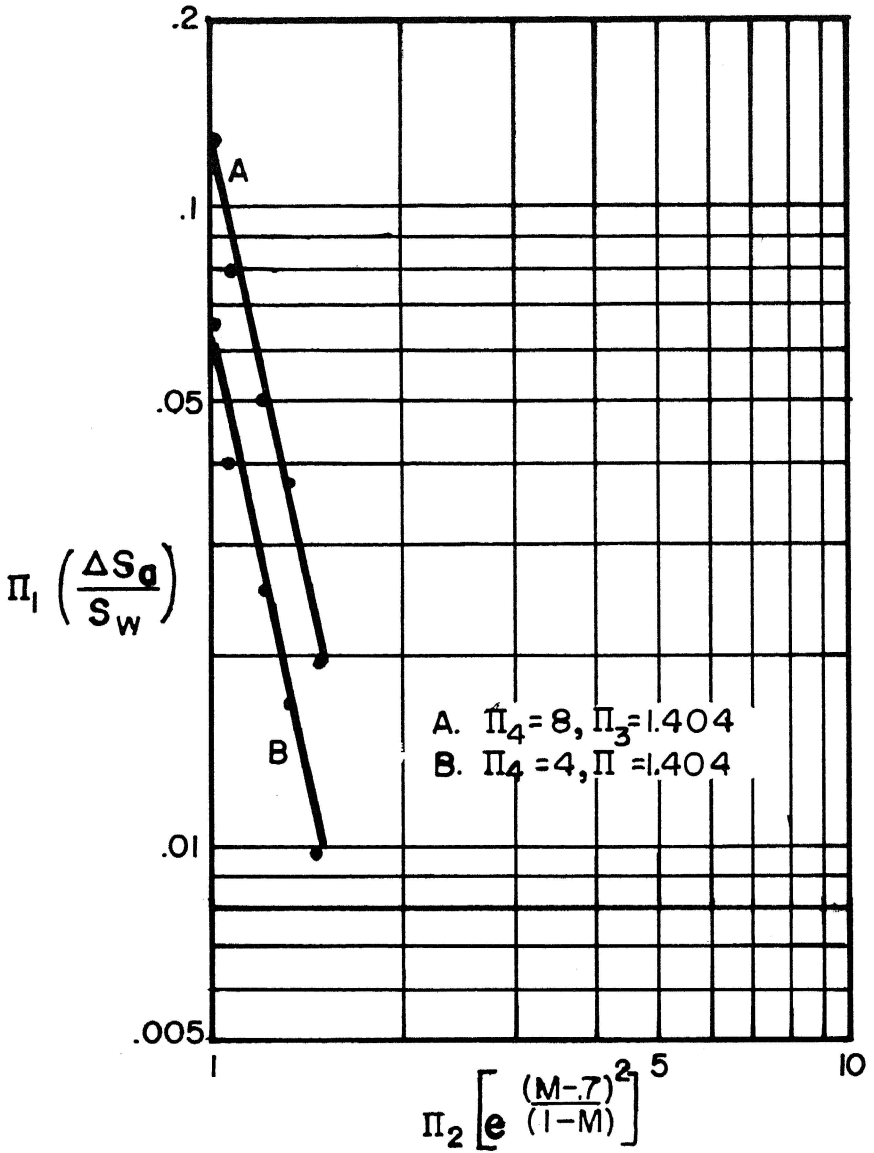(The values of the Pi terms are calculated from Figure 5). Again the relations between π_1 and π_3 at $\pi_2 = 1.48$ for two values of π_4 ($\pi_4 = 4$ and $\pi_4 = 8$) are shown in Figure 15. The equivalent mathematical equations are

$$(\pi_1)_{\overline{4}\,\overline{2}} = 0.0128\ \pi_3^{-0.72},\ \pi_{\overline{4}} = 4, \pi_{\overline{2}} = 1.48 \qquad\qquad \text{Equation 9e}$$

