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M. F. MILLER, *Director*

Growth and Development

With Special Reference to Domestic Animals

LVI. The Influence of Dairy Merit, Body Size, and Plane of Nutrition on the Economy of Milk Production

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FOREWORD

The special investigation on growth and development is a cooperative enterprise in which the departments of Animal Husbandry, Dairy Husbandry, Agricultural Chemistry, and Poultry Husbandry have each contributed a substantial part. The parts for the investigation in the beginning were inaugurated by a committee including A. C. Ragsdale, E. A. Trowbridge, H. L. Kempster, A. G. Hogan, and F. B. Mumford. Samuel Brody served as Chairman of this committee and has been chiefly responsible for the execution of the plans, interpretation of results and the preparation of the publications resulting from this enterprise.

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ABSTRACT

The concept *dairy merit* is defined quantitatively and a table and a formula are presented for its evaluation for individual cows to serve as a yardstick for selecting cows of high lactational ability regardless of live weight. The milk yield is lactationally significant only in relation to the maintenance cost, or to the size, of the animal, and the proposed method for evaluation of *dairy merit* thus gives quantitative meaning to lactational aptitude which absolute milk yield alone does not give. The concept *lactationally-effective* body size is defined quantitatively and a table is presented for its evaluation for individual cows. The relation of the *dairy merit* and *lactationally-effective* body size indices to dairy cattle improvement and to potential dairy profit is somewhat analogous to the relation of the Babcock butter-fat test to dairy cattle improvement and to potential dairy profit. A theory is adjusted to function as a useful tool. It is shown that if dairy merit and other conditions are equal, the monetary profit per unit milk produced, per cow, and for the herd increases very rapidly with increasing size of cow. Other conditions are not, however, equal and the sources of inequality are discussed critically. The influence of dairy merit and plane of nutrition on the profit of animals of equal body size are also discussed analytically with reference to war-time applications.

Growth and Development

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LVI. The Influence of Dairy Merit, Body Size, and Plane of Nutrition on the Economy of Milk Production

Samuel Brody

The dairy farmer would be very glad indeed to function more abundantly in the support of national health if an economic framework could be devised.—H. D. Kay.

Milk-production economics depend on many factors of which dairy merit, body size, and plane of nutrition are more or less under the dairyman's control and a study of which is, therefore, appropriate in this critical period of milk scarcity.

1. DAIRY MERIT: QUANTITATIVE DEFINITION

"Dairy merit" is usually understood to mean "milk producing efficiency" and is often judged by conformation, temperament, and similar qualitative characteristics which are thought to condition milk production efficiency. For the purpose of the present study we shall quantitize these qualities by defining dairy merit or *dairy-merit ratio* by the ratio of milk-energy production to TDN (total digestible nutrients) energy consumption. This definition may be represented by the equation

$$\begin{matrix}\text{Lactational efficiency} \\ \text{or} \\ \text{Dairy-merit ratio}\end{matrix} = \frac{\text{milk-energy production}}{\text{TDN-energy consumption}} = \frac{340 \times \text{lb. FCM produced}}{1814 \times \text{lb. TDN consumed}}$$

assuming that 1 lb. FCM (4%-fat milk) has an energy equivalent of 340 Calories and 1 lb. TDN has an energy equivalent of 1814 Calories. Dairy merit of the *animal* is numerically equal to the energetic efficiency of the lactation *process*.

The upper limiting value of this dairy-merit ratio is 50%; not over one-half of the consumed TDN energy can be converted into milk energy. "Superior" dairy animals convert about a third, about 33%, of the consumed TDN energy into milk energy. "Good" dairy animals convert about a fourth, about 25%, of the consumed TDN energy into milk energy.[1]

[1]Brody, S., J. Nut. 17, 235, 1939; Science 95, 485, 1942.

This dairy-merit ratio appears to be independent of body size *as such;* its upper limiting value is approximately the same in rats, goats, cows, and even in humans.[1] Approximately the same percentage of digestible dietary nutrients consumed may be converted into milk in all these species, large or small.* The profit on milk production, however, does vary with body size of the animal because, if other conditions are equal, the overhead expense per unit milk production declines with increasing size of animal. Let us, therefore, next define body size quantitatively.

2. LACTATIONALLY-EFFECTIVE BODY SIZE: QUANTITATIVE DEFINITION

Since the feed is converted into milk by the body, the quantity of such conversion of feed to milk should, other conditions being equal, increase with increase in the size of the body. Body size must be an important factor in the quantity of milk production. It is true that some large cows yield no more, even less, than small ones, but this is because the *dairy merit ratio,* the lactational drive, of the large cow is inferior to that of the small one; because other conditions are not equal. Thus dairy cows produce more milk than beef cows independently of similarities or differences in body size because other conditions are not equal. But when other conditions are equal, a small dairy cow should yield more milk than a dairy goat; a large dairy cow should yield more than a small dairy cow, and since there is a maintenance cost to every pound of live weight, every pound of live weight necessarily counts for or against the dairy merit and profit of the animal yielding a given quantity of milk energy, depending on whether or not each pound produces milk in proportion to its maintenance cost.

Dairymen, of course know this. Why, then, do they report milk records without reference to body weight? With the modern chest-girth tape measure which is graduated to read in pounds, estimating weights of cows is simpler than estimating butterfat production; and, as will be presently explained, for estimating dairy merit or milk-production efficiency and profit, it is almost as important to have body weights as milk yield.

The major reason for the general neglect of the body weight datum in reporting milk yield is that milk production does not increase *directly* with *simple* body weight but in a more complex manner which appears to be confusing. For instance, we know of a 700-lb. cow (Stone-

*There is no reason for assuming that different amounts of consumed feed energy, above the maintenance needs, should be required to produce unit milk energy in, for example, 700 and 1400-lb. cows; there is no reason why the energy cost of producing unit milk above the maintenance cost, should be different in the two animals. If the maintenance cost is included the efficiency will be the same if the ratio of milk-energy production to maintenance-energy cost is the same in large and small animals, and this appears to be the case.

hurst Patrician's Lily) that produced at the rate of 70 lbs. FCM a day or 26,000 lbs. FCM a year, but it is probably impossible for a 1400-lb. cow to produce at the rate of 140 lbs. FCM a day or 52,000 lbs. FCM a year. Milk production evidently does not increase directly with simple weight. This gives the superficial impression that body weight may not be an important factor in milk production, and as it is difficult to think through this peculiarity, it is ignored. But the body weight factor is important, very important indeed and we must, therefore, discuss the involved relationships although they are repugnant in their complexity.

Milk is not produced by the body as a whole, but only by the visceral (internal) organs and by the surfaces that participate in the digestive, assimilatory, respiratory, excretory, and secretory, including endocrine, processes. The supporting structures (skeletal muscles and bones) do not participate in the milk-production process; and it so happens, for reasons explained below, that these non-participating supporting structures increase at a relatively more rapid rate or the visceral and surface structures increase at relatively less rapid rate than the body as a whole.

In other words, a 1200-lb. cow cannot produce ten-fold the milk energy of a 120-lb. goat at its upper limit, and a 1400-lb. cow cannot produce twice the milk energy of a 700-lb. cow at its upper limit, because large animals have relatively larger supporting structures and, therefore, relatively smaller visceral organs and areas than small animals. The explanation for the relatively larger supporting structures in larger animals is given in the following two paragraphs.

As animals increase in size, the pull of gravity increases directly with body weight (body weight is the pull of gravity!), that is, with the cube of the linear dimensions; surfaces, however, increase not with the cube but with the square of the linear dimensions.* The result is that the surfaces increase not with simple body weight, $W^{1.0}$, but with, approximately, the 2/3 power of body weight, with $W^{2/3}$.

As animals increase in size, the pull of gravity increases directly with body weight but the strength of the supporting structures, such as of the legs, tend to increase with the 2/3 power of the body weight (that is, with the cross-section areas of the supporting structures);

*Geometrical derivation of this argument:

Surface area, S, varies with the square of linear size, L.

$$S \propto L^2$$

Volume or, what is virtually the same, weight, W, varies with the cube of linear size, L,

$$W \propto L^3$$

or linear size is the cube root of weight

$$L \propto W^{1/3}$$

and surface area, S, is, therefore, the 2/3 power of weight

$$S \propto L^2 \propto (W^{1/3})^2 \propto W^{2/3}$$

On converting the proportionality to an equality we have

$$S = aW^{2/3}$$

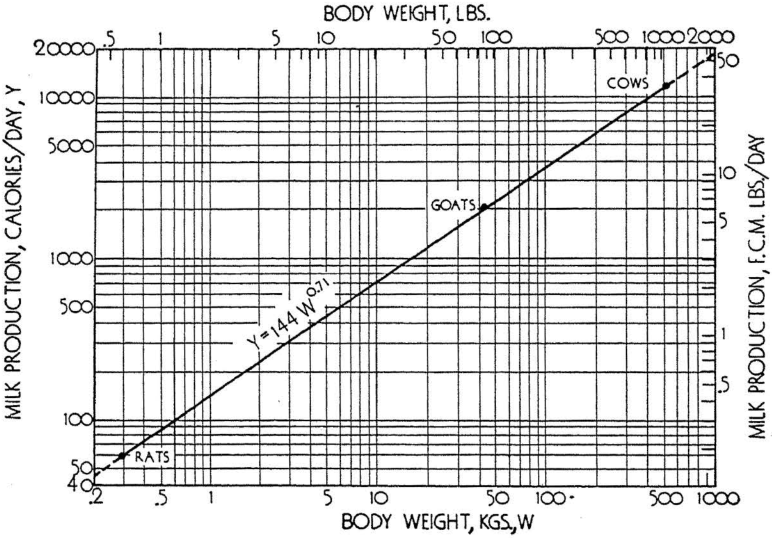


Fig. 1.—The relation of milk-energy production to body weight in mature animals of different species: averages of 368 "good" cows, 7 good goats, 5 excellent rat mothers. Plotted on a logarithmic grid. Y represents milk-energy production per day for body weight, W.

hence, to retain stability the supporting structures must grow more rapidly than the visceral organs, or the visceral organs must grow less rapidly than the body as a whole, approximately in proportion to $W^{2/3}$, which they do[2]; and, as previously explained, it is the metabolism-supporting visceral organs and the nutritive and excretory surfaces that condition and limit the functional rates including milk production; hence milk production tends to increase not directly with simple weight, $W^{1.0}$, but with $W^{2/3}$.

