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GROWTH AND DEVELOPMENT

With Special Reference to Domestic Animals

VI. Growth Rates During the Self-Inhibiting Phase of Growth

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GROWTH AND DEVELOPMENT

With Special Reference to Domestic Animals

VI. Growth Rates During the Self-Inhibiting Phase of Growth.

Samuel Brody

Abstract.—The period of growth may be divided into two phases: (1) a *self-accelerating phase* during which the time rate of growth increases with the increase of the size of the organism; and (2) a *self-inhibiting phase* during which the time-rate of growth decreases while the size of the organism increases. This bulletin presents an analysis of growth curves of the *self-inhibiting phase* of growth. This analysis demonstrates the fact that following the major inflection in the growth curve the time-rates of growth decline by a constant percentage. Sixteen charts are presented including the growth curves of cattle, horses, swine, sheep, fowls, rabbits, guinea pigs, rats, mice, pigeons, and doves; of oat and maize plants; of the fruit of *C. pepo*; of populations of man, fly, and yeast. The observed data points as well as the theoretical curves are given and also the numerical values of the constants (mature weight and percentage decline in the time-rate of growth) for each of the curves.

INTRODUCTION

In the third bulletin of this series (Missouri Agricultural Experiment Station Research Bulletin 97) it was explained that the period of growth of multicellular organisms, and of populations of organisms, may be divided into two fairly distinct phases: (1) a *self-accelerating* phase during which the time rate, or velocity, of growth increases with the increase in the size of the organism or the population; and (2) a *Self-inhibiting* phase during which the time-rate of growth decreases with the increase in size of the organism.

The question of detailed mechanisms bringing about the similarities in the course of growth of animals, plants, and populations cannot be gone into except to note that the course of growth is in all these cases governed, directly or indirectly, by the same two primal forces: (1) the force inherent in all organisms to reproduce at a constant percentage-rate; and (2) the growth inhibiting forces resulting from the finite nature of the universe in which the organisms find themselves.

Thus in the early history of growth of organisms or populations, the food supply is relatively unlimited, as is also space, and there is nothing to keep the reproducing units from reproducing at constant intervals characteristic of their kind. In other words, growth tends to take place at a constant *percentage-rate* or at an increasing *time-rate*. Thus it comes about that the larger the organism or population, the more rapid is its *absolute* growth. In brief, *growth is a self-accelerating process.*

In the later history of the organism or population, the situation is changed. The increase in the membership of the given community in the given limited universe (the "universe" may be within the confines of a multicellular organism) causes the reproducing units to "get into one another's way". There may be a shortage of food either because there is not enough of it to go around, or because the food-transportation facilities become inadequate (thus, the surfaces of the alimentary tract through which food is supplied to the cells of a multicellular animal, or of respiratory tracts through which air is supplied to the cells, decrease in proportion to the whole body with increasing weight). Again, by-products of growth may limit the course of growth. A case in point is the growth of lactic-acid producing bacteria in milk, when the accumulating acid retards and finally inhibits growth as illustrated in Fig. 1. Whatever the growth-inhibiting mechanisms (and these are but rarely

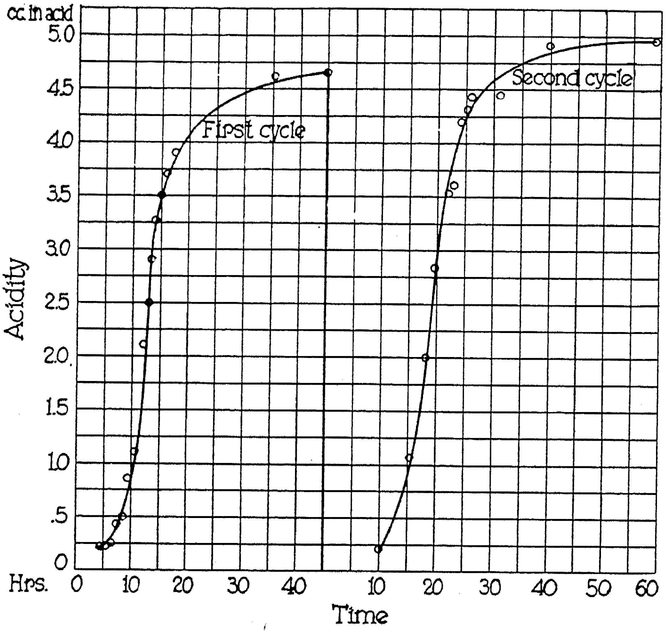

Fig. 1—Growth of lactic acid producing bacteria in whey as measured by the course of increase in acidity. The acidity is evidently the limiting growth factor, because neutralizing the medium has a rejuvenating effect on the growth of the population.

The lettering in this and in the following charts was done by Mr. Raymond Hase and Mr. James Boden, students in the University of Missouri.

known), sooner or later the percentage-rate and, later, the time-rate of growth declines, and the older (i. e. the larger) the organism, the less the amount of growth. In other words, growth becomes a self-inhibiting process.

The presence of these two phases in the course of growth gives the time-curve of growth a sigmoid appearance (resembling the letter S.) The segment of increasing slope of the letter S represents, of course, the self-accelerating phase of growth, while the segment of decreasing slope represents the self-inhibiting phase of growth. The junction between the two segments occurs at the time of puberty in animals and the time of flowering in plants.

The functional, or causal, relation between puberty and inflection in the curve is not clear. Dr. Murneek found (see Missouri Agr. Exp. Sta. Research Bulletin 90) that the tomato plant will grow indefinitely at a constant rate if its fruiting organs are not permitted to develop. In the tomato plant it therefore appears that its decline in growth (and finally its death) is due to a deflection of the food from the mother plant to its fruit. But this is not true of many other plants, and certainly not of animals.

The purpose of these bulletins is not to attempt an exposition of the detailed mechanisms bringing about these changes in growth rate, but rather to formulate "laws", i. e. quantitative rational descriptions of the course of growth. The preceding two bulletins (Missouri Agr. Exp. Sta. Research Bulletins 98 and 99) were concerned with such a detailed consideration of the segment of the growth curve preceding the major inflection (puberty). This bulletin is concerned with the segment following the inflection in the growth curve.

The principal property of the segment of the curve following the inflection is that it can be represented by the equation

$$W = A - Be^{-kt} \qquad (1)$$

or

$$\frac{W}{A} = 1 - e^{-k(t-t^*)} \qquad (2)$$

in which W is the weight (or size) of the organism (or population) at the age t, and k is the fraction of decline in the time-rate of growth. A is the weight (or size) at maturity, e is the base of natural logarithms, B is an age-parameter used to correct for the fact that while age is counted from birth or conception, this equation holds true only during the phase of growth following the major inflection. Instead of using equation (1) containing the constant B, equation (2) may be used, which frankly represents the fact that the equation begins at a certain age designated by t^*. Since the significance of B and t^* was discussed in detail in the third bulletin of this series, it need not be further considered here except to recall that t^* is that point on the age-axis where it is met by the curve of equation (1), and for the present purpose it may be considered as a more or less empirical constant or parameter.

