WINTER ROAD MAINTENANCE RESOURCE ALLOCATION MODELS

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by

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The undersigned, appointed by the Dean of the Graduate School, have examined the dissertation entitled:

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ABSTRACT

Winter snow storms could cause serious disruptions to traffic and transportation. Because resources for winter road maintenance, such as snow removal trucks, are limited, using them properly would improve the efficiency and effectiveness of the winter maintenance work. However, a fixed resource allocation plan among service regions may not work well in several situations because of different types and intensity of winter storms. Therefore, reallocation of resources among service regions is often needed. The objective of this research is to develop a resource reallocation model that minimizes the total cost of reallocation operations and provides equitable resources to service regions. Road and weather condition factors, such as road classes, weather forecasts, and service levels, are taken into account in the model.

Chapter 1

Introduction

Resource allocation operation is the process of distributing limited resources to satisfy various demands in different locations. This kind of problem is especially important when an emergency happens. When natural disasters, such as earthquakes, tsunamis and blizzards, strike certain areas, it is urgent to find the best assignment of available rescue resource so that more people can be saved. On the other hand, if the threat is partially predictable, for example, by forecasting the storm or hurricane movement, it would be more efficient and economical to reallocate the resource before the emergency happens.

In winter, snow storms could cause serious disruptions to traffic transportation, because it is hard to drive on slick roads which are covered by ice and snow. According to the statistics of the Federal Highway Administration, 24 percent of weather-related vehicle crashes occur on snowy, slushy, icy roads and 15 percent happen during snowfall or sleet each year. Therefore, winter maintenance work, including spreading salts and abrasives, snow plowing, loading snow into snow removal vehicles and hauling snow to disposal sites, is of great importance to decrease traffic accident risk in winter. One of the most important winter maintenance resources is the snow removal truck, which is used to clear snow-clad roads. Since the quantity of snow removal trucks is always limited due to fiscal constraints, deploying them properly would help improve the level of winter maintenance service.

In this thesis, we consider the situation where a certain area is under the threat of winter storms. There are several districts in this area, and each district owns a road maintenance depot which carries a certain number of snow removal trucks. Based on the characteristics of districts, such as road condition, number of trucks, snowfall intensity, etc, it is reasonable to imagine that some of the districts would have the capability to maintain a high level of service during a snow storm, while others not. Hence, reallocating the snow removal trucks in this area is effective in improving the winter maintenance service quality in the whole area.

In the state of Missouri, more than 1,800 vehicles are available for snow removal work in winter on the state's 32,000-mile highway system. The work is divided among 10 districts. The traditional method of solving these maintenance problems is highly empirical in nature. Most of the decisions on deploying snow removal trucks are typically made by district supervisors, based on the first hand reports and personal experience. It is hard to adjust the amount of snow removal trucks in each district in a global perspective, and decisions made too late delay the dispatch of snow removal vehicles, decreasing the efficiency of winter road maintenance work.

Our objective of this research is to develop a resource allocation model for winter maintenance work in the considered area. In this model, reallocation operation is performed before a storm strikes, and a certain level of winter maintenance service is maintained in each district after reallocation. The goal of the model is to minimize the total reallocation operation cost under service level constraints.

Chapter 2

Literature Review

2.1 Typical Winter Road Maintenance Models

Winter road maintenance operations include spreading of salts and abrasives, snow plowing, loading snow into snow removal vehicles and hauling snow to disposal sites. Due to the complexity of the operations, various models have been introduced to the planning and management of winter road maintenance work. Although these models emphasize different aspects of winter road maintenance operations, their objectives are typically the same, i.e. minimizing the sum of the operational costs. Most of the existing models can be classified into three major fields: vehicle routing, depot location and fleet sizing.

2.1.1 Vehicle Routing and Scheduling

Within these models, vehicle routing problems received the most attention because these operations are common to snow fighting in all urban and rural regions. These problems are practical examples of the Chinese Postman Problem and related arc routing problems, and are similar to other arc routing problems such as garbage collection and street sweeping.

There are three kinds of vehicle routing problems: spread routing, plow routing and snow disposal routing. Spread routing problems concern the operations of spreading chemicals and abrasives, while plow routing problems focus on the removal of snow from the road, and snow disposal routing problems deal with loading snow and hauling snow to the disposal sites. The first two problems both consist of determining a set of routes, each performed by a snow-fighting vehicle that starts and ends at its own depot, so that all districts are served, every operational constraint is satisfied, and more importantly the global cost is minimized. However, the snow disposal routing problem is more complicated, which determines the best set of itineraries for the trucks filled with snow that travel from the assigned snow blower site, to disposal sites, and back to the snow blower site such that the total cost is minimized. Both spread routing and plow routing problems can be generally formulated as arc routing problems, the snow disposal routing problem is a more difficult shortest path problem.

Marks and Sticker (1971) modeled the plow routing problem as a multiple vehicle undirected Chinese Postman Problem, and proposed two approaches – a route first-cluster second approach and a cluster first-route second approach – to solve it. Eiselt et al. (1995) presented a review on arc routing problems, and gave the algorithmic results for Chinese Postman Problems under different conditions. A typical scheduling problem is presented by Lu, et al. (2009), who described a routing problem of winter road maintenance, considering the operating costs, quality of service and weather condition factors, and then established a linear model to find out the optimized schedule for assigning routs, service type and corresponding start time.

2.1.2 Depot Location and Sector Design

Depot location and sector design involve "partitioning the geographic region into sectors for efficient operations, locating the necessary facilities, and assigning the sectors obtained from the partitioning to various facilities" (Pierrier, et al. 2006). Similarly to the vehicle routing problems, sector design and depot location problems can be classified into two kinds of problems: sector design for spreading and plowing and sector design for snow disposal. The sector design problem for spreading and plowing consists of partitioning a spreading or plowing route network into non-overlapping subnetworks, and assigning vehicle depots to these sectors, such that the transport costs and vehicle depot costs are minimized. It is similar to the arc partitioning problem in the context of postal delivery and districting problems for arc routing applications. On the other hand, given a road network and a set of planned disposal sites, the sector design problem for snow disposal consists of determining a set of non-overlapping subnetworks, and assigning each sector to a single snow disposal site in order to minimize the relevant variable and fixed costs. Solution approaches for both problems are similar. Korhonen et al. (1992) developed a decision support system allowing managers to select vehicle depots and their corresponding sectors such that variable transport cost and fixed vehicle depot costs are minimized. The model was solved by a construction heuristic that opens depots sequentially until no further savings are realized. Perrier et al. (2008) provided a mathematical model of sector design and assignment of sectors to disposal sites, and proposed two constructive approaches – the assign first, partition second method and the partition first, assign second method – to solve it. The result of Perrier's experimentation showed that the assign-partition heuristic could result in substantial savings compared to the partition-assign approach.

2.1.3 Fleet Sizing and Replacement

Fleet sizing problems consist of determining the number of winter maintenance vehicles from depots such that the total operational and depot depreciation costs are minimized, while a specified level of service for each road class is satisfied. According to the types of winter maintenance operations, the fleet sizing problems can be divided into two classes: fleet sizing for plowing and fleet sizing for hauling snow to disposal sites. The difference between the two problems is that fleet sizing for plowing balances the total costs and the length of time to plow each class road, while fleet sizing for snow disposal balances the total costs and the length of time for the snow loading and hauling operations.

Fleet replacement or fleet design considers the cost of purchasing, operating, maintaining and replacing vehicles in a fleet. These kind of problems determine a replacement schedule (i.e. how many replacement groups the fleet should have, how large each replacement group should be, the age at which each group is replaced and the relative distribution of the groups overtime), so that the total costs of operating, maintenance and net replacement are minimized. Jones (1993) considered a general fleet design problem in a simplified economic environment, and developed the first formal model that determines optimal steady-state fleet design. The research showed that all replacement groups must be equally sized in the optimal steady-state fleet design.

2.1.4 Integrated Winter Road Maintenance Model

Although the winter maintenance models above are most often solved separately, there are still strong interactions between them. For example, in the vehicle routing problem, each route starts from a depot and ends at a depot, and a set of routes complete the service in a sector. Therefore, vehicle routing problems always correlate with depot location or sector design problems. Hayman and Howard (1972) generated a compound model of sector design, depot location and fleet sizing. Lotan, et al. (1996) discussed a problem combining the depot location and routing problems for spreading. Zhang, et al (2006) developed an integrated system which considers the optimization models and solution algorithms for the routing of snow removal trucks and the location of road maintenance depot in Boone County, MO, and proposed a route first-cluster second approach based on Marks and Sticker's (1971) method.

2.2 Resource Allocation Problems

Resources are meant to be limited in every aspect of life, however, there are always various demands among different functional parts of a system that need to be satisfied. Therefore, how to allocate the limited amount of resources so as to achieve a high performance of the system would be a problem that every organization encounters. In winter road maintenance problems, snow-fighting vehicles such as snow plowing trucks might be the most important resource. Recall the typical winter maintenance models; almost every model considers the number of snow-fighting vehicles available as the primary factor. Then the winter road maintenance resource allocation problem becomes determining a plan that allocates the limited number of snow-fighting vehicles in order to achieve a high level of service. Efficiency and effectiveness are the two parameters that measure the primary aspects of winter maintenance work.

2.2.1 Efficiency-oriented model

Efficiency generally concerns the cost associated with doing business. In other words, the objective of an efficiency-oriented resource allocation model is minimizing the total operational cost, while several effective constraints may need to be satisfied. One of the most solved efficiency-oriented problems is the Hitchcock transportation problem. A typical transportation problem aims to find the best strategy of fulfilling the demands of a set of destinations using the supplies of a set of sources. While trying to find the best way, a variable cost of shipping the commodity from one source to a destination, as well as the capacity of supply in each source and the minimum demand in each destination should be taken into consideration. The objective of a typical transportation problem is to minimize the sum of all incurred transportation costs. Due to its special mathematical structure, several efficient solution approaches based on the simplex method have been developed. These methods differ in how to calculate the necessary simplex-method information.

In 1951, the primal simplex transportation method was proposed by Danzig, who adapted the simplex method to the transportation problem. Charnes and Cooper (1954) developed the stepping- stone method, in which unused cells in transportation tableaus were referred to as "water" and used cells as "stones" – from the analogy of walking on a path of stones half-submerged in water. The stepping-stone method is much easier to comprehend, because most of the solution procedures are conducted on transportation tables. However, it is not applicable to all types of linear programming problems. Besides, it is not even suitable for transportation problems with a large number of origins or destinations. Arsham and Kahn (1989) provided a simplex-type tableau-dual algorithm which is an alternative to the stepping-stone method for general transportation problems. This algorithm only has one operation – Gauss Jordan pivoting in transportation tableau, and all operations can be performed in a single working tableau. In addition to these simplex-type methods, some heuristics were also introduced: Vignaux and Michalewicz (1991) used a genetic algorithm to solve the linear transportation problem; Adlakha and Kowalski (2003) presented a simple heuristic algorithm to solve the fixed-charge transportation problem.

2.2.2 Effectiveness-oriented model

Effectiveness generally concerns the resulting outcome, which means the objective of an effectiveness-oriented model is maximizing the outcome after allocation, while several efficiency constraints may be included. There are quite a few resource allocation models with the objective functions that minimize the total cost of allocation operations, however, when the loss incurred by inadequate resources is invaluable, compared with the cost of allocation operations, the objective of this kind of resource allocation models changes to the optimization of the outcome: minimization of the loss caused by the unavailable resources, in many cases.

For instance, when natural disasters like earthquakes happen, it is urgent to send rescue teams and equipments to the affected areas in the first few days, otherwise more people would be killed. The maximum time of survival lies between four and seven days, and the probability of survival decreases to zero in one day if the trapped person is injured. Since the quantity and quality of the rescue resources are limited, the emergency manager has to find the best assignment of available resources, so that more people can be saved in the shortest time. Fiedrich et al. (2000) introduced a dynamic resource allocation model for the search-and-rescue operations after earthquakes, of which the goal is to minimize the total number of fatalities. In his model, three different rescue tasks – rescuing people out of the collapsed buildings, stabilizing work to prevent second disasters and rehabilitation of transportation lifelines to improve the accessibility of important areas, are taken into consideration. Thus, rescue resources are classified into corresponding categories, and the maximum volume of each resource is used as the restriction that should be fulfilled in the model. Two heuristic methods – Simulated Annealing and Tabu Search are introduced to solve this resource allocation problem.

