COLOMBIAN TEACHERS' EXPECTATIONS OF POOR AND BLACK STUDENTS' ABILITY TO LEARN ALGEBRA

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by

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The undersigned, appointed by the Dean of the Graduate School, have examined the [thesis or dissertation entitled

COLOMBIAN TEACHERS' EXPECTATIONS OF POOR AND BLACK STUDENTS'

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A Rosa, mi madre y mejor maestra

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ABSTRACT

Underachievement in algebra along lines of race, class, and gender is a striking and pervasive phenomenon in the Colombian educational system. Black and poor students persistently lag behind their mestizo and wealthier peers. Explanations for the low mathematics performance of these student populations are usually grounded in essentialist perspectives that mainly attribute the students' failure to their racial, ethnic, and economic status. Adopting a sociopolitical perspective, the present study investigated the relationships between student social background, teacher expectations, and teaching practices. In particular, the study focuses on the relationships between the forms wherein student social backgrounds nurture and shape teacher expectations and influence the teaching practices implemented to help student build meaning of algebraic objects and procedures.

Using a comparative method and an interpretative approach, the expectations and teaching practices of three different mathematics teachers in three different social contexts were studied for four weeks. Their classes were filmed and the students wore sunglass cameras to record their interactions with their peers and teachers. The teachers were also interviewed and debriefed once a week during the observation time. The main findings point out a pronounced relationship between teachers' expectations and the students' backgrounds. The teachers hold cultural and class deficit views about black and poor students and position them as incapable of learning complex and meaningful algebraic knowledge. Such expectations translate into poor teaching practices, practices of exclusion and marginalization within the mathematics classrooms, and low quality interactions between teachers and students that impede the acquisition of importance competences needed to successful

participate in the democratic and political life within the Colombian society. Implications for research and teacher education are suggested.

CHAPTER 1. INTRODUCTION

Rationale

Algebra is acknowledged as one of the most important subjects in school (Bolea, 2003; Moses & Cobb, 2001). On the one hand, it is thought that algebra equips its users with powerful tools of thinking that other areas of mathematics such as arithmetic and geometry do not provide. On the other hand, algebra is considered the gateway for students to access higher education and the labor world; those who are successful in its learning are more likely to move upwards in society. However, several studies (Booth, 1984; Gallardo & Rojano, 1988; Tabach, Arcavi, & Hershkowitz, 2008; Van Ameron, 2003) have demonstrated the complexity in teaching and learning algebra and the high rates of student failure that result, a situation that largely affects non-white and low-socioeconomic status (SES) students (Kaput, 2008). The current situation demonstrates that rather than a gateway, algebra is a gatekeeper (Gonzålez, 2007), a selecting tool that grants some students and denies others access to post-secondary studies as well as to participate socially, economically, and politically in the world. In this regard, it seems that algebra acts as a *social filter* for certain groups of students in school and society.

Because the nation of Colombia is a society characterized by a sharp unequal distribution of wealth (Programa de las Naciones Unidas para el Desarrollo [PNUD], 2011) and with an educational system that seems to reinforce social inequalities (Viáfara & Urrea, 2006), understanding how algebra teaching practices might contribute to the marginalization of particular groups of students is a priority endeavor. An important goal of the Colombian educational system is to provide support and opportunities to learn algebra for *all* students

(Ministerio de Educación Nacional [MEN], 1998). However, that is not the current situation. As the students' outcomes in national and international large-scale assessment show [e.g., Saber test[1] and Third International Mathematics and Science Study (TIMSS)], the failure in school algebra affects mainly students of marginalized and poor communities (Instituto Colombiano para el Fomento de la Educación Superior [ICFES], 2009; ICFES 2010), which are largely inhabited by Afro Colombians[2] and displaced people, and in which the scarcity of both governmental services and resources are common. Although it appears that the intersection of race and class shapes Colombian students' opportunities to learn mathematics in general and algebra in particular, little is known about how teacher expectations and teaching practices contribute to this reality. Hence, studying the teaching practices that Colombian teachers use to teach algebra contributes to a better understanding of whether or not and how teaching practices influence the marginalization of certain groups of students. This study contributes to the field of teacher education and the improvement of mathematics classroom practices so that all Colombian students regardless of their race, class, ethnicity, and gender have enhanced mathematical learning opportunities.

Statement of Purpose

This study lies in the domain of mathematics teaching practices and their relations to issues of power in school. In particular, its focus falls under two issues: a) algebra teaching practices that marginalize or privilege certain ways of being (identity), doing and thinking (praxes) in school and, b) the intersection of race and class in determining and shaping algebra teaching practices. Historically, cognitive and sociocultural frameworks have

[1] The Saber 9 test is a large-scale assessment that is administered each year to ninth graders in Colombia.

[2] I use Blacks or Afro Colombians to refer to people with African heritage.

dominated research in the learning and teaching of algebra. Studies in these perspectives have contributed to understanding students' difficulties when learning algebraic concepts and procedures. In contrast, studies focused on teachers' beliefs about algebra and their teaching practices related to algebra have been limited (Doerr, 2004). Thus, using a sociopolitical perspective to mathematics education, the main purpose of this study is to analyze the mechanisms by which students' racial and social backgrounds shape teachers' expectations and practices that either hinder or promote students' opportunities to learn algebra.

Statement of the Problem

In spite of different curricular efforts to improve the teaching and learning of algebra in Colombia during the last 20 years, student failure has been persistent and enduring. The students' outcomes in algebra in the TIMSS (2007) reveal significant differences in performance based on gender (boys outperformed girls by 32 points), the public-private nature of schools (students attending private schools outperformed students in public schools by 58 points), and the schools' location (students in urban schools outperformed students in rural school by 45 points).

Confirming this trend in students' performance in algebra, the Saber 9 test provides evidence of students' difficulties in reaching the basic goals as set up in the school mathematics curriculum. In particular, the results of the Saber 9 test administered in 2012[3] show that in Cali, the city in which this study was conducted, ninth graders mainly perform in the below basic and basic levels[4].

[3] www.icfes.gov.co

[4] Appendix A describes these levels of performance as well as the criteria used by the ICFES to locate students and schools in a given SES.

Seventy-nine percent of students in Cali perform in these levels, while only 21% achieve the goals established in the satisfactory and proficient level. In fact, only 2% of students can accurately respond to items corresponding to the proficient level. The Saber 9 test results also indicate significant differences in ninth graders' performance based on SES, gender, the public-private nature of schools, and their location. Thus, for instance, 82% of girls perform in the below basic and basic levels compared with 68% of boys who do so. Only 18% of girls perform in the two highest levels while 27% of boys reach these two levels[5].

Although the majority of ninth graders perform in the basic level across different SES, the main differences occur in the extreme levels of performance: the below basic and proficient levels. The information displayed in Figure 1.1 shows that the highest percent of students performing below basic attend the more economically depressed schools (i.e., SES 1). In addition, students who attend SES 1 types of schools are barely found among those who reach the two highest levels of performance (i.e., satisfactory and proficient).

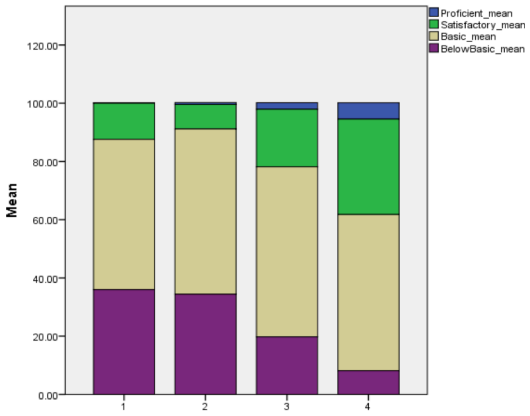

Figure1.1 Ninth graders' mathematics performance based on schools' SES in Cali

[5] Information retrieved from www.icges.gov.co

The urban area of Cali is organized in 22 *comunas* or groups of neighborhoods that usually share similar racial and economic compositions and that have relative autonomy to administer funding that come from the municipal government. The rural area of the city Cali comprises 15 small towns whose populations range between 500 and 10,000 people. When the Saber 9 test performance data is further disaggregated by *comunas*, the information reveals some interesting relationships between ninth graders' performance and the SES and race represented by schools. Figure 1.2 shows that the significant differences in students' performance occur again in the below basic and satisfactory levels. The 7th, 13th, 14th, 15th, 20th and 21st *comunas* exhibit the highest percentages of students who perform below basic. These *comunas* also have the lowest percentage of students with satisfactory levels and an almost nonexistent percentage of students at the proficient level. When compared to the rest of the *comunas,* these six evidence the more troublesome and alarming results.

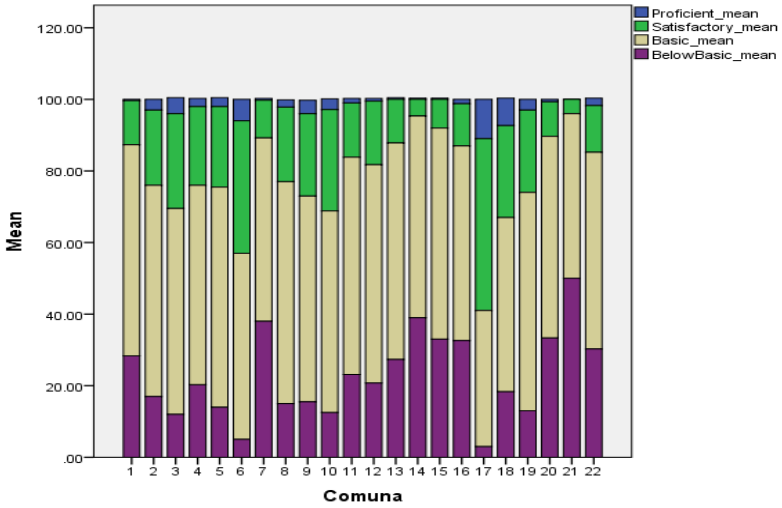

Figure 1.2 Ninth graders' performance in mathematics in the Saber 9 test 2012 by *comunas* in Cali

One interesting aspect of these results to take into account is how race and social class interact in these *comunas*. Although different studies (e.g., Alonso, Arcos, Solano, Llanos, & Gallego, 2007; Rodriguez-Garavito, Alfonso, & Cavelier, 2008; Urrea, Viáfara, Ramírez, & Botero, 2007) indicate different percentages of the black population inhabiting these *comunas*, they do coincide with the fact that Blacks in Cali are mainly concentrated in the 7th, 13th, 14th, 15th, and 21st *comunas* which, all but one, also evidence the highest levels of poverty and percentages of overcrowding houses (Viáfara & Urrea, 2006). The black population in these *comunas* comprises 65% of overall Blacks living in Cali (Viáfara & Urrea, 2006).