II. A TABLE OF CONSTANTS

The weight A, of an animal at maturity is, of course, a more or less definite characteristic of the animal; it may be considered as a genetic constant of the animal. So is k, the fractional decline in the time-rate of growth of the animal. t^*, as pointed out, may for the present purpose, be considered as a more or less empirical constant although, obviously, it does express a relationship between the slope of the curve preceding the inflection and that following it. As these three constants have been evaluated, the segment of the curve during the self-inhibiting phase of growth is completely defined—provided that equation (1), or (2), represents the course of growth during this period. In Tables I and IA is given a series of numerical values of these three constants for a series of curves to which the equation was fitted.

III. GROWTH CURVES

The principal purpose of this bulletin is to present the series of growth constants (A, the mature weight; k, the fractional decline in the time-rate of growth; and t^* a more or less empirical constant) for domesticated animals for which growth data are available. But the theoretical and practical values of such constants are conditioned on the applicability of equations (1) and (2) to the growth data. It is in consequence necessary to give a detailed demonstration that the values computed from the given equation agree, within the limits of experimental error, with the observed data.

The criterion employed for judging the degree of agreement between observed and computed values depends on the point of view of the writer (the present writer's point of view was given in Research Bulletin 97, the third bulletin of this series) and on the character of the available data (especially as it relates to the *relative* changes of the magnitudes). For the present purpose, it seems best to plot the data, then to draw curves computed from the equation, and to judge the degree of agreement between observed and computed values by inspection. The character of change in shape of the curves, and the scale on which the curves are drawn, is such that a small *percentage* deviation is represented by a large *absolute* deviation. In other words, a reasonably good absolute agreement between observed and computed values indicates a relatively negligible percentage deviation. Employing this criterion, the agreement between observed and computed values appears satisfactory.

And if the agreement between the observed values and the curve computed from the equation is satisfactory, then the curves and equations may be of practical value for interpolation purposes. This is a second reason for presenting the charts rather than numerical criteria of agreement between observed and computed values.

A third reason for presenting the growth curves, is to bring out in concrete manner, the differences with respect to the mature weight (A) and the rapidity (k) of approach to the mature weight of different groups of animals of the same species. Attention is called to this very important matter in several legends to the charts.

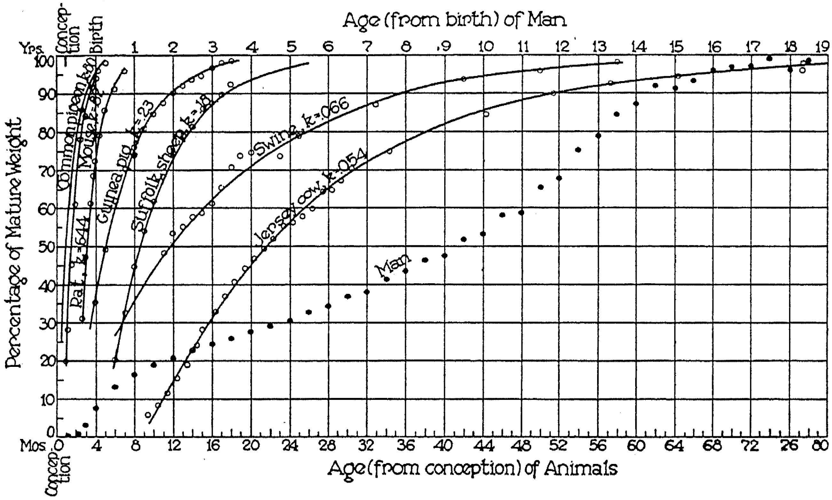

Fig. 2.—The relation between the numerical values of the velocity constant k and the steepness of the growth curves illustrated graphically. The mature weights of the animals are represented by 100 percent, and the weights at other ages by the percentages of the mature weights. This chart also shows the satisfactory agreement between observed values represented by circles and the smooth curves computed from equation (1); also the striking difference between the curve of man and of other forms.

It should be clearly understood that the values of k give a numerical measure of the rapidity of approach to the mature weight. The rapidity of approach to the mature weight is indeed directly proportional to the numerical value of k. This is shown in a qualitative way in Fig. 2, in which the mature weight is represented by 100% and other weights are represented as percentages of the mature weight.

The other charts in the bulletin, when taken with their legends, are self-explanatory and they need not be further discussed. It is perhaps only desirable to emphasize the fact that t^* is a more or less empirical point and that while the theoretical curves were extrapolated in most cases to meet the age-axis at t^*, it is not meant to imply that the equation fits the data below the inflection in the curve. It is claimed that equations (1) and (2) fit the data points only during the phase of growth following the major inflection in the curve.

While this series of bulletins is concerned primarily with the growth of animals, it seemed desirable, in order to recall the fact noted in the introduction that the course of growth is in all cases governed by the same two general forces, to present several charts on growth of plants and of populations of organisms. Accordingly, Figs. 13 to 15 are presented showing the course of growth of common plants, and Fig. 16 showing the course of growth of several types of populations during the self-inhibiting phase of growth. We do not wish to discuss further these charts in the text as we feel that the charts are the important contributions of this bulletin and not anything that may be said about them in the text. The text is presented merely as an introduction to the study of the curves themselves.

IV. SUMMARY AND CONCLUSION

The present bulletin is merely an extension of the third bulletin of this series. Its purpose is to demonstrate in some detail the statements made in the third bulletin of this series (Mo. Res. Bul. 97) relating to the properties of the segment of the growth curve following the major inflection.

The principal property of this segment of the curve is that the velocity, or time-rate of growth (i. e. gains in weight per unit of time), declines by a constant fraction, or percentage, which is principally a genetic characteristic of the given organism or population and to a less extent a result of the environmental conditions.

A table of growth constants for a series of animals is presented.

A series of curves are presented indicating the agreement between the observed values and values computed with the aid of the given growth constants.

Other items of interest are pointed out in the legends of the curves.

SOURCES OF DATA AND BIBLIOGRAPHY

The sources of data are indicated in Table 1 and in the first bulletin of this series (Missouri Agricultural Experiment Station Research Bulletin 96). The references to the literature are given in the first bulletin of this series (Missouri Agricultural Experiment Station Research Bulletin 96).