$$(\pi_1)_{\overline{4}\,\overline{2}} = 0.0225\ \pi_3^{-0.69},\ \pi_{\overline{4}} = 8, \pi_{\overline{2}} = 1.48 \qquad\qquad \text{Equation 9f}$$
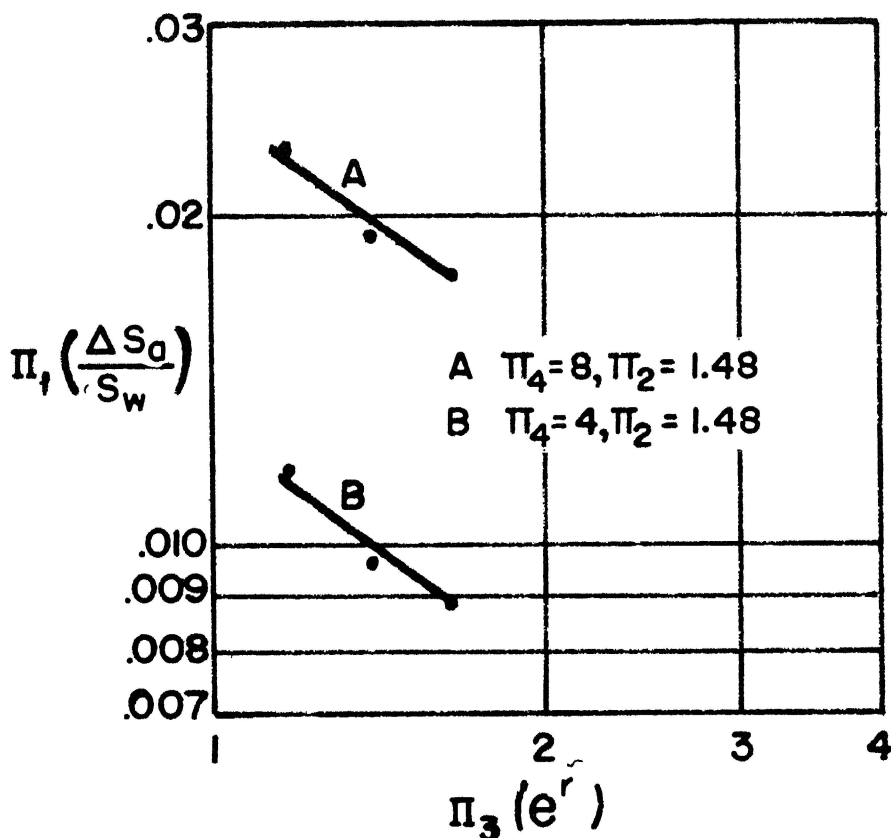

Figure 15. $\dfrac{\Delta S_a}{S_w}$ Versus e^r for 11 Percent Moisture Content at Different Depths.

(The values of the Pi terms are calculated from Figure 8). As all of the three sets of relations plot as straight lines on log paper and lines of each set are approximately parallel to each other, the Pi terms can be combined by multiplication. The resulting equation is

$$\pi_1 = \frac{0.065\ \pi_2{}^{-4.8} \times 0.0024\ \pi_4 \times 0.01275\ \pi_3{}^{-0.72}}{(0.0096)^2}\ .$$ Equation 10a

On substitution of the values of Pi terms, the equation becomes

$$\Delta S_a = C\,Z\,e^{-\frac{4.8}{1-M}(M-0.7)^2 + 0.72r}$$ Equation 10b

where C is a constant. The value of the constant C was calculated directly from Equation 10a and found to be 1.36. However, it is considered more appropriate to evaluate it directly from the data. The values of C for different moisture contents, length ratios, and depths are given in Table IV. A fair degree of constancy

TABLE IV

VALUES OF CONSTANT C IN THE EQUATION 10 b FOR HAY WITH
MOISTURE CONTENTS OF 11% TO 66%, LENGTH RATIOS
OF 0.167 TO 0.5 AND DEPTHS OF
4 FT. TO 20 FT.