As mentioned above, time seems to be the most important factor that affects the total number of fatalities. Therefore, instead of the total number of fatalities, time could also be used as the measure of allocation performance. In Gong and Batta's (2007) ambulance allocation and reallocation model, the objective becomes minimization of makespan, which is the maximal finish time of the rescue operation in each cluster, or minimization of the total flow time, which means the summation of the finish time of rescue operations in all clusters. Gong and Batta first formulated the deterministic model that concerned the allocation of the correct number of ambulances at the beginning of the rescue operations. Moreover, since the situation in each cluster changes during the rescue period, the former ambulance plan may not be optimal in the new period. As a result, a reallocation model with the objective of minimizing the makespan over discrete time period was proposed. Both the ambulance allocation and reallocation problems can be solved by an iterative procedure presented by Gong and Batta.

2.2.3 Fair Allocation

Efficiency and effectiveness are common measures of a resource allocation system performance, nevertheless, only minimizing the total cost or maximizing aggregate outcome may be extremely unfair in systems which serve many different demands. One of the widely used fair allocation models is the so-called Min-Max model, which tries to find the best allocation of limited resources to a given set of demand sites, so that the maximum of the profit or loss differences between the demand sites is minimized.

Much research has been conducted on the Min-Max fair allocation model. Zeitlin (1981) first considered the Min-Max resource allocation problem with discrete resources. Katoh et al. (1985) presented a more general model, whose constraints include the fixed size of discrete commodity and lower and upper bounds on the amount of the commodity to be allocated to each demand site. Tang (1988) formulated several manufacturing problems into a max-min resource allocation model, and developed a procedure that finds an optimal integer solution. Lee et al. (1994) extended the min-max discrete resource allocation problem by considering multiple types of resources. Luss (1999) introduced the lexicographic min-max problem, which determines the lexicographically smallest vector whose performance function values are sorted in non-increasing order. Karabati et al, (2001) proposed a new min-max model, in which the system performance measure function consists of multiple components and is equal to the sum of these components. He suggested different ways to efficiently solve it, according to the size of the problem.

Chapter 3

Winter Road Maintenance Model

The literature review of the past research on winter road maintenance models shows that the former models always assumed static requirements or capacities of the winter road maintenance resources, such as the total distance of the roadways that need to be served, the service rate of the snow removal trucks on the road, the number of available snow removal trucks in a depot, etc. However, in reality, the winter road maintenance workload increases while the service speed decreases as the storm becomes severe, and the number of available trucks in each depot tends to change when cooperation exists among districts. Considering these dynamic aspects, we proposed a new resource allocation model for winter road maintenance operation, whose objective is to find the best resource reallocation plan that minimizes the total reallocation cost while each district needs to be fully served.

In this chapter, the important components that constitute our model are discussed first. Then parameters necessary to the model are set according the these components. The procedure of modeling is presented at the end of this chapter.

3.1 Relevant Components

Three main components are related with the requirements and capacities of the winter road maintenance resource: the service area where winter road maintenance operations are conducted, the storm factors that affect the speed of the snow removal trucks, and the basic information about the service resource.

3.1.1 Service Area

Suppose there are m districts. In each district, there are n classes of roads with different service targets. In this model, service frequencies are used to indicate the different service targets. Let f^j denote the the service frequency of class j roads during unit duration, or one working shift which is often 12 hours. That is, during unit duration, class j roads should be served f^j times. For example, $f^3 = 2$ means that class 3 roads need to be served twice during unit duration. Assume that $f^i > f^j$ if $i < j$ without loss of generality.

In addition, let l_i^j denote the length of class j roads in district i. Then $l_i^j f^j$ denotes the absolute service length of class j roads in district i that needs to be served during unit duration. Assume that l_i^j is given as lane miles, which means that the service length of a road with multiple lanes is considered as the product of the number of lanes and the length of a single lane.

3.1.2 Storm Factors

The intensity of the storm obviously impacts the winter maintenance work. In our model, storm intensity is indicated by discrete multiple levels: from level 1 to level K. In practice, K would be the number between 3 and 10. Any natural number between 1 and K indicates a storm intensity level. The greater number indicates higher intensity of storm. For instance, level 1 storm indicates no storm, while the level K storm has the extreme intensity.

The impact of storm can be viewed as a decrease of winter maintenance efficiency. Therefore, let $\alpha_j \in (0, 1]$ denote the service efficiency under storm level j. For example, $\alpha_3 = 0.7$ means that the service speed of a snow removal truck under a level 3 storm deceases to 70% of that in a normal situation. Assume $\alpha_1 = 1$, and $\alpha_i > \alpha_j$ if $i < j$.

Because resource is reallocated before a storm strikes, forecast of storm intensity is needed. Let p_{ik} denote probability of having a level k storm in district i. Then a vector $P_i = [p_{i1}, \ldots, p_{iK}]$ denotes the probability of storm intensity in district i, where $\sum_{k=1}^{K} p_{ik} = 1$, $p_{ik} \in [0, 1]$, for all district $i \in [1, m]$.

3.1.3 Service Resource

Assume there is only one type of snow removal trucks in this model, and the original number of trucks in district i is n_i, which is a constant. Let s^j denote the normal service speed of the snow removal truck in class j roads. Hence, the service speed of the truck in class j roads during a level i storm should be $\alpha_i s^j$, where α_i is the service efficiency under a level i storm. In addition, assume that the service speed is given as miles per unit duration, so that s^j represents the service distance of a snow removal truck during unit duration.

3.2 The Model

With the parameters defined in the previous section, we can formulate the realistic winter road maintenance problem as follows. Consider class j roads in district i in a normal condition. As shown in the previous section, $l_i^j f^j$ denotes the service length of class j roads in district i that needs to be served during unit duration, and s^j is the normal service speed (equivalently, service distance during unit duration) in class j roads. Hence, the the minimum number of trucks needed for class j roads in district i in a normal condition during unit duration is $\frac{l_i^j f^j}{s^j}$. The minimum number of trucks needed for all the roads in district i in normal condition during unit duration is given as

$$\sum_{j=1}^{n} \frac{l_i^j f^j}{s^j}.$$

Because there should be enough number of trucks to operate in a normal condition, we assume

$$\sum_{j=1}^{n} \frac{l_i^j f^j}{s^j} \leq n_i \quad \text{for} \quad 1 \leq i \leq m, \tag{3.1}$$

where n_i is the number of trucks originally assigned to district i.

When there is a storm, the service speed in class j road under a level k storm decreases to $\alpha_k s^j$, and the expected number of trucks needed to serve class j roads in district i becomes

$$\sum_{k=1}^{K} \frac{p_{ik} l_i^j f^j}{\alpha_k s^j},$$

where p_{ik} is the probability of having a level k storm in district i.

Therefore, the expected number of trucks needed for all the roads in district i under a potential storm during unit duration is given as

$$\sum_{j=1}^{n}\sum_{k=1}^{K}\frac{p_{ik}l_i^j f^j}{\alpha_k s^j}. \tag{3.2}$$

Clearly, the number of trucks needed under a storm in (3.2) is larger than the number under a normal condition given in the left hand side of (3.1). Observe that

$$\sum_{j=1}^{n}\sum_{k=1}^{K}\frac{p_{ik}l_i^j f^j}{\alpha_k s^j} > \sum_{j=1}^{n}\sum_{k=1}^{K}\frac{p_{ik}l_i^j f^j}{s^j} = \sum_{j=1}^{n}\frac{l_i^j f^j}{s^j}.$$

The first inequality holds because $\alpha_k < 1$ when $k > 1$. The second equality holds because $\sum_{k=1}^{K} p_{ik} = 1$. From (3.1) and (3.2), it is possible that for some i, $1 \le i \le m$

$$\sum_{j=1}^{n}\sum_{k=1}^{K}\frac{p_{ik}l_i^j f^j}{\alpha_k s^j} > n_i.$$

That is, there may not be enough number of trucks in certain districts when a storm arrives. In this situation, it is desirable to reallocate the snow removal trucks among districts so that every district can be served fairly and efficiently.

Let x_{ij} be the number of trucks reallocated from district i to district j. These are the decision variables having integer values such that $x_{ij} \in [0, n_i]$ for all $i = 1, \ldots, m$.

Let c_{ij} denote the cost of moving one truck from district i to district j. The cost can be proportional to the distance between districts i and j. That is, the longer the distance between the districts, the higher the moving operation cost.

In addition, the inefficiency of the snow removal operation of reallocated trucks is considered. Truck drivers who are unfamiliar with roads and environment in a reallocated district cannot work as efficient as truck drivers who have worked in the district long time.

This reallocation inefficiency is expressed by a constant factor $\beta \in [0, 1]$. Hence, the product of β and the number of moving-in trucks gives the actual utility of reallocated trucks.

The number of trucks in district i after reallocation is given by

$$n_i + \sum_{j=1}^{m} \beta x_{ji} - \sum_{j=1}^{m} x_{ij},$$

which is the original number of trucks in the district plus the practical number of moved-in trucks minus the number of moved-out trucks.

Hence, district i has enough trucks if following equation holds:

$$\sum_{j=1}^{n} \sum_{k=1}^{K} \frac{p_{ik} l_i^j f^j}{\alpha_k s^j} \leq n_i + \sum_{j=1}^{m} \beta x_{ji} - \sum_{j=1}^{m} x_{ij}. \tag{3.3}$$

The goal of the problem can be to minimize the total reallocation cost while satisfying above constraint (3.3), if possible. Before developing a mathematical model, we identify a property of an optimal policy, which holds under the triangular inequality in reallocation cost, i.e. $c_{ik} \leq c_{ij} + c_{jk}$.

Lemma 1 *There exists an optimal reallocation policy that does not allow a district to both send and receive trucks.*

Proof. Suppose that a truck is moved from district i to district j and then district j to district k. The reallocation of one truck incurs the cost of $c_{ij} + c_{jk}$, decreases truck capacity by 1 in district i and by $1-\beta$ in district j, and increase truck capacity by β in district k.

If the truck is moved directly from district i to district k, the associated cost is c_{ik}. District i loses truck capacity by 1 and district k earns truck capacity by β, while there is no capacity change in district j.

The second policy costs less and performs better or equally in every district compared to the first policy. Because this argument can be generalized for multiple trucks and multiple districts, the proof is complete. ■

The above Lemma simplifies our modeling. Let I_1 be the set of districts with enough number of trucks to serve their own districts, and I_2 be the set of those without enough number of trucks. That is, if $i \in I_1$, then

$$\sum_{j=1}^{n}\sum_{k=1}^{K}\frac{p_{ik}l_i^j f^j}{\alpha_k s^j} \leq n_i,$$

and if $i \in I_2$, then

$$\sum_{j=1}^{n}\sum_{k=1}^{K}\frac{p_{ik}l_i^j f^j}{\alpha_k s^j} > n_i.$$

Note that $I_1 \cup I_2$ shows all the districts in the area, and $I_1 \cap I_2 = \emptyset$.