TABLE 1

No.	Animal.	A (Mature weight.)		B	100k[1] (Percentage of monthly decline in growth.)	t*	Age[2] (from conception) at One-half mature weight.[3]	Three-fourths mature weight.	98 per cent mature weight.
		kg.	*lbs.*			*mos.*[2]	*mos.*	*mos.*	*mos.*
	Beef cattle.								
	(Moulton *et al.* Missouri data)								
1	Hereford-Shorthorn, castrated males	1100	2425	1600	3.47	10.7	31	51	124
	Dairy cattle.								
	(Eckles *et al.* Missouri data)								
2	Holstein-Friesian, females	550	1215	805	4.6	8.3	23	39	93
3	Ayrshire, females	460	1014	725	5.0	9.1	23	37	87
4	Jersey, females	420	926	680	5.4	8.9	22	35	81
	Dairy cattle.								
5	Register of Merit Jersey Cattle, females, Missouri compilation,	436	961	441[4]	5.0	0.2[4]	14[4]	28	78
	Swine								
	(Mumford *et al.*, Missouri data)								
6	Duroc-Jersey, females (early breeding data)	200	441	260	6.2	4.4	15.0	26.0	67.0
	Sheep.								
7	Hampshire, males (Trowbridge *et al.*, Missouri data	90	198	170	13.6	4.7	9.8	15.0	33.2
8	Suffolk females (Murray)	80	176	200	18.5	5.0	8.7	12.5	26.0
9	Shropshire × Merino, females (Murray)	50	110	120	18.8	4.7	8.3	12.2	25.3
	Rabbit.[5]								
	(Minot.)								
10	Females	3.9	6.5	4.6	15.3	1.1	5.6	10.1	26.6
11	Males	3.0	6.5	3.9	25.3	1.0	3.9	6.6	16.6
	Rabbit.								
	Males and females (Castle).								
12	Flemish	3.2	7.1	8.0	43.4	2.7	3.7	5.3	11.1
13	F_1 Himalayan × Flemish	2.8	6.2	9.2	44.7	2.2	4.2	5.8	11.4

TABLE 1 (Continued)

No.	Animal.	A (Mature weight.)		B	100k[1] (Percentage of monthly decline in growth.)	t*	Age[2] (from conception) at		
							One-half mature weight.[3]	Three-fourths mature weight.	98 per cent mature weight.
		kg	*lbs.*			*mos.*i	mos.	*mos.*	*mos.*
14	F_1 Polish × Flemish	2.5	5.5	11.0	49.2	3.0	4.4	5.8	11.0
15	F_1 Himalayan × Polish	2.0	4.4	7.0	47.4	2.6	4.1	5.6	10.9
16	Polish	1.4	3.1	3.3	43.4	2.0	3.6	5.2	11.0
	Domestic fowl.								
	(Kempster *et al*, Missouri data)								
	Females								
17	Rhode Island Red	3.56	7.85	5.2	16.6	2.28	6.5	10.6	25.9
18	Plymouth Rock	3.40	7.50	5.43	19.5	2.40	6.0	9.5	22.5
19	Rhode Island White	3.00	6.61	4.65	20.6	2.10	5.5	8.9	21.1
20	White Leghorn	2.20	4.85	3.25	18.0	2.17	6.0	9.9	23.9
21	Ancona	1.65	3.64	4.0	34.4	2.57	4.6	6.6	15.0
	Guinea pig.								
	(Wright.)								
22	Males	.81	1.78	1.4	24.9	2.1	5.3	8.3	19.3
	Guinea pig.								
	(Castle.)								
23	F_1, Arequipa × Race B, males	1.30	2.87	1.95	19.8	2.05	5.6	9.1	21.8
24	F_2, Arequipa × Race B, males	1.04	2.29	1.46	20.9	1.62	4.9	8.3	20.5
25	F_1, *Cavia cutleri* × Race B, males	.930	2.05	1.60	27.4	2.00	4.5	7.0	16.3
26	Race B, males	.870	1.92	1.45	21.7	2.35	5.55	8.76	20.4
27	Race B, females	.800	1.76	1.59	24.7	2.6	5.7	8.4	18.5
28	F^1, *C. cutleri* × Race B, females	.785	1.73	1.26	26.1	1.8	4.5	7.1	17.0
29	F^2, *C. cutleri* × Race B, males	.725	1.60	1.10	25.1	1.66	4.4	7.2	17.5
30	F^2, *cutleri* × Race B, females	.590	1.30	1.45	40.5	2.22	3.9	5.6	11.9
31	*C.*, *cutleri*, males	.400	.882	1.7	52.9	2.74	4.0	5.4	10.1
32	*C.*, *cutleri*, females	.333	.734	.780	43.4	1.96	5.95	6.95	11.0

	Norway rat.								
	(King.)								
33	Females	.290	.638	.415	12.9	.89	6.3	11.7	31.2
34	Males	.385	.849	.43	12.2	.906	6.6	12.2	32.4
	Albino rat.								
35	Specially well fed and cared for males (Greenman and Duhring)	.350	.772	.65	35.0	1.77	3.75	5.73	12.9
	Inbred, 7-15 generation series (King).								
36	Males	.335	.738	.57	38.3	1.4	3.21	5.00	11.5
37	Females	.215	.474	.42	49.0	1.4	2.6	4.2	9.3
	On a whole milk and whole wheat diet (Sherman and MacLeod).								
38	Males	.330	.728	.80	52.0	1.7	3.03	4.37	9.3
39	Females	.223	.492	.88	70.7	1.94	2.92	3.90	7.5
	Inbred, 16-25 generation series (King).								
40	Males	.320	.705	.470	29.1	1.33	3.70	6.08	14.7
41	Females	.217	.478	.350	38.3	1.25	3.05	4.87	11.3
	Stock rats (Donaldson *et al.*)								
42	Males	.280	.617	.59	40.0	1.86	3.59	5.33	11.6
43	Females	.203	.447	.75	64.4	2.03	3.11	4.18	8.1
	Stock rats (Ferry).								
44	Males	.270	.595	.450	34.1	1.50	3.55	5.56	13.0
45	Females	.172	.379	.280	41.8	1.17	2.8	4.5	10.5
	Control rats for inbreeding experiments (King).								
46	Males	.255	.562	.400	33.0	1.36	3.46	5.57	13.1
47	Females	.189	.417	.280	35.7	1.10	3.0	5.0	10.9
	Stock rats (King).								
48	Females	.210	.463	.39	48.5	1.28	2.7	4.1	9.3
	Stock rats (Hoskins).								
49	Males	.230	.507	.86	81.0	1.63	2.5	3.3	6.5
50	Females	.166	.366	.60	88.5	1.5	2.2	3.0	5.8
	"Runt" (Series 1, No. 1) (King).								
51	Female	.147	.324	.33	62.1	1.3	2.4	3.5	7.5
	Normal litter mate to runt (Series 1, No. 3).								
52	Female	.170	.375	.460	70.7	1.4	2.4	3.4	6.9
	White mouse.								
	(Robertson.)								
53	Males	.0275	.061	.044	62.0	.76	1.88	3.01	7.10
54	Females	.0235	.052	.050	82.0	.92	1.76	2.60	5.70

TABLE 1 (Continued)

No.	Animal.	A (Mature weight.)		B	$100k$[1] (Percentage of monthly decline in growth.)	t^*	Age[2] (from conception) at One-half mature weight.[3]	Age[2] (from conception) at Three-fourths mature weight.	Age[2] (from conception) at 98 per cent mature weight.
	Pigeon.								
	(Riddle and Frey.)								
55	Common pigeon, male and female	.340	.750	1.40	80	.139	2.6	3.5	6.7
56	Ring dove, male and female	.160	.353	.006	102[6]	.06	.74	1.4	3.9

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[1] The percentage of monthly persistency of growth, $100P$, may be obtained by subtracting the percentage decline, $100k$, from 100; *i.e.*, $100P = 100 - 100k$.