In an analysis of milk-energy production in relation to body weight in different species, rats, goats, and cows, milk production[2] was observed to vary with, approximately, the 0.7 power of body weight, with $W^{0.7}$ as shown in Fig. 1. This means that increasing body weight 1% tends to be associated with an increase in milk production of 0.7%. More concretely, a 1400-lb. cow tends to produce not 100% more milk than a 700-lb. cow, but only 70% more, and this only when the dairy-merit ratios are the same in the 1400- and 700-lb. cows.

The reference base $W^{0.7}$ is, of course, very close to that of $W^{2/3}$, the conventional reference base for the surface-weight relation ($S = aW^{2/3}$); so that one may say, if one prefers, that milk-energy production tends to vary directly with surface area.

An important fact in this connection is that the basal energy metabolism and resting maintenance needs for energy and protein, also vary with, approximately, the 0.7 power of body weight, with $W^{0.7}$. The energy cost of moving the body during walking and similar activities varies directly with body weight, with $W^{1.0}$; however, voluntary activities of animals tend to decline with increasing body weight; large animals tend to make fewer and slower movements than small ones, and the decline appears to be in such manner that the total maintenance cost tends to vary with $W^{0.7}$, in the same manner as does milk-energy production.

Teachers of livestock feeding appear to feel that the maintenance cost does not increase in direct proportion to simple body weight. Thus, according to Morrison's book, a 120-lb. sheep needs 2 lbs. TDN a day for maintenance and a 1200-lb. cow needs not ten-fold that of the sheep or 20 lbs., but only 10 lbs. TDN a day.

If the maintenance-energy cost and the milk-energy production vary in the same manner with increasing body weight, that is with $W^{0.7}$, the dairy-merit ratio, that is, the ratio of milk-energy production to feed-energy consumption, must be the same in small and large animals because, as previously noted, there is no reason for assuming that animals differing in size differ in their ability of feed utilization for

[3]Missouri Res. Buls. 222, 1935; 238, 1936; 291, 1938, 285, 1938.
[2]Missouri Agr. Exp. Sta. Res. Bulls. 328 & 335, 1941.

milk production, and we virtually demonstrated[1,3] that this is the case in animals of different species, rats, goats, cattle, and even in humans; in all these species, regardless of body weight, the upper limiting value of the *dairy-merit ratio* is 50%, and superior animals have a ratio near 33%. We thus have the important conclusions that (1) if other conditions are equal, dairy merit is independent of body size; it is the same in small and large animals; (2) milk-energy production, as maintenance cost, varies not with simple body weight, $W^{1.0}$, but with $W^{0.7}$. The lactationally-effective body size is represented not by $W^{1.0}$ but by $W^{0.7}$.

3. EVALUATION OF DAIRY MERIT

Dairy merit is defined by the ratio of milk-energy production to TDN-energy consumption. It is easy to obtain the milk-energy production* but difficult to obtain the TDN-energy consumption. Because of this difficulty indirect dairy-merit *indices* may be used.

The dairy-merit index proposed by Gaines[4] is the ratio of milk production to live weight, FCM/W, as for example, milk production per 1000 lb. live weight.

The FCM/W dairy-merit index is the easiest to compute and to understand. It is, perhaps, satisfactory for comparing animals close together in body weight, such as those within a homogeneous breed of cattle. But, as previously explained, the FCM/W index is not satisfactory when animals differ widely in weight, as, for example, Carnation Ormsby Butter King Daisy and Stonehurst Patricians Lily, 1700 lbs. and 700 lbs. respectively. The 700-lb. cow produced at the average rate of 70 lbs. FCM a day, while it is probably physically impossible for the 1700-lb. cow to produce $70 \times \frac{1700}{700} = 170$ lbs. FCM a day. Milk production does not increase directly with simple body weight, with $W^{1.0}$.

As previously explained, the ratio of milk production to body weight, in common with the ratios of the surfaces, visceral organs, and most functions to body weight, including maintenance cost, tends to decline with increasing body weight as shown in the following table (p. 11) representing cattle, goats, and rats.

The first three columns in this table show that the ratio milk Cal. to simple body weight, Kg., declines from 150 for rats to 50 for goats to 22 for cows; the FCM lbs. per 1000 lbs. body weight ratio declines

*Milk-energy production (according to Gaines) = 340 x lb. FCM. FCM represents milk corrected to 4% fat.

[4]Gaines, W. L., J. Dairy Sc. 23, 71, 259, 1031, 1940, and preceding papers. Gaines suggested the use of a special weight, the initial weight, I. W., at the beginning of the lactation period, shortly after freshening.

MILK PRODUCTION IN RELATION TO VARIOUS REFERENCE BASES IN CATTLE, GOATS, AND RATS

	Average of 368 "good" cows	Average of 7 "good" goats	Average of 12 "good" white rats	A "champion" Holstein cow[1]	A "champion" Jersey cow[2]
Live weight, lbs.	1130	95	0.662	1700	700
Live weight, kg.	513	43	0.3	771	318
Milk yield, FCM lbs./day	33.6	6.2	0.1765	100	71
Milk yield, Cal/day	11,440	2,114	60.	34,000	24,140
Ratio FCM lbs. per 1000 lbs. live wt.	30	66	267	59	101
Ratio milk Cal. per kg. live weight	22	50	150	44	63
Ratio Milk Cal. to estimated basal-met. Cal.	1.7	1.9	2.1	3.8	5.1
Milk Cal. per $kg.^{0.70}$ live weight[3]	145	152	139	324	428
Milk Cal. per $kg.^{0.71}$ live weight[4]	136	146	141	303	404
Milk Cal. per $kg.^{0.73}$ live weight[5]	120	137	147	266	360
Ratio FCM lbs. to $lb.^{0.7}$ live weight[6]	24.5	25.6	23.6	54.8	72.4
Dairy merit = gross energetic efficiency[7]	31%	34.9%	44%	44%	48%

[1]This 1700-lb. cow, Carnation Ormsby Butter King "Daisy" (Holstein-Friesian World, 33, Feb. 22, 1936), produced in 365 days 38,607 lbs of 3.63%-fat milk containing 1402 lbs. butterfat.

[2]This 700-lb. cow, Stonehurst Patrician's Lily (Jersey Bull. & Dairy World, 54, No. 15, April 10, 1935), produced in 365 days 24,094 lbs. of 4.5%-fat milk containing 1087 lbs. butterfat.

[3]The $kg^{0.70}$ values are respectively: 78.9, 13.9, 0.4305, 105, 56.5.

[4]The $kg^{0.71}$ values are respectively: 84.0, 14.5, 0.4253, 112.1, 59.8.

[5]The $kg^{0.73}$ values are respectively: 95.1, 15.6, 0.4152, 128.1, 67.1.

[6]The $lb.^{0.7}$ values are respectively: 137.1, 24.2, 0.7492, 182.5, 98.1.

[7]Dairy merit of a 120-lb. goat is 44% when producing 15 lb. FCM/day; 47.5% when producing 20 lbs./day; 41% when producing 12 lbs. FCM, 39% for 10 lbs., 40.5% for 11.2 lbs.

from 267 for rats, to 66 for goats, to 30 for cows. On the other hand, the ratio of FCM lbs. to $(\text{lb.})^{0.7}$ body weight is virtually the same, about 25 for rats, goats, and cows. The ratio of milk Cal. to basal metabolism Cal. is also quite constant, 2.1 for rats, 1.9 for goats, 1.7 for cows. The ratios of milk Cal. to $(\text{Kg})^{0.71}$, $(\text{Kg})^{0.70}$, and to $(\text{Kg})^{0.73}$ are also quite constant.

The two last (right) columns in this table show, as might be expected, that the ratios of milk energy to estimated basal metabolism or to $(\text{Kg})^{0.7}$ in the "champion" cows are much above that for the "good" cows, about three-fold in the champion Jersey than that for the "good" cows. The ratio of milk-energy to $(\text{Kg})^{0.7}$ is likewise three times as high in the champion Jersey as in the "good" cows.

Note that by Gaines' FCM/W dairy-merit index the champion Jersey is about 70% "better" than the champion Holstein (101:59); by the milk Cal/$(\text{Kg})^{0.71}$, the champion Jersey is about 33% "better" than the champion Holstein (404:303); by the milk Cal/basal metabolism Cal, the champion Jersey is likewise about 33% "better" than the Holstein (5.1:3.8).

According to equation (3) below, a 1700-lb. cow should produce 138 lbs. FCM a day to have the same dairy merit or milk-producing efficiency as a 700-lb. cow producing 70 lbs. FCM a day. Actually as shown in the above table, the 1700-lb. cow produced only 100 lbs. FCM a day. The 1700-lb. cow producing 100 lbs. FCM a day is thus inferior, from the dairy merit viewpoint, to the 700-lb. cow producing 70 lbs. FCM a day.

Summarizing, if the range in live weight of animals is considerable, the relative dairy merit of the animals under comparison is best given by the ratio FCM/$W^{0.7}$ or, perhaps, FCM/$W^{0.73}$. The FCM values are computed from the milk and fat percentage production (Table 1, appendix) and the $W^{0.7}$ values may be read from a chart or table (Table 3 in appendix) for given live weights.

If milk-energy production tends to vary with $W^{0.7}$, how does it happen that Gaines reported that FCM tends to vary more nearly with $W^{1.0}$? A clue to this puzzle is given in Figs. 2A and B, which represent the same data on logarithmic and arithmetic grids respectively.