From Lemma 1, the reallocation of trucks can happen only from I_1 to I_2. Hence, the math model minimizing the total allocation cost is written as

$$\begin{aligned}
\min & \sum_{i\in I_1}\sum_{j\in I_2} c_{ij}x_{ij} & & \\
\text{s.t.} & \sum_{j=1}^{n}\sum_{k=1}^{K}\frac{p_{ik}l_i^j f^j}{\alpha_k s^j} \leq n_i - \sum_{j\in I_2} x_{ij} & \text{for all} \quad i \in I_1 & \qquad (3.4)\\
& \sum_{j=1}^{n}\sum_{k=1}^{K}\frac{p_{ik}l_i^j f^j}{\alpha_k s^j} \leq n_i + \sum_{j\in I_1} \beta x_{ji} & \text{for all} \quad i \in I_2 & \qquad (3.5)\\
& x_{ij} \in [0, n_i], \quad x_{ij} \in Z. & i, j = 1, \ldots, m &
\end{aligned}$$

The existence of a feasible solution can be easily determined as follows:

From equation (3.4), the number of available trucks to send in district $i \in I_1$ is at most

$$\left\lfloor n_i - \sum_{j=1}^{n} \sum_{k=1}^{K} \frac{p_{ik} l_i^j f^j}{\alpha_k s^j} \right\rfloor .$$

The sum over all the districts with sufficient trucks becomes

$$\sum_{i \in I_1} \left\lfloor n_i - \sum_{j=1}^{n} \sum_{k=1}^{K} \frac{p_{ik} l_i^j f^j}{\alpha_k s^j} \right\rfloor . \tag{3.6}$$

Similarly from (3.5), the number of trucks needed in district $i \in I_2$ is

$$\left\lceil \frac{1}{\beta} \left(\sum_{j=1}^{n} \sum_{k=1}^{K} \frac{p_{ik} l_i^j f^j}{\alpha_k s^j} - n_i \right) \right\rceil .$$

Again, the sum over all the districts which need trucks becomes

$$\sum_{i \in I_2} \left\lceil \frac{1}{\beta} \left(\sum_{j=1}^{n} \sum_{k=1}^{K} \frac{p_{ik} l_i^j f^j}{\alpha_k s^j} - n_i \right) \right\rceil . \tag{3.7}$$

Therefore, from (3.6) and (3.7), a feasible solution for the optimization model exists when

$$\sum_{i \in I_1} \left\lfloor n_i - \sum_{j=1}^{n} \sum_{k=1}^{K} \frac{p_{ik} l_i^j f^j}{\alpha_k s^j} \right\rfloor \geq \sum_{i \in I_2} \left\lceil \frac{1}{\beta} \left(\sum_{j=1}^{n} \sum_{k=1}^{K} \frac{p_{ik} l_i^j f^j}{\alpha_k s^j} - n_i \right) \right\rceil . \tag{3.8}$$

In other words, the number of all trucks that can be reallocated should be greater than or equal to the total number of trucks that are needed. When (3.8) is satisfied, at least one feasible solution for this model exists. The optimization problem can be rewritten as

$$\min \sum_{i \in I_1} \sum_{j \in I_2} c_{ij} x_{ij} \tag{3.9}$$

$$\text{s.t.} \quad \sum_{j \in I_2} x_{ij} \leq \left\lfloor n_i - \sum_{j=1}^{n} \sum_{k=1}^{K} \frac{p_{ik} l_i^j f^j}{\alpha_k s^j} \right\rfloor \qquad \text{for all} \quad i \in I_1 \tag{3.10}$$

$$\sum_{j \in I_1} x_{ji} = \left\lceil \frac{1}{\beta} \left(\sum_{j=1}^{n} \sum_{k=1}^{K} \frac{p_{ik} l_i^j f^j}{\alpha_k s^j} - n_i \right) \right\rceil \qquad \text{for all} \quad i \in I_2 \tag{3.11}$$

$$x_{ij} \in [0, n_i], \quad x_{ij} \in Z.$$

Chapter 4

Solution Approaches

The proposed model (3.9) does not always have feasible solutions. In the worst situation, when the whole area faces a high-intensity storm, each of the districts in the area requires more trucks to maintain the service level, and none of them has any additional trucks to be reallocated. Then there would be no feasible solution in this model because of (3.8), which asks the total number of trucks that can be reallocated to be greater than the total number of trucks that are needed. Therefore, different solution approaches are needed, depending on whether feasible solutions exist.

4.1 When Feasible Solutions Exist

We first consider a case when a feasible solution to model (3.9) exists.

To simplify the model, in (3.10) and (3.11), we define

$$a_i = \left\lfloor n_i - \sum_{j=1}^{n} \sum_{k=1}^{K} \frac{p_{ik} l_i^j f^j}{\alpha_k s^j} \right\rfloor \qquad \text{if} \quad i \in I_1$$

$$b_i = \left\lceil \frac{1}{\beta} \left(\sum_{j=1}^{n} \sum_{k=1}^{K} \frac{p_{ik} l_i^j f^j}{\alpha_k s^j} - n_i \right) \right\rceil \qquad \text{if} \quad i \in I_2$$

and suppose there are m_1 supply districts in I_1, m_2 demand districts in I_2. $m_1 + m_2 = m$, where m is the total number of districts that we need to serve. We can label the supply districts from 1 to m_1, and the demand districts from $m_1 + 1$ to $m_1 + m_2$

without loss of generality. Then, the model can be simplified as

$$\min \sum_{i=1}^{m_1} \sum_{j=m_1+1}^{m_1+m_2} c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{j=m_1+1}^{m_1+m_2} x_{ij} \leq a_i \qquad i = 1, \ldots, m_1 \tag{4.1}$$

$$\sum_{i=1}^{m_1} x_{ij} = b_j \qquad j = m_1 + 1, \ldots, m_1 + m_2 \tag{4.2}$$

$$x_{ij} \in [0, n_i], \quad x_{ij} \in Z.$$

We also have

$$\sum_{i=1}^{m_1} a_i \geq \sum_{j=m_1+1}^{m_1+m_2} b_j, \tag{4.3}$$

which comes from (3.8), so that at least one feasible solution exists. It is now easy to see that this model is a typical transportation model. Then we can solve it with LP method mentioned in the literature review chapter.

4.2 When Feasible Solutions Do Not Exist

Suppose that feasible solutions satisfying all demands do not exist, that is, the quantity of trucks that can be reallocated is less than the total demand as follows

$$\sum_{i=1}^{m_1} a_i < \sum_{j=m_1+1}^{m_1+m_2} b_j. \tag{4.4}$$

In this case, there are different ways of approach the problem. A straightforward extension transforms the problem back to a balanced transportation problem by adding a dummy supply district that satisfies the shortage. Suppose we add a dummy supply district labeled $i = m_1 + m_2 + 1$, and set its maximum supply to be

$$a_{m_1+m_2+1} = \sum_{j=m_1+1}^{m_1+m_2} b_j - \sum_{i=1}^{m_1} a_i,$$

and its reallocation unit cost to be

$$c_{m_1+m_2+1,j} = 0 \qquad \text{for all} \quad j = m_1 + 1, \ldots, m_1 + m_2.$$

Then the problem becomes a balanced transportation model:

$$\begin{aligned}
&\min \sum_{i=1}^{m_1} \sum_{j=m_1+1}^{m_1+m_2} c_{ij} x_{ij} \\
&\text{s.t.} \sum_{j=m_1+1}^{m_1+m_2} x_{ij} = a_i \qquad && i = 1, \ldots, m_1, m_1 + m_2 + 1 \qquad (4.5) \\
&\sum_{i=1}^{m_1} x_{ij} + x_{(m_1+m_2+1)j} = b_j \qquad && j = m_1 + 1, \ldots, m_1 + m_2 \qquad (4.6) \\
&\left(\sum_{i=1}^{m_1} a_i \right) + a_{m_1+m_2+1} = \sum_{j=1}^{m_2} b_j \qquad && x_{ij} \in [0, n_i], \quad x_{ij} \in Z.
\end{aligned}$$

This problem can be solved easily by the same LP method. Although the total operation cost is minimized in the result, this approach does not guarantee that all the districts will maintain a certain level of service. In fact, some districts which are in great need of trucks may even receive no trucks at all in the end, because the dummy supply district dose not exist actually. Hence, their level of service remain the same as that in the beginning. Consider the following example:

There are four districts in the considered area, in which districts 1 and 2 are supply districts, each with the maximum of two trucks to supply, while districts 3 and 4 are demand districts, each with the minimum of four trucks to receive. Table 5.1 lists the unit reallocation cost between districts:

Table 4.1: Unit Reallocation Cost

	District 3	District 4
District 1	8	3
District 2	6	2

Then the problem can be modeled as follows, which dose not have a feasible solution:

$$\begin{aligned} \min \quad & 8x_{13} + 3x_{14} + 6x_{23} + 2x_{24} \\ \text{s.t.} \sum_{j=3}^{4} x_{1j} = 2 & \qquad \sum_{j=3}^{4} x_{2j} = 2 \\ \sum_{i=1}^{2} x_{i3} = 4 & \qquad \sum_{i=1}^{2} x_{i4} = 4 \\ x_{ij} \in [0, n_i], \quad & x_{ij} \in Z. \end{aligned}$$

It is easy to find that the total demand $(4+4)$ is greater than the total supply $(2+2)$ in this example. Then we can add a dummy supply, labeled district 5, whose maximum supply is the difference between the total demand and the total supply. The problem becomes:

$$\begin{aligned} \min \quad & 8x_{13} + 3x_{14} + 6x_{23} + 2x_{24} \\ \text{s.t.} & \sum_{j=3}^{4} x_{1j} = 2 \\ & \sum_{j=3}^{4} x_{2j} = 2 \\ & \sum_{j=3}^{4} x_{5j} = 4 \\ & \sum_{i=1}^{2} x_{i3} + x_{53} = 4 \\ & \sum_{i=1}^{2} x_{i4} + x_{54} = 4 \\ & x_{ij} \in [0, n_i], \quad x_{ij} \in Z. \end{aligned}$$

The optimal solution for this balanced problem is

$$x_{14} = 2 \quad x_{24} = 2 \quad x_{53} = 4.$$

District 1 and 2 will send all their trucks to district 4, because the cost of moving into district 4 is less than that of moving into district 3 in the objective function. As a result, district 3 receives no truck at all actually, since district 5 does not really exist. Therefore, this method may lead to very low level of service in certain districts – district 3 in this example, which is not fair for those districts, and the allocation plan will not be accepted.

To solve this unfairness, we introduce another policy called Fair Allocation. In a Fair Allocation model, the objective remains the same: finding the best allocation plan that minimize the total reallocation operation cost. However, we restrict each district to maintain the same level of service after reallocation.

Considering the way we calculated the expected number of trucks needed in each district, it is reasonable to define the level of service as the number of trucks needed to be fully serve a district. Our objective is trying to find a reallocation plan, which maintains the same service level for all districts with minimal cost.

Let N_i denote the expected number of trucks needed in district i to fully serve the district. Recall that n_i denotes the number of trucks that district i originally has.Then, from (3.2),

$$N_i = \sum_{j=1}^{n} \sum_{k=1}^{K} \frac{p_{ik} l_i^j f^j}{\alpha_k s^j}. \tag{4.7}$$

Let ΔN_i denote the level of service in district i before reallocation, and $\Delta N_i'$ the level of service in district i after reallocation:

$$\Delta N_i = n_i - N_i \tag{4.8}$$

$$\Delta N_i{}' = (n_i - \sum_{j \in I} x_{ij} + \sum_{j \in I} \beta x_{ji}) - N_i. \tag{4.9}$$

Where I denotes the set of all districts.

Then, the Fair Allocation model can be modeled as follows:

$$\begin{aligned} \min & \sum_{i \in I} \sum_{j \in I} c_{ij} x_{ij} \\ \text{s.t. } & \Delta N_i{}' = \Delta N_j{}' \qquad \text{for any} \quad i, j \in I \\ & x_{ij} \in [0, n_i], \quad x_{ij} \in Z. \end{aligned} \tag{4.10}$$

Lemma 1, which argues that a district both sending and receiving trucks is not allowed in an optimal solution, still holds in this Fair Allocation model. The proof is similar as what we did before:

Suppose that a truck is moved from district i to district j and then district j to district k. The reallocation of one truck incurs the cost of $(c_{ij} + c_{jk})$, decreases service level by 1 in district i and by $(1 - \beta)$ in district j, and increase service level by β in district k.

If the truck is moved directly from district i to district k, the associated cost is c_{ik}. District i decreases level of service by 1 and district k improve its level of service by β, while there is no change of service level in district j.

The second policy costs less and performs better or equally in every district compared to the first policy. Because this argument can be generalized for multiple trucks and multiple districts, the proof is complete.