[2] By age is meant, in all cases, age as counted from conception. The following ages of the animals at birth are given should the reader desire to convert the conceptional ages to birth ages: Cattle 9.4 mos., horse 11 mos., swine 4.0 mos., sheep 5.0 mos.; rabbit 1.0 mos.; fowl 0.7 mos. (21 days); guinea pig 2.2 mos. (67 days); rat 0.7 mos. (22 days); mouse 0.66 mos. (20 days); pigeon 0.6 mos. (18 days); man 0.79 yrs. (9.5 mos.)

[3] Ages at one-half mature weight, three-fourths mature weight, 98 per cent mature weight, etc. are evaluated from equation (1) by solving for t after replacing W by $1/2A$, $3/4A$, $98\%A$, etc.

[4] Data for Register of Merit Jersey Cattle are available only after the age of 2 years. For this reason we are certain of the applicability of equation (2) based on the given constants only after the age of 2 years.

[5] The rabbit appears to differ from other domestic animals by the fact that the female tends to be larger than the male and apparently also tends to require a longer period to reach maturity than the male. Castle calls attention to the fact that such difference in mature weight is observed only in the larger breeds of rabbits.

[6] $100k$ is the monthly rate of depreciation of the growth velocity. But how is it possible to have a monthly rate of depreciation of growth velocity of 102 per cent for the dove when this bird required 3 months to reach only 98 per cent of its mature weight? This paradox is due to the fact that this is a rate of depreciation of a quantity $(A - W)$ which declines from moment to moment in a geometric progression and that the absolute "interest" on this quantity, that is the velocity of growth, also declines from moment to moment in a geometric progression. It follows from this fact that the time required to deplete $(A - W)$ is longer than it would have been if the value of $(A - W)$ continued constant over the entire month, and the per centage persistency, $100P$, which is merely 100 per cent minus $100k$, is negative. Hence, strictly speaking, when employing finite units of time, k and p are only relative indices of the decline in the velocity of growth with age; that is, relative indices of the time required to reach maturity.

TABLE 1A

No.	Animal	A (Mature weight)		100k (Percentage of monthly decline in growth)	t* (from conception)	Age (from conception) at		
						One-half mature weight	Three-fourths mature weight	98 per cent mature weight
		kg.	*lbs.*					
	Horses							
1	Trowbridge and Chittenden (Missouri data) Percheron, Females	678	1493	8.2	11.0	19.5	28.0	58.7
	Males, castrated	636	1400	7.8	11.0	19.9	28.8	61.2
	Domestic Fowl							
	(Jull, U. S. D. A.)							
2	Rhode Island Red, Males	4.6	10.1	21	1.8	5.1	8.4	20.4
3	Rhode Island Red, Males	4.2	9.3	24	1.2	4.1	7.0	17.5
4	Rhode Island Red Castrated Males	4.2	9.3	24	1.2	4.1	7.0	17.5
5	Rhode Island Red Females	3.6	7.9	17	2.3	6.4	10.5	23.2
	(May R. I. Station)							
6	Cornish, males	2.8	6.2	24	2.1	5.0	7.9	18.4
7	Cornish, females	2.1	4.6	22	1.9	5.1	8.2	19.7
8	Hamburg, males	1.7	3.7	21	1.5	4.8	8.1	20.1
9	Hamburg, females	1.5	3.3	17	1.4	5.5	9.6	24.4
	(May & Waters, R. I. Station)							
10	Brahma, Males	4.1	9.0	25.1	2.7	5.5	8.2	18.3
11	Brahma, Females	3.4	7.5	24.8	2.9	5.7	8.5	18.7
12	Leghorn, Males	2.2	4.9	33.4	2.4	4.5	6.6	14.1
13	Leghorn, Females	1.8	3.9	35.0	2.3	4.3	6.3	13.5
14	L X B, Males	3.3	7.2	45.0	3.2	4.7	6.3	11.9
15	B X L, Males	3.1	6.9	35.6	2.8	4.7	6.7	13.8
16	L X B, Females	2.3	5.0	57.5	3.1	4.3	5.5	9.9
17	B X L, Females	2.5	5.4	35.5	2.6	4.6	6.5	13.6
	Albino Mouse							
	(Robertson & Ray, 1925)							
18	Males	.026	.057	67.2	1.03	2.1	3.1	6.9
19	Females	.0228	.050	63.9	.88	2.0	3.1	7.0
	Pigeon and Dove							
	(Riddle)							
20	Pigeon, males and females	.340	.75	1.64	.59	1.0	1.4	3.0
21	Ring dove	.151	.33	1.91	.52	0.9	1.2	2.6

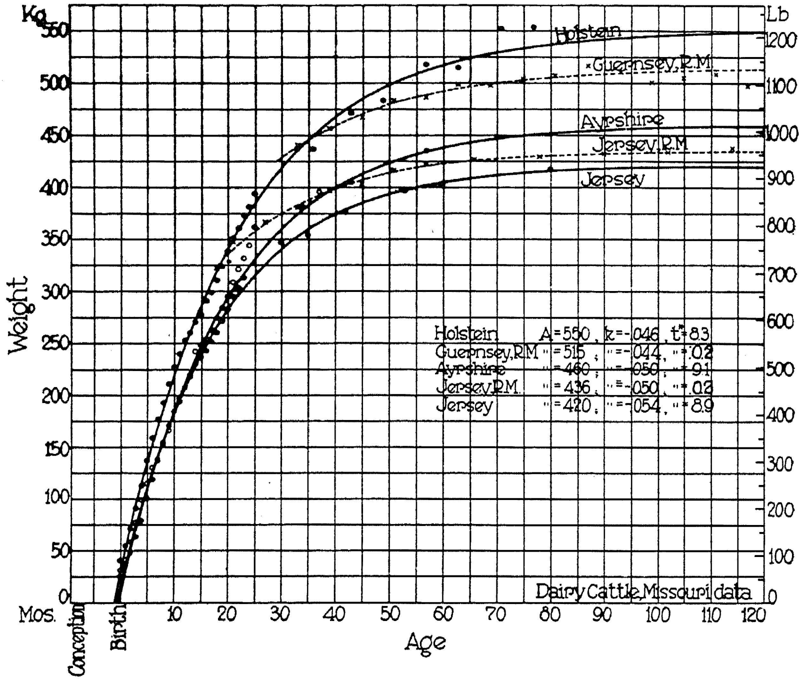


Fig. 3.—Growth curves of pure-bred dairy cattle. The circles and crosses represent observed values. The curves represent values computed from the given constants employing equation (2). The value of t* represents the age at which the curve meets the age-axis. t* represents the age as counted from conception (note: birth in dairy cattle occurs 9.4 months after conception). Compare with Fig. 15 of Research Bulletin 98.