The slopes of the curves in the two charts (the slopes represent the exponents of W, the value of b in W^b) range from 0.3 for well-fed animals of almost exactly the same age to 1.3 and 2.1 for animals including all ages and all states of nutrition, animals classified by live weight regardless of age and state of nutrition. These differences in

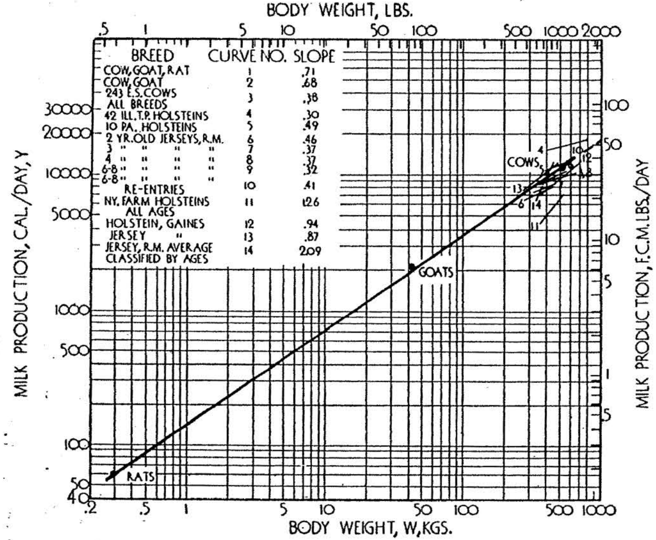


Fig. 2A.—Milk production as function of body weight on a logarithmic grid. The heavy line represents mature rates, goats, and the 368 "good" cows of all breeds, and has a slope of 0.71 (i.e. $Y = aW^{0.71}$). The other curves represent various groups of cows as shown. Note the wide variations in the slopes of the cattle data due to differences in grouping by ages, nutritional status, and dairy merit.

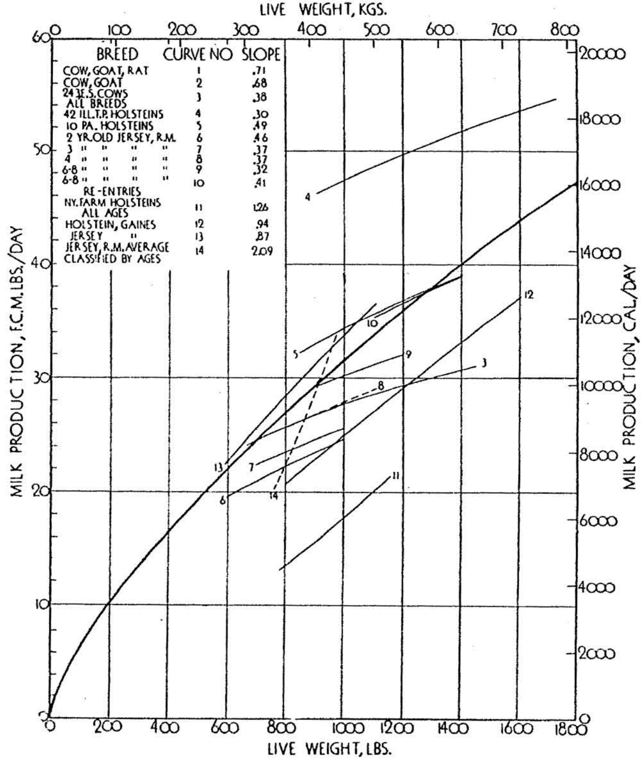


Fig. 2B.—Milk production as function of body weight on an arithmetic grid. The same data as in Fig. 2A. The longest heavy line represents the mature rats, goats, and 368 cows. The other curves are numbered as in Fig. 2A.

slope for the various cow populations thus reflect differences in the composition of the population.

If animals different in age are included in the population classified by weight (regardless of age) the increase in milk production associated with increasing body weight is fortified by increase in milk production associated with maturation, with increasing age during growth.

If animals in different nutritional conditions are included, increase in weight as result of fattening will not increase milk production; on the other hand if undernourished animals are included, the light animals may have a lower yield than they should and when mixed with well-fed animals will give the curve a greater slope.

Summarizing, unless a cow population is homogeneous with regards to age, state of nutrition and dairy merit, the slope relating milk yield to body weight is ambiguous. Thus curves 14 (Figs. 2A and B) of very high slope represents the same data as curves 2 to 10, each of low slope. Curves 14 and 11 have high slopes because they have animals of different ages, and so on. The differences in slope reflect differences in classification of data.

Instead of employing dairy-merit *indices*, such as $FCM/W^{0.7}$ or $FCM/W^{0.73}$, which are numerically removed from the actual values of the *dairy merit* ratio, defined by the percentage of consumed TDN energy that is converted to milk energy, a table or graph may be constructed giving dairy-merit estimates, *estimates of the percentages of consumed TDN energy that is converted into milk energy.*

Such a table, Table 2 in the appendix, or Fig. 5, was constructed on the basis of the following considerations.

First, it was assumed[5] that the feed cost of maintenance varies with the 0.73 power of weight, with $W^{0.73}$.

Second, the following equation was set up, giving the distribution of the TDN consumed between milk production and maintenance[2] cost.

$$TDN = aFCM + bW^{0.73}$$

The TDN represents the amount TDN consumed, FCM the amount FCM produced, and W the live weight. Solving the equation on the basis of data for 368 "good" dairy cows, involving the usual range in live weight on all major breeds of dairy cattle[6] and employing the pound unit, this relation (when changes in body weight and in composition are insignificant) was obtained.

$$TDN = 0.305FCM + 0.053W^{0.73} \quad \text{.......................(1)}$$

meaning that 0.305 lbs. TDN is used for producing 1 lb. FCM, not counting maintenance, and 0.053 lb. TDN per unit $W^{0.73}$ is used for maintenance.

[5]Missouri Res. Bul. 220.
[6]Missouri Res. Bul. 222, and 238.

Assuming an energetic equivalence of 340 Cal. per pound FCM (Gaines), and 1814 Cal. per pound TDN, and remembering the definition of dairy merit as the percentage TDN that is converted to FCM, the relation is

$$\text{Dairy merit} = 100 \frac{340 \times FCM}{1814 \times TDN} \quad \text{..........(2)}$$

The TDN is used for two purposes, for producing FCM at the rate of 0.305 lb. TDN for 1 lb. FCM and for maintenance at the rate of 0.053 lb. TDN for one unit $W^{0.73}$. Therefore, equation (2) becomes, on substituting the values from equation (1),

$$\text{Dairy merit} = 100 \frac{340\ FCM}{1814(0.305\ FCM + 0.053W^{0.73})}$$

$$= 100 \frac{340\ FCM}{553FCM + 96.1W^{0.73}}$$

$$= \frac{61\ FCM}{FCM + 0.173W^{0.73}} \quad \text{..........(3)}$$

Factor 61, of course, represents the percentage of TDN energy converted to FCM energy above the maintenance level, not counting the maintenance cost. Factor 61 means that the net energetic efficiency of milk production (not including the maintenance cost) is 61%.

The dairy merit may thus be computed from equation (3) even though the TDN consumption is not known. The FCM production and the body weight are the only needed data.

Instead of going thru the computation of dairy merit by substituting in equation (3) for FCM and W, a table may be set up giving the dairy merit for different live weight, W. Such is Table 2 or Fig. 5 in the appendix, from which the dairy merit (the percentage of TDN energy converted to milk energy) may be read directly if milk production and live weight are known.

By way of further elucidation of Table 2, the following figures indicate how FCM production increases with body weight, assuming

FCM PRODUCTION IN RELATION TO BODY WEIGHT WHEN 30% OF TDN ENERGY IS CONVERTED TO MILK ENERGY

Body weight lbs.	FCM production lbs./day	Body weight lbs.	FCM production lbs./day	Body weight lbs.	FCM production lbs./day	Body weight lbs.	FCM production lbs./day
1800	39.5	1100	27.5	400	13.0	10.0	0.88
1700	38.0	1000	26.0	300	10.5	5.0	0.53
1600	36.0	900	24.0	200	7.8	1.0	0.16
1500	34.0	800	21.0	150	6.4	0.7	0.126
1400	33.0	700	19.6	100	4.7	0.1	0.030
1300	31.5	600	17.5	75	3.8		
1200	29.0	500	15.3	50	2.9		

that the dairy merit is 30%, that 30% of the consumed TDN energy is converted to milk energy. The values are extrapolated low enough to include dogs, rabbits, rats, etc. It is hoped someone will check these figures (they have been checked on rats and goats).

Summarizing, it is suggested that the dairy value of cattle be estimated not by the absolute amounts of milk production because this varies with body weight, not by the ratio of milk production to body weight (unless the range in weight be narrow) because this ratio necessarily declines with increasing body weight, but by the ratio FCM/$W^{0.7}$ or, preferably, by the dairy merit ratio, defined as the percentage of TDN energy consumed that is converted into milk. Dairy merit in the animal is identical with gross energetic efficiency of the milk production process. Numerical values of $W^{0.7}$ are given in Table 3. Estimates of the numerical values of dairy merit for various live weights and milk production are given in Table 2, computed from equation (3) above.

4. DAIRY MERIT AND PROFIT ON ANIMALS OF EQUAL BODY WEIGHT

It is evident that as the production level increases the maintenance tax per unit milk production decreases. The labor per unit milk production also decreases because it does not take more time to milk, clean, feed, bookkeep, sell, manage, rear, a superior than a mediocre cow. It is thus evident that the more milk a cow produces the greater the profit per head, per herd, and per pound of milk produced.

It is popularly assumed that the total milk-production cost is double the feed cost for both high and low milking cows. This is exemplified by the following values based on New York State Dairy Herd Improvement data.*

INFLUENCE OF MILK YIELD ON COST OF MILK PRODUCTION

Milk fat production, lbs./yr.	150	200	250	300	350	400	450
4%-milk production, lbs./yr.	3750	5000	6250	7500	8900	10000	11250
Estimated feed cost, $	52	61	68	74	81	88	94
Estimated total cost, $	104	122	136	148	162	176	188
Cost per 100 lbs. of 4%-milk	2.77	2.44	2.18	1.97	1.85	1.76	1.67

The above constant-relation assumption is probably fair. As the milk yield increases, the feed cost *per unit milk* becomes less, because of the saving on the feed cost of maintenance per unit milk production. The decrease in feed cost *per unit milk* is probably paralleled by decrease in labor cost *per unit milk* because it does not take more time

*Reported in chart form in the American Dairyman, Aug. 5, 1941. The values (except for the 4%-milk row computed by the writer) were read from the chart by permission of Leland W. Lamb, who obtained the feed figures from the Dairy Records office, Cornell University.

to milk, feed, clean, etc. a superior than an inferior animal. It is therefore, concluded that profits per animal and per unit milk increase with increasing milk production in the approximate manner given in the above table.