Moisture Content (Percent) (M)	Length Ratio (r)	Depth (ft)	S_a	Value of C
66	0.34	4	4.4	1.43
66	0.34	8	8.2	1.35
48	0.34	5	3.0	1.20
48	0.34	12	7.3	1.22
30	0.34	5	1.9	1.45
30	0.34	16	6.1	1.46
22	0.34	10	2.65	1.40
22	0.34	20	5.15	1.35
11	0.34	10	1.5	1.26
11	0.34	20	2.95	1.24
11	0.50	10	1.3	1.205
11	0.50	20	2.7	1.25
11	0.167	10	1.9	1.39
11	0.167	20	3.3	1.21

Mean Value of C = 1.32

of the values of C further substantiates the validity of Equation 10a. The mean value of C is 1.32, and on substitution in Equation 10a, we get

$$\Delta S_a = 1.32\,e^{-\frac{4.8}{1-M}(M-0.7)^2 + 0.72r}$$ Equation 10b

Now Equation 1, i.e. $S_a = S_{ai} + \Delta S_a$ can be written by combining Equation 7 and 10b as

$$S_a = 2.69\,e^{2.14M-1.77r} + 1.32\,Z\,e^{-\frac{4.8}{1-M}(M-0.7) + 0.72r}$$ Equation 11a

Generally, the length of cut of chopped hay varies from 2 to 6 inches. For a bin

eight feet wide, the length ratio becomes 0.02083 to 0.06525. The contribution of the term (0.7r) to the value of S_a is insignificant and can be dropped out of Equation 11a. Again, for all practical purposes the term $e^{-1.77r}$ can be taken as

$$e^{-1.77 \times .04} = 0.9315.$$

With these modifications, the Equation 11a can be written for bins eight feet or wider as

$$S_a = 2.5\, e^{2.14M} + 1.32\, Z\, e^{-\frac{4.8}{1-M}(M-0.7)^2} \qquad \text{Equation 11b}$$

Substituting

$$S_{ai} = 2.5\, e^{2.15M}$$

and

$$q = 1.32\, e^{-\frac{4.8}{1-M}(M-0.7)^2}$$

we have

$$S_a = S_{ai} + q\, Z \qquad \text{Equation 12}$$

The weight W of a column of hay of one square foot cross section is

$$W = \int_{Z_1}^{Z_2} S_a\, dZ = \int_{Z_1}^{Z_2} (S_{ai} + qZ)\, dZ$$

$$= S_{ai}\,(Z_2 - Z_1) + q/2\,(Z_2{}^2 - Z_1{}^2) \qquad \text{Equation 13}$$

Comparison of Predicted Results of Field Tests

Equation 13, which predicts the weight of a column of hay, is more convenient to use for a comparison of results than Equation 12 which predicts bulk density, since bulk density cannot be directly evaluated in bins. A comparison

TABLE V

PREDICTED WEIGHT OF HAY BY EQUATION 13 AND COMPARISON WITH
THE OBSERVED DATA GIVEN IN TABLE I FOR JULY 25, 1962

Moisture Content (%)	Depth From Top (ft)	Predicted Weight by Eq. 13 (lbs/ft^2)	Observed Weight (lbs/ft^2)
21	0.0 – 3.438	15.221	13.438
22	3.438 – 6.50	16.975	19.500
21.5	6.50 – 8.917	15.230	14.938
25	8.917 –10.833	14.813	17.375
Total		62.239	65.25

of the predicted and observed results is shown in Tables V, VI, VII, VIII, and
IX for five different tests.