The Advanced Register cattle indicated by R. M. and broken curves in the chart, represent data as sent by breeders to the Cattle Clubs at the *end* of the year, while the ages of the animals are recorded at the time of entry at the *beginning* of the year. Hence there may be a difference of one year between the given and actual ages. This may explain the difference in values between t* of the R. M. and the other curves. The R. M. Guernsey curve represents unpublished data by Mr. C. W. Turner of this department. The other data were obtained in this Department principally by C. H. Eckles and associates.

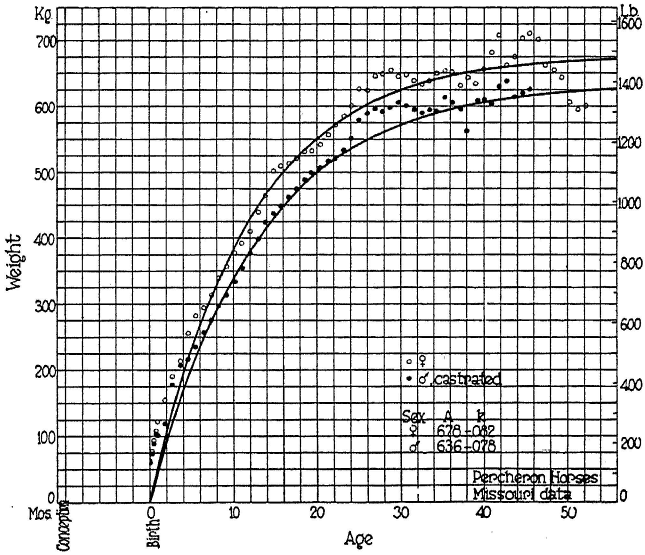


Fig. 4.—Growth of Percheron horses (data by E. A. Trowbridge and D. W. Chittenden). The curve for the castrated males is evidently below the curve for the females, a fact not entirely in agreement with the prevalent ideas on the effect of castration on growth.

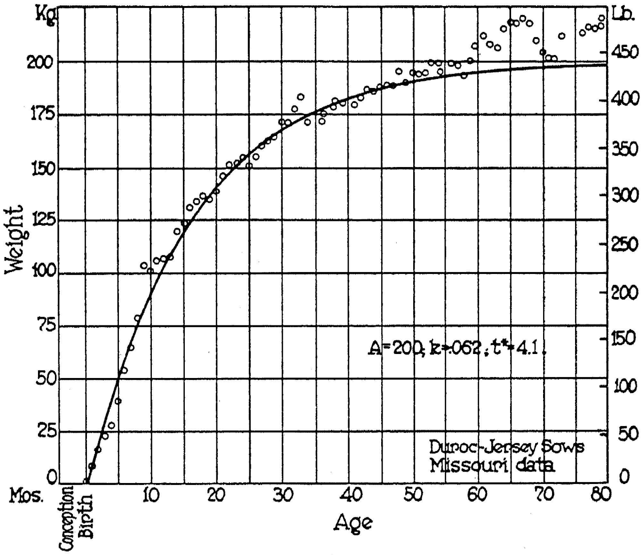


Fig. 5.—Growth of Swine. The data were obtained by F. B. Mumford in his investigation on the effect on growth of breeding immature animals. (cf. Missouri Agricultural Experiment Station Research Bulletin 45). It is possible that the rate of approach to the mature weight (i. e. the value of k) is not as great in the case of these animals as with animals under average conditions. It is probable that swine reach the maximum weight at an earlier age than that represented in this chart. The increase in the observed values as compared to the computed values following the age of 50 months, is no doubt due to excessive fattening which the equation employed does not consider.

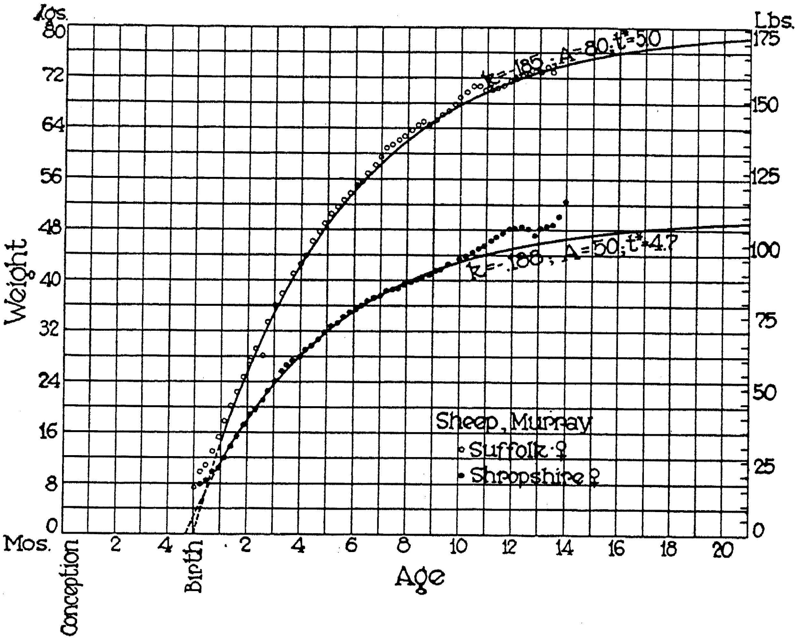


Fig. 6.—Growth of Sheep. Compare with Fig. 16 of Research Bulletin 98.

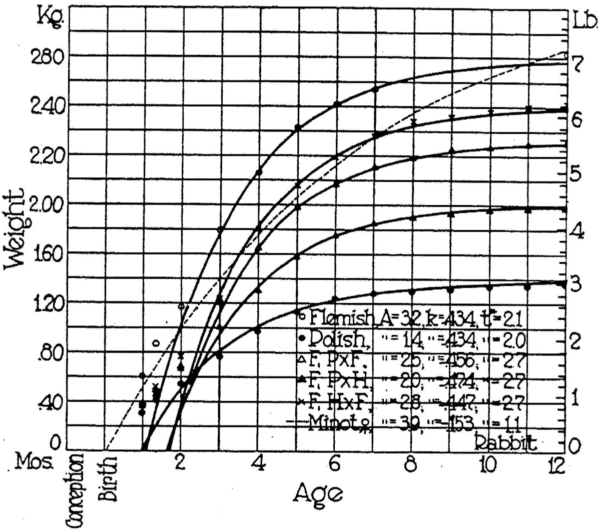


Fig. 7.—Growth of the Rabbit. The smooth curves represent the data of Castle. The broken curve represents the data of Minot. The difference between the two types of curves is remarkable, yet no more remarkable than the difference between the curves of the albino rat and the Norway rat (Fig 10). Both of these probably represent cases of retarded growth due to faulty nutrition. The animals of Minot correspond to the Norway rat with respect to the shape of their growth curves while those of Castle correspond to the albino rat in shape.