The 14th *comuna* for instance, has the highest concentration of the black population in Cali with 24.9 points higher than the total percentage in Cali. In addition, its population mainly belongs to the lowest SES characterized for its lack of economic resources, joblessness, high levels of violence, and lack of access to health services. While about 22% of the Cali population is classified in the SES 1, the percentage of population in the 14th *comuna* in this SES is about 3 times more than the overall percent in the city. The intersection of race and class is also apparent in the 15th *comuna* where about 50% of its population self-identifies as Black and 80% classifies in the lowest SES, as many as 4 more times the percentage of the city. This combined effect of race and class is notorious in the case of the 21st *comuna* where the percentages of the black population and low-SES individuals widely exceed the corresponding percentages in Cali.

Table 1.1

Percentages of Blacks by SES in six comunas in Cali

Comuna	Population 2013	Percentage of Blacks	SES 1	SES 2	SES 3	SES 4	SES 5	SES 6
Cali	**2.319.655**	**26.2%**	**21.6%**	**31.7%**	**30.6%**	**7.1%**	**7.1%**	**1.9%**
7	72.360	37.8%	7%	41%	52%	-	-	-
13	176.827	37.9%	26%	64%	10%	-	-	
14	169.091	51.1%	68.4%	31.6%	-	-	-	-
15	153.144	49.9%	41%	39%	20%	-	-	-
20	68.631	12.6%	83%	14%	3%	-	-	-
21	106.113	44.7%	80.7%	19.3%	-	-	-	-

In this trend, the 20th *comuna* differs in one aspect. Although in this *comuna* the percentage of black population is lower when compared with the overall percentage of the city, it is an economically depressed *comuna* in which 83% of its population belongs to the lowest SES level. Indeed, this is the *comuna* in Cali with the highest percentage of individuals living in the SES 1 level (Alonso et al., 2007). As may be noted in the map of Cali (see Figure 2), Blacks and low-income people inhabit the periphery of the city. The racial segregation in the city is evident and expresses itself in educational segregation that affects these populations.

In summary, three social factors converge to configure the students' performance in algebra in Cali. First, Blacks in Cali—the Colombian urban center with the highest concentration of Blacks—are overrepresented in the more economically depressed sectors of the population (Viáfara & Urrea, 2006). Second, the relationships between race and class in this case are not surprising. Rodriguez-Garavito, Alfonso, and Cavelier, (2008) show that Blacks in Colombia experience more levels of poverty and indigence when compared to the mestizo population; this is an indicator of a structural although subtle racism (Wade, 1993) against Blacks particularly installed and palpable in Cali. And third, the *comunas* with the

lowest levels of eighth and ninth graders' performance in mathematics evidence either a combined effect of poverty and highest presence of black population or a single effect of poverty. Thus, although it is not possible to draw conclusive results from this analysis, it is possible to identify a trend revealing that the failure in mathematics during the schooling period in which students are learning algebra in Cali is mainly affecting low-income and black pupils.

In spite of these disturbing results, they are neither challenged nor questioned by the Colombian government and society. It seems that Colombian society expects that its poorest students fail in algebra, and mathematics in general. Moreover, this failure is assumed as a natural occurrence, as is conveyed by ICFES (2010) in its analysis of the relationships between students' SES and their performance in mathematics and science in the TIMSS (2007). In this report, it is affirmed "*as was expected* [emphasis added], the students' scores in both areas [mathematics and science] are higher in schools attended by wealthier students" (p. 146, my translation). Although some studies (e.g., Banco Mundial [BM], 2008) have shown significant correlations between the students' social class and their learning outcomes in both elementary and secondary school in Colombia, little attention is paid to the reasons that make this situation persistent over time. We can ask *why is the failure of economically depressed students in mathematics expected? How has this situation become naturalized in the official discourses such as those expressed by the ICFES?*

One way to interpret the lack of questioning of the beliefs about low-SES students is related to what different researchers have called *essentialism* (Boaler, 2002a; Gutiérrez, 2002). Essentialism refers to the tendency of attributing students' learning outcomes to the

individual characteristics of people and social groups. Thus, for instance, girls' failure in mathematics is explained on the basis of their gender (they fail because they are girls) (Boaler, 2002a) and, as in the situation of poor students in Colombia, they fail because they are poor. This is a form of naturalization of discrimination and marginalization in society as well as a form of diverting the focus from the nature of the social praxes that might produce such marginalization to superficial and unquestionable factors that not controvert the established social order. However, from the sociopolitical stance assumed in this study, it is important to analyze the different forms in which issues of power in the macro structural level are expressed in micro, institutional contexts. In particular, it is critical to understand and interpret how social issues such as racism and poverty shape the social praxes of teaching mathematics and their effects not only in student learning but also in student identity as doers and learners of mathematics. Although the complexities of analyzing the interplay between race, class, and mathematics learning are well known, it is also widely recognized the racialized nature of the mathematics classrooms (Ladson-Billings & Tate, 1995; Lave, 1996; Martin, 2009; Spencer, 2006) as well as the effects of social class in students' mathematics learning (Baird, 2012). Thus, this is the context in which the research questions of the study were framed.

Research Questions

Regarding the relationships between students' racial and social backgrounds and teachers' expectations, the study addressed the following question:

How are Colombian teachers' expectations about eighth graders' ability to learn algebra related to the students' racial and social backgrounds?

Regarding the relationship between teacher expectations and practices, the study addressed the following question:

How are these teacher expectations expressed in their teaching practices?

A Sociopolitical Perspective to Mathematics Education

A sociopolitical perspective to mathematics education, as proposed in this study, is grounded in the premise that individuals are constituted in power relations as they occur within the social world (Popkewitz & Brennan, 1998). In particular, a sociopolitical perspective to mathematics education focuses on both the mechanisms through which marginalization of teachers and learners is constructed in the context of mathematics education practices and the ways in which power operates at school to produce such marginalization. Two assumptions underpin this theoretical perspective.

The first assumption relates to the role of schools in the unequal distribution of knowledge and dispositions along lines of race, class, gender, and ethnicity (Apple, 1990). The second assumption relates to the recognition that the practices associated to the teaching of mathematics are political (Gutiérrez, 2010; Valero, 2009). As a highly valued form of knowledge and rationality within the Western world (Bishop, 2008; Ernest, 1991), mathematics gives their users a value (Pais, 2010) that translates into social mobility. The social status of mathematics contributes to the fundamental role that the teaching practices of this subject play in the processes of exclusion in educational systems.

School and Power

If power is considered as "a capacity of some people—or groups of people—to keep others in their condition of excluded" (Valero, 2004, p. 10) from accessing certain social

goods, knowledge, and practices, it is possible to affirm that schools are not neutral institutions in relation to issues of marginalization and discrimination (Apple, 1990; Darder, 1991; Giroux, 1981). Indeed, schools are considered as one of the main social agencies that contribute to the reproduction of specific forms of consciousness, beliefs, values, and practices that frame and signify people's ways of being and acting. Giroux (1981) calls *hegemonic ideologies* these sets of representations of the social world.

Hegemonic Ideologies

Through hegemonic ideologies people assume daily and ordinary events as natural and taken-for-granted. Ideologies provide people with an intuitive awareness that enable them to anticipate the occurrence and causes of these events. As held by Williams (1981), ideologies constitute a sense of reality from which our interpretations of society and man become natural. A hegemonic ideology is not simply an imposition of a dominant class over marginalized groups but a saturating set of meanings and values that constitutes people consciousness (Williams, 1981). Hegemonic representations are not the result of individual choices. They are deeply embedded in our ways of acting and thinking and help us make sense of the social world.

In the context of the sociopolitical stance as assumed in this study, two aspects are considered in order to deepen into the forms in which ideologies emerge at school. The first one is called by Bonilla-Silva (2010) *frameworks.* Frameworks constitute the sources that nurture the representations teachers elaborate about particular group of students. They allow teachers to explain and justify, among other things, the school failure or success of students. The second aspect is *the discursive strategies* used to convey hegemonic representations.

Everyday conversations and institutional talks and texts are the key mechanisms in the reproduction of prejudices (Bonilla-Silva, 2010; van Dijk, 1992) and in their naturalization. Van Dijk (1992) argues that individuals usually display different communicative strategies to convey messages about the speaker's viewpoints and stances about how the world is or should be. Frameworks and discursive strategies are fundamental in understanding how power operates at school.

Ideological Reproduction at School

In educational systems, ideologies as embodied in rules, discourses, and procedures associated to schooling provide meaning to everyday school practices (Popkewitz, 1988). For instance, representations about academic failure and success become naturalized over time; they are part of official discourses that allow the school community to explain and justify student outcomes without questioning such discourses. Thus, the school failure of particular groups of students such as blacks, Latinos, or girls is foreseen and viewed as natural and not the result of school and social practices that reproduce marginalization and disadvantage. In this sense, the power of a hegemonic ideology relies, precisely, in the "successful attempt to establish its view of the world as all inclusive and universal" (Giroux, 1981, p. 23). Schools and other social institutions serve the purposes of maintaining a particular social order by turning ideologies in the form of particular consciousness, beliefs, attitudes, values, and practices into natural occurrences (Apple, 1990).

Apple (1990) emphasizes this role of schools in forging "a form of consciousness" (p. 3) that helps individuals realize what their places and roles are within the social world; such places and roles are usually aligned to the general social expectations based on students'

social backgrounds. Schools contribute to give meaning to the roles that individuals are expected to perform in society and, in this way, to maintain an already established social order without the use of force or coercion (Apple, 1990). Schools arise as one of the "main agencies of transmission of a dominant culture" (Williams, 1981; p. 6) that result in a differential positioning of individuals within the social world, legitimating rather than ameliorating the injustices of the larger society (Giroux, 1981). Oakes (2005) points out school practices as contributing to unequal educational opportunities and outcomes and in this sense, they constitute a "black box" in need of being revealed and understood. Even though the study of school practices is fundamental from a sociopolitical perspective, Anyon (2009) recalls us that the study of such practices is incomplete without exploring the broader social, political, and historical contexts in which they take place. Anyon argues that rather than isolating the school practices, "one needs to situate schools and districts, policies and procedures, institutional forms and processes in the larger social contexts in which they occur, in which they operate and are operated upon" (p. 3). Trying to understand the mathematics teaching practices that might contribute to the marginalization of poor and racial minority students without taking into account how the social constructed categories of race and class operate in the larger society is, in Anyon's view, a failed endeavor.