With this lemma in mind, we can easily identify another property of an optimal policy for our Fair Allocation model. Let these m districts be ordered according to non-increasing level of service before reallocation, without loss of generality:

$$\Delta N_m \geq \Delta N_{m-1} \geq \ldots \geq \Delta N_1. \tag{4.11}$$

Let δ denote the fairness level, i.e. final level of service that every district reaches after reallocation. We have

$$\begin{aligned}
\delta &= (n_i - \sum_{j\in I} x_{ij} + \sum_{j\in I} \beta x_{ji}) - N_i \qquad \text{for any} \quad i \in I \\
\sum_{i\in I} \delta &= \sum_{i\in I} n_i - \sum_{i\in I}\sum_{j\in I} x_{ij} + \sum_{i\in I}\sum_{j\in I} \beta x_{ji} - \sum_{i\in I} N_i \\
\delta &= \frac{\sum_{i\in I} n_i - \sum_{i\in I} N_i - (1-\beta)\sum_{i\in I}\sum_{j\in I} x_{ij}}{m}.
\end{aligned} \tag{4.12}$$

When $\beta = 1$, which means the best reallocation efficiency,

$$\delta = \frac{\sum_{i\in I} n_i - \sum_{i\in I} N_i}{m} = \frac{\sum_{i\in I} \Delta N_i}{m};$$

when $\beta = 0$, which means the worst reallocation efficiency,

$$\delta = \Delta N_1.$$

Therefore,

$$\Delta N_1 \leq \delta \leq \frac{\sum_{i\in I} \Delta N_i}{m} \leq \Delta N_m. \tag{4.13}$$

Lemma 2 *In an optimal reallocation policy, if district i send trucks to other districts, then district $(i+1)$ would not receive any trucks. On the other side, if district j receive trucks from other districts, then district $(j-1)$ would not send any trucks.*

Proof. If district i need to send trucks in optimal reallocation plan, then it can not receive any trucks because of Lemma 1. That means the fairness level

$$\delta \leq \Delta N_i. \tag{4.14}$$

Also from (4.8),

$$\Delta N_i \leq \Delta N_{i+1}. \tag{4.15}$$

From (4.14) and (4.15), we have

$$\delta \leq \Delta N_{i+1}. \tag{4.16}$$

Therefore, district $i+1$ should also only send trucks to other districts, otherwise, it will not reach the fairness level.

On the other side, If district j need to receive trucks to reach the fairness level, then it can not send any trucks because of Lemma 1. That means fairness level

$$\delta \geq \Delta N_j. \tag{4.17}$$

Also from (4.8),

$$\Delta N_j \geq \Delta N_{j-1}. \tag{4.18}$$

From (4.17) and (4.18), we have

$$\delta \geq \Delta N_{j-1}. \tag{4.19}$$

Therefore, district $j-1$ should only receive trucks, otherwise, it will not reach the fairness level. ■

Corollary 1 *If the original service level of district i is greater than the fairness level δ, then it can only send trucks in an optimal reallocation plan; if the original service level of district j is less than the fairness level δ, then it can only receive trucks in an optimal reallocation plan.*

Proof. From Lemma 1, if district i receive trucks, it can not send trucks at the same time. Therefore, the service level of district i will exceed the fairness level more.

Similarly, if district j send trucks, it can not receive trucks at the same time. Therefore, the service level of district i will decrease, and it can not reach the fairness level. This completes the proof. ■

From Lemma 2 and Corollary 1, there is a certain district i_0 that satisfies:

(1) For any $i \geq i_0$, District i either only sends trucks or dose not receive any trucks in optimal reallocation policy;

(2) For any $j < i_0$, District j only receives trucks in optimal reallocation policy;

(3) Fairness level δ lies between ΔN_{i_0} and ΔN_{i_0-1}, that is: $\Delta N_{i_0-1} \leq \delta \leq \Delta N_{i_0}$.

Our solution procedure that will solve the Fair Allocation problem includes the following stages: first, we find the specific district i_0 according to the properties of the model; then we can calculate the fairness level by its definition; finally, we solve the integer problem.

Stage One: Find District i_0

For any $i \geq i_0$, $(\Delta N_i - \delta)$ denotes the number of trucks that move out of district i. Then the total number of trucks moving out should be

$$\sum_{i_0 \leq i \leq m} (\Delta N_i - \delta). \tag{4.20}$$

For any $j < i_0$, $\frac{1}{\beta}(\delta - \Delta N_j)$ denotes the number of trucks that move into district j.

Then the total number of trucks moving in should be

$$\frac{1}{\beta} \sum_{1 \le j < i_0} (\delta - \Delta N_j) \tag{4.21}$$

.

Since the total number of trucks being sent out should equal the total number of trucks being received, we have:

$$\sum_{i_0 \le i \le m} (\Delta N_i - \delta) = \frac{1}{\beta} \sum_{1 \le j < i_0} (\delta - \Delta N_j). \tag{4.22}$$

According to the third property of district i_0, we have:

$$\sum_{i_0 \le i \le m} (\Delta N_i - \Delta N_{i_0}) \le \frac{1}{\beta} \sum_{1 \le j < i_0} (\Delta N_{i_0} - \Delta N_j), \tag{4.23}$$

and

$$\sum_{i_0 - 1 \le i \le m} (\Delta N_i - \Delta N_{i_0 - 1}) \ge \frac{1}{\beta} \sum_{1 \le j < i_0 - 1} (\Delta N_{i_0 - 1} - \Delta N_j). \tag{4.24}$$

Define N_k^{out} and N_k^{in} as follows:

$$\begin{aligned} N_k^{out} &= \sum_{k \le i \le m} (\Delta N_i - \Delta N_k), \\ N_k^{in} &= \frac{1}{\beta} \sum_{1 \le j < k} (\Delta N_k - \Delta N_j). \end{aligned}$$

Lemma 3 *N_k^{out} decreases in k; N_k^{in} increases in k.*

Proof. For any $1 \le k < m$,

$$\begin{aligned} N_k^{out} &= \sum_{k \le i \le m} (\Delta N_i - \Delta N_k) \\ &= (\Delta N_k - \Delta N_k) + \sum_{k+1 \le i \le m} (\Delta N_i - \Delta N_k) \\ &= \sum_{k+1 \le i \le m} (\Delta N_i - \Delta N_k). \end{aligned}$$

Since $\Delta N_k \leq \Delta N_{k+1}$, then

$$\begin{aligned}
(\Delta N_i - \Delta N_k) &\geq (\Delta N_i - \Delta N_k) \\
\sum_{k+1 \leq i \leq m} (\Delta N_i - \Delta N_k) &\geq \sum_{k+1 \leq i \leq m} (\Delta N_i - \Delta N_{k+1}) \\
N_k^{out} &\geq N_{k+1}^{out}.
\end{aligned}$$

Therefore, N_k^{out} decreases in k.

On the other hand, for any $1 < k \leq m$,

$$\begin{aligned}
N_{k+1}^{in} &= \frac{1}{\beta} \sum_{1 \leq j < k+1} (\Delta N_{k+1} - \Delta N_j) \\
&= \frac{1}{\beta} \left(\sum_{1 \leq j < k} (\Delta N_{k+1} - \Delta N_j) + (\Delta N_{k+1} - \Delta N_k) \right)
\end{aligned}$$

Since $\Delta N_k \leq \Delta N_{k+1}$ and $\Delta N_{k+1} - \Delta N_k \geq 0$,

$$\begin{aligned}
(\Delta N_k - \Delta N_j) &\leq (\Delta N_{k+1} - \Delta N_j) \\
\frac{1}{\beta} \sum_{1 \leq j < k} (\Delta N_k - \Delta N_j) &\leq \frac{1}{\beta} \sum_{1 \leq j < k} (\Delta N_{k+1} - \Delta N_j) \\
\frac{1}{\beta} \sum_{1 \leq j < k} (\Delta N_k - \Delta N_j) &\leq \frac{1}{\beta} \left(\sum_{1 \leq j < k} (\Delta N_{k+1} - \Delta N_j) + (\Delta N_{k+1} - \Delta N_k) \right) \\
\frac{1}{\beta} \sum_{1 \leq j < k} (\Delta N_k - \Delta N_j) &\leq \frac{1}{\beta} \sum_{1 \leq j < k+1} (\Delta N_{k+1} - \Delta N_j) \\
N_k^{in} &\leq N_{k+1}^{in}.
\end{aligned}$$

Therefore, N_k^{in} increases in k. The proof is complete. ■

With Lemma 3 and (4.23), we have

$$\sum_{k \leq i \leq m} (\Delta N_i - \Delta N_k) \leq \frac{1}{\beta} \sum_{1 \leq j < k} (\Delta N_k - \Delta N_j) \quad \text{for any} \quad i_0 \leq k \leq m. \tag{4.25}$$

With Lemma 3 and (4.24), we have

$$\sum_{k \leq i \leq m} (\Delta N_i - \Delta N_k) \geq \frac{1}{\beta} \sum_{1 \leq j < k} (\Delta N_k - \Delta N_j) \quad \text{for any} \quad 1 \leq k \leq i_0 - 1. \tag{4.26}$$

Therefore, we can find i_0 through the following steps:

Step 0 Set $k = 1$.

Step 1 Calculate N_k^{out}, N_k^{in}, N_{k+1}^{out} and N_{k+1}^{in} .

Step 2 Check

if $N_k^{out} = N_k^{in}$, then $i_0 = k$, STOP;

if $N_{k+1}^{out} = N_{k+1}^{in}$, then $i_0 = k + 1$, STOP;

if $N_k^{out} > N_k^{in}$ and $N_{k+1}^{out} < N_{k+1}^{in}$, then $i_0 = k + 1$, STOP;

otherwise, set $k = k + 1$ and go back to **Step 1.**

Stage Two: Find Fairness Level δ

When we find i_0, the fairness level δ can be calculated by (4.22):

$$\begin{aligned} \beta \sum_{i_0 \le i \le m} (\Delta N_i - \delta) &= \sum_{1 \le j < i_0} (\delta - \Delta N_j) \\ \delta &= \frac{\sum_{j=1}^{i_0-1} \Delta N_j + \beta \sum_{i=i_0}^{m} \Delta N_i}{i_0 - 1 + \beta(m - i_0 + 1)}. \end{aligned} \tag{4.27}$$

Stage Three: Solve the Fair Allocation Integer Model

The Fair Allocation model becomes

$$\begin{aligned} &\min \sum_{i=i_0}^{m} \sum_{j=1}^{i_0-1} c_{ij} x_{ij} \\ &\text{s.t. } (n_i - \sum_{j=1}^{i_0-1} x_{ij}) - N_i = (n_j + \beta \sum_{i=i_0}^{m} x_{ij}) - N_j \quad \text{for any} \quad i_0 \le i \le m, \quad 1 \le j < i_0 \\ &\quad x_{ij} \in [0, n_i], \quad x_{ij} \in Z. \end{aligned} \tag{4.28}$$

Since $\delta = (n_i - \sum_{j=1}^{i_0-1} x_{ij}) - N_i = (n_j + \beta \sum_{i=i_0}^{m} x_{ij}) - N_j$, then the model can be written as

$$\min \sum_{i=i_0}^{m} \sum_{j=1}^{i_0-1} c_{ij} x_{ij}$$

$$\text{s.t. } (n_i - \sum_{j=1}^{i_0-1} x_{ij}) - N_i = \delta \qquad \text{for any} \quad i_0 \leq i \leq m \tag{4.29}$$

$$(n_j + \beta \sum_{i=i_0}^{m} x_{ij}) - N_j = \delta \qquad \text{for any} \quad 1 \leq j < i_0 \tag{4.30}$$

$$x_{ij} \in [0, n_i], \quad x_{ij} \in Z.$$

Or

$$\min \sum_{i=i_0}^{m} \sum_{j=1}^{i_0-1} c_{ij} x_{ij}$$

$$\text{s.t. } \sum_{j=1}^{i_0-1} x_{ij} = (n_i - N_i) - \delta \qquad \text{for any} \quad i_0 \leq i \leq m \tag{4.31}$$

$$\sum_{i=i_0}^{m} x_{ij} = \frac{1}{\beta}(\delta - (n_j - N_j)) \qquad \text{for any} \quad 1 \leq j < i_0 \tag{4.32}$$

$$x_{ij} \in [0, n_i], \quad x_{ij} \in Z.$$

The model above dose not have any feasible solution, because integer numbers would never satisfy the non-integer constraints. To solve this problem, we can round the supply constraint (4.31) to be less than a larger integer number but greater than a smaller integer number, and also round the demand constraint (4.32) to be greater than a smaller integer number but less than a larger integer number:

$$\min \sum_{i=i_0}^{m} \sum_{j=1}^{i_0-1} c_{ij} x_{ij}$$

$$\text{s.t.} \quad \lfloor (n_i - N_i) - \delta \rfloor \leq \sum_{j=1}^{i_0-1} x_{ij} \leq \lceil (n_i - N_i) - \delta \rceil \qquad \text{for any} \quad i_0 \leq i \leq m \tag{4.33}$$

$$\lceil \frac{1}{\beta}(\delta - (n_j - N_j)) \rceil \geq \sum_{i=i_0}^{m} x_{ij} \geq \lfloor \frac{1}{\beta}(\delta - (n_j - N_j)) \rfloor \quad \text{for any} \quad 1 \leq j < i_0 \tag{4.34}$$

$$x_{ij} \in [0, n_i], \quad x_{ij} \in Z.$$

Then, the integer problem becomes a variant of the transportation problem. It can be solved by transforming to a standard transportation problem, according to the method presented by Dahiya and Verma. Therefore, we can solve it similarly as a transportation model.