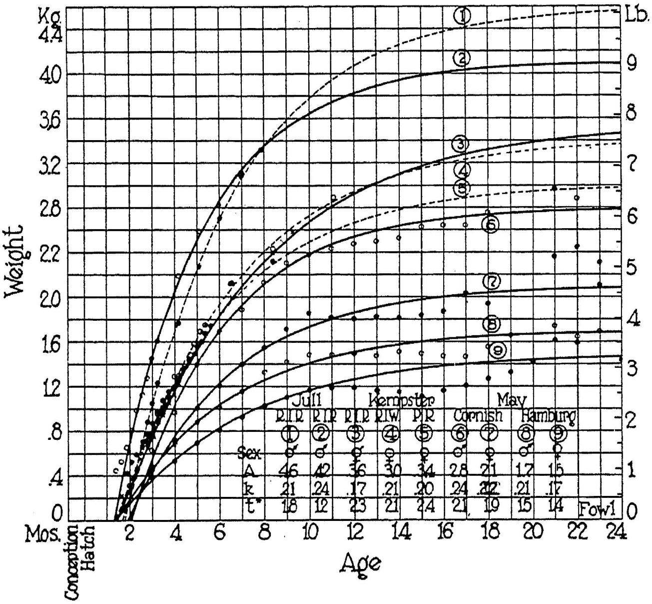


Fig. 8.—Growth of chickens. The values of A and k may be in error on account of the paucity of data for the later ages.

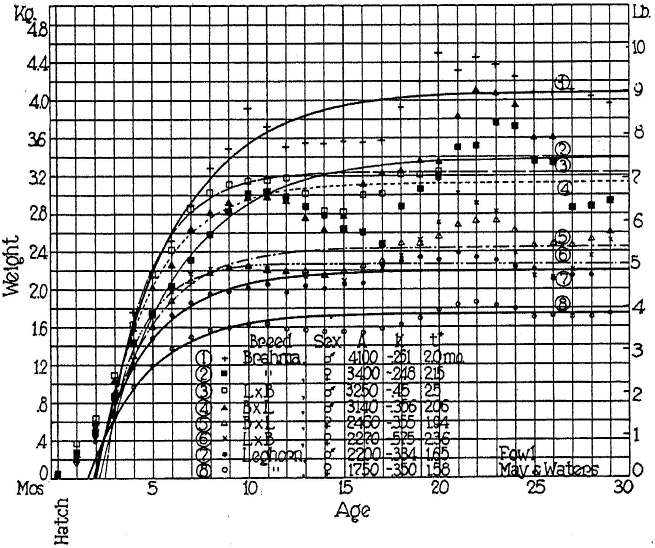


Fig. 8a.—Additional curves for the growth of the domestic fowl including Brahmas, Leghorns, and crosses of these two breeds. (t^* is counted from hatching.)

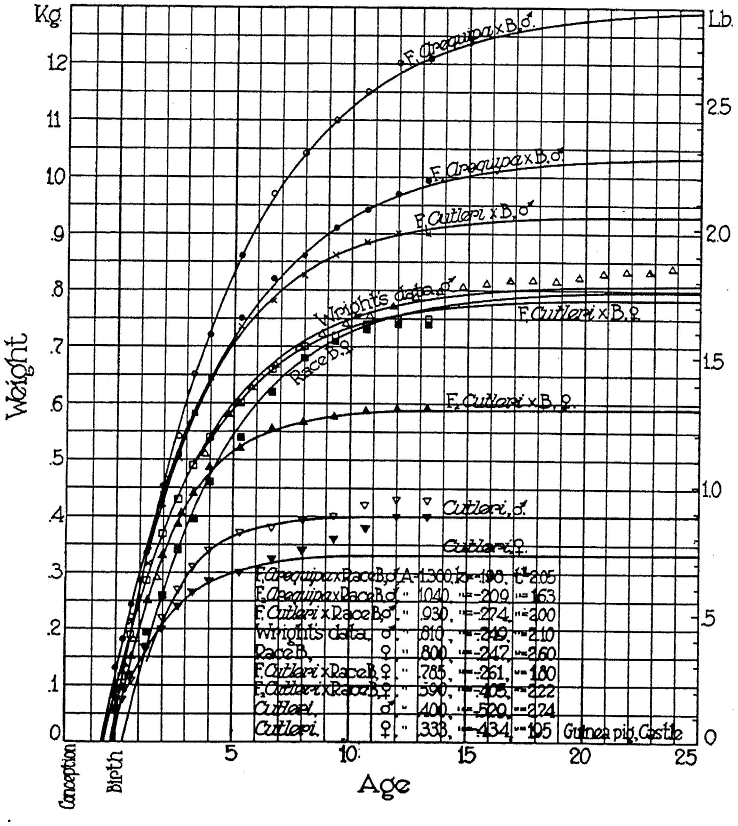
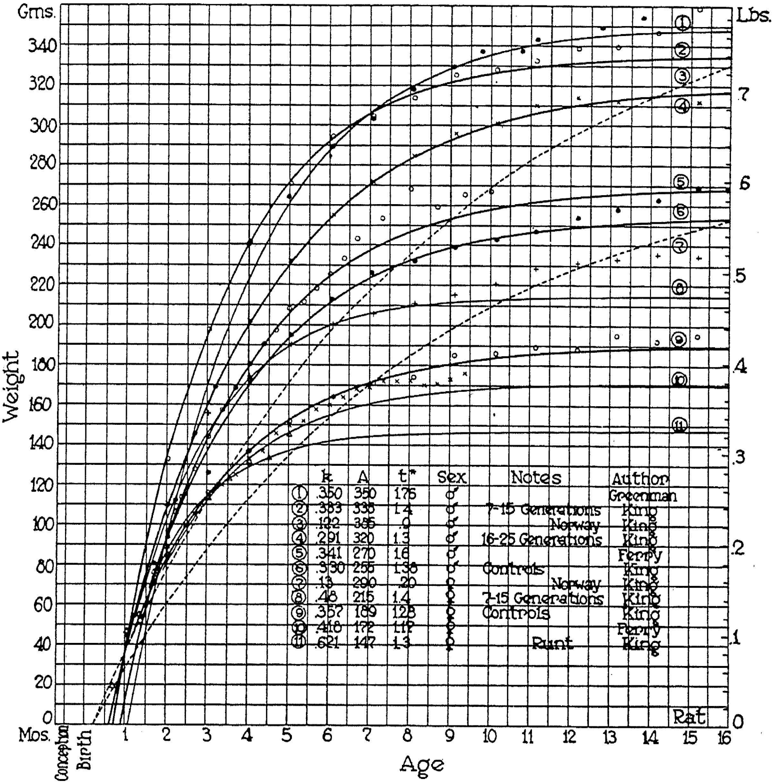