Institutional Practices and Individual Agency

Social practices, such as mathematics teaching, are political to the extent that "they are implicated in the functioning and distribution of power in social relations" (Valero, 2012; p. 376). It could be thought that the relationship between social practices, as they take place in social institutions, and the broader sociopolitical and economic contexts is unidirectional,

or in other words, that the macro-structures of society completely determine and shape social practices. Several researchers have contested such determinism (e.g., Apple, 1990). In fact, the relationship between the macro and institutional levels is dialectical (Williams, 1981). Social and cultural practices are not simply reflections of an already constituted social order but rather they contribute to its constitution, or in Giroux's words, "structures not only constitute the subject but are themselves shaped by human actions" (p. 15). Giroux notes that neglecting the dialectical relation between the macro and institutional levels would prevent us from explaining and understanding institutional transformations and consciousness formation.

Hence, social practices, as they take place in institutional settings, arise as critical elements in the comprehension of the functioning of mechanisms of exclusion and inclusion of different social actors. In the case of educational institutions, Darder (1991) emphasizes the importance of analyzing school practices and pedagogies in order to understand minority and poor students' performance and positioning at school. A focus on teaching practices, as implemented in a particular social and cultural context during a given historical moment, would enable researchers go beyond essentialist stances to school inequality. The analysis of teaching practices would also contribute to illuminate the ways in which their organization and implementation in schools lead to the unequal distribution of specific types of knowledge and dispositions. Although the study of teaching practices is fundamental to understand issues of power at school, this study would be incomplete without including the analysis of the relationships between practices and broader social, cultural, and political contexts.

It is worth noticing that recognizing the role of school practices in reinforcing social inequalities should not conceal the active role of teachers and students in contesting and resisting the imposition of a dominant culture. Instead of considering teachers and students as passive value receivers and cultural dupes (Giroux, 1981), a socio political stance recognizes their agency in reacting against situations of exclusion and injustice that may end up in taking actions to transform certain realities of schools. Popkewitz and Brennan (1998) underline the relation between resistance and power by arguing that, "resistance…is a methodological approach to continually making problematic the stories we are given and those we tell" (p. 27). Let us consider, for example, students' responses to hostile learning environments at school. Teachers' practices might prevent particular groups of students from engaging in learning activities. The classroom environment created by teachers can also undermine students' self esteem and their participation in the learning process (Turner et al., 2002). As a reaction, students might respond by misbehaving in the classroom, withdrawing from the learning process, and even vandalizing schools (Anyon, 1995). These behaviors are seen as students' responses to a series of practices that overtly position them as incapable of learning and doing well at school. In contrast, teachers might also create inclusive and caring learning environments that promote the active participation and positive positioning of minority students (Pinnow & Chval, 2014). In this sense, teachers and students are not only subjects of the school but also active agents that participate either in exclusion and avoidance practices or in processes of contestation and reconstruction; in this sense, their actions shape the dynamics inside schools.

Mathematics Education Practices

Regarding the previous discussion, and in order to understand how issues of power operate in the context of the school mathematics, it is important to focus on the mathematics education practices at different social levels (Valero, 2009). Usually, research in the field has centered on the students-teacher-mathematics triad leaving unexplored the influence of broader contexts that seem to shape teaching and learning practices. In this sense, mathematics education practices are defined as those "that contribute giving meaning to the activity of people when thinking, learning, and teaching mathematics, as well as when engaging in situations where mathematical elements are present" (Valero, 2009, p. LXVIII). These practices are not only restricted to the school as an institution. They comprise the complex network of social practices carried out by different actors interested in instituting certain meanings on the teaching and learning of mathematics within society. For example, the social practices of textbook production, teacher education, and international and national assessment design and implementation are some of the contexts that, along with the labor market dynamics, shape mathematics teaching and learning although not always in coherent and harmonious ways.

In addition to the focus on mathematics educational practices, a sociopolitical perspective points to the critical role of mathematics in the school curriculum. As a highly valued form of knowledge in society, the inclusion of mathematics in the school curriculum is not exempt of interests but rather is part of a *selective tradition* (Williams, 1981), a process wherein certain knowledge and practices are designated to be transmitted whereas others are neglected. The selective tradition enables us to understand how a mathematical rationality–

developed by the ancient Greeks–was imposed in the Western world while others–developed by Egyptians, Indians, and Africans among others–were ignored (Powell, 2002). At a more local level, the selective tradition also allows us to analyze how the mathematics knowledge of certain cultural groups have been systematically ignored at school while privileging the representation of particular groups. Understanding these processes and the valuation of mathematics is critical to comprehend their role in the exclusion of some students at school.

Value and Knowledge in Mathematics

In broadly accepted characterizations of mathematics, "abstract is valued over concrete, formal over informal, objective over subjective, justification over discovery, rationality over intuition, reason over emotion, general over particular, theory over practice, the work of the brain over the work of the hand, and so on" (Ernest, 1991, p. 259). Bishop (2008) also suggests rationalism, objectivity, control, progress, openness and mystery as the main values associated with mathematics in the Western world. Mathematics is considered as a model of rationality in our societies (Popkewitz, 2002).

In educational systems, mathematics functions as a critical filter to advanced mathematics at school as well as to the most well-paid jobs in society which are usually in mathematics-related fields (e.g., engineering and business) (Ernest, 1991). In terms of Bourdieu's theory, mathematics might be seen as part of the cultural capital that guarantees school success to dominant groups in society; this success is materialized in the form of academic qualifications (Bourdieu, 2011) such as diplomas and certificates that, in turn, "makes possible to establish conversion rates between cultural capital and economic capital

by guaranteeing the monetary value of a given academic capital" (Bourdieu, 2011, p. 88). This is one form in which the role of mathematics as a tool of power can be understood.

Discussing the mechanisms and logics that underlie this role of mathematics, Pais (2010) asserts that, beyond empowering people in different forms (Skovsmose & Valero, 2008), mathematics mainly "gives people a value" (p. 139). He suggests that:

> [T]he reasons why they need it [mathematics] are not related with mathematics knowledge of competences, *but with the school valorization that mathematics gives to people* [emphasis added]. People need school mathematics not because they will use it directly in the democratic participation (as knowledge or competence), but to continue having success in school, to take a university course and to find a stable job, so that they become workers and citizens. I argue that the importance of mathematics must be discussed not just in the field of knowledge but also in the field of value. (p. 140)

We have then, two sides of the same coin. On the one hand, mathematics is a highly valued knowledge in society and this appreciation translates into institutional practices of both recognition and marginalization. On the other hand, and because of the result of these institutional practices, mathematics valorizes people, a status that positively positions them in society and translates into social and economic advantages (D'Ambrosio, 2005). Mathematics is used as an instrument of social selection and marginalization.

Algebra Teaching Practices

In this study, I focus on the social practice of teaching algebra. In part, this is because algebra is highly valued in the field of mathematics education. Algebra is commonly

associated with generality, abstractness, analytic ways of solving problems, and formalism, among other characteristics (Agudelo-Valderrama, 2004; Bell, 1996; Lins & Kaput, 2004; Gascón-Pérez, 1993; Mason, 1996; Radford, 1996; Ursini, 2001). It is also thought that algebra is necessary for *all* individuals to understand a world that becomes every day more technologized. In this regard, Moses and Cobb (2001) depict this critical role of algebra in individuals' lives when affirming that "now [algebra] is the gatekeeper for citizenship; and people who don't have it are like the people who couldn't read and write in the industrial age" (p. 14). In terms of individuals' democratic participation, algebra is considered a required tool to ensure the "active participation in democratic societies" (Agudelo-Valderrama, 2004, p. 1). To the extent that algebra is fundamental not only for the development of science but also for citizenship, its role in the school curriculum and the forms through which access to algebraic knowledge is denied or allowed are critical features to understand issues of power at school.

In summary, the importance of considering sociopolitical approaches rather than theoretical frameworks traditionally focused on psychology and cognition is justified by the fact that the learning of mathematics takes place in classrooms, and these are essentially social, cultural, and political spaces (Pais & Valero, 2012). The study of the practices that disempower students in the mathematics classroom and condition their access to democratic forms of participation in society is one of the main endeavors from a sociopolitical perspective. This is the theoretical stance assumed in the present study that is intended to disentangle the practices that are used to teach algebra to marginalized students.

CHAPTER 2. Literature Review

Researchers in the field have broadly recognized considerable differences in achievement, the quality of mathematics instruction, and mathematics learning among students from different racial, ethnic and social backgrounds (Boaler, Altendorff, & Kent, 2011; Martin, 2006, 2009; Reyes & Stanic, 1988; Secada, 1995; Zevenbergen, 2003). It is well known that performance in mathematics is strongly tied to both in-school factors and an interplay of social and cultural forces that frame students' experiences and their mathematics identities in schools.

Despite this recognition, it is also accepted that understanding the interactions between the students' learning and their backgrounds is a complex endeavor that faces multiple challenges and obstacles. Some researchers argue that the social practices of teaching and learning cannot be effectively addressed without considering issues of identity, agency, and power (Valero, 2012) and in this regard, traditional psychological and sociocultural frameworks have proved limited in the attempts to account for the complexity of such practices (Lewic, Enciso, & Moje, 2007). A lack of robust theories and methodological tools exist for researchers to conduct studies that shed light on these issues. The marginalization of discourses about power and identity in mathematics education (Parks & Schmeichel, 2012), as well as the persistence of using race and class simply as demographic variables in research (Ladson-Billings & Tate, 1995) constitute additional obstacles for researchers interested in these problems.

In the present review, I focus on research that has explored the interplay of race, class, and mathematics learning highlighting the relationships between mathematics teachers' expectations and their practices in their links to students' racial and social backgrounds.

Race and Social Class in Mathematics Education Research

One of the main difficulties in conducting research related to issues of identity, agency, and power in the teaching and learning of mathematics is defining and characterizing the theoretical constructs of race and class. From different perspectives, researchers incorporate definitions of both constructs with significant implications for the research process, findings, and implications. Below, I discuss some of the ways in which class and race have been approached in mathematics education research.

Social Class in Mathematics Education Research

There is not agreement among researchers about the meaning of social class and its main differences with SES (Apple & Whitty, 2002; Archer, 2003; Bourne, 2002; Lubienski, 2000; 2002; McLaren & Farahmandpur, 2002; Reyes & Stanic, 1988). Archer (2003) argues that the stance assumed by researchers in relation to social class has important effects in both the research process and the conclusions achieved. In her analysis of the impact of social class in college students' performance, Archer identifies four different definitions of social class grounded on functionalist approaches, Marxists and post-Marxists theories, and poststructuralist perspectives (p.8). Despite such variety, Archer stresses the critical role of social class in student learning. Class has real, concrete effects particularly for low-income students to the extent that "institutional, social, and economic factors and inequalities interact

in complex ways with multiple identities to render particular educational routes 'unthinkable' for diverse young working-class people" (p. 19).