Chapter 5

Case Study

This chapter includes an illustration of the mathematical models and solution approaches proposed in the research. The presented problem in this chapter is based on a series of hypothetical storm situations in the seven service regions in central Missouri. Our objective is to determine the best resource (snow removal trucks) reallocation plan that maintains the same service level for all service regions with the minimal cost.

5.1 Case Background

5.1.1 Overview

This section provides a case study which motivated this research. As shown in Figure 5.1 and Figure A.1, the area considered for this case study is the seven service regions in Missouri, which include 13 counties, 453,000 people, 7,802 sq. miles and 3,625 road miles to be maintained by Missouri Department of Transportation (MoDOT). As previous mentioned, winter road maintenance work includes the operations of salt and abrasives spreading, snow plowing, loading snow into snow removal vehicles and hauling snow to disposal sites, and presents a variety of decision-making problems, such as routing problem, sector design and fleet sizing, which are extremely complex.

Therefore, we assume that the winter road maintenance network and policy in each region have been established by a set of predefined routes. The depot location and service range in each district are fixed; the operating and maintenance cost of the snow removal resources is not considered. We focus on the operational objectives and constraints, which are used for the modeling of winter road maintenance resource allocation.

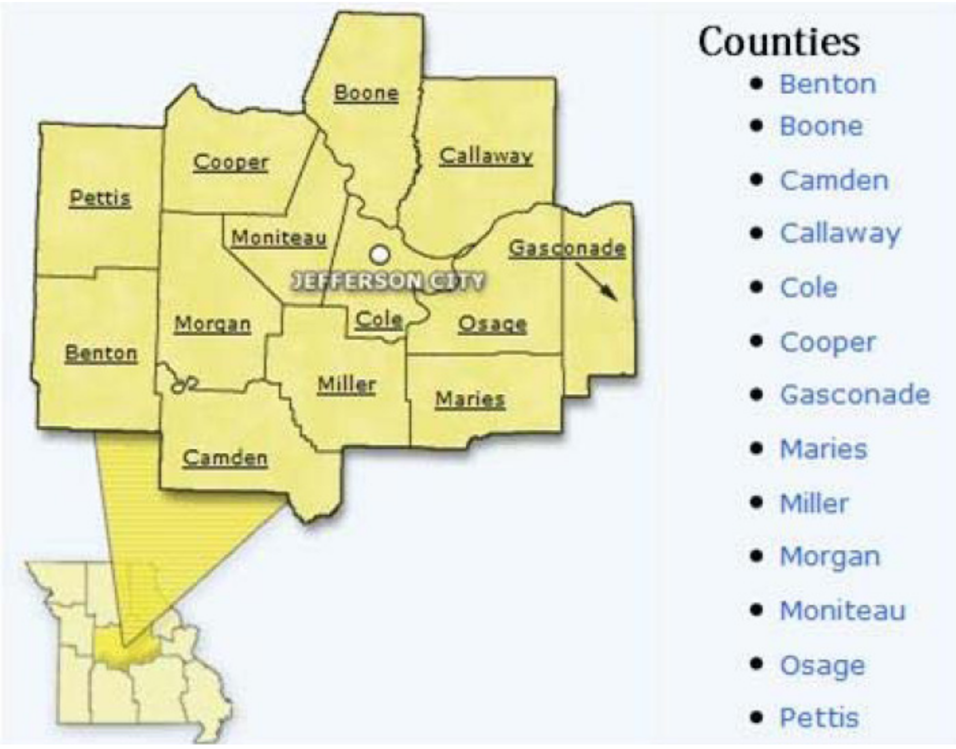

Figure 5.1: Counties in Central Missouri

5.1.2 Winter Road Maintenance Operations

MoDOT conducts three major winter road maintenance operations in the seven districts: pre-treatment before storm, spreading and plowing during storm, and after-storm cleanup. Pre-treatment operation includes spreading abrasives or chemicals over the roadway in order to prevent formation of ice or pack - snow compacted by traffic action that becomes nearly as tightly bonded to pavement as ice, before or in the early stages of a storm event. This pre-treatment can be conducted in all types of roadways: highway, bridges, hills and curves. But depending on the storm conditions, it is possible that only part of the roadways need to be pre-treated. Spreading-and-plowing plays the most important role in winter road maintenance operation. Snow plowing operation removes as much snow and loose ice as possible in order to keep the road surface clear, while spreading operation tries to melt ice and improve traction during a storm event in order to keep the road surface from slick. Although the speed of plowing is lower than that of spreading, spreading operation requires more frequent return trips for replenishment than plowing. After-storm cleanup operation is the process of plowing the remaining snow from the roadways. The inner and outer shoulders of highways and major roads need to be served once a storm has ended. Then the remaining snow over any other roadways and bridges which is built up as a result of previous plowing operations can be removed.

5.1.3 Winter Road Maintenance Network

MoDOT is responsible for serving all state roads within the seven districts in the center of Missouri, including interstate highways, state highways and other state roadways. Hence, snow removal vehicles are restricted to the state road network while providing winter road maintenance service.

Since many roadways have multiple lanes and the snow removal vehicle can only serve one pass per lane, we calculate the total service distance of a road with multiple lanes by the multiplication of centerline distance and the number of lanes it has.

Different types of roads have different service frequencies during unit service duration. For example, interstate highways should be served more often than normal state roadways. According to the survey completed by the managers in major depots, all the state roadways that are served by MoDOT can be classified based on the historical average daily traffic (ADT) data. The three-class hierarchy is shown in Table 5.1. Class A1, Class A2 and Class A3 roadways should be served within 2, 6 and 12 hours respectively per 12-hour shift, which means that Class A1, A2 and A3 roadways needs to be served 6, 2, and 1 times respectively per 12-hour shift. The frequency is considered ideal because replenishment and other operational time between service runs are not considered.

Table 5.1: Three-Class Hierarchy

Class	**ADT**
A1	ADT $>$ 2500
A2	2500 $>$ ADT $>$ 1000
A3	1000 $>$ ADT

Besides, there are total 37 existing winter road maintenance depots operated by MoDOT within the seven districts in central Missouri. Depot locations and associated routes have evolved as a result of annual decisions and adjustment made by MoDOT's managers and planners based on their operational experience. The proximity to the highways and other major state roadways, as well as the accessibility to nearby roadways and storage space for maintenance materials and equipment is mostly considered when locating a winter road maintenance depot.

5.1.4 Winter Road Maintenance Resources

Snow removal vehicles are the most important resources in winter road maintenance operations. There are two types of snow removal vehicles that MoDOT has: heavy-duty single-axle trucks and extra heavy-duty tandem-axle trucks. The difference between these two types of vehicles is that the tandem-axle trucks can hold more abrasive or chemical material than the single-axle trucks. Monitors are used in both trucks in order to control the rate of material spreading. Normally, the spreading rate is $200lbs$ per lane mile, but could increase to $400lbs$ per lane mile depending on the intensity of the storm. Besides, both could be equipped with 10-, 12-, or 14-foot-wide plow for snow plowing operation. Any of the three types of plow could serve one traffic lane by adjusting the angle of the plow, while a larger-size plow would clear the road more thoroughly. The average serving speed is 40 miles per hour on Class A1 roadways, and 30 miles per hour on Class A2 and A3 roadways.

5.2 Model Parameters

5.2.1 Considered Area

We consider the seven service regions in central Missouri determined by MoDOT, and number them from 1 to 7. Each service region may consist of multiple counties. The roadways in each service region can be classified by the three-class hierarchy which is defined in the last chapter. The service frequency associated with the roadway class is shown in Table 5.2.

The length of a road is defined in lane miles, which means the length of a road with multiple lanes is the product of the centerline distance in a single lane and the

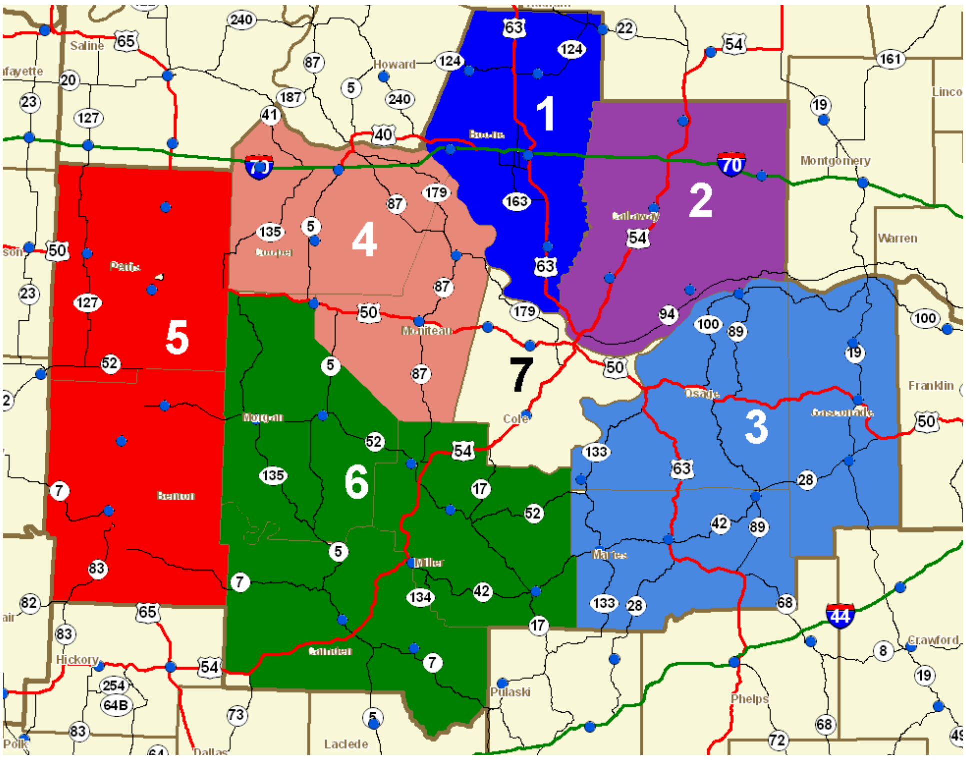

Figure 5.2: Service Regions in Central Missouri

Table 5.2: Service Frequency by Class

Class	ADT	Service Frequency per Unit Duration
A1	ADT>2500	6
A2	2500>ADT>1000	2
A3	1000>ADT	1

number of lanes it has. Table 5.3 illustrates the road information in each region:

Note that since Class A2 roads are mostly scattered in Cole County, we combine Class A2 and A3 roads together. Same operation is conducted in Cooper and Moniteau counties.

The distances between service regions are computed by the location of the major depot in each region where the regional supervisor works. All distances between these major depots can be found using the on-line mapping site MapQuest.

Table 5.3: Roadway Information

Region Number	Counties	Road Length in Lane Miles Class A1	Class A2	Class A3
1	Boone	566.64	125.52	337.35
2	Callaway	308.8	117	500.7
3	Osage, Maries, Gasconade	124.4	201.6	1147.8
4	Cooper, Moniteau	265.87	0	848.66
5	Benton, Pettis	271.76	149.39	989.75
6	Morgan, Miller, Camden	314.11	374.19	863.3
7	Cole	216.02	255.41	0

5.2.2 Storm Factors

We assume the hypothetical storm has three levels of intensity: from level 1 to level 3. Each storm intensity level has a different discount on the normal average service speed of the snow removal vehicle. The greater number indicates the higher intensity of a storm and a lower service speed. As described in table 5.4, in a level 1 storm, snow removal trucks serve the roads at the normal speed, i.e. 40 miles per hour on Class A1 roadways and 30 miles per hour on Class A2 and A3 roadways; while in a level 2 storm, the service speed is reduced to 60% of the normal speed, i.e. 24 miles per hour on Class A1 roadways and 18 miles per hour on Class A2 and A3 roadways; in a level 3 storm, the snow removal trucks can only serve the roads at 30% of the normal speed, that is 12 miles per hour on Class A1 roadways and 9 miles per hour on Class A2 and A3 roadways.