Fig. 9.—Growth of Guinea Pigs. On comparing the successive values of A and k one is impressed with the fact that the smaller the animals the more rapidly is the mature weight approached (i. e. the larger the values of k). However, there are exceptions to this general regularity. In general, however, it appears that the differences in size of animals of a given species may be due to differences in the duration of the period of growth.



	k	A	t*	Sex	Notes	Author
①	.350	350	176	♂		Greenman
②	.383	335	1.4	♂	7-15 Generations	King
③	.122	385	.9	♂	Norway	King
④	.291	320	1.3	♂	16-25 Generations	King
⑤	.341	270	1.6	♂		Ferry
⑥	.330	255	1.38	♂	Controls	King
⑦	.13	290	.20	♀	Norway	King
⑧	.48	215	1.4	♀	7-15 Generations	King
⑨	.357	189	1.28	♀	Controls	King
⑩	.418	172	1.12	♀		Ferry
⑪	.621	147	1.3	♀	Runt	King

Fig. 10.—Growth of rats. This chart brings out the remarkable differences in the shape of growth curves of the albino rat, and the wild Norway rat. A similar situation was met in the case of growth of the rabbit (Fig 7). Are these differences due to environmental or genetic factors? Dr. King, who is investigating the shape of growth curves of the Norway rat in successive generations, finds that the curves of the Norway rat tend to approach more and more those of the albino rat with the passage of successive generations as bred under laboratory conditions. This chart also serves to bring out in a striking manner the remarkable differences with respect to the mature weight, A, and the rapidity of approach, k, to the mature weight of the different groups of animals.

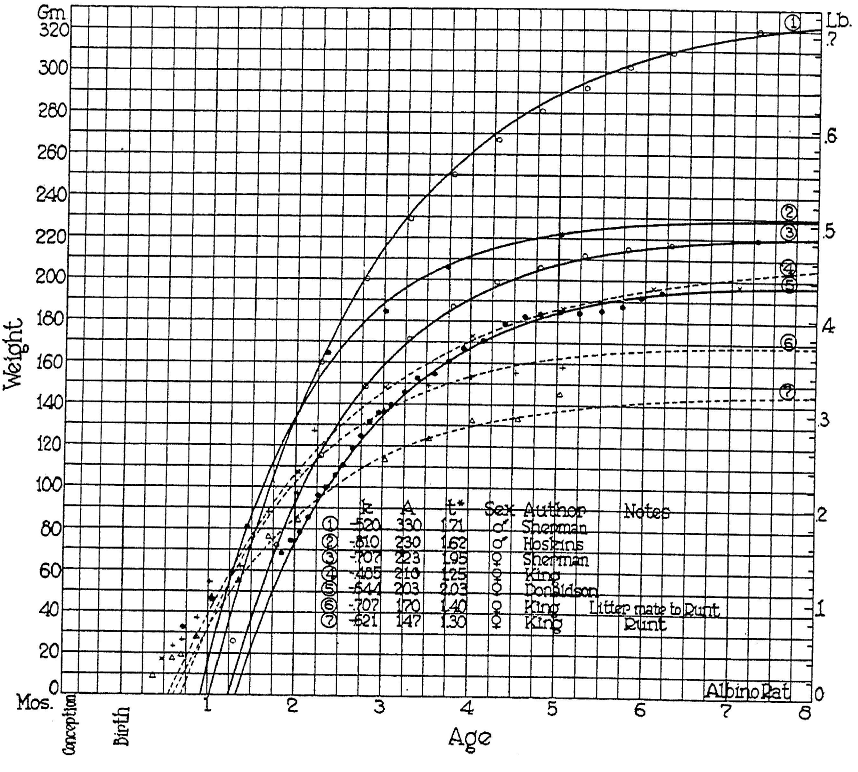


Fig. 10a.—Additional Curves for the Growth of the Rat.

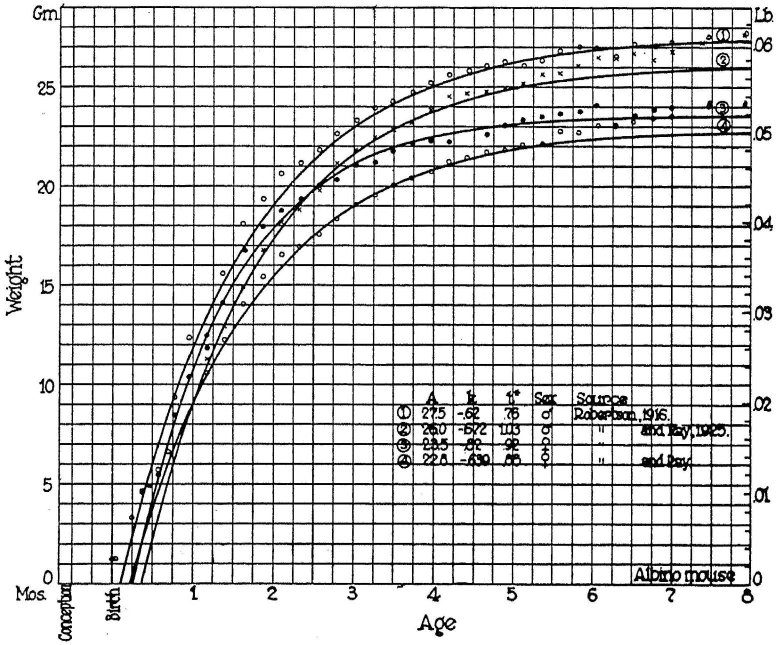


Fig. 11.—Growth of the Albino Mouse. Following 8 months the observed values exceed to an increasing extent the computed values. But increasing weight in the mouse following the age of 8 months can no more be considered to be growth than increasing weight in man following the age of 30 years.

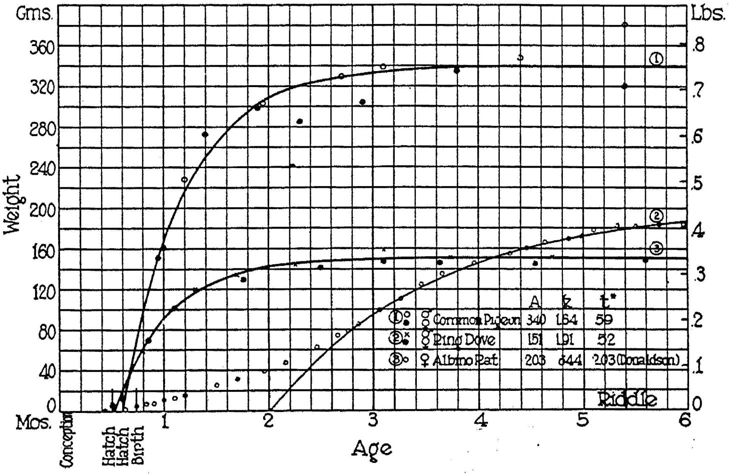


Fig. 12.—Growth of the Pigeon and Dove (from unpublished data by Dr. Oscar Riddle). For the purpose of comparison data for the albino rat were included. While the mature weight of the pigeon is greater than that of the rat, yet the mature weight in the pigeon is reached before that in the rat. The pigeon is larger than the rat not because it grows longer, but because it grows faster. The embryonic, "infantile", and "juvenile" periods are extremely short in the pigeon as compared to mammals of the same weight.