The field of mathematics education has not been immune to the difficulties involved in the study of the links between class and learning. Calling for more studies that inform researchers, practitioners, and policymakers about the relationships between class and mathematics learning, Reyes and Stanic (1988) underscore the difficulties in this endeavor. When highlighting the struggles in distinguishing social class and SES, Reyes and Stanic selected the latter because they did not feel yet "prepared to deal with all the consequences of using the term *social class*" (p. 28) in their research. Lubienski (2000) also opts for SES as an indicator of class to the extent that it "can be thought as an approximation for one's social class, which connotes more permanence, shared group values, and beliefs about roles in society and relationships to power" (p. 381). However, critics of identifying class and SES argue that the different variables used to measure SES such as parents' schooling, jobs' classifications, and so on, can be problematic because of their arbitrariness, relativism, and circumstantial character (Bolaer, Altendorff, & Kent 2011). In addition, social class is a more complex construct that "carries with in an overlay of shared group values concerning a range of social issues, differentiated roles in our society, notions of oppression, and struggles among members of differential social classes for power" (Secada, 1992, p. 626), and therefore, SES does not capture the intricate nature of class.

Race in Mathematics Education Research

Researchers recognize the role of race as a sociopolitical marker (Parks & Schmeichel, 2012) that has concrete and real consequences in peoples' lives. According to

Bonilla-Silva (2010), race is not an abstract concept but a "socially constructed category that has a social reality" (p. 9). As a result of being racialized as Blacks or Whites, individuals experiment social, political and economic effects largely represented either in marginalization or in advantages. These processes of racialization are grounded on both *racial structures* understood as the sum of social relations and practices that sustain white privilege (Bonilla-Silva & Glover, 2004), and *racial ideologies* that nourish discourses and practices about race classification and discrimination (Bonilla-Silva, 2010; Twine, 2006; Wade, 1993). In this sense, race is institutionally embodied and permeates individuals' social relations through largely subtle institutional practices and discourses.

Coinciding with this perspective, Ladson-Billings and Tate (1995) argue that race is a social category helpful in the comprehension not only of social inequality at large, but also of educational inequality in particular. Although research has consistently showed educational inequalities based on race, this construct remains undertheorized and the mechanisms that perpetuate such inequalities in school are unexplored. In other words, the theoretical importance of race in the explanation of educational inequities has not typically been salient in the research community.

Mathematics education researchers have criticized the sole use of race as a statistical variable helpful to describe differences in achievement (Martin, 2006; 2009; Spencer, 2006). Beyond statistical approaches, the construct of race needs to be considered in the historical and political contexts in which racial relations are constructed. Race is a contingent concept (Parks & Schmeichel, 2012) opened to "contestation, resistance, and agency" (Martin, 2006, p. 201), and therefore, understanding how institutions such as schools shape individuals' live

experiences based on their race in particular historical moments is an important step in the goal of achieving equality in society.

The disregard of race in the analysis of inequalities in mathematics learning is grounded on narrowed views of mathematics and mathematics education as culture-free and ahistorical processes (Martin, Gholson, & Leonard, 2010) that occur in a vacuum. Martin, Gholson, and Leonard contend such views and highlight the role of the students' social realities and the forces that affect such realities in the form taken by their mathematics learning and participation in school. For these researchers, overlooking the sociopolitical forces that determine and condition the students' processes of schooling is the main reason of the failure of the mainstream research community in its efforts in improving the mathematics learning of underachieving students who largely come from historically marginalized groups. In this sense, "understanding how and why children interact with mathematics content in the ways that they do as well as how and why they learn is not a question of mathematics content alone" (Martin, Gholson, & Leonard, 2010, p. 17) but a matter of deeply rooted and complex sociopolitical factors in which race and issues of racism seem to be relevant.

In spite of these multiple approaches to social class and race and the lack of consensus about their definitions, researchers do agree in the existence of a still murky relationship between the students' racial and class backgrounds and their experience in learning mathematics at school. Qualitative and quantitative studies consistently show that poor and racial minority students are disproportionally overrepresented in the lowest levels of performance in mathematics and barely pursue mathematics-related careers. However, how class and race influence, determine, and shape the experiences of students as doers and

learners of mathematics remains a challenging and pervasive issue in the field of mathematics education. When analyzing this issue, researchers have focused either on the differential performance in mathematics from the perspective of the achievement gap or on the macro social factors that shape the micro level of the mathematics classroom.

Race, Class, and the Achievement Gap in Mathematics

Despite the improvements in the quality of curriculum, instruction, and assessment due to research efforts in the field, the well-known achievement gap in mathematics is widespread and studies that address differences in mathematics achievement in large-scale tests systematically find that the learners' social backgrounds anticipate their performance in mathematics (Boaler, Altendorff, & Kent, 2011; Reyes & Stanic, 1988).

Several analyses of students' test scores in studies such as National Assessment of Educational Progress (NAEP), the National Education Longitudinal Study (NAELS), and the Education Longitudinal Study (ELS), have revealed the persistent disparities in mathematics achievement along race/ethnicity (Lee, 2002; Lubienski, 2002; Ogbu, 1988; Riegle-Crumb & Grodsky, 2010; Tate, 1997) and class (Baird, 2012; Frankenstein, 1995; Lim, 2008; Payne & Biddle, 1999). A critical examination of the so-called racial and ethnic achievement gap over the past 30 years in the U.S. evidences that although there were some improvements in scores, the knowledge gains among Blacks and Latino/as were mainly at the basic level skills but Whites surpassed them at the advanced level skills (Lee, 2002). Tate (1997) also found that white students outperformed African American and Hispanic students "on a more standards-based examination, and on college entrance and AP examination" (p. 662). Although the racial and ethnic gap in mathematics achievement seems to be narrowing, it is a

persistent phenomenon, and over time, the patterns of performance in mathematics of different student population are changing. For instance, and even though all group of students have made progress in different degrees, Asian students have notably improved their scores in mathematics outperforming white students in particular, and the other racial groups in general. Black students are still the racial group that shows the lowest level of performance according to the results of the NAEP from 1973 to 2008; and Latino/a students have started outperforming black students in different large-scale tests (e.g., PISA). These transformations express the economic, social, cultural, and racial changes that our societies experience (Lee, 2002; Tate 1997) and, as a result, the comprehension of the phenomena related to the dynamics of the racial and ethnic achievement gap is more challenging.

Regarding social class, low-income students consistently lag behind their middle- and upper middle class peers in mathematics performance (Baird, 2012; Riegle-Crumb & Grodsky, 2010). Highlighting the high level of child poverty in the U.S. and challenging the results of quantitative studies that downplayed the effects of poverty and school funding in student mathematics achievement, Payne and Biddle (1999) found that these two factors *did* impact pupils' performance. They used the U.S. eighth graders scores in mathematics in the Second International Mathematics Study (SIMS) and the School District Data Book (SDDB) to study the relationships between poverty and mathematics achievement in a sample of 205 classes in 32 states. Payne and Biddle found that "school achievement in America *is* tied significantly to differences in school funding and child poverty at the district level [italics in the original]" (p. 10) and that "the net effects of child poverty are substantial and largely independent of those of race" (p. 11). In other words, when curriculum, race, school funding,

and child poverty are used as independent variables to predict mathematics performance, the two latest have substantial and statistically significant effects in eighth graders' mathematics achievement.

In her qualitative analysis of mathematics performance differences in the TIMSS 2003 between low-income and high-income student differences in wealthy countries, Baird (2012) found that a significant percentage of the class achievement gap "can be attributed to characteristics of low SES students rather than to differences in school resources" (p. 506). This is characteristic of countries with larger achievement gaps such as Taiwan, the U.S., Korea, and Belgium, whereas in other countries school resource differences matter. Hence, the relationship among class, school characteristics, and mathematics achievement is not consistent across countries and it is not possible to draw compelling conclusions about how social class and mathematics achievement interact. However, the individual characteristics of students (e.g., motivation, interest, and effort) that disadvantage them in their schools are, according to Baird, disproportionately found in low-income pupils, and their failure in mathematics "can be traced to a school system that succeeds in providing equal opportunity for students as long as students bring certain characteristics with them; and students without these characteristics are disproportionately found among low SES students" (p. 506). Although the risk with this conclusion is that of blaming students for lacking the needed characteristics to success in school mathematics, Baird's emphasis in the organization of school as a factor that might prevent low-income student success in mathematics is worth noticing and I return to it later. In short, although research consistently points out that differences in mathematics achievement are tied to the students' racial and social

backgrounds, little is known about how race and class shape students' performance and how they interact with other factors to shape mathematics achievement.

Rather than considering race and class as isolated factors, some researchers have analyzed how they combine to influence achievement (Diamond, Randolph, & Spinalle, 2004). The main reason of this relies on the striking fact that racial minorities are overrepresented in the lowest SES (Milner, 2013), and therefore, the achievement gap seems to express a compound effect of race and class in students' performance in mathematics.

The Intersection of Race and Class in Mathematics Achievement

The conclusions of studies that seek for the combined effects of race and class in mathematics achievement reveal discrepancies regarding the weight of each one of these factors in the differential achievement in mathematics.

Lubienski (2002) found that if SES and race are taken together to analyze U.S. students' outcomes across the 1990, 1996, and 2000 NEAP results in mathematics, SES fails in explaining the racial achievement gap and that "the *lowest* SES white students consistently scored equal to or higher (often significantly so) than the *highest* SES black students across the three grades [4^{th}, 8^{th}, and 12^{th}] in both 1990 and 1996 [italics in the original]" (p. 276). Other studies also evidence that middle-class white students outperform their middle class African American counterparts, and therefore, race rather than class seems to be the determinant factor in the students' outcomes (Ogbu, 1988). In contrast, Frankenstein (1995), Lim (2008), and Hoadley (2007) argue that class differences in society structure and might determine the students' schooling experiences, and therefore, "[the students'] socioeconomic background is the major predictor of educational attainment" (Frankenstein, 1995, p. 167).

Social class is the most critical factor determining student membership in racial and class groups (Hoadley, 2007) and therefore its effects are more significant in mathematics achievement.

In a different perspective, and drawing on a meta-analysis of quantitative studies that address the achievement gap in the U.S., Darling-Hammond (2004) did not find conclusive arguments to assign a determinant role to student social backgrounds in student achievement but to school resources. Rather than racial or social differences producing disparities in achievement, educational outcomes of minority students are a "function of unequal access to key educational resources, including skilled teachers and quality curriculum" (p. 213). Darling-Hammond recognizes that the structure of schools in the U.S. allocates unequal learning opportunities to students based on their social backgrounds; however, she does not address the reasons that persistently sustain and naturalize the uneven distribution of high quality instruction to students based on race and class. How this happens remains a "black box."