TABLE VI

PREDICTED WEIGHT OF HAY BY EQUATION 13 AND COMPARISON WITH
THE OBSERVED DATA GIVEN IN TABLE I FOR JULY 19, 1963

Moisture Content (%)	Depth From Top (ft)	Predicted Weight by Eq. 13 (lbs/ft^2)	Observed Weight (lbs/ft^2)
25.6	0 - 2.917	14.130	13.563
26.1	2.917 - 5.875	17.686	19.313
25.1	5.875 - 8.833	20.306	18.750
25.2	8.833 -10.333	11.507	15.375
Total		63.629	67.001

TABLE VII

PREDICTED WEIGHT OF HAY BY EQUATION 13 AND COMPARISON WITH
THE OBSERVED DATA GIVEN IN TABLE 1 FOR SEPTEMBER 6, 1963

Moisture Content (%)	Depth From Top (ft)	Predicted Weight by Eq. 13 (lbs/ft^2)	Observed Weight (lbs/ft^2)
42.3	0 - 3.00	21.79	23.25
39.3	3.00 - 6.50	30.75	25.625
41.7	6.50 - 8.625	24.03	22.313
Total		76.57	71.188

TABLE VIII

PREDICTED WEIGHT OF HAY BY EQUATION 13 AND COMPARISON WITH
THE OBSERVED DATA GIVEN IN TABLE I FOR JUNE 2, 1964

Moisture Content (%)	Depth From Top (ft)	Predicted Weight by Eq. 13 (lbs/ft^2)	Observed Weight (lbs/ft^2)
56.7	0 - 2.021	19.138	17.50
55.4	2.021 - 3.458	15.765	16.00
56.2	3.458 - 5.458	25.908	27.188
56.8	5.458 - 6.771	19.531	20.313
57.2	6.771 - 8.188	23.336	19.188
Total		103.678	100.189

TABLE IX

PREDICTED WEIGHT OF HAY BY EQUATION 13 AND COMPARISON WITH
THE OBSERVED DATA GIVEN IN TABLE I FOR JULY 15, 1964

Moisture Content (%)	Depth From Top (ft)	Predicted Weight by Eq. 13 (lbs/ft^2)	Observed Weight (lbs/ft^2)
30.1	0 - 4.00	22.466	17.125
27.6	4.00 - 6.667	17.560	16.938
24.8	6.667 - 9.750	21.900	18.750
31.6	9.750 -11.500	17.064	19.100
Total		78.99	71.913

SUMMARY AND CONCLUSIONS

The bulk density of chopped alfalfa hay was found to vary directly with the moisture content and depth of storage. This is consistent with the findings of Davis and Baker (2). The following expression for the bulk density at any depth was developed

$$S_a = 2.5\ e^{\,2.14M} + 1.32\ Z\ e^{\frac{-4.8}{1-M}\ (M - 0.7)^2}$$

where M is the moisture content of the hay expressed as a decimal and Z is the vertical distance from the top surface of the hay (feet).

The following table shows some values calculated from the above expression. Bulk densities are given in pounds per cubic foot.

Depth	Moisture Content When Placed in Storage			
	15%	25%	40%	60%
Top Layers	3.4	4.3	5.9	8.8
5 Feet	4.6	6.1	9.1	14.7
10 Feet	5.7	7.9	12.3	20.5
20 Feet	8.1	10.8	18.8	32.2

The above table indicates that a value of 8 to 10 pounds per cubic foot for the density of hay is suitable for structural design purposes as long as the moisture content of the hay is 25% or less. When placed in storage at higher moisture contents (as is usually the case when forced air drying is employed), a design based on an average density of 8 to 10 pounds per cubic foot may result in structural failure if depths are in excess of 15 feet since the average density is likely to exceed these values.

The information on density of hay is not only valuable for structural design purposes, but is also extremely important in the design of drying systems. Previous investigations have shown that resistance of hay to air flow increases approximately as the third power of its density in pounds per cubic foot. Thus, if the density of hay is *twice* as high at the bottom of the mow as it is at the top of the mow, the resistance to air flow will be *eight* times as high at the bottom when air flow through the hay is in a vertical direction.

The weight W of a column of hay one square foot in cross section may be determined from the expression

$$W = S_{ai}\ (Z_2 - Z_1) + q/2\ (Z_2 - Z_1)^2$$

where

$$S_{ai} = 2.5\ e^{\,2.15M}$$

and

$$q = 1.32\ e^{\frac{-4.8}{1-M}\ (M - 0.7)_2}$$

Weights predicted from the above expression were found to conform quite closely with weights determined experimentally as shown in Tables V through IX.