5. BODY WEIGHT AND PROFIT ON ANIMALS OF EQUAL DAIRY MERIT

The cost of milk production is usually divided into feed 50%, labor and management 30%, miscellaneous 20%. Since the labor cost is so important and its importance is rapidly mounting during this war, it is useful to discuss the influence of body weight on the labor-cost aspect of milk production.

If *dairy merit is equal in large and small animals,* the larger the animals the more milk she will produce and the greater profit she will yield.

Since it does not take more time to milk, feed, clean, rear, and manage a large than a small cow, it is obvious that the labor and management costs *per unit milk* should be less for large than for small animals.

By way of illustration let us assume that it is desired to produce the equivalent of 1000 lbs. of 4%-milk a day at an efficiency, or dairy merit, of 30%. How many animals of different weights would be required to produce this milk? The following table gives the answer.

Weight of animal lbs.	4% milk produced daily by the herd lbs.	TDN consumed daily by the herd lbs.	Number of animals required to produce milk at 30% efficiency.
1700	1000	625	26
1400	1000	625	30
1000	1000	625	38
900	1000	625	42
700	1000	625	53
100	1000	625	200

Obviously it takes more labor to milk, feed, etc. 200 goats than 53 cows; 53 than 38 cows; 38 than 26 cows. Therefore, *per 1000-lb.* milk yield and when dairy merits are equal, the larger the animals, the fewer required to produce the milk, consequently the less the overhead, especially the less the labor costs per unit milk produced.

Using different wording, about twice as much milk (at nearly half the labor cost) may be produced from *a given number* of 1700-lb. cows than from 700-lb. cows, provided that the dairy merits of the two are equal; and a given number of large animals constitute a large business with larger profits than small animals of the same dairy merit.

The following example illustrates how a difference in body size of an order frequently found in dairy herds, in animals of the **same dairy merit,** affects the monetary profit.

Assume that each of 2 groups of cows produces 1000 lbs. 4%-milk a day, selling at $2.00 per 100 lbs. The feed cost is the same, $1.50 per 100 lbs. TDN; dairy merit of the cows, or the efficiency of milk production, is the same, 30% of the TDN is converted into milk. The only difference is that in one group the animals weighed 900 lbs., in the other 1400 lbs. How do the two herds compare from the profit viewpoint?

Solution:

	900-lb. cows	1400-lb. cows
No. cows needed to produce 1000 lbs. 4%-milk daily at 30% efficiency	42	30
TDN needed to produce 1000 lbs. 4%-milk daily at 30% efficiency	625 lbs./day	625 lbs./day
Milking time	13.6 hrs.	10 hrs.
Housing, records, taxes, etc.	42x	30x

(x=10c per cow per day)

Cost for 900-lb herd=		Cost for 1400-lb. herd=	
feed	$9.37	feed	$9.37
labor	2.72	labor	2.00
records, etc.	4.20	records, etc.	3.00
	16.29		14.37

Return from milk = $20 per day

Profit for the herd of 900-lb cows $20.00—$16.29 = $3.71 per day
Profit for the herd of 1400-lb. cows $20.00—$14.37 = $5.63 per day

Relative profit = $\frac{5.63}{3.71}$ = 1.5 as much for the herd of 1400-lb. cows as for the 900-lb. cows.

The above computations indicate that if dairy merit and price per unit 4%-milk are the same for milk of the small and large cows, the profit on **a given amount of 4%-milk** produced is 50% greater when produced by the 1400-lb. than by the 900-lb. cows. The profit differences, of course, increase with increasing body-size differences. This type of reasoning is not applicable with the same force to pasture fed cattle where the housing and management items are of a different order.

The above estimates are based on the assumption that dairy merit and other conditions are equal in large and small cows, which is not, however, usually the case.

Thus small cows tend to be more efficient than large ones. Smaller cows have to be more efficient than large ones to survive. This is because the basis for selection has been the production per cow. To stay in the herd the small cow has had to produce nearly as much milk as the big cow; but if the small cow produces as much as the big one, she is more efficient, she has the higher dairy merit ratio, she converts a greater percentage of the total TDN into milk, because she has a smaller body to maintain she expends less of her feed for maintenance.

The fact that the larger animals are, on the average, probably less efficient than the small presents the greater opportunity for raising the efficinecy of the large cows, and Table 2, or Fig. 5, furnishes the necessary (tentative) yardstick for measuring dairy merit of cows, regardless of their body size, thus enabling raising the lactation capacity of larger cows with corresponding increase in profit.

Then, too, there is usually a marked price difference for milk produced by different-sized cows. Some milks are especially in demand for fluid consumption; others for butter production; still others for cheese production. Table 4 (in the appendix) shows why some milks are preferred, are more profitable, for butter and others for cheese—some are relatively richer in fat, others in casein. Color is also a factor.

Topography and climate may favor one or another sized cow. Thus large cows have greater difficulty in grazing on steeply-rolling pastures than small ones, especially on poor pastures which involve much movement. Large animals have a smaller surface (for dissipating heat) per unit weight than small animals, and in hot weather large animals may, perhaps, not be able to keep as comfortably cool as small ones (however, the extent of surface area per unit weight is only one of many factors involved in keeping cool).

There are other factors, no doubt, such as the clumsiness of larger cows and their greater probability to injury, and so on, which may counteract the obviously desirable features of large animals. Indeed some conditions favor animals such as small cows and goats. However, when *conditions are equally favorable for both*, the large cows are more profitable for large-scale milk production; in a barn with a given number of stanchions you can produce more milk with large cows than with small ones and reduce the production cost per unit milk proportionately.

In summary, we should like to modify the phrase used by McDowell[7] "within the breed big dairy cows excell," to "within a dairy-merit class big dairy cows excell." McDowell was only partly right in his conclusion. Large cows may or may not excell within or without the breed, depending on the relative dairy merits of the large and small cows.

6. PLANE OF NUTRITION, EFFICIENCY, AND PROFIT ON ANIMALS OF EQUAL BODY SIZE AND DAIRY MERIT

Milk production is a function of many variables, two of which, dairy merit and body size, have been discussed. We shall next discuss a third variable, *plane of nutrition*. How does the plane of TDN consumption influence the efficiency of feed utilization and the conversion of the nutrients into milk? How does it affect the profit on milk production?

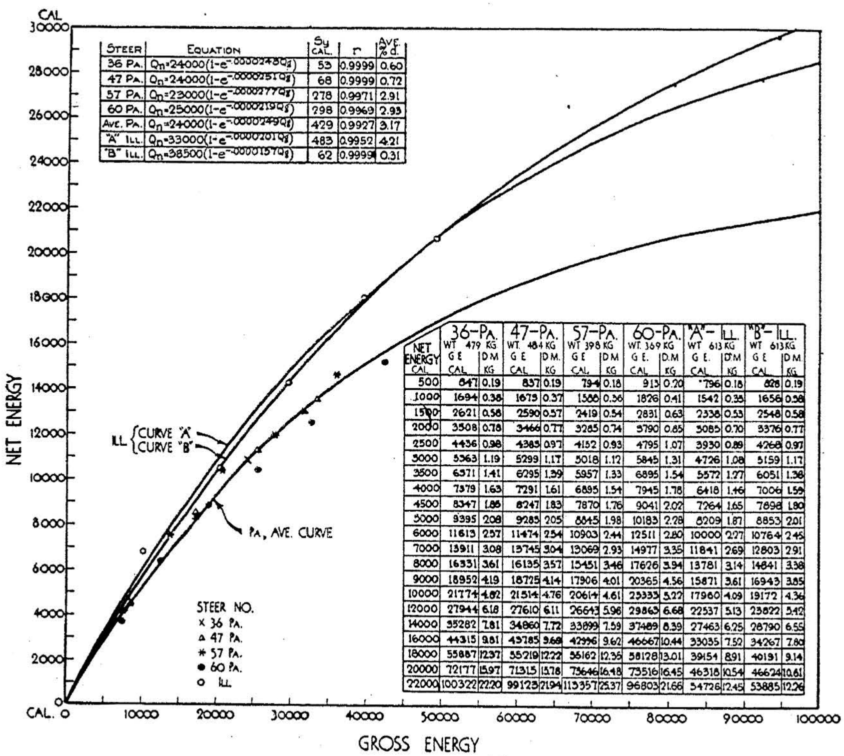
[7] McDowell, J. C., U.S.D.A. Circ. 114, 1930.

A given plane of nutrition is compared with a standard plane, such as Haecker's feeding standard, taken as 100%.

Usually dairy cows are allowed good roughage *ad libitum*. The plane of nutrition is controlled by the grain supplement. Since grain is much more digestible, available, and concentrated than roughage, a small amount of grain is equivalent in TDN and net energy to a relatively large amount of roughage. The amount of roughage a cow consumes is usually limited by her physical capacity to take it; the amount of grain a cow receives is usually limited by grain prices since grain is more expensive per unit TDN than roughage.

By controlling the grain allowance milk production may be increased or decreased by 20% from the Haecker-Standard-fed cows.