In summary, it is still unclear for researcher *why* and *how* poor, black students fail in mathematics. The persistence of this phenomenon unveils an unequal distribution in opportunities to learn mathematics among students based on their race and class (Boaler, Altendorff, & Kent, 2011; Borman & Overman, 2004; Hoadley, 2007; Lim, 2008; Lubienski, 2000; Oakes, 2005; Nasir & Hand, 2006; Viáfara & Urrea, 2006) that requires looking beyond the statistical analysis of the patterns of the achievement gap. Race and class are sociopolitical markers (Parks & Schmeichel, 2012) that might structure and organize the students' mathematical experiences in school in a process that seems to mirror what happens

within the macro-level of society with racial minorities and poor populations. However, more research is needed to disentangle the mechanism throughout which the educational system unequally distributes opportunities to learn mathematics to students.

Race, Class, and Opportunities to Learn Mathematics in School

The search for the sources of differential opportunities to learn mathematics has comprised the study of micro- and macro-level of factors and forces that might either marginalize or empower the students in the educational system. On the one hand, the micro-level of schools and communities in which the learning and teaching of mathematics takes place is seen as the main source of both impediments and assistance to student learning. In this perspective, instruction, curriculum, and teachers-as part of the school organization and resources-are the main focus of interest. On the other hand, the macro-level of societal structures and institutions with their discourses and practices about ability and success are considered as the sources of marginalization and discrimination for the students in the school. Recently, researchers have begun to examine the interactions between these two levels of analysis, as I discuss in the following sections.

The Micro-Level forces Influencing Minority Students' Mathematics Learning

Research on the characteristics of high-achieving minority students in mathematics shows that the quality of curriculum, teacher expectations, and high-quality instruction aligned to the students' cultural experiences are important factors in explaining these students' success (Berry, Thunder, & Mcclain, 2011; Boaler, 1999; 2002b; Borman & Overman, 2004; Howard, 2001; Walker, 2006). Culturally relevant pedagogies (Ladson-Billings & Tate, 1995) in the form of teaching practices consistent with the language used by

students in their homes and communities, their norms and values, and that attend to their moral, academic, and social competencies have positive effects on student learning of mathematics (Howard, 2001). In this sense, this research underscores school resources and practices as the factors that make the difference between success and failure in the mathematics learning of marginalized students. Teaching that engages students, connects them with the outside world, encourages the development of positive mathematics identities, and promotes the development of high-level thinking, is not only characterized as "good teaching" but also "equitable teaching" (Boaler, Altendorff, & Kent, 2011, p; 480) to the extent that favors *all* students' learning.

The family and community environments have also been underscored as an important factor in the academic accomplishment of historically marginalized students (Walker, 2006; Berry, 2008). Walker (2006) found that supportive peers, communities, and families that share high expectations for Latino/as and black students positively influence the students' desire to success in mathematics. Home environments where students find positive role models, parents' support, and strong academic identities may also play an important role in the success of marginalized students (Berry, 2008).

The Macro-Levels Forces Influencing Minority Students' Mathematics Learning

The risk with these types of approaches rests on attributing the responsibility of school mathematics failure to students and their families, favoring cultural deprivation perspectives. Failure is explained in terms of students' laziness, lack of effort, absence of positive role models and family support, and undervaluation of education. And although the studies above shed light on important in- and out-school factors that might influence the

marginalized students' learning of mathematics, the striking reality is that minority and low-SES students are more likely to be exposed to local, everyday knowledge rather than specialized forms of mathematics knowledge (Hoadley, 2007; Oakes, 2005); to traditional forms of instruction that focus on memorization of facts, the acquisition and mastering of routine procedures, and the development of low-order skills (Lubienski, 2000); and to less challenging content (Lee, Smith, & Croninger, 1997).

Such differences are neither due to individuals' characteristics nor to the willingness of the school administrators and teachers, but must be contextualized in broader societal struggles that frame the students' school experiences. Zevenbergen (2003) argues that the differential teaching practices and school organizations that marginalize students are a reflection of common forms of racial and social discrimination that minority groups face in society. School reproduces social inequalities and privileges certain values, norms, and ideologies that usually correspond with middle-class based cultural values and dispositions (Lim, 2008; Rist, 1970; Zevenbergen & Niesche, 2000) that poor students usually lack. In this sense, the teaching and learning of mathematics in school not only involve communicating and acquiring content, but also participating in a social world that "contains standards of reason, rules of practice and conceptions of knowledge. The social patterns of school conduct are not neutral but related to the larger social and cultural differentiation that exist in our societies" (Popkewitz, 1988; p. 221). This perspective reveals that there exist different types of schooling for different students and that those who do not possess particular cultural and social characteristics of the dominant groups in society are more likely to be marginalized in and eventually expulsed from the educational system.

Thus, in order to understand these different types of schooling, it is not enough to look for individual characteristics of students and schools, but to analyze how discrimination by race and class are installed and expressed in the mathematics classroom.

Beyond Learning Mathematics: Racism and Classism in the Mathematics Classroom

The realization that the failure in school mathematics consistently affect racial minority and low-income students led some researchers to consider that the teaching and learning of mathematics transcend the traditional relationships of students and teachers around content (Martin, Gholson, & Leonard, 2010). In school, besides mathematics, the students also learn about race and class, how to treat "the others," and their position in the social world, or, in other words, "they learn to *be* [italics in the original]" (Boaler & Greeno, 2000, p. 188).

Among the practices that configure teacher-student interactions, tracking is highlighted as an engine of reproducing social inequalities in schools. Oakes (2005) points out that tracking frequently places white and middle-class students in the high track mathematics courses whereas marginal groups of students are overrepresented in low-track courses. In her study of twenty-five junior and senior high schools, Oakes found that students in low-track mathematics and English courses were more likely to be exposed to less time allocated for instruction and lower teachers' expectations. The development of critical thinking, reasoning, and creativity were some goals set up by teachers for high-track students in mathematics, whereas filling out insurance forms and income tax returns, and the learning of practical mathematics skills for everyday living were some of the goals established for

low-track students. In this sense, these schools seemed to prepare high-track students for leadership and autonomy whereas low-track students learned compliance and obedience (Nasir & Hand, 2006). The students not only experience mathematics learning in markedly different ways but also build a social identity aligned with their own places in school.

Racialization is an additional process that students usually experience in schools and mathematics classrooms. Black and Latino/a students are frequently positioned as incapable of doing well in mathematics and academically inferior (Ladson-Billings & Tate, 1995; Martin, 2006; 2009; Spencer, 2006) even before coming into the classroom. Martin (2009) argues that learning mathematics as many other social practices is a racialized endeavor in which black students learn that their failure in mathematics is not the result of poor instructional practices and learning environments, but the consequence of their lack of discipline and effort. As held by Spencer (2006), low scores and performance in mathematics serve as indicators that black students are "unmotivated or incapable, parents are uninvolved and unconcerned and that the community does not value education" (p. 239). In this sense, the students learn about the meaning of being black when learning mathematics leading them to build negative mathematics identities. In his research about mathematics identities, Martin (2006) explored the school experiences of middle-class black adults–who had children at school in the moment of the study–as learners of mathematics. He found that despite the effort and discipline they demonstrated in their mathematics classrooms, they were consistently denied access to advanced mathematics courses. Adults in the school were completely convinced that "forces within the educational systems worked against African American children" (p. 219) and that race does matter to success in school mathematics.

Spencer (2006) also points out the framing role of race in the students' school experiences and asserts that there exists "overarching structures" (p. 272) at schools that contribute to create negative stereotypes of Blacks as doers and learners of mathematics and that block their access to high-quality mathematics instruction.

Low-income students also seem both to experience the learning of mathematics in different ways than their middle- and upper middle-class counterparts and build negative mathematics identities. The dominant cultural and social context of schools that seem to be aligned with middle-class values and dispositions are seen as excluding spaces for poor and working-class students whose behaviors do not respond to such values and dispositions. Lim (2008) asserts that schools in our societies usually disregard and undervalue the culture and communication styles of working-class people, portraying them as inferior and worthless. In her comparative study of two high achieving though racially and socially different girls in mathematics and the way in which there were positioned by their teacher, Lim found that the teacher's expectations about these students were strongly related to their racial and class backgrounds. Although the two girls were academically successful in mathematics, their experiences in the classroom were significantly different to the extent that they were mediated by the teacher's beliefs that usually expressed and favored middle class values about the social world. The ways of talking and preferred styles of learning of the black, working class girl were "rarely respected in the classroom space" (p. 92) and considered by the teacher as obstacles to her future achievement in school. Thus, regardless the students' performance, other critical factors did affect the learners' mathematics experiences in school, and Lim (2008) stresses social class as the critical factor. Her findings are consistent with

other studies that underscore the role of students' social class in their learning of mathematics (Rist, 1970; Frankenstein, 1995; Diamond, Randolph, & Spillane, 2004).

Although my review, and for analytic reasons, presents race and class separately in their shaping of student mathematics learning and identity, it is worth noticing that in reality, they intersect and produce a compounded effect in the students' opportunities to learn mathematics. Low-income and racial minority students face, in this way, a double jeopardy (Lim, 2008) in school and little is known about how the intersection of race and class shape the pupils' mathematics learning.

Among the in-school elements framing student mathematics learning and experiences, and as the above analysis reveals, teachers' expectations are underscored as critical factors in the perpetuation of unequal access of marginalized students to high-quality mathematics instruction as well as in the creation of learning environments that welcome and support diversity. Teachers' expectations of students seem to be powerful filters that can either prevent or encourage their learning of mathematics. This is particularly true in the case of low income and black students to the extent that an important number of culturally and socially grounded beliefs about how these students learn are expressed in the expectations that teachers usually hold, as I discuss in the following section.

Mathematics Teacher Expectations and Practices

The relationships between teachers' beliefs and practices have been the focus of increasing interest among researchers in the field. The main assumption underlying this interest is the recognition of teacher agency and the teacher's role as an active decision maker, whose instructional practices significantly frame students' learning and outcomes.