The potential snow storm may have a certain pattern, such as high intensity in the center and low intensity on the edge of the storm. In our case, according to the snowfall 1971-2000 averages data in Appendix C provided by the Midwestern Regional

Table 5.4: Storm Impact on Service Speed

Storm Intensity Level	**Discount on Service Speed**
3	0.3
2	0.6
1	1

Climate Center, we found that the northeastern part in central Missouri always has much more snowfall than the southwestern part during winter. For example, the annual average snowfall level in Boone and Callaway counties is around 22inches, which is much higher, compared with 11.6inches in Pettis, 5.7inches in Morgan and 11.2inches in Miller. To demonstrate this pattern, we assume that in a hypothetical storm, the probabilities of having high levels of intensities in the northeastern regions, including Callaway and Boone, would be relatively higher, while the probabilities of having high levels of intensities in the southwestern regions, including Cole, Benton, Pettis, Morgan, Miller and Camden, would be relatively lower, and medium in Cooper, Moniteau, Osage, Maries and Gasconade. Therefore, we partition the service regions by the possibilities of having higher levels of storm intensities, however, the value of probability of having each level of storm in a single service region is picked randomly.

The illustrations given in this chapter include three scenarios of hypothetical storm situations in central Missouri. In the first scenario, the intensity of the hypothetical storm is very weak, and all the service regions have low probabilities of facing high intensity levels. In the second scenario, the intensity of the hypothetical storm is stronger than the first one, thus, all the service regions have higher probabilities of facing high intensity levels. In the third scenario, the hypothetical storm becomes very strong, causing the greatest risk of intense storms and need for snow removal trucks. Although the intensity of the hypothetical storm changes in each scenario, the same storm pattern holds. That is, in any scenario, the northeastern regions 1 and 2 would have relatively higher probabilities of facing strong storms than others, while the southwestern regions 5, 6 and 7 would have relatively lower probabilities of facing them.

The sets of probabilities of having different levels of intensities in each service region for each scenario are shown in the following tables:

Table 5.5: Storm Probabilities in Each Region (Scenario 1)

	Storm Probability		
Service Region	**Level 3**	**Level 2**	**Level 1**
1	0.2	0.5	0.3
2	0.3	0.3	0.4
3	0.1	0.3	0.6
4	0.1	0.4	0.5
5	0.0	0.1	0.9
6	0.0	0.2	0.8
7	0.0	0.2	0.8

Table 5.6: Storm Probabilities in Each Region (Scenario 2)

	Storm Probability		
Service Region	**Level 3**	**Level 2**	**Level 1**
1	0.4	0.4	0.2
2	0.5	0.4	0.1
3	0.3	0.4	0.3
4	0.3	0.5	0.2
5	0.0	0.3	0.7
6	0.2	0.3	0.5
7	0.1	0.3	0.6

Table 5.7: Storm Probabilities in Each Region (Scenario 3)

	Storm Probability		
Service Region	**Level 3**	**Level 2**	**Level 1**
1	0.6	0.3	0.1
2	0.7	0.2	0.1
3	0.4	0.5	0.1
4	0.5	0.4	0.1
5	0.2	0.4	0.4
6	0.3	0.4	0.3
7	0.3	0.3	0.4

5.2.3 Service Resource

Since there is little difference in service speed and service duration between single-axle trucks and tandem-axle trucks, we consider only one type of trucks. Assume that the vehicle crew could work 8 hour shift a day. Then the service distance for one shift is 320 miles on Class A1 roadways, and 240 miles on Class A2 and A3 roadways.

Table 5.8 lists the number of trucks available in each region before reallocation.

Table 5.8: Original Number of Trucks

Service Region	Original Number of Trucks
1	21
2	18
3	30
4	19
5	13
6	14
7	17

To find the reallocation cost, we suppose the MPG of a snow removal truck is 10 miles per gallon, and the gas price is 2.4 dollars per gallon, then the fuel cost is 0.24 dollars per mile. In addition, operating cost, repair cost and depreciation cost are considered based on John Siebert's report "Truckers must not be flying by the seat of their pants", which is posted on Owner-Operator Independent Drivers Association Website, and the vehicle replacement cost analysis in Appendix D.

We estimate the total reallocation cost as the total cost of fuel, operating, repair and depreciation, which is (1.2 + 0.24) dollars per mile. We also assume that the reallocation inefficiency discount is 0.8, which means the reallocated trucks are only able to complete 80% of the regular workload in the new service region.

5.3 Case Results and Analysis

First, we coded the proposed model by Matlab to determine whether feasible solution exists in this problem. The result for each scenario is shown below:

Table 5.9: Number of Trucks (Scenario 1)

	Number of Trucks			
Service Region	**Original**	**Expected**	**Supply**	**Demand**
1	21	24	0	4
2	18	17	1	0
3	30	13	17	0
4	19	13	6	0
5	13	12	1	0
6	14	17	0	4
7	17	8	9	0
Total	132	104	34	8

The "Expected" column shows the total expected number of trucks needed to fully serve each region. The "Supply" column shows the quantity of extra trucks that can be reallocated in each region that has enough trucks to meet the total expected number. The "Demand" column gives the actual number of trucks needed to fully serve a region, in addition to what it has.

Table 5.10: Number of Trucks (Scenario 2)

	Number of Trucks			
Service Region	**Original**	**Expected**	**Supply**	**Demand**
1	21	29	0	10
2	18	22	0	5
3	30	18	12	0
4	19	18	1	0
5	13	13	0	0
6	14	22	0	10
7	17	9	8	0
Total	132	131	21	25

Table 5.11: Number of Trucks (Scenario 3)

	Number of Trucks			
Service Region	**Original**	**Expected**	**Supply**	**Demand**
1	21	34	0	17
2	18	25	0	9
3	30	20	10	0
4	19	21	0	3
5	13	19	0	8
6	14	25	0	14
7	17	12	5	0
Total	132	156	15	51

For scenario 1, the total number of trucks that could be reallocated was 34, which was much greater than the number of total demand, which was 8. Therefore, a number of feasible solutions would exist. Then the problem could be solved as a typical transportation model that we mentioned in the previous chapter, that is:

$$\min \sum_{i=2,3,4,5,7} \sum_{j=1,6} c_{ij} x_{ij}$$

$$\text{s.t. } x_{21} + x_{26} \leq 1 \tag{5.1}$$

$$x_{31} + x_{36} \leq 17 \tag{5.2}$$

$$x_{41} + x_{46} \leq 6 \tag{5.3}$$

$$x_{51} + x_{56} \leq 1 \tag{5.4}$$

$$x_{71} + x_{76} \leq 9 \tag{5.5}$$

$$x_{21} + x_{31} + x_{41} + x_{51} + x_{71} = 4 \tag{5.6}$$

$$x_{26} + x_{36} + x_{46} + x_{56} + x_{76} = 4 \tag{5.7}$$

$$x_{ij} \in [0, n_i], \quad x_{ij} \in Z.$$

On the contrary, the total number of trucks that could be reallocated was 21 in scenario 2, 15 in scenario 3, while the total demand was 25 in scenario 2 and 51 in scenario 3. There was not enough trucks to fully serve all the regions in either scenario 2 or scenario 3, hence, feasible solutions did not exist. The Fair Allocation policy needs to be employed to solve these two scenarios.

The next step was to find district i_0 and the fairness level. The program used to implement the method finding district i_0 and the fairness level could be found in Appendix A. The Matlab results for scenarios 2 and 3 are shown in Table 5.12 and Table 5.13 respectively. For scenario 2, service region 5 was district i_0, and the fairness level was -0.516. For scenario 3, service region 4 was district i_0, and the fairness level was -4.156.

The "Current" column shows the service level in each region before reallocation. The "Fairness" column shows the theoretical level that all the regions will reach after

Table 5.12: Level of Service (Scenario 2)

	Level of Service			
Service Region	**Current**	**Fairness**	**Change**	**Round**
1	-8		-9.355	(-10,-9)
2	-4		-4.355	(-5,-4)
3	12		12.516	(12,13)
4	1	-0.516	1.516	(1,2)
5	0		0.516	(0,1)
6	-8		-9.355	(-10,-9)
7	8		8.516	(8,9)

Table 5.13: Level of Service (Scenario 3)

	Level of Service			
Service Region	**Current**	**Fairness**	**Change**	**Round**
1	-13		-11.055	(-12,-11)
2	-7		-3.555	(-4,-3)
3	10		14.156	(14,15)
4	-2	-4.156	2.695	(2,3)
5	-6		-2.305	(-3,-2)
6	-11		-8.555	(-9,-8)
7	5		9.156	(9,10)

applying the Fair Allocation policy. The "Change" Column shows the number of trucks that need to be reallocated in each service region in order to reach the fairness level. The integer numbers in the "Round" Column were based on the numbers in the "Change" Column, according to the rounding policy proposed in Chapter 4. In "Round" columns, positive numbers mean the quantity of sending-out trucks, while negative numbers mean the quantity of moving-in trucks. Hence, these two problems became the following transportation problems.

For Scenario 2,

$$\min \sum_{i=3,4,5,7} \sum_{j=1,2,6} c_{ij} x_{ij}$$

$$\text{s.t. } 12 \leq x_{31} + x_{32} + x_{36} \leq 13 \tag{5.8}$$

$$1 \leq x_{41} + x_{42} + x_{46} \leq 2 \tag{5.9}$$

$$0 \leq x_{51} + x_{52} + x_{56} \leq 1 \tag{5.10}$$

$$8 \leq x_{71} + x_{72} + x_{76} \leq 9 \tag{5.11}$$

$$10 \geq x_{31} + x_{41} + x_{51} + x_{71} \geq 9 \tag{5.12}$$

$$5 \geq x_{32} + x_{42} + x_{52} + x_{72} \geq 4 \tag{5.13}$$

$$10 \geq x_{36} + x_{46} + x_{56} + x_{76} \geq 9 \tag{5.14}$$

$$x_{ij} \in [0, n_i], \quad x_{ij} \in Z.$$

For Scenario 3,

$$\min \sum_{i=3,4,7} \sum_{j=1,2,5,6} c_{ij} x_{ij}$$

$$\text{s.t. } 14 \leq x_{31} + x_{32} + x_{35} + x_{36} \leq 15 \tag{5.15}$$

$$2 \leq x_{41} + x_{42} + x_{45} + x_{46} \leq 3 \tag{5.16}$$

$$9 \leq x_{71} + x_{72} + x_{75} + x_{76} \leq 10 \tag{5.17}$$

$$12 \geq x_{31} + x_{41} + x_{71} \geq 11 \tag{5.18}$$

$$4 \geq x_{32} + x_{42} + x_{72} \geq 3 \tag{5.19}$$

$$3 \geq x_{35} + x_{45} + x_{75} \geq 2 \tag{5.20}$$

$$9 \geq x_{36} + x_{46} + x_{76} \geq 8 \tag{5.21}$$

$$x_{ij} \in [0, n_i], \quad x_{ij} \in Z.$$

The final step was to solve all the three transportation problems using the LINGO optimization software package. A Branch-and-Bound procedure was implemented by LINGO to determine the best transportation plan that results the minimized transportation cost. Table 5.14, 5.15 and 5.16 show the results, including the reallocation plan and optimal reallocation cost, while Figure 5.3, 5.4 and 5.5 show the directions of reallocation movements on the map for scenario 1, 2 and 3 respectively.