The effect of changing the plane of nutrition on milk production depends on the cow. A good environment is useful only in relation to the animal's ability to benefit thereby. An inferior milk cow gets all she can use from the usual standard allowance. It is only superior



Steer	Equation	Sy cal.	r	Ave. % d.
36 Pa.	$Q_n = 24000(1-e^{-.0000248 Q_g})$	53	0.9999	0.60
47 Pa.	$Q_n = 24000(1-e^{-.0000251 Q_g})$	68	0.9999	0.72
57 Pa.	$Q_n = 23000(1-e^{-.0000277 Q_g})$	278	0.9971	2.91
60 Pa.	$Q_n = 25000(1-e^{-.0000219 Q_g})$	298	0.9969	2.93
Ave. Pa.	$Q_n = 24000(1-e^{-.0000249 Q_g})$	429	0.9927	3.17
"A" Ill.	$Q_n = 33000(1-e^{-.0000201 Q_g})$	483	0.9952	4.21
"B" Ill.	$Q_n = 38500(1-e^{-.0000157 Q_g})$	62	0.9999	0.31

Net energy	36-Pa. WT 479 KG		47-Pa. WT. 484 KG		57-Pa. WT 398 KG		60-Pa. WT. 369 KG		"A"-Ill. WT 613 KG		"B"-Ill. WT 613 KG	
Cal.	G.E. Cal.	D.M. Kg.	G.E. Cal.	D.M. Kg.	G.E. Cal.	D.M. Kg.	G.E. Cal.	D.M. Kg.	G.E. Cal.	D.M. Kg.	G.E. Cal.	D.M. Kg.
500	847	0.19	837	0.19	794	0.18	913	0.20	796	0.18	828	0.19
1000	1694	0.38	1673	0.37	1588	0.36	1826	0.41	1542	0.35	1656	0.38
1500	2621	0.58	2590	0.57	2419	0.54	2831	0.63	2338	0.53	2548	0.58
2000	3508	0.78	3466	0.77	3285	0.74	3790	0.85	3085	0.70	3376	0.77
2500	4436	0.98	4385	0.97	4152	0.93	4795	1.07	3930	0.89	4268	0.97
3000	5363	1.19	5299	1.17	5018	1.12	5845	1.31	4726	1.08	5159	1.17
3500	6371	1.41	6295	1.39	5957	1.33	6895	1.54	5572	1.27	6051	1.38
4000	7379	1.63	7291	1.61	6895	1.54	7945	1.78	6418	1.46	7006	1.59
4500	8347	1.85	8247	1.83	7870	1.76	9041	2.02	7264	1.65	7898	1.80
5000	9395	2.08	9285	2.05	8845	1.98	10183	2.28	8209	1.87	8853	2.01
6000	11613	2.57	11474	2.54	10903	2.44	12511	2.80	10000	2.27	10764	2.45
7000	13911	3.08	13745	3.04	13069	2.93	14977	3.35	11841	2.69	12803	2.91
8000	16331	3.61	16135	3.57	15451	3.46	17626	3.94	13781	3.14	14841	3.38
9000	18952	4.19	18725	4.14	17906	4.01	20365	4.56	15871	3.61	16943	3.85
10000	21774	4.82	21514	4.76	20614	4.61	23333	5.22	17960	4.09	19172	4.36
12000	27944	6.18	27610	6.11	26643	5.96	29863	6.68	22537	5.13	23822	5.42
14000	35282	7.81	34860	7.72	33899	7.59	37489	8.39	27463	6.25	28790	6.55
16000	44315	9.81	43785	9.68	42996	9.62	46667	10.44	33035	7.52	34267	7.80
18000	55887	12.37	55219	12.22	55162	12.35	58128	13.01	39154	8.91	40191	9.14
20000	72177	15.97	71315	15.78	73646	16.48	73516	16.45	46318	10.54	46624	10.61
22000	100322	22.20	99123	21.94	113357	25.37	96803	21.66	54726	12.45	53885	12.26

Fig. 3.—Net energy (utilized for productive purposes) as function of gross-energy in the feed. The greater the gross-energy consumption the more net energy but the increase in net energy occurs at diminishing increments (from Missouri Res. Bul. 193, 1933).

animals that respond to superior nutritional opportunities, and we are concerned with such "good" to "superior" animals.

The quantitative response of plants or animals to additional nutrients is generalized by the "principle of diminishing increments" on which there is a considerable literature.[8] According to this principle, illustrated graphically by Fig. 3, the higher the plane of nutrition the more nutrients may be utilized in total, the more milk may be produced, but the increases in milk production or other desired product occurs at decreasing rate. The amount of extra product for each added feed unit grows ever smaller and finally stops or even declines.

The above statement as it bears on milk production is illustrated by the values in the following tables recomputed and rearranged from the interesting bulletins by Jensen and by Borland.[9]

INFLUENCE OF PLANE OF NUTRITION ON THE PERCENTAGE CONVERSION OF TDN TO MILK AND ON THE MILK (FCM) YIELD PER POUND OF GRAIN FED (In Addition to *Ad Libitum* Roughage)

I. *After Jensen et al.*

Percentage of Haecker's feeding standard	Percentage[1] TDN Cal. converted to milk Cal.	Lbs. milk (FCM) yield per lb. grain fed	Lbs. grain consumed	Lbs. milk (FCM) produced	Lbs. TDN consumed	Lbs. milk (FCM) per lb. TDN consumed
96	25.3	4.4	1722	7626	2366	3.2
101	25.1	3.9	2098	8184	2829	2.9
105	25.1	3.2	2777	8824	3287	2.7
110	24.7	2.6	3666	9400	3844	2.4
114	24.3	2.4	4132	9780	4243	2.3
116	23.7	1.9	5304	9965	4611	2.2

1A lb. FCM is assumed to be equivalent to 340 Cal. and a lb. TDN to 1814 Cal.

II. *After Borland et al.*

Percentage of Haecker's feeding standard	Lbs. milk (FCM) yield per lb. grain fed	Lbs. grain consumed	Lbs. milk (FCM) produced	Feed cost per cow dollars	Feed cost per lb. milk cents	Value of milk over feed cost per cow dollars
70	5.3	1511	7993	95	1.19	113
80	3.9	2248	8816	107	1.22	122
90	3.0	3455	10253	119	1.16	148
100	2.7	4221	11518	148	1.29	151
110	2.3	4751	10879	151	1.39	132
120	2.0	6221	12170	174	1.43	142
123	1.7	7300	12756	192	1.43	140

Grain cost 1.8c per lb.
Milk sold 2.6c per lb.
No other costs included.

Jensen's data indicate that the greater the feed consumption the greater the milk production, but the *milk production per unit grain* or *per unit TDN consumption*, or the *percentage of TDN Calories con-*

8Liebig, J., Die Grundsatze der Agrikulturchemie, 1855. Die Chemie in ihrer Anwendung auf Agrikultur, 1876. Mitscherlich, E. A.; Das Gesetz des Minimums. Landw. Jahrb. 38, 537, 1909; 53, 130, 167, 1919; Fuhlings Landw. Z. 68, 130, 1919, and many other papers. Wiegner, G., & Ghoneim, A., Die Tierernahrung, 2, 193, 1930. Spillman, W. J., U.S.D.A. Tech. Bul. 348, 1933.

9Jensen, E., Klein, J. W., Rauchenstein, E., Woodward, T. E., & Smith, R. H., Input-output relationships in milk production. U.S.D. Agriculture, Tech. Bul. 815, 1942. Borland, A. A., Bean, A. L., & Jones, P. D., The relation of grain feeding to milk production. Pa. Agr. Exp. Sta. Bul. 242, 192.

verted into milk Calories (apparent dairy merit) declines with increasing plane of nutrition.

Since, however, the production per cow is increased, the profit per cow, per stanchion, and therefore for the whole herd tends to increase as shown by Borland's data. The profit per cow in Borland's table refers to that of milk income over feed expenditures (not counting the other 50% of the expenses, for labor, etc.). The profit would increase more rapidly with increasing plane of nutrition if the other expenses were included, especially labor, since the labor cost of feeding, milking, cleaning, etc., is no greater for milking the larger than the smaller amounts of milk.

The exact profit, of course, depends not only on the dairy merit of the cow, overhead costs of labor, housing, etc., but also on the prices of milk and feeds, on the economic framework in which the business operates.

The high feeding level may, sometimes, have an unfavorable long-range effect, such as development of mastitis, milk fever, and so on. The actual situation is of course more complex than can be indicated by a table or graph.

Summarizing, dairy animals produce about 80% as much milk energy on an exclusive, good, roughage ration as they do on such a ration supplemented with about one lb. grain per six lbs. milk (FCM), as called for by the Haecker Standard.[10] On the other hand, milk production may be increased to 20% above the level attained by the Haecker Standard by additional grain. The increased TDN consumption, brought about by increased grain allowance, tends to increase the milk yield, but at decreasing increments with successive feed units in accordance with the principle of diminishing increments.

7. GENERAL DISCUSSION AND SUMMARY

Dairy merit, lactationally-effective body size, and plane of nutrition in their relation to milk yield and to monetary profit have been defined and discussed.

1. *Dairy merit* is defined and measured by gross energetic efficiency of milk production, by the percentage of the consumed TDN energy that is converted into milk energy, as indicated by the equation

$$\text{Dairy merit ratio} = \text{gross energetic efficiency of milk production} = \frac{\text{milk-energy production}}{\text{TDN-energy-consumption}} = \frac{340\text{xlbs. FCM produced}}{1814\text{xlbs. TDN consumed}}$$

suming tat 1 lb. FCM (4%-fat milk) has an energy equivalent of 340 Calories and 1 lb. TDN has an energy equivalent to 1814 Calories.

10See also Woodward, et al., U.S.D. Agr. Misc. Publ. 179, 1933; Sherwood & Dean, Oreg. Agr. Exp. Sta. Bul. 380, 1940; Headley, Nev. Agr. Exp. Sta. Bul. 140, 1935. Graves, et al., U.S.D. Agr. Tech. Bul. 724, 1940.

The dairy merit of "good" dairy cows is approximately 25%; one-fourth of the consumed TDN energy is converted into milk energy. This 25% dairy-merit level pays, approximately, for the dairyman's work, feed, and other expenses at the current rate. "Making money" on cows involves higher dairy merit. What are usually called "superior cows" have a dairy merit of about 33%. The following table indicates in round numbers the milk (FCM) production for cows of different weight at dairy-merit levels of 25% and 33%.

Body weight, lbs.	600	700	800	900	1000	1100	1200
Lbs. FCM/day for 25% dairy merit	13	14	16	17	19	20	21
Lbs. FCM/yr. for 25% dairy merit	4700	5100	5800	6200	6900	7300	7700
Lbs. FCM/day for 33% dairy merit	21	24	26	29	32	34	36
Lbs. FCM/yr. for 33% dairy merit	7700	8700	9500	10600	11600	12400	13100

Body weight, lbs.	1300	1400	1500	1600	1700	1800
Lbs. FCM/day for 25% dairy merit	22	24	25	26	27	28
Lbs. FCM/yr. for 25% dairy merit	8000	8700	9100	9500	9800	10200
Lbs. FCM/day for 33% dairy merit	38	40	42	44	46	48
Lbs. FCM/yr. for 33% dairy merit	13800	14600	15300	16000	16800	17500

The writer believes that dairy merit is independent of body weight as such. Mammals rearing large litters of helpless young, such as rats, mice, hamsters, rabbits, cats, dogs, and so on, were developed in the course of evolution for high milk production during the relative short nursing periods. Dairy cattle do not naturally belong to this high-lactation class; they were selected *by man* for high performance and for long periods of performance.