In the 1980's, cognitive approaches dominated attempts to describe the interplay between the teachers' conceptions about mathematics, its teaching, and student learning (Ernest, 1988; Thompson, 1984). Process-product approaches (Good & Grouws, 1979; Khoeler & Grouws, 1992) that mainly considered a straightforward relationship between teachers' actions and students' learning were prevalent. Researchers looked for evidence of teachers' professed beliefs in their observable behaviors when teaching mathematics and started an important way of approaching and studying teachers' practices (Ernest, 1988; Nathan & Koedinger, 2000). An important underlying assumption of these approaches was that "the practice of teaching mathematics depends on a number of key elements, most notably the teacher's mental contents or schemas, particularly the system of beliefs concerning mathematics and its teaching and learning" (Ernest, 1988, p. 249). Over time, researchers developed and refined different models to interpret and describe the interactions between teachers' beliefs and practices (Guskey, 2002; Nathan & Knuth, 2003); there is agreement about an essential connection between teachers' systems of beliefs and their instructional decisions; however, how teachers' beliefs and practices interact to shape students' learning remains an under-researched area.

From a sociopolitical stance, a fundamental criticism is made about this way of studying teachers' beliefs and practices. Beliefs and practices do not exist in a vacuum but are grounded and framed in broader cultural, political, and social contexts, and therefore, they are contingent and express the ways of doing and thinking in specific historical moments. Teachers' beliefs are not only related to the mathematics content and its teaching

and learning, but they also embody and enact comprehensions of the social world, individuals, and social practices (Agudelo-Valderrama, 2004).

Nevertheless, researchers use indistinctly the words beliefs and expectations and they generally are left undefined in research (Philipp, 2007), the latter is usually linked to the set of anticipations that teachers make about students' learning and performance based on different types of indicators such as previous achievement, race, class, and gender. Research shows that teacher expectations are important predictors of student learning and outcomes, particularly for marginalized students.

Teacher Expectations of Marginalized Student Learning of Mathematics

When teachers maintain high expectations for marginalized students' achievement and learning, their instruction is more oriented towards their students' success through the establishment of meaningful interchanges in the mathematics classroom (Archambault, Janosz, & Chouinard, 2012). However, several studies that explore teacher expectations of marginalized students have shown that they hold deficit views about black and poor students and consider these students as culturally inferior and incapable of success in school (Brantlinger, 2003; Battery, 2013; Bucley, 2010; Martin, 2009; Rist, 1970; Diamond, Randolph, & Spillane, 2004; Dunne & Gazeley, 2008; Gillborn, Rollock, Vicent, & Ball, 2012; Rumberger & Palardy, 2005; Zevenbergen, 2003). In her study of mathematics teachers' beliefs and expectations about economically disadvantaged students, Zevenbergen (2003) found that teachers view these students as lacking particular attributes or dispositions required for success in the learning of mathematics. In general, the participating teachers suggested that, "students from socially disadvantaged backgrounds had poverty in their

experiences that influenced their capacity to undertake and/or understand the concepts that were being developed in mathematics" (p. 139). Thus, the student background is perceived as a powerful influence in her capacity to learn; moreover, poverty and racial and cultural minority status are synonymous of lower ability for learning mathematics. In this sense, the entire responsibility of the failure in mathematics learning is transferred from the school practices to the student characteristics.

Teacher expectations frame both the interactions among teachers and students as they take place in the mathematics classroom and the instructional practices. Secada (1992) argues that teachers establish "differential patterns of interactions with their students that vary based on student demographic characteristics as well on their expectations of student success" (p. 644). The findings of the Rist's (1970) longitudinal study of preschool and elementary black teachers' expectations of black students in urban schools support this assumption. Rist found that the learners' status as high or low ability student determined his or her location in the classroom. High ability students–mainly from middle class backgrounds–were located closer to the blackboard and the teachers' desks provided them a better view of explanations and facilitated communicational interchanges, whereas low-ability students–poor and darker skin color students–were located in the rear of the classroom and barely communicated with teachers. In this sense, the teacher created in her classroom a geography of inclusion and exclusion that ended up shaping the students' learning and identities. Interestingly, the teachers in this study did not use academic criteria for grouping the students but the physical appearance of the children, their language (standard American English), and family background. The attributes and behaviors valued by the middle class people became the basis

for the evaluation of the children and then, "those who possessed these particular characteristics were expected to succeed while those who did not could be expected not to succeed" (Rist, 1970, p. 276). Battery (2013) also found that teachers expressed low expectations and deficit views of poor and black students in a variety of ways. For example, they used sarcasm and unamiable facial and corporal gestures, withheld instruction, treated students as invisibles, and ignored their contributions because of language issues. Battery observes that these are common gestures and ways of interactions between elementary teachers and marginalized students in the mathematics classroom.

Researchers have highlighted the critical role played by the opportunities the students have to participate in class as well as the characteristics of such participation in their learning. Muijs and Reynolds (2003) approach the notion of classroom climate as the arrangement of interactions, dispositions, and physical environment that influence student achievement. The authors shed light on the role played by the ways in which teachers interact with the students, their acts of positioning the students in the classroom, the organization of instruction, and student behavior management. These elements constitute critical components of the process of understanding the mechanisms wherein knowledge and dispositions are unequally distributed at school. Turner et al. (2002) draw upon a discursive perspective to approach the notion of classroom climate as the teachers' discourses and settlement of learning goals and expectations. According to these authors, the classroom climate play a critical role in the development of the students' mathematics identities as well as in the structuration of trustful relationships that allow the students to safely participate in the joint construction of mathematics knowledge. When classroom climate undermines students' self-

esteem and restrict their participation in the learning process, practices of avoidance, withdrawing, and resistance might take place during instruction lessening the students' chances of meaningfully engaging the learning community.

Research on classroom climate shows that low-income and minority students are more likely to be engaged in less motivating, less demanding, and less supportive practices of participation in mathematics classroom (Darling-Hammond, 2004). Usually, their participation is limited to Initiation-Reply-Evaluation models of interactions (Mehan, 1978) that reduce their chances to engage in meaningful process of construction of mathematics knowledge. Straehler-Pohl, Fernández, Gellert, and Figueiras (2014) found systematic differences in the ways in which teachers engaged students in meaningful participations across different social contexts. Students in upper, middle, and low class were exposed to different types of communicative interactions that in the case of the wealthier students fostered their learning of mathematics whereas for the poorest students emerged as obstacle in their learning. According to the authors, the reproduction of social inequalities is mediated and manifested in classroom communication.

Deficit views of students influence the ways wherein teachers construct a classroom environment to support the mathematics learning. In their study, Atweh, Bleicher and Cooper (1998) found that differences in classroom environment were consistent with the differences in teachers' perceptions of the students. Each participating teacher conducted the classes in a manner that was consisted with the perceptions of the students' abilities and needs. In this sense, poor and minority students engaged in less demanding forms of participation.

Teachers' expectations also seem to translate into particular teaching practices that end up disadvantaging low-income and black students. Research has pointed out that these student populations are more likely to be exposed to low-quality mathematics instruction that focuses on the development of lower-order skills, drill and practice, recitation, and basic knowledge (Lee, Smith & Croninger, 1997). According to Boaler and Greeno (2000), these narrowed teaching practices produce learning environments in which "most students must surrender agency and thought in order to follow predetermined routines" (p. 171) and in this sense, they not only impact student learning but also identity. Anyon (1980) found striking differences in teaching practices among teachers in wealthy and poor schools. In these elementary schools, low-income students were exposed to teaching practices that focus on showing disconnected and unexplained mathematics procedures; presenting mathematics vocabulary out of context; and assessing correct/incorrect answers and procedures. Zucker (1995) also found that traditional practices of teaching that favor memorization of facts, procedural skill and drills are more likely to be implemented in high-poverty mathematics classrooms. In addition, the teachers of these students used fewer manipulatives and curriculum materials, although they were always available for teachers to use as they desired. Other studies confirm that black students are less likely to receive the type of mathematics instructions that the National Council of Teachers of Mathematics (NCTM) advocates (McKinney, Chappel, Berry, & Hickman, 2009; Buckley, 2010) and to be involved in solving challenging and rich mathematics problems because their teachers think this type of knowledge is "what these [black] kids need" (Diamond, Randolph, & Spillane, 2004). Thus, these teaching practices that coincide with what Haberman (1991) called "the pedagogy of

poverty," disadvantaging marginalized students in terms of learning and achievement in mathematics. Although different studies consistently show differences in expectations and learning opportunities that low-income and racial minority students experience in the mathematics classrooms, a gap exists in the literature concerning how teachers elaborate expectations about their students, the sources that nurture such expectations, and how they are negotiated with broader institutional ideologies about ability and learning in mathematics.

In addition to these gaps in the literature, it is worth noting that most of the studies in this review approach the mathematics content in a general way. Mathematics is, sometimes, just the backdrop of the classroom and we are left without information about whether and how the content might contribute to frame teacher expectations and practices. For instance, *are teachers' expectations about students' learning of arithmetic different from their expectations about students' learning of algebra? If so, how are such differences expressed in their practices?*

In the case of algebra, although some researchers have explored teachers' beliefs of algebra, its teaching, and learning (Agudelo, 2008; Chazan, Yerushalmy, & Likin, 2008; McCrory, Floden, Ferrini-Munday, Reckase, & Senk, 2012); the ways in which teachers interpret students' algebraic work (Nathan & Koedinger, 2000; Tirosh, Even, & Robison, 1998); and instruction that seems to enhance the algebra learning of marginalized students (Boscardin et. al, 2005; Carpenter, Franke, & Levi, 2003; Chazan, 2000; Moses & Cobb, 2001; Stein, Kaufman, Sherman, & Hillen, 2011), research has barely examined the links among expectations, practices, and opportunities to learn algebra in the case of low income and black students.

The few studies in this perspective have explored the teachers' knowledge for teaching algebra to diverse students (Brown, Davis, & Kulm; 2011) and the relationships between teachers' beliefs about low SES students' ability in algebra courses and their final grades (Schullo & Alperson, 1998). These studies have focused on measuring beliefs with predesigned surveys and tests (e.g. Mathematics Teaching Efficacy Beliefs Instrument [MTEBI] and the Knowledge for Algebra Teaching for Equity [KATE] test) rather than on what occurs in the mathematics classroom, and then, little is known about the interplay among teachers' beliefs, practices, and low-income and racial minority students' learning of algebra. It is important to ask, for instance, *how do teachers' beliefs about low-income and black students' abilities to learn algebra influence their teaching practices? What are the characteristics of the practices that teachers use to teach algebra to low-income and black students? Are they different from the teaching practices used in other socioeconomic and racial contexts? How do teachers interact with low-income and black students in their algebra classes? What type of identities do low-income and black students build during their learning of algebra?* In order to effectively address these questions, it is important to take a close look into the dynamics of the mathematics classroom.