Table 5.14: Reallocation Plan (Scenario 1)

From	**To**	**Number of Trucks**	**Total Cost ($)**
7	6	4	
7	1	3	443.82
2	1	1	

Table 5.15: Reallocation Plan (Scenario 2)

From	**To**	**Number of Trucks**	**Total Cost ($)**
4	1	1	
7	1	5	
7	2	4	1509.07
3	6	9	
3	1	3	

Table 5.16: Reallocation Plan (Scenario 3)

From	**To**	**Number of Trucks**	**Total Cost ($)**
4	5	2	
7	1	5	
7	2	4	1750.30
3	1	6	
3	6	8	

For scenario 2 and 3, Table 5.17 and 5.18 show the comparison between the service level after reallocation operations and the fairness level. The "Level of Service"

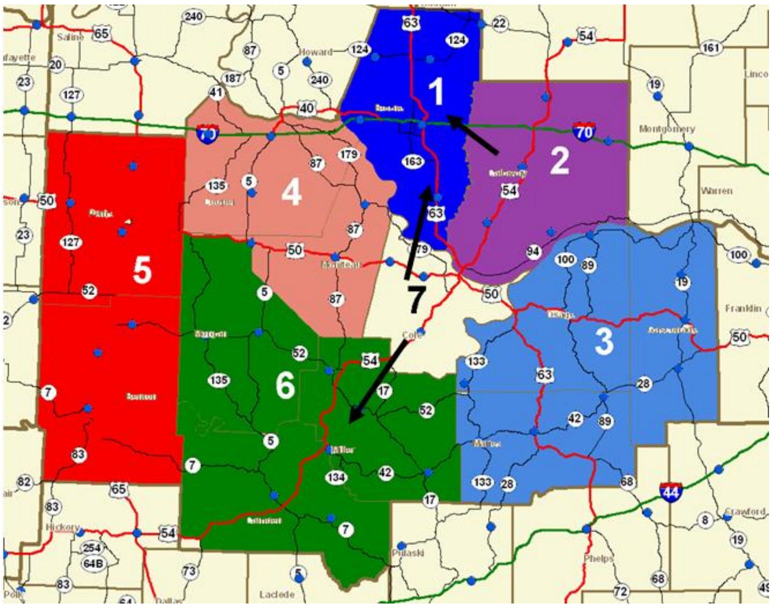

Figure 5.3: Reallocation (Scenario 1)

column describes the service level after reallocation, while the "Fairness Level" is the same theoretical value we mentioned before. The difference between the service level and the fairness level is less than one in most of the regions. However, region 3, containing the counties of Osage, Maries and Gasconade, has the largest difference for both scenarios. One reason is that the total number of supply is greater than the total number of demand after rounding, thus there would be some region with higher service level than the fairness level after reallocation. Another reason is that region 3 faces medium intensity, but owns too many trucks. As a result, region 3 is the region with the largest number of supply in both scenarios. The last reason is that region 1 is the regions with the greatest demand in both scenarios, but it is not close to region 3. Therefore, region 3 always has extra trucks after reallocation, which results in a relatively higher service compared with fairness level.

The results show that when the winter maintenance resource allocation model becomes a typical transportation model, reallocation operations are always chosen

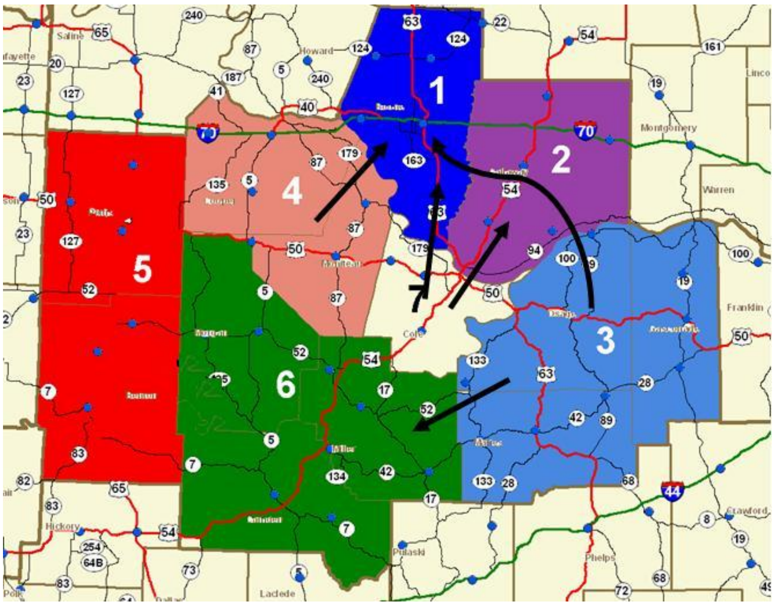

Figure 5.4: Reallocation (Scenario 2)

Table 5.17: Level of Service after Reallocation (Scenario 2)

Region Number	Counties	Level of Service	Fairness Level
1	Boone	-0.8	
2	Callaway	-0.8	
3	Osage, Maries, Gasconade	0	
4	Cooper, Moniteau	0	-0.4194
5	Benton, Pettis	0	
6	Morgan, Miller, Camden	-0.8	
7	Cole	-1	

between service regions that are close to each other in the optimal reallocation plan. There are two reasons: First, the objective function is trying to find the minimum reallocation operation cost, which means the reallocation operation with less cost is preferred; second, the reallocation cost is proportional to the distance between service regions, thus reallocation between regions that are close to each other is preferred. However, reallocation operations may happen between service regions that are far from each other. For example, in scenario 2, three trucks were reallocated from

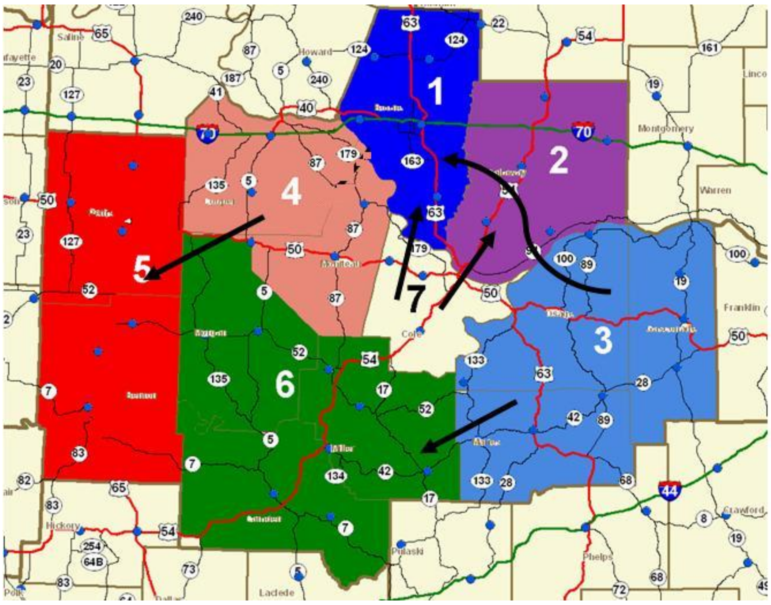

Figure 5.5: Reallocation (Scenario 3)

Table 5.18: Level of Service after Reallocation (Scenario 3)

Region Number	Counties	Level of Service	Fairness Level
1	Boone	-4.2	
2	Callaway	-3.8	
3	Osage, Maries, Gasconade	-4	
4	Cooper, Moniteau	-4	-4.154
5	Benton, Pettis	-4.4	
6	Morgan, Miller, Camden	-4.6	
7	Cole	-4	

region 3 to region 1, although region 7, 2 and 4 are closer to region 1 than region 3. But further study shows that region 7 and 4 had no capacity to send more trucks to region 1, and region 2 was the region that needed more trucks to reach the fairness level. Then the reallocation operation between region 3 and region 1 was reasonable, because region 3 was the closest to region 1 in all the regions that were able to send trucks to region 1 in this scenario.

Another feature of the Fair Allocation model can be found by the comparison between scenario 2 and 3. For scenario 2, service region 7, 3, 4 were the districts capable of sending trucks, and the others were in lack of more trucks to reach the fairness level except region 5. This partition of the service regions almost stayed the same for scenario 3. More importantly, a comparison between Table 5.15 and Table 5.16 shows the similarity of the reallocation plans for both scenarios: the origins and the destinations of the reallocation routs were nearly the same, only with a slight change in the number of trucks that were reallocated. The reason for this resemblance is the pattern of the hypothetical storms were fixed, that is relatively high probabilities of high intensity levels in the northeast, low probabilities of high intensity levels in the southwest, and medium in the rest, even though the hypothetical storm in scenario 3 was much more intense than that in scenario 2. This feature reveals the possibility of better preparation before storms. If the pattern of the storm is predictable, a rough reallocation operation can be laid out without knowing the actual intensity of the storm. Moreover, adjustment of the existing resources in each service region at the beginning of each fiscal year would be possible, with the help of the historical snowfall data in the area.

Chapter 6

Conclusions

In our research, we have attempted to model the resource allocation process in the winter road maintenance operation, which is not considered in most typical winter road maintenance models. For example, the routing and scheduling problems assume that the number of snow removal trucks in a depot is fixed; the sector design and depot location models consider a constant snow removal rate in each sector; the fleet sizing and replacement models try to determine the optimal number of snow removal trucks that balances the total cost and maintenance operation rate in each depot. Our model consists of two dynamic aspects within the winter road maintenance operation: the probabilistic nature of a snow storm and the cooperation between depots or sectors. The benefit of considering these two factors is clear. Since different districts face different probabilities of having a snow storm, the capacity of designated winter road maintenance resources in some may not be enough to fully serve their districts, while the capacity in others may exceed their needs. Therefore, reallocating the surplus resources to the districts whose capacity is insufficient will improve not only the level of service in those districts, but also the efficiency of resource utilization in the whole area. This resource allocation model can be extended to many fields other than winter road maintenance, as long as the problem is affected by various, but predictable, demands and relocatable resources.

Two solution approaches were presented to solve this resource allocation model, depending on whether all the regions can be fully served by existing resources. Both solutions are based on the lemma that there exists an optimal reallocation policy that does not allow a district to both send and receive trucks. This allows the division of the districts into two parts, which simplifies the modeling of the problem. When the total number of trucks that can be reallocated is greater than the total demand, feasible solutions exist for the original optimization problem (3.9). This reduced the integer model to a typical transportation problem. It could be solved by many LP methods, including a branch-and-bound approach, which is implemented by the LINGO program. When there are not enough trucks that can be reallocated to the demand districts, feasible solutions do not exist. This requires a new operation policy, hence, the Fair Allocation policy was introduced, and an iterative approach was proposed to find the fairness level. Then the districts could be divided into two groups according to that level, and the problem returns to a typical transportation problem. The key point in the Fair Allocation policy is that both reallocation cost and quality of service requirement are considered. A case study is conducted to illustrate the benefit of the proposed resource allocation model, which maintains a fair level of service in all the service regions with minimal cost.

One of the most important factors that needs to be included in the future resource allocation model is the time factor. Taking either the time spent in reallocating the trucks from one region to the other, or the time of completing the winter maintenance task in a region into consideration will improve the model and the solution approaches.

For instance, it is not reasonable to receive trucks from a region far away with a time constraint on the service completion time, even though that region has extra trucks which can be reallocated. That is, the time wasted during reallocation might be more valuable than the savings. Lemma 1 will not hold any more, since there could be a situation where a service region may first send trucks to fulfill the demand of a nearby region, and then receive trucks from a farther region to fully serve itself. Another extension of the time factor could be determining the optimal resource allocation plan not a for a single time duration, but for a number of successive time periods. In this case, the probabilities of storm intensity change from period to period, which makes the resource allocation model more complicated.

Appendix A

Matlab Program Code

```
clc;
clear;

m=7;                              %number of district
                                  %service distance [A1 A2 A3]
l1=[216.02 255.41 0];             %cole
l2=[308.8 117 500.7];             %callaway
l3=[124.4 201.6 1147.8];          %osage, maries, gasconade
l4=[265.87 0 848.66];             %cooper, moniteau
l5=[271.76 149.39 989.75];        %benton,pettis
l6=[314.11 374.19 863.3];         %morgan, miller, camden
l7=[566.64 125.52 337.35];        %boone

f=[6 2 1];                        %service frequency [A1 A2 A3]
s=[40*8 30*8 30*8];               %service speed in unit duration

alpha=[0.3 0.6 1.0];              %storm impact
beta=0.8                          %reallocation inefficiency discount
p=[0.4 0.4 0.2;
   0.3 0.4 0.3;
   0.5 0.4 0.1;
   0.2 0.1 0.7;
   0.1 0.3 0.6;
   0.3 0.3 0.4;
   0.0 0.1 0.9];                  %storm probability

l=[l1; l2; l3; l4; l5; l6; l7];   %road length matrix(n*3)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
N=zeros(1,m);
N0=[17  18  30  9  13  14  21]    %original number of trucks
for i=1:m
    N(i)=ceil((sum((l(i,:).*f)./s)*sum(p(i,:)./alpha)));
end;
N                                 %expected number of trucks

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
out=zeros(1,m);
in=zeros(1,m);
for i=1:m
    if N0(i)>=N(i)
        out(i)=floor(N0(i)-N(i));
    else if N0(i)<N(i)
            in(i)=ceil((N(i)-N0(i))/beta);
        end
    end
end
out                               %number of trucks that can be sent out
in                                %number of trucks that needs to be
moved in

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Figure A.1: Matlab Code 1

```
deltaN=N0-N                          %level of service (# of trucks lack)
deltaNs=sort(deltaN)                 %sorted level of service