Small dairy cows, especially within the breed, usually have a higher dairy merit than large ones. This is because of the understandably human tendency to evaluate performance by absolute standards. Cows are judged by *absolute* milk production rather than by the dairy merit ratio, the *ratio* of milk produced to feed consumed; so that only such small cows survive in the herd as produce almost as much as the large ones; and if a small animal produces as much as a larger one she is more efficient because the small one used less feed for the maintenance of her smaller body.

It is indicated in the text that the 1700-lb. cow that produced the fabulous 1400 lbs. butterfat in a year, equivalent to about 42000 lbs. of 3.3%-fat milk, has an appreciably lower dairy merit than the 700-lb. cow that produced 26000 lbs. FCM in a year. Assuming that large cows are biologically capable of developing and ultimately will be developed, to the same dairy-merit level as small ones, we shall have 1700-lb. cows producing the equivalent of about 50,000 lbs. FCM or 2000 lbs. butterfat in a year, equivalent in terms of gross energetic efficiency of milk production, or dairy merit of a 700-lb. cow producing 26000 lbs. FCM or 1040 lbs. butterfat in a year.

From the viewpoint of dimensional analysis, dairy merit may be thought of as the *intensity factor* in the lactational process.

2. The *capacity factor* in the lactational process is body size, not simple weight, W, but *lactationally effective* body size, which appears to be best represented by $W^{0.7}$. If the ratio FCM/$W^{0.7}$ is constant then the ratio FCM/W declines rather rapidly with increasing weight (Table 3); increasing body weight 100% increases (according to this formula and the data cited in the text) not by 100% but only by about 70%. This is theoretical. The actual value of the exponent on observed data ranges from 0.3 to 2.0, depending on the composition of the population with reference to age, nutritional status, and dairy merit. The writer however, feels that the ratio FCM/$W^{0.7}$ is superior to the ratio FCM/W as an index of dairy merit.

A formula is presented $\left(\text{dairy merit ratio} = \frac{61\ \text{FCM}}{\text{FCM} + 0.173\ W^{0.73}}\right)$ and a table based thereon, from which the dairy merit ratio may be estimated if FCM and W (body weight) are known. It is suggested that this formula or table be employed for estimating dairy merit, rather than the absolute milk yield, or FCM/W, or FCM/$W^{0.7}$.

If this reasoning is soundly realistic, a yardstick is then available for measuring lactational ability, a method for selecting the superior animal in *relation* to its size as the Babcock method is used for selecting the animal for *absolute* butterfat yield.

3. Attention is called to the literature indicating that a good roughage, without grain supplement, fed *ad libitum* to "good" dairy cows, yields about 20% less milk than if the roughage is supplemented with about 1 lb. grain per 6 lbs. milk (FCM). Supplementing this Haecker-Standard ratio by still further grain, increases the milk production to about 20% above the Haecker-Standard-fed cows; but the yield increase is not directly proportional to the extra grain fed but it occurs at decreasing increments in accordance with the principle of diminishing returns (Fig. 3). The grain allowance is governed by the economic framework, by the relative costs of feed, labor, and miscellaneous expenses on one hand and the return for milk on the other.

4. The basic profit aspects relating to milk production are: (1) the higher the milk production in animals of *given size* the smaller the feed fraction used for maintenance and the larger the feed fraction used for production; (2) the higher the milk production the lower the overhead cost of labor, management, housing, etc., per unit milk and, therefore, the greater the profit per unit milk, per stanchion, and for the herd. Fig. 4 shows how the profit of milk over *feed cost* only (not

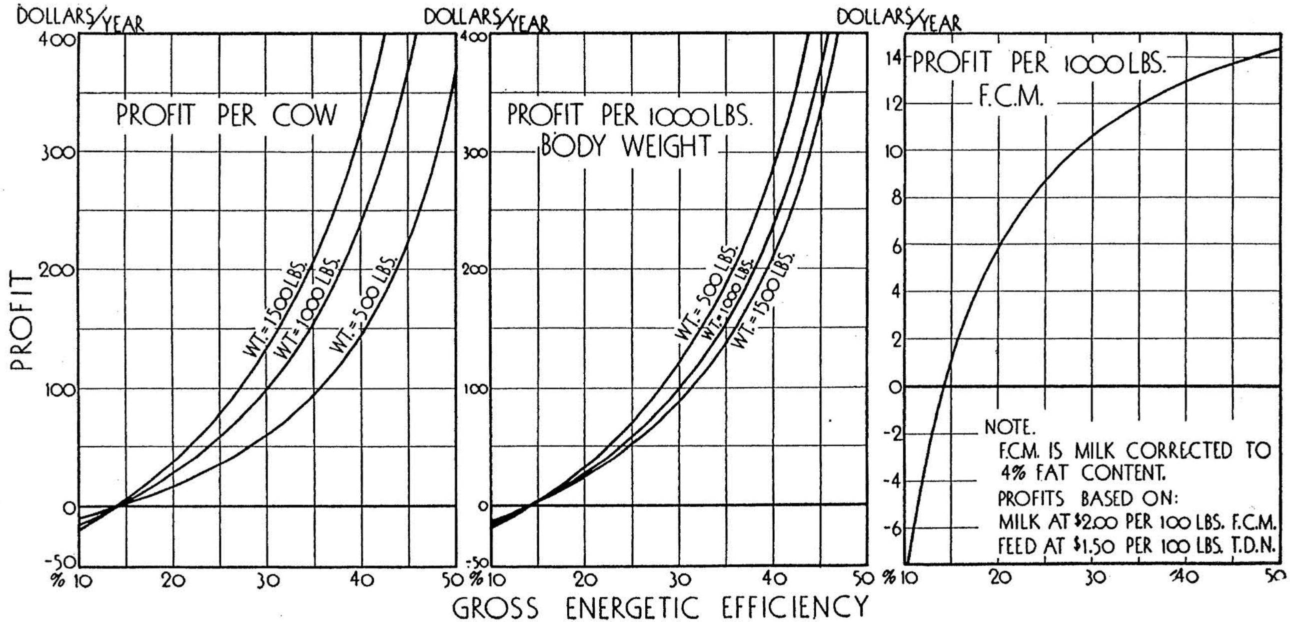


Fig. 4.—The profit of income from milk over cost of feed as function of dairy merit of the cow (gross energetic efficiency of milk production). The two left curves could be much steeper if the other expenses (labor, etc.) were included in the cost, and the curve on the right would bend upwards instead of downwards.

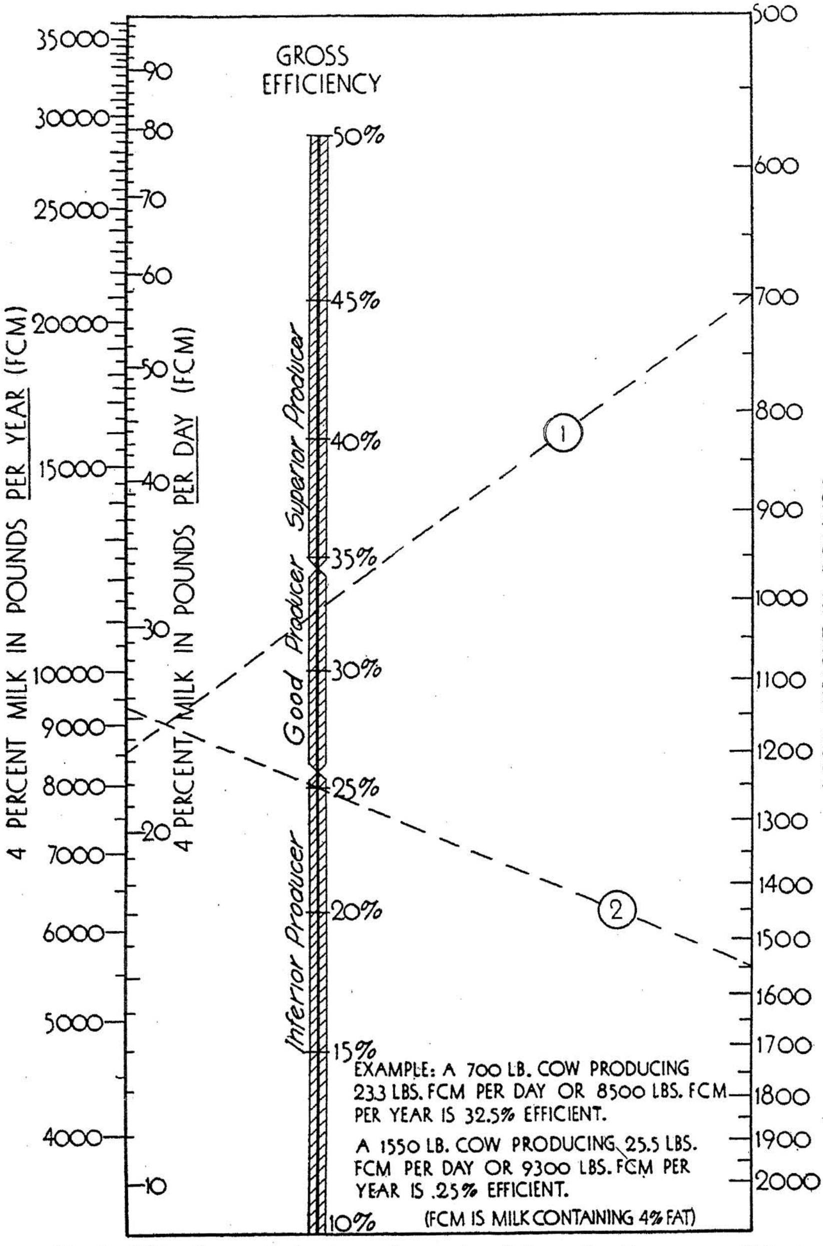


Fig. 5.—This alignment chart may be employed for estimating gross efficiency of milk production, that is the apparent dairy merit of the cow. First, the pounds of the given milk must be converted to pounds "FCM," that is, to milk containing 4% fat (see conversion table 1.). The efficiency of milk production is then read from this chart. Thus if it is desired to find the efficiency, or dairy merit, of a 700-pound cow producing 8500 pounds yearly or on the average 23.3 pounds daily of FCM (4%-milk), place a straight edge between 23.3 on the left (or milk) scale, and 700 on the right (or body-weight) scale, and read the answer 32.5 on the center (or efficiency) scale.

including labor costs, etc.) rises with increasing gross energetic efficiency of milk production, that is, dairy merit. The profit increases much more rapidly if the labor and other overhead items are included. In brief, the profit on the cow increases much more rapidly than her dairy merit.