Summary

Research has consistently shown that teachers' expectations are usually grounded in student racial and class backgrounds rather than in academic achievement. Teachers' low expectations translate in poor instructional practices and differential patterns of interactions that reinforce and mirror social inequities in school and prevent poor and black students to success in their learning of mathematics. In this sense, teacher expectation and practices

creates literal and symbolic spaces of exclusion and inclusion in the mathematics classrooms to the extent that some students have access to high quality opportunities to learn mathematics, whereas other are prevented to do so. The studies presented show that the origin and the nature of teacher expectations and their interplay with practices and other gestures are far from being easily understood. The understanding of how teachers construct ideas, beliefs, and expectations about ability and learning in mathematics, and how race and class feed such beliefs requires additional research efforts. Thus, for instance, it is important to ask *to what extent do mathematics teachers' practices systematically exclude particular student populations in school? How do mathematics teachers build academic expectations about students? To what extent are such expectations linked to students' social backgrounds? How do teacher, administrators, and staff justify and explain the failure of historically marginalized students in mathematics? How do broader cultural assumptions about marginalized students permeate teachers' expectations? What are the characteristics of the teaching practices that exclude students and limit their chances of learning mathematics? How do the interactions between teachers and students differ based on students' background? How do such interactions affect mathematics learning?*

Researchers need to address these issues if our societies have a genuine interest and commitment in reaching the goal of equity in mathematics education.

CHAPTER 3. Methodology

The object of interest in this study lies in the domain of mathematics teaching practices in their relations to issues of identity, agency, and power. In other words, the study investigated how racism and poverty express themselves within the mathematics classroom and shape both students' identities and opportunities to learn. In particular, the study focused on (a) teachers' expectations about low income and racial and ethnic minority students' ability to learn algebra, and (b) teachers' practices in relation to such expectations. This research involved the study of the participating teachers' classroom dynamics and the comprehension of the complexity of teacher-students interactions as they took place during instruction.

According to Nickson (1992), educational research at large–and mathematics education in particular–has considered important transformations due to, among other reasons, an increasing interest in social, political, and interpersonal aspects of the classroom. This growing interest has required researchers to search alternative epistemological perspectives and theoretical frameworks usually borrowed from anthropology, sociology, linguistic, and political sciences to approach these issues. Critical theory, Marxism, poststructuralism, and feminism are now less foreign terms in educational theory and their concepts and developments more familiar to researchers. This tendency has also brought to education research new methodological tools and ways to explain educational phenomena. However, this process is not exempt of challenges and questions.

From a sociopolitical stance, Valero (2004) asks for the features that make a piece of research "sociopolitical" (p.14) in mathematics education. She contests traditional ways of

conducting research in mathematics education and calls for "constructing alternative discourses about the research process itself" (Valero, 2004, p. 14). In this regard, Valero offers some clues of what research from a socio political perspective would look like. First, the visibility of researchers is an important starting point. Knijnik (2004, as cited by Valero, 2004) states that making the researcher "real" "reveals the subjectivity of the researcher, her/his political stance, and the ways of interpreting the world; all of which imprint the topics and the methodologies that the researcher chooses within the research process" (p. 15). The importance of disclosing the researchers' stances has to do with the belief that all knowledge production is, in nature, political and therefore, the positions that the researchers occupy in the social world shape the phenomena they observe, the methods they use to approach them, and the interpretations they construct about such phenomena. Second, although the political nature of the social practices that take place in the mathematics classroom is recognized as critical from sociopolitical perspectives, the analysis of such praxes cannot be made in disconnection with the broader cultural, social and historical context in which they occur (Anyon, 2009). Thus, understanding and interpreting the social practices in their sociopolitical contexts is an important goal from a sociopolitical stance.

Therefore, I begin discussing my position as a researcher and present the rationale for using qualitative methods in this study. I describe the sampling process, the participants and the data analysis process.

Researcher Positioning

My personal interest in issues of race, class and mathematics education relates to my own experiences as a black, low-income student trying to succeed in the Colombian

educational system. Exploring teachers' beliefs about race and poverty in their relations to mathematics learning, I face my own story as a student, who many times saw her teachers expressing surprise at finding a "successful" poor, black student in the classroom. After years of working as a mathematics teacher in elementary and secondary schools located in marginalized neighborhoods, and as a teacher educator in pre and in-service education programs in local universities in my hometown in Colombia, I recognize the significant influence that the students' racial and class backgrounds have on teachers' beliefs and ways of constructing narratives about their practices and student learning. These experiences have highlighted that not only teachers' ideas about the subject matter influence their practices, but also their interpretations of who students are and other social factors seem to shape their mathematics instruction.

An additional experience marked my research interest in issues of power and mathematics education. My work as a graduate research assistant on a National Science Foundation funded project that investigated minority children's mathematics instruction in U.S. classroom, provided first-hand experiences of the powerful influences that ideological representations about minority students might exert in the practices implemented by mainstream teachers during mathematics instruction. From my involvement in the project, I learned that teachers not only teach mathematics but also convey meanings about the students' places in the social world by the positioning acts that they effectuate. I focused my academic interest in topics related to the forms in which school practices contribute to marginalize students and to forge particular identities for already marginalized learners.

Thus, my interpretations of different phenomena in this study might be influenced by such experiences. I am aware of the need to approach these issues in a country, as Colombia, in which the failure of racial minority and poor students seems to be expected and therefore, it is an unquestionable phenomenon. I reveal the conditions in which marginalized students, as I was, are currently taught and to foster a community of scholars interested in issues of equity and social justice in Colombia through my research. I hope that, through my research, I help give voice to students who endure hardships in the Colombian educational system based on their race and class.

Rationale for Using Qualitative Research

Naturalistic Experiments

Qualitative research is considered useful for studies aimed to gain detailed and deeper understandings of individuals' interaction, participation, and collective processes of making meaning. This type of research allows researchers to carefully observe and interpret the phenomena of interest through the observation of the natural settings in which human interactions and practices occur. It also enables researchers to investigate the participants' perspectives and capture the complexity of the social phenomena under study. In this regard, the strength of qualitative research relies precisely on the close understanding of both social practices and people's stances through their detailed and focused observation (Bernard, 2006).

The interest in deeper understandings of the individuals' interactions in natural settings leads researchers to get genuinely involved in the process of data collection. As a matter of fact, in the context of naturalistic research, researchers are "the main data gathering

instrument" (Hatch, 2002, p. 7) and make the data happen (Bernard, 2006). The researcher constructs the data in an interactive process that involves engaging closely with the participants, recording their stances and interactions, and questioning their narratives. Hence, qualitative research is pertinent and helpful when:

> We need a complex, detailed understanding of the issue. This detail can only be established by talking directly with people, going to their homes or places of work, and allowing them to tell the stories unencumbered by what we expect to find or what we have read in the literature (Creswell, 2007, p. 40).

To the extent that the purpose of this study sought for making sense of specific phenomena as they occur in mathematics classrooms, a qualitative approach was suitable to conduct the present investigation. The study of the teachers' expectations and practices when teaching algebra as well as the interactions between and among teachers and students requires a close observation of the classroom and school environments. In this process of understanding, it is important to give voice to historically marginalized students allowing them to elaborate and enact their narratives about personal stories, their work, their teaching, and in general, their experiences as participants in the social practices of teaching and learning mathematics. It is also important to deeply observe the school and community environments in order to make sense of the type of factors influencing the instructional decisions that teachers make. In order to do so, it is necessary to collect data directly in the site where teaching takes place: the participating teachers' schools and classrooms. It also means to take into account the institutional conditions of teaching and learning to understand how such conditions shape teachers' practices.

Comparative Research

The present study used a comparative approach. In general terms, comparative research refers to qualitative and quantitative studies that compare social entities (Mills, van de Bunt, & Bruijn, 2006) to find the forms wherein an issue is determined, influenced, and shaped by different social and cultural settings. Although comparative research has mainly been used to examine a given process in different cultural contexts, in what has been called *cross-cultural research*, it is also a powerful method for inquiring phenomena across categories and social groups (Mills, van de Bunt, & Bruijn, 2006). Accordingly, the underlying goal of comparative research is to search for similarity and variance (Ragin, 2006) in order to construct and advance theory in the social sciences. In fact, Bernard (2006) precisely emphasizes the power of comparative research in theory production. This type of research, in Bernard's stance, enables the researchers to go beyond merely describing the social phenomena by capturing their shared and divergent attributes. Considering that the present study sought for analyzing a social practice–teaching algebra–as taking place in different social, ethnic and racial contexts, comparative research was a suitable method. In the following paragraphs, I describe the research design.

Research Design

Schools' Sampling Process

In order to make sense of differences in teachers' expectations and practices in relation to student class, ethnic, and racial backgrounds, I selected three schools based on the following criteria:

a. The school principals and teachers' willingness to participate in the study.
b. Schools located in three different socioeconomic neighborhoods within the city of Cali. In particular, I sought one school situated in a low-income neighborhood, one in a working-class neighborhood, and the last one in a middle-class neighborhood.
c. The nature of the schools. I sought a public low-income and a working class school. I also sought a middle-class private school. The inclusion of this criterion was not willful but responded to the characteristics of the Colombian educational system and the demographic composition of Cali. Public schools serve poor and working class students whereas private schools serve middle and upper middle-class students. So, in order to observe the teachers' expectations and practices about middle class students' abilities in algebra, I selected a private school.
d. The racial composition of schools. I particularly sought the presence of black students among the schools' student population, specifically in the eighth grade.

Based on the specificities of the study that included in-depth observation of a cultural practice carried out in a natural setting, I used a nonprobabilistic sample method (Bernard, 2006) to ensure that each school fulfilled the racial and class composition the study required. Nonprobabilistic sampling is more suitable for studies that approach cultural and social practices requiring expert informants rather than randomly selected respondents (Creswell,

2007). In particular, I used a purposive or judgment sampling (Bernard, 2006) that allowed me to select the schools based on the proposed criteria. As Bernard argues, purposive sampling methods are useful in situations in which it is intended to deeply describe and characterize a cultural phenomenon, as it is the case of this study.

To select the schools, I contacted the principals by email and met with them to explain the study. I also visited the schools when the principals showed interest in the study in order to get a sense of the schools' organization and the racial composition of the student population.

In order to select the school located in a low-income neighborhood, I first emailed the principals of the seven public schools located in the 14th, 15th, and 21st *comunas.* I was particularly interested in these *comunas* because of the combined effect of race and class that characterizes their populations including higher percentages of black and poor people (see Table 1.1 in chapter 1). Two principals from different *comunas* replied showing interest in the study. I visited the schools, explained to the principals the research purposes and scope, and invited them to join the study. I did not find exceptional differences among the schools regarding their organization, weekly time allocated to the teaching of algebra, and so forth. I chose the school with the highest presence of black students taking into account that the population in these *comunas* included the same levels of extreme poverty.