Nkin=zeros(1,m);
Nkout=zeros(1,m);
for k=1:m
    for i=1:k
        Nkin(k)=Nkin(k)+(deltaNs(k)-deltaNs(i));
    end
    for j=k:m
        Nkout(k)=Nkout(k)+beta*(deltaNs(j)-deltaNs(k));
    end
end;
for i=1:m
    if Nkout(i)==Nkin(i)
        i0=i
    else if (Nkout(i)>=Nkin(i))&(Nkout(i+1)<=Nkin(i+1))
        i0=i+1, break
        end
    end
end;                                %find i0

delta1=0;
delta2=0;
for i=1:(i0-1)
    delta1=delta1+deltaNs(i);
end
for j=i0:m
    delta2=delta2+beta*deltaNs(j);
end

delta=(delta1+delta2)/(i0-1+beta*(m-i0+1))       %fairness level

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
trans=zeros(1,m);
for i=1:(i0-1)
    trans(i)=((delta-deltaNs(i))/beta);
end
for j=i0:m
    trans(j)=(deltaNs(j)-delta);
end
trans
```

```
in                                   %number of trucks that needs to be
moved in

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Figure A.2: Matlab Code 2

Appendix B

Service Regions and Depot Locations

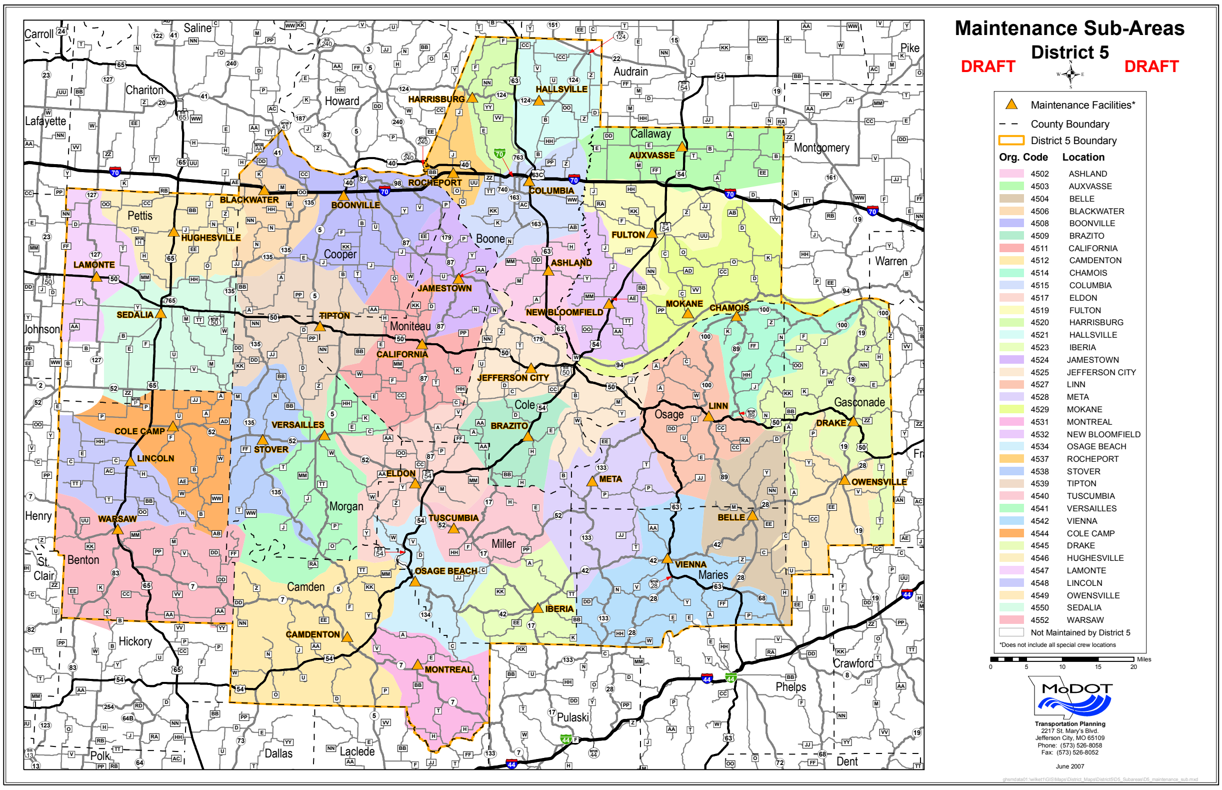

Figure B.1: Service Regions and Depot Locations

Appendix C

Snowfall Averages

Snowfall 1971-2000 Averages

Midwestern Regional Climate Center

County	Element	JAN	FEB	MAR	APR	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC	ANN
Cole	Snow(in)	5.6	3.6	1.9	0.2	0	0	0	0	0	0	1	2.7	15
Callaway	Snow(in)	7.2	5.8	3.4	0.6	0	0	0	0	0	0	1.6	4.7	23.3
Osage	Snow(in)	4.9	2.6	2.4	0	0	0	0	0	0	0	0.9	3.5	14.3
Maries	Snow(in)	5.7	3.6	2.6	0.3	0	0	0	0	0	0	1.7	3.2	17.1
Gasconade	Snow(in)	5.8	5.3	3.2	0.4	0	0	0	0	0	0	2.1	4.5	21.3
Cooper	Snow(in)	7.3	6	3	0.3	0	0	0	0	0	0	1.5	4.2	22.3
Moniteau	Snow(in)	4.2	3.9	1.6	0.2	0	0	0	0	0	0	1.1	2.4	13.4
Benton	Snow(in)	6.1	5.5	2	0.4	0	0	0	0	0	0	1.8	3.2	19
Pettis	Snow(in)	4.2	3.3	0.9	0.3	0	0	0	0	0	0	1.1	1.8	11.6
Morgan	Snow(in)	6.1	4.4	2.6	0.4	0	0	0	0	0	0	1.9	3.3	18.7
Miller	Snow(in)	4	3.2	1.2	0.1	0	0	0	0	0	0	0.7	2	11.2
Camden	Snow(in)	5.3	4.4	2.4	0.1	0	0	0	0	0	0	1.2	3.3	16.7
Boone	Snow(in)	7.3	5.2	3	0.5	0	0	0	0	0	0	1.8	4.4	22.2

Figure C.1: Snowfall Averages

Appendix D

Vehicle Replacement Cost Analysis

Vehicle Replacement Cost Benefit Analysis

For: CIP\Council Committee	MAKE: INTERNATIONAL
By: Jack Stucky	AGE: 1991 (15 YEARS)
Project: Snow Plow Deicer	CONDITION: FAIR TO POOR
Unit #: 125 Single Axle Dump Truck	USAGE: SNOW REMOVAL \ DEICER

Existing Vehicle or Equipment		New Vehicle or Equipment	
Unit 125 Single Axle Dump With Plow De-Icer Unit		**2005 or New Single Dump With Plow and Sand Spreader	
Total Miles	New Meter 85,000+	Total Mileage	<1,000
Total Hours (estimated)	12,000.00	Total Hours	New
Operating Costs Per Hour (year to date)	$14.58	Operating Costs Per Hour	$7.06
Repair Costs Per Hour (year to date)	$23.61	Repair Costs Per Hour	$13.68
Depreciation Costs Per Hour Unit is fully depreciated.	$0.00	*Depreciation Costs Per Hour	$8.45
Total Cost Per Hour	$38.19	Total Cost Per Hour\Hour	$29.19

Expected Cost Savings per Hour	-$9.00	Based on <> Total Cost Per Hour	
Estimated Annual Usage Rate	133	Based on last years usage rates	
Estimated Annual Expense Reduction	-$1,197.00		
Estimated 2nd Year Expense Reduction	-$1,244.53	Based on Life To Date Increase Trend	0.03971
Estimated 3rd Year Expense Reduction	-$1,283.26	Based on Life To Date Increase Trend	0.03112

*Based $87,500 cost less $3,000 residual over 10,000 hour life.

**Composite average (year to date) of operation and repair costs of 4 new units less set up costs.

Figure D.1: Vehicle Replacement Cost Analysis

Bibliography

[1] Marks, H. D., and R. Stricker. (1971). Routing for public service vehicles. *Journal of the Urban Planning and Development Division*, 97, 165-78.

[2] Eiselt HA, Gendreau M, Laporte G. (1995). Arc routing problems. Part I: the Chinese postman problem. *Operations Research*, 43, 231-42.

[3] Eiselt HA, Gendreau M, Laporte G. (1995) Arc routing problems. Part II: the rural postman problem. *Operations Research*, 43, 399-414.

[4] Fu, L., et al. (2009). Optimizing winter road maintenance operations under real-time information. *European Journal of Operations Research*, 332-341.

[5] Perrier, N., et al. (2008). The sector design and assignment problem for snow disposal operations. *European Journal of Operational Research*, 189, 508-525.

[6] Jones, P C, J L Zydiak. (1993). The fleet design problem. *The Engineering Economost*. 38, 83-98.

[7] Perrier, N. et al. (2006). A survey of models and algorithms for winter road maintenance. Part I: system design for spreading and plowing. *Computer and Operations Research*, 33, 209-238.

[8] Perrier, N. et al. (2006). A survey of models and algorithms for winter road maintenance. Part II: system design for snow disposal. *Computer and Operations Research*, 33, 239-262.

[9] Perrier, N. et al. (2006). A survey of models and algorithms for winter road maintenance. Part III: Vehicle routing and depot location for spreading. *Computer and Operations Research*, 34, 211-257.

[10] Perrier, N. et al. (2006). A survey of models and algorithms for winter road maintenance. Part IV: Vehicle routing and fleet sizing for plowing and snow disposal. *Computer and Operations Research*, 34, 258-294.

[11] Hayman RW, Howard CA. (1972) Maintenance station location through operations research at the Wyoming State Highway Department. *Highway Research Record*, 391, 17-30.

[12] Lotan T, et al. (1996) Winter gritting in the province of Antewerp: a combined location and routing problem. *Belgian Journal of Operations Research, Statistics and Computer Science*, 36,141-57.

[13] Zhang, Y. et al. (2006). An integrated systems approach to the development of winter maintenance/management systems. *Transportations Scholars Conference 2006*.

[14] Deck R, et al. (2001). Measuring efficiency of winter maintenance practices. *Transportation Research record*, 1741, 167-175.

[15] Danzig, G. B. (1951). Application of the simplex method on a transportation problem. *Activity Analysis of Production and Allocation*, Chap 23, Wiley, New York.

[16] Charnes, A. and W. W. Cooper. (1954). The stepping stone method of explaining linear programming calculations in transportation problems. Management Science, 1, 49-69.

[17] Arsham, H. and A. B. Kahn. (1989). A simplex-type algorithm for general transportation problems: an alternative to stepping-stone. *Journal of Operational Research Society*, 40, 581-590.

[18] Vignaux, G. A. and Z. Michalewicz. (1991). A genetic algorithm for linear transportation problem. *IEEE Transactions on Systems, Man, and Cybernetics*, 21, 445-452.

[19] Adlakaha, V. and K. Kowalski. (2003). A heuristic for solving small fixed-charge transportation problems. *Omega, International Journal of Management Science*, 27, 381-388.

[20] Fiedrich, F. et al. (2000). Optimized resource allocation for emergency response after earthquake disasters. *Safety Science*, 35, 41-57.

[21] Gong, Q. and R. Batta. (2007). Allocation and reallocation of ambulances to casualty clusters in a disaster relief operation. *IIE Transactions*, 39, 27-39.

[22] Zeitlin, Z. (1981). Minimization of maximum absolute deviation in integers. *Discrete Applied Mathematics*, 3, 203-220.

[23] Katoh, N. et al. (1985). An algorithm for the equipollent resource allocation problem. *Mathematics of Operations Research*, 10, 44-53.

[24] Tang, C. S. (1988). A max-min allocation problem: its solution and applications. *Operations Research*, 36, 359-367.

[25] Lee, W. J., et al. (1994). A branch and bound algorithm for solving separable convex integer programming problems. *Computer and Operations Research*, 21, 1011-1024.

[26] Luss, H. (1999). On equitable resource allocation problems: a lexicographic minimax approach. *Operations Research*, 47, 361-378.

[27] Karabati, S. et al. (2001). A min-max-sum resource allocation problem and its applications. *Operations Research*, 49, 913-922.

[28] Dahiya, K. and Verma, Vanita. (2007). Capacitated transportation problem with bounds on RIM conditions. *European Journal of Operational Research*, 178, 718-737.