Similar reasoning holds for the influence of body size on profit. For *equal dairy merit,* the profit rises very rapidly per unit milk, per animal, and for the entire herd with increasing size of the cow. This is because the time and overhead costs—such as the time required for milking, feeding, cleaning, etc.—are virtually the same for large as for small cows, and since the large cow yields the most milk the cost per unit milk is least and the profit greatest for the largest cow.

This conclusion, relating profit to size, however, holds only when all other conditions are equal, and they seldom are. This conclusion is, then, in the nature of a general principle which, like other rules, must be modified to suit the conditions.

Having these principles and methods relating dairy merit, live weight, plane of nutrition to milk yield to labor costs and to profit, the problem is how to apply them, how to modify them under the unique conditions of this day.

APPENDIX

TABLE 1.—TABLE FOR CONVERTING MILK OF GIVEN FAT PERCENTAGE TO "4 PER CENT MILK" BY MEANS OF TABLE 1.*

A Per cent Fat in Milk	B Factor for Converting to 4% Milk	A Per cent Fat in Milk	B Factor for Converting to 4% Milk
2.5	0.775	5.0	1.150
2.6	0.790	5.1	1.165
2.7	0.805	5.2	1.180
2.8	0.820	5.3	1.195
2.9	0.835	5.4	1.210
3.0	0.850	5.5	1.225
3.1	0.865	5.6	1.240
3.2	0.880	5.7	1.255
3.3	0.895	5.8	1.270
3.4	0.910	5.9	1.285
3.5	0.925	6.0	1.300
3.6	0.940	6.1	1.315
3.7	0.955	6.2	1.330
3.8	0.970	6.3	1.345
3.9	0.985	6.4	1.360
4.0	1.000	6.5	1.375
4.1	1.015	6.6	1.390
4.2	1.030	6.7	1.405
4.3	1.045	6.8	1.420
4.4	1.060	6.9	1.435
4.5	1.075	7.0	1.450
4.6	1.090	7.1	1.465
4.7	1.105	7.2	1.480
4.8	1.120	7.3	1.495
4.9	1.135	7.4	1.510

Column A gives fat percentages, column B corresponding conversion factors, which when multiplied by pounds of milk produced, will convert the given milk to 4% milk. Thus if a cow produces 10,000 pounds of 3% milk multiply 10,000 by 0.850 and get the answer 8500 pounds of 4% milk. In other words 10,000 pounds of 3% milk contains the same amount of energy as 8500 pounds of 4% milk.

*The conversion factors in this table were computed from Gaines' formula "FCM = .4M + 15F, where FCM (fat-corrected milk) is gross energy value in terms of normal average cows' milk of 4 per cent fat content, M is actual milk and F is fat, all in the same unit of weight." (W. L. Gaines, Univ. Ill. Agric. Expt. Station Bulletin 308, 1928.)

TABLE 2.—ESTIMATING PER CENT EFFICIENCE OF MILK PRODUCTION FROM BODY WEIGHT OF COW AND MILK PRODUCTION (4% MILK)

4% milk, pounds per year. (FCM)	Body Weight, Pounds													4% milk pounds per day
	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	1700	1800	
3000	18.9	17.4	16.2	15.2	14.4	13.6	13.0	12.4	11.8	11.4	10.9	10.6	10.2	8.2
3500	21.0	19.5	18.2	17.1	16.1	15.3	14.6	14.0	13.4	12.9	12.4	12.0	11.6	9.6
4000	22.9	21.2	19.9	18.7	17.8	16.9	16.2	15.5	14.8	14.3	13.8	13.3	12.9	11.0
4500	24.6	22.9	21.6	20.4	19.3	18.4	17.6	16.9	16.2	15.6	15.1	14.6	14.1	12.3
5000	26.1	24.6	23.1	21.7	20.8	19.9	18.9	18.2	17.5	16.9	16.3	15.8	15.3	13.7
5500	27.6	25.9	24.4	23.2	22.1	21.0	20.2	19.5	18.7	18.1	17.5	17.4	16.4	15.1
6000	28.9	27.2	25.7	24.4	23.2	22.3	21.5	20.6	19.9	19.1	18.6	18.0	17.5	16.4
6500	30.2	28.3	26.8	25.7	24.6	23.4	22.5	21.7	21.0	20.2	19.7	19.1	18.5	17.8
7000	31.3	29.6	28.1	26.8	25.5	24.6	23.6	22.9	21.9	21.4	20.6	20.1	19.5	19.2
7500	32.2	30.6	29.1	27.7	26.6	25.5	24.6	23.8	23.0	22.3	21.6	21.0	20.4	20.5
8000	33.4	31.5	30.2	28.7	27.7	26.6	25.5	24.7	24.0	23.2	22.5	21.9	21.4	21.9
8500	34.3	32.6	31.1	29.6	28.5	27.4	26.4	25.7	24.7	24.0	23.4	22.7	22.1	23.3
9000	35.0	33.4	31.9	30.6	29.4	28.3	27.4	26.4	25.7	24.9	24.2	23.6	23.0	24.7
9500	35.8	34.1	32.8	31.3	30.2	29.2	28.1	27.4	26.4	25.7	25.1	24.4	23.8	26.0
10000	36.7	34.9	33.5	32.2	30.9	30.0	29.1	28.1	27.2	26.4	25.9	25.1	24.6	27.4
10500	37.3	35.6	34.2	33.0	31.7	30.7	29.6	28.9	27.9	27.2	26.4	25.9	25.3	28.8
11000	38.0	36.4	35.0	33.5	32.4	31.5	30.4	29.4	28.7	27.9	27.2	26.6	25.9	30.1
11500	38.6	37.1	35.6	34.3	33.2	32.0	31.1	30.2	29.4	28.5	27.9	27.2	26.6	31.5
12000	39.4	37.7	36.2	35.0	33.7	32.8	31.7	30.9	30.0	29.2	28.5	27.7	27.2	32.9
12500	39.9	38.2	36.7	35.6	34.3	33.4	32.4	31.5	30.7	30.2	29.2	28.5	27.9	34.2
13000	40.5	38.8	37.5	36.2	35.0	33.9	33.0	32.0	31.3	30.6	29.8	29.0	28.5	35.6
13500	40.9	39.4	37.9	36.7	35.6	34.7	33.5	32.6	31.9	31.1	30.3	29.6	29.1	37.0
14000	41.4	39.9	38.4	37.3	36.2	35.0	34.1	33.2	32.4	31.7	30.9	30.2	29.6	38.4
14500	42.0	40.5	38.8	37.9	36.5	35.6	34.7	33.7	33.0	32.2	31.5	30.7	30.2	39.7
15000	42.4	40.9	39.5	38.4	37.1	36.2	35.2	34.3	33.5	32.8	32.1	31.3	30.6	41.1
15500	42.7	41.2	39.9	38.8	37.7	36.5	35.6	34.9	33.9	33.2	32.4	31.7	31.1	42.5
16000	43.1	41.8	40.5	39.2	38.0	37.1	36.2	35.2	34.2	33.5	32.7	32.2	31.5	43.8
16500	43.7	42.2	40.9	39.5	38.4	37.5	36.5	35.6	34.7	34.0	33.2	32.6	32.1	45.2
17000	44.0	42.5	41.2	40.1	39.0	37.9	37.1	36.2	35.2	34.4	33.7	33.2	32.6	46.6
17500	44.2	42.9	41.6	40.5	39.4	38.4	37.5	36.5	35.6	34.8	34.1	33.7	33.0	47.9
18000	44.6	43.3	42.0	40.9	39.7	38.8	37.9	36.9	36.0	35.3	34.5	34.1	33.4	49.3
18500	45.0	43.7	42.4	41.2	40.1	39.2	38.2	37.5	36.4	35.7	34.9	34.5	33.7	50.7
19000	45.4	43.9	42.7	41.6	40.5	39.5	38.6	37.9	36.8	36.0	35.3	34.9	34.3	52.0
19500	45.5	44.2	42.9	41.9	40.9	39.9	39.0	38.2	37.2	36.4	35.7	35.2	34.7	53.4
20000	45.9	44.6	43.3	42.2	41.2	40.1	39.4	38.6	37.5	36.8	36.1	35.6	35.0	54.8