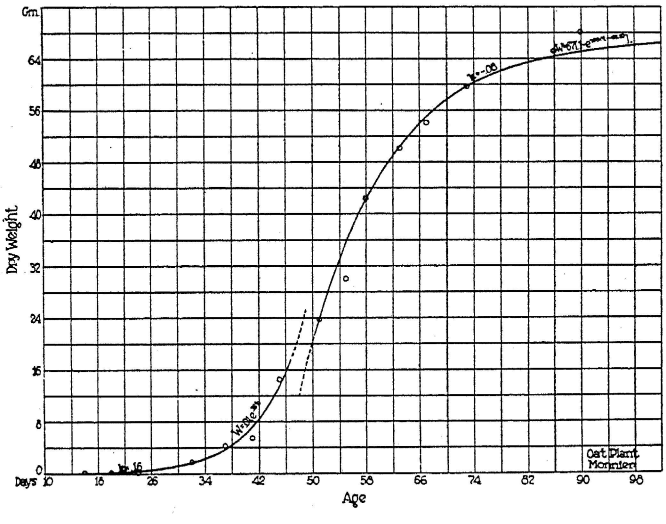


Fig. 13.—Growth of the Oat Plant. Age is counted from the time of seeding. The velocity of growth is at a maximum at about 7 weeks. Up to this time the velocity of growth *increases* at the rate of 16 percent per day. Following this time the velocity of growth *decreases* at the rate of 8 per cent per day. The curve is, therefore, not symmetrical about its center. Compare to Fig. 21 of Research Bulletin 68 of this series.

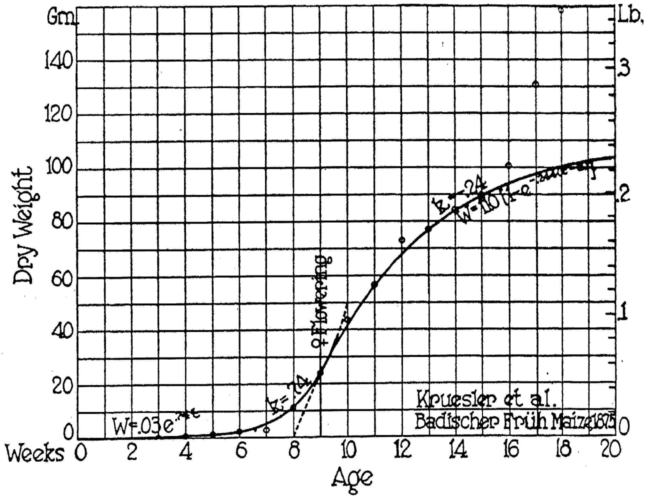


Fig. 14.—Growth of the Corn Plant. The inflection (maximum growth velocity) occurs at the time of flowering which is 9 weeks after seeding. Up to this time the velocity of growth *increases* at the rate of 74 percent per week; following this the velocity of growth *decreases* at the rate of 12 percent. Compare to Fig. 22 of Research Bulletin 98 in this series.

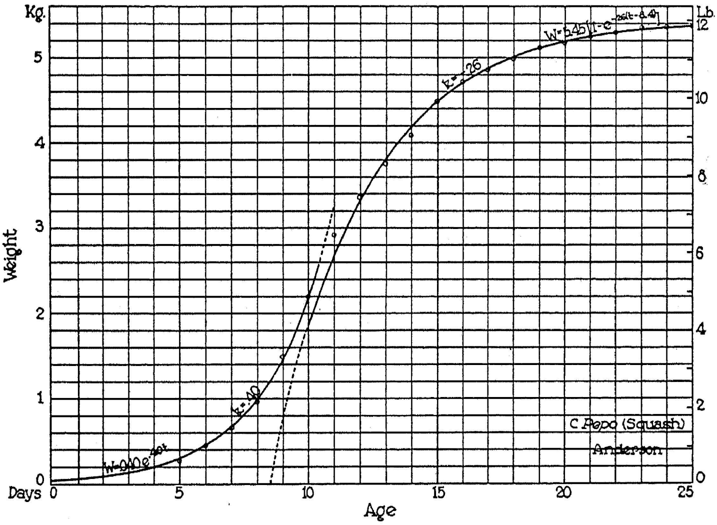


Fig. 15.—Growth of the Pumpkin (or squash) Fruit. Time is counted from pollination. The maximum velocity of growth occurred at 10 days. Up to this age the velocity *increased* at the rate of 40 percent per day. Following this time the velocity *decreased* at the rate of 26 percent per day. Compare to Fig. 20 of Research Bulletin 98 of this series and to Fig. 17 of T. B. Robertson's monograph on "The Chemical Basis of Growth and Senescence".

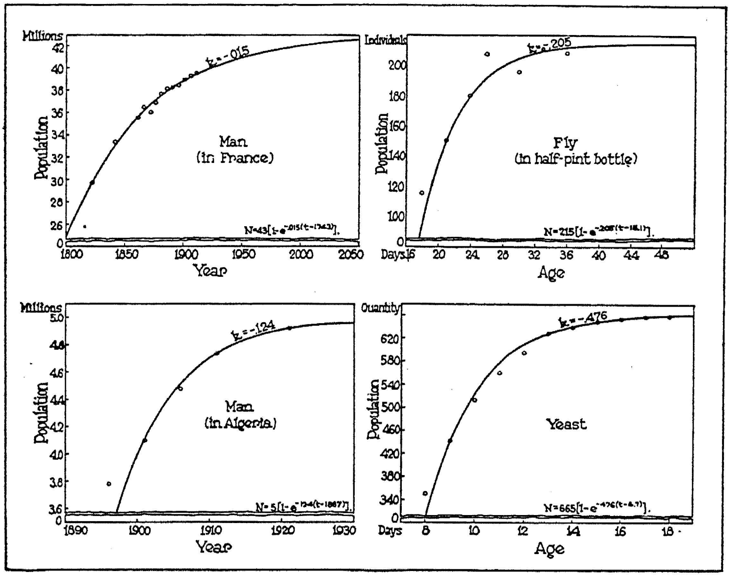


Fig. 16.—Growth of Several Kinds of Populations During the Self-inhibiting Phase of Growth. (Plotted from data cited by Raymond Pearl in "The Biology of Population Growth", published by A. A. Knopf, New York, 1925.). The Velocity of growth of the human population in France *declines* at the rate of 1.5 per cent per year; of the population in Algeria, at the rate of 12.4 per cent per year; of a fly population under the given conditions, at the rate of 20.5 percent per day; of a yeast population under the given conditions, at the rate of 47.6 percent per day.