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APPENDIX

derivation of a depth-density relationship from
a load-density curve

Data

1. Moisture content of hay (wet basis) = 66%
2. Initial bulk density (density of hay without any superimposed load) = 5.8 lbs/ft^3.
3. Load-density relationship. Figure 3. Curve A.

Assumptions

Consider a column of hay of one square foot cross section. The bulk density of one foot depth of hay is numerically equal to the weight of it. Again, let the column be divided into one foot layers, starting from the top. Let the topmost layer be designated as first layer, the one next to it as second layer and so on.

Calculations

  I. *Bulk density of first layer.* Bulk density of first layer = initial bulk density = 5.81 lbs/ft^3.
 II. *Bulk density of second layer.*
     A. The superimposed load on the second layer is the same as the weight of first layer (5.8 lbs).
     B. The load in Figure 3 includes the weight of the hay itself. Hence calculation of total load, i.e. sum of superimposed load and weight of hay of the layer under consideration, is necessary to evaluate bulk density from Figure 3. As the weight of the second layer of hay, which is numerically equal to its bulk density, is not known it is to be estimated by successive approximations as illustrated in the following steps.
         1. The first approximation.
             a. Let the first estimate of the bulk density of the second layer be 5.8 lbs/ft^3.
             b. Hence the weight of second layer is 5.8 lbs.
             c. The total load is (5.8 + 5.8)lbs = 11.6 lbs.
             d. From Curve A., Figure 3, bulk density = 6.3 lbs/ft^3. As the value of bulk density obtained from the curve is different from that estimated, the process of approximation must be repeated (until the two values are approximately the same).
         2. The second approximation.
             a. Let the second estimate of the bulk density of the second layer be slightly more than 6.3 lbs/ft^3 (6.4 lbs/ft^3).
             b. Hence the weight of the second layer = 6.4 lbs.
             c. The total load is (5.8 + 6.4) lbs = 12.2 lbs.

      d. From Curve A., Figure 3, bulk density $= 6.4$ lbs/ft^3. As the value of the bulk density obtained from the curve is same as that estimated, this is accepted as the value of bulk density of the second layer.

III. *Bulk density of third and successive layers.* The superimposed load for for any given layer is the sum of weights of all the layers above it. The weight and bulk density of hay itself is calculated in the same way as explained under II B. The calculation is carried out for superimposed loads up to 90 pounds.

The bulk densities of hay for 66 percent moisture content for different depths, calculated according to the method described above, are given in the following table.

TABLE X

DETAILS OF CALCULATIONS FOR DERIVATION OF DEPTH – DENSITY RELATIONSHIP
FROM LOAD – DENSITY RELATIONSHIP

Depth (ft)	Super-imposed Load (lbs)	First Approximation of Bulk Density (lbs/ft)	Total Weight (lbs)	Bulk Density From Fig. 3 (lbs/ft)	Second Approximation of Bulk Density (lbs/ft)	Total Weight (lbs)	Bulk Density From Fig. 3 (lbs/ft)
0–1	0	5.8	5.8	5.8	5.8	5.8	5.8
1–2	5.8	5.8	11.6	6.3	6.4	12.2	6.4
2–3	12.2	6.4	18.6	7.6	7.8	20.0	7.8
3–4	20.0	7.8	27.8	8.8	9.0	29.0	9.0
4–5	29.0	9.0	38.0	10.0	10.2	39.2	10.2
5–6	39.2	10.2	49.4	11.1	11.2	50.4	11.2
6–7	50.4	11.2	61.6	12.0	12.1	62.5	12.1
7–8	62.5	12.1	74.6	13.0	13.05	75.55	13.05
8–9	75.55	13.05	88.60	14.0	14.0	89.55	14.0
9–10	89.55	14.0	103.55	----	----	-----	-----