20500	46.3	44.8	43.7	42.5	41.6	40.5	39.7	38.8	37.9	37.2	36.5	36.0	35.4	56.2
21000	46.5	45.2	44.0	42.9	41.8	40.9	40.1	39.2	38.2	37.5	36.8	36.4	35.8	57.5
21500	46.7	45.5	44.2	43.1	42.2	41.2	40.5	38.5	38.6	37.9	37.2	36.7	36.2	58.9
22000	47.0	45.7	44.6	43.5	42.5	41.6	40.7	39.9	38.9	38.2	37.5	37.1	36.5	60.3
22500	47.2	45.9	44.8	43.9	42.7	41.8	41.0	40.1	39.2	38.5	37.8	37.5	36.7	61.6
23000	47.4	46.3	45.0	44.0	43.1	42.8	41.2	40.5	39.5	38.8	38.1	37.7	37.1	63.0
23500	47.8	46.5	45.4	44.2	43.3	42.4	41.6	40.9	39.8	39.1	38.4	38.0	37.5	64.4
24000	48.0	46.7	45.6	44.6	43.7	42.7	41.8	41.0	40.1	39.4	38.7	38.2	37.9	65.8
24500	48.2	47.0	45.7	44.8	43.9	42.9	42.2	41.4	40.4	39.7	39.0	38.6	38.0	67.1
25000	48.4	47.2	46.1	45.0	44.0	43.3	42.4	41.6	40.7	40.0	39.3	38.8	38.2	68.5
25500	48.5	47.4	46.3	45.4	44.2	43.5	42.7	41.8	40.9	40.3	39.6	39.2	38.6	69.9
26000	48.7	47.6	46.5	45.5	44.6	43.7	42.9	42.2	41.2	40.5	39.8	39.5	38.8	71.2
26500	48.9	47.8	46.7	45.7	44.8	44.0	43.1	42.4	41.4	40.8	40.1	39.7	39.2	72.6
27000	49.1	48.0	46.9	45.9	45.0	44.2	43.3	42.7	41.7	41.0	40.4	39.9	39.5	74.0
27500	49.3	48.2	47.0	46.3	45.4	44.4	43.7	42.9	41.9	41.3	40.6	40.1	39.7	75.3
28000	49.5	48.4	47.4	46.5	45.5	44.6	43.9	43.1	42.2	41.5	40.9	40.3	40.0	76.7
28500	49.7	48.5	47.6	46.7	45.7	44.8	44.0	43.3	42.4	41.8	41.1	40.5	40.3	78.1
29000	49.9	48.7	47.8	46.7	45.9	45.0	44.2	43.7	42.9	42.2	41.6	41.0	40.5	79.4
29500	50.0	48.9	48.0	47.0	46.1	45.4	44.6	43.9	43.1	42.4	41.8	41.2	40.7	80.8
30000	50.2	49.0	48.2	47.2	46.3	45.5	44.8	44.0	43.3	42.7	42.0	41.4	40.9	82.2
30500	50.2	49.1	48.3	47.4	46.5	45.7	45.0	44.2	43.7	42.9	42.4	41.6	41.1	83.6
31000	50.4	49.3	48.4	47.6	46.7	45.8	45.2	44.4	43.8	43.1	42.5	42.0	41.4	84.9
31500	50.6	49.5	48.5	47.7	46.9	46.1	45.4	44.6	43.9	43.3	42.7	42.2	41.6	86.3
32000	50.7	49.7	48.7	47.8	47.0	46.3	45.5	44.8	44.0	43.5	42.9	42.4	41.8	87.7
32500	50.8	49.9	48.9	48.0	47.2	46.5	45.7	45.0	44.2	43.7	43.1	42.5	42.0	89.0
33000	51.0	50.0	49.0	48.2	47.4	46.6	45.9	45.2	44.6	43.9	43.3	42.7	42.2	90.4
33500	51.1	50.1	49.1	48.3	47.5	46.7	46.1	45.4	44.7	44.0	43.5	42.9	42.4	91.8
34000	51.2	50.2	49.3	48.4	47.6	46.8	46.3	45.5	44.8	44.2	43.7	43.1	42.5	93.2
34500	51.4	50.4	49.4	48.5	47.8	47.0	46.4	45.7	45.0	44.4	43.9	43.3	42.7	94.5
35000	51.5	50.5	49.5	48.7	48.0	47.2	46.5	45.9	45.3	44.6	44.0	43.5	42.9	95.9
35500	51.5	50.6	49.7	48.9	48.2	47.4	46.7	46.1	45.4	44.8	44.2	43.7	43.1	97.3
36000	51.7	50.8	49.9	49.0	48.3	47.6	46.9	46.2	45.5	45.0	44.4	43.9	43.3	98.6
36500	51.9	50.9	50.0	49.1	48.4	47.7	47.0	46.3	45.7	45.2	44.6	44.0	43.5	100.0
37000	51.9	51.0	50.1	49.3	48.5	47.8	47.2	46.5	45.9	45.4	44.7	44.2	43.7	101.4
37500	52.	51.2	50.2	49.5	48.6	47.9	47.3	46.7	46.0	45.5	44.8	44.3	43.8	102.7
38000	52.1	51.3	50.4	49.6	48.7	48.0	47.4	46.9	46.1	45.6	45.0	44.4	43.9	104.1
38500	52.3	51.4	50.5	49.7	48.9	48.2	47.5	47.0	46.3	45.7	45.2	44.6	44.0	105.5
39000	52.3	51.4	50.6	49.8	49.1	48.4	47.6	47.1	46.5	45.9	45.3	44.8	44.2	106.8
39500	52.5	51.5	50.8	49.9	49.2	48.5	47.8	47.2	46.6	46.0	45.4	45.0	44.4	108.2
40000	52.5	51.5	51.0	50.0	49.3	48.7	48.0	47.4	46.7	46.1	45.5	45.2	44.6	109.6

TABLE 3.—NUMERICAL VALUES OF WEIGHTS RAISED TO THE POWERS 0.70, 0.73, 2/3, AND 3/4 FOR COMPUTING THE EFFICIENCY INDEX $\frac{FCM}{(Wt.)^{0.70}}$ ETC., FOR CATTLE AND GOATS.

Weight	$(Wt.)^{0.70}$	$(Wt.)^{0.73}$	$W^{2/3}$	$W^{3/4}$
1800	190.0	238	148.0	276
1700	182.5	228	142.4	265
1600	174.9	218	136.8	253
1500	167.2	208	131.0	241
1400	159.3	198	125.1	229
1300	151.3	188	119.1	217
1200	143.0	177	112.9	204
1100	134.6	166	106.6	191
1000	125.9	155	100.0	178
900	116.9	143	93.2	164
800	107.7	131.6	86.2	150
700	98.1	119.4	78.8	136
600	88.1	106.7	71.1	121
150	33.4	38.8	28.2	42.9
125	29.4	33.9	25.0	37.4
100	25.1	28.8	21.5	31.6
75	20.5	23.4	17.8	25.5
50	15.5	17.4	13.6	18.8

TABLE 4.—THE RELATION BETWEEN THE PERCENTAGES OF FAT, PROTEIN AND LACTOSE[1]

Fat, %	Protein, %						Lactose, %						Ratio fat to protein						Ratio fat to lactose						Ratio protein to lactose					
	Ayrshire	Brown-Swiss	Holstein	Guernsey	Jersey	Goats	Ayrshire	Brown-Swiss	Holstein	Guernsey	Jersey	Goats	Ayrshire	Brown-Swiss	Holstein	Guernsey	Jersey	Goats	Ayrshire	Brown-Swiss	Holstein	Guernsey	Jersey	Goats	Ayrshire	Brown-Swiss	Holstein	Guernsey	Jersey	Goats
3.0	3.1	3.3	3.0			2.9	4.6	5.1	4.9			4.3	.97	.90	.98			1.0	.66	.58	.61			.69	.68	.65	.62			.67
3.5	3.3	3.4	3.4		3.4	3.1	4.6	5.1	4.9			4.4	1.0	1.0	1.0		1.0	1.1	.76	.69	.72			.80	.72	.67	.70			.69
4.0	3.5	3.6	3.7	3.5	3.5	3.2	4.7	5.1	4.8	4.9	5.0	4.5	1.1	1.1	1.1	1.2	1.2	1.2	.85	.79	.83	.81	.80	.89	.74	.71	.77	.71	.69	.72
4.5	3.7	3.7	4.1	3.7	3.6	3.4	4.7	5.0	4.8	4.9	5.0	4.5	1.2	1.2	1.1	1.2	1.2	1.3	.95	.90	.94	.91	.90	.99	.78	.73	.85	.74	.72	.74
5.0	3.9	4.1	4.4	3.9	3.8	3.5	4.8	5.0	4.7	4.9	5.0	4.6	1.3	1.2	1.1	1.3	1.3	1.4	1.0	1.0	1.0	1.0	1.0	1.1	.81	.82	.93	.80	.76	.77
5.5	4.1	4.4	4.7	4.2	4.0	3.7	4.8	4.9	4.7	4.9	5.0	4.7	1.3	1.2	1.2	1.3	1.4	1.5	1.1	1.1	1.2	1.1	1.1	1.2	.84	.90	1.0	.84	.80	.79
6.0		4.9	5.1	4.4	4.1	3.8		4.9	4.7	4.9	5.0	4.8	1.2	1.2		1.4	1.5	1.6		1.2	1.3	1.2	1.2	1.3		1.0	1.1	.90	.83	.81
6.5		5.4		4.6	4.2					4.8	4.9			1.2		1.4	1.6					1.3	1.3					.95	.86	
7.0				4.8	4.3					4.7	4.9					1.5	1.7					1.5	1.4					1.0		

McDowell[2]

Influence of fat percentage in milk on cheese production

Fat %	Casein %	Casein to fat ratio	Cheese/lb. fat, lbs.	Cheese/100 lbs. milk, lbs.	Fat in whey,% (Cheddar cheese)
3.0	2.3	0.75	2.8	8.5	0.3
3.5	2.4	0.70	2.6	9.5	0.25
4.0	2.6	0.65	2.5	10.2	0.38
4.5	2.7	0.60	2.4	10.9	0.34
5.0	2.8	0.55	2.3	11.7	0.35
5.5	2.9	0.52	2.4	12.5	0.46
6.0	3.1	0.50	2.4	13.2	

Eckles & Shaw[3]

Month Lactation	Fat	Casein %	Casein to fat ratio
1	4.0	2.7	.67
2	3.9	2.4	.61
3-7	3.8	2.5	.69
8	3.9	2.7	.71
9	4.0	2.9	.72
10	4.1	3.1	.75
11	4.2	3.2	.76
12	4.5	3.4	.75
13	4.7	3.6	.78

[1]From smoothed curves of data obtained from many sources especially from H. C. Lythgoe "Composition of goat milk of known purity" J. Dairy Sc., 23 1097, 1940, and Overman, O. R., Garrett, O. F., Wright, K. E., & Sanmann, F. P., "Composition of milk of Brown Swiss cows." Univ. Ill. A.E.S. Bul. 457, 1939.

[2]McDowell, F H., J. New Zealand J. Sci. & Tech. 18, 137, 1936.

[3]Eckles, C. H., & Shaw, R. H., U.S.B. Animal Industry, Buls. 155, 156, 157, 1913.