To select the second public school, I emailed five principals whose schools were located in working class neighborhoods in three different *comunas*. Two principals replied showing interest in the study. I met the principals and visited the schools to obtain information about their racial composition. Few black students attended both schools, so I

selected the school with the highest number of black students to increase the likelihood that there would be black students at the eighth grade level.

Getting access to the private schools was a more complicated process. Following the same process that I used with the public schools, I emailed the principals of four well known and prestigious schools in Cali attended by middle and upper-middle class students. I had the opportunity to visit one of the schools that replied and talked with the mathematics coordinator. Although he and the vice principal seemed interested, they never confirmed their intention to participate in the study. I met with the principal of the second private school that replied as well as a group of school personnel including the academic and mathematics coordinators and the school psychologist. After a process that included submitting different documents, the principal accepted to participate and I was able to start the data collection process about three weeks later than planned. As the principal told me, private schools are very suspicious of external individuals entering in their daily routines because of security matters and institutional rules.

In the following section, I briefly describe the racial and class compositions of the neighborhoods and general characteristics of the three schools. The names of the participating schools, students, and teachers are pseudonyms.

Hope Middle School

Hope Middle School (HMS) is situated on the east area of Cali. The school neighborhood is part of one of the six comunas that constitute the Whitewater district, *distrito de Aguablanca.* Extreme levels of poverty, overcrowding, unemployment, and violence characterize this sector of the city. A highway separates the *distrito* from the rest of

Cali. The Cauca River is its limit to the east. The changing dynamics of drug traffic in Cali has facilitated the perfect place for housing "gang bands" and hitmen offices that are dedicated to paid homicides, drug deals, and robbery in the *distrito*. Young men are easy prey of criminal organizations that offer them "easy and quick" money.

The *distrito* started as small slums built in illegally occupied areas that over time evolved to legally constituted barrios. Describing the populating dynamics in the *distrito*, Urrea (2012) identifies different migratory flows in different periods during the last 80 years. Motivated for economic aspirations, better education, and even because of a tsunami that took place in 1979 in the south Pacific coast, poor and working class Blacks and indigenous populations from the south of Colombia moved to Cali and settled in the *distrito*. Poor mestizos looking for cheap housing also migrated to this area of the city. Currently, due to the high rates of violence in areas inhabited by black communities, a new migratory flow keeps bringing this population from the Pacific coast to Cali (Urrea, 2012).

The comuna in which HMS is located comprises ten barrios and is one of most crowded in the city. Its population belongs mainly to the two lowest SES groups and is largely inhabited by Blacks and mestizos. HMS is one of the seven public schools located in the district and one of the three in this comuna. Approximately 1200 students attend the school distributed in two periods during the school day[6]. At nights, the school also offers educational services to adults who want to get a high school certificate.

[6] The first period goes from 6:30 a.m. to 12:30 p.m. and it is attended by sixth, seventh, and eighth graders. The second period starts at 12:45 p.m. and goes until 6:30 p.m. and it is attended by ninth, tenth, and eleventh graders. This is a common school arrangement in public schools in Colombia due to a lack of schools that offer the educational service to all students in just one period.

HMS was founded in 1991 by a community initiative that ended up with the local government providing the school personnel. Currently, a principal, two coordinators, and 34 teachers comprise the school personnel. Seventy-five percent of the students are black and all come from low-income families. The school website details some of the problems identified by the school community such as building deterioration, the scarcity of curriculum materials, and a lack of space for students' physical education.

The school has a vocational program in electronics, computers, and electricity for tenth and eleventh graders that leads to certifications in these areas. The school also develops an ethno educational project that is aimed to foster the recognition of the racial and cultural diversity among students and within the community. Different cultural activities take place over the school year to raise awareness among the school staff, students, and parents about the role and place of Afro Colombians in the culture and identity of the country.

Spring Middle School

Spring Middle School (SMS) is located in a *comuna* on the northwest part of the city comprising 25 barrios. It is one the few *comunas* in Cali in which it is possible to find low income and upper middle class neighborhoods in the same area. Thus, 10.5% of its population belongs to the lowest SES, whereas 89.5% is classified as middle and upper middle class. The *comuna* is a traditional place in Cali inhabited by the social, political, and economic elites of the city (Urrea, 2012) and it is full of fancy and expensive restaurants, malls, and hotels. However, built on a hill, there are some poor barrios located in the westernmost area of the comuna. Some of them were the result of the migratory flow from the pacific coast and the south part of the country that chose this area rather than the *distrito*.

So, its population is mainly black and indigenous who came to Cali fleeing from the violence in their communities and looking for better life opportunities.

The racial composition of the *comuna* matches the class composition that follows the pattern of race and class intersection in Cali. Thus, 89.6% of the population is mestizo, 10% is black, and 0.4% is indigenous. Urrea (2012) holds that a few Blacks in this comuna are professionals and businessmen, but most of them come down from the hill everyday to work in the rich houses as housekeepers and watchmen.

SMS is a traditional public school in the city founded in 1960. It is well known because, until recently, it was one of the few exclusive all-female schools in Cali. Although the number of male students has been increasing, girls still outnumber them. The school is located in one of the few working-class neighborhoods in the *comuna.* The school social and racial diversity reflect the *comuna* social composition, so its student population is mainly mestizo, but working and low middle class. The students coming from the poorest barrios on the hill are bussed to the school in the context of a program funded by the local government. SMS has two buildings separated by one block. The first one is attended by ninth, tenth and eleventh graders, whereas seventh and eighth graders attend the second one. Vocational education in accounting and secretarial skills is offered to students in the two last years of schooling. The school also offers adult education and has a similar school day arrangement as HMS.

The school staff comprises a principal, two coordinators, and 36 teachers. According to the principal, the main difficulties they face are the building deterioration, flooding during

the two rainy seasons of the year, and a lack of economic resources to better equip the school.

St. John School

St. John School (SJS) is a private school located in the same comuna of Spring Middle although in an upper middle class neighborhood. As many of the private schools in the city, St. John is part of a network of schools administered by a religious community. Even though its staff is completely secular, in the past priests of the religious order administered the school.

SJS is one of the oldest private schools funded in the city as an initiative of a group of parents looking for high quality education for their children. It is well known because of the students' performance in national standardized tests. It began offering education to 131 male students in 1897 and recently started accepting girls. According to its website, the school functioned in a normal house; after a few years, a new building was erected using "imported materials from Europe and following a Renaissance design." Currently, the school functions in two buildings, one for preschoolers and early elementary graders, and the other one for the fourth and upper graders. St. John has wide and open areas with trees and benches for students, a gym, and several soccer and basketball fields.

St. John offers three educational levels (preschool, elementary, and secondary) as established by Colombian legislation. The school develops a mathematics project from preschool to sixth grade aimed to enhance student learning. The teachers use different games and a variety of curriculum materials as part of their instruction in order to provide students with multiple opportunities to build meaning for mathematics concepts and procedures.

According to the principal, one of the main challenges they currently face is to assure the continuity of the project in the upper grades.

About 1000 students comprise the SJS population. They are mainly mestizo and middle and upper middle class, with less than 20 black and none indigenous students attending it. Table 3.1 shows the social and racial compositions of the schools' *comunas.*

Table 3.1

Class and racial compositions of the schools' comunas

Comunas	**Population 2013**	**Racial Composition**			**Class Composition**					
		Black	**Indigenous**	**Mestizo**	**SES 1**	**SES 2**	**SES 3**	**SES 4**	**SES 5**	**SES 6**
Cali	**2,319,655**	**26.2%**	**0.5%**	**73.3%**	**21.6%**	**31.7%**	**30.6%**	**7.1%**	**7.1%**	**1.9%**
SMS and SJS *Comuna*	110,879	10.5%	0.37%	89.13%	3.9%	-	6.6%	36.2%	44.0%	9.3%
HMS *Comuna*	151,544	51.1%	0.43%	48.47	68.4%	31.6%	-	-	-	-

Participants

Teachers

After selecting the schools, I began the process of choosing the participating teachers. In particular, I established the following criteria:

a. Teachers willing to participate.

b. Nonblack teachers. This criterion does not imply that no racial and social tensions exist in the context of interactions between black teachers and black and nonblack students (Rist, 1970). However, selecting nonblack teachers was pertinent based on the study purposes.

c. Teachers with more than five years of mathematics teaching experience.

d. Teachers with similar years of experience in teaching algebra.

I selected Diana, Juan, and Pedro, as the participating teachers.

Diana

Diana is an electrical engineer who taught mathematics and electricity for 13 years in a private university in Cali. For personal reasons and changes in her schedule at the university, she obtained a university certification in education and applied for a job in the public educational system. Diana was assigned to HMS three years ago and started teaching geometry and algebra to seventh and eighth graders. She enjoys her work at Hope Middle. Teaching there has been a rewarding experience full of personal achievements for her. Diana identifies herself as middle class and mestizo.

Juan

Juan studied in one of the two normal schools in Cali and taught in the same elementary school for 30 years. He pursued a bachelor degree in education with emphasis in mathematics and physics 15 years ago. Juan had the opportunity to move to SMS three years ago and since then, he has been teaching algebra to eighth graders. He has not regretted the decision to shift to middle school because he loves teaching mathematics to teenagers. Juan identifies himself as middle class and mestizo.

Pedro

Pedro started teaching in elementary schools in a nearby city 30 years ago. He pursued an undergraduate degree in education with an emphasis on mathematics and physics. Juan moved out to Cali and started teaching high school mathematics in a public school. He

came to St. John 13 years ago while keeping his job in the public school. He has always taught high school mathematics because he feels more comfortable teaching calculus and trigonometry. However, the St. John's administration made changes and assigned him to eighth grade four years ago. Pedro believes that it is a challenge to teach eighth grade algebra because it is a hard subject for the students. Pedro identifies himself as middle class and mestizo. Table 3.2 shows the participating teachers' profiles and Table 3.3 the social and racial compositions of the student population in each classroom.

Table 3.2

Profile of the participating teachers

Name	**School**	**Mathematics teaching experience (in years)**	**Experience teaching algebra (in years)**	**Undergraduate degree**
Diana	HMS	16	3	Electrical engineer (Certification in pedagogy)
Juan	SMS	18	3	Bachelor in Education (Mathematics and Physics)
Pedro	SJS	20	4	Bachelor in Education (Mathematics and Physics)

Table 3.3

The students' racial and social compositions in the three classrooms

Name	School	Students' racial composition			Students' class composition		
		Black	**Indigenous**	**Mestizos**	**Low**	**Working and low middle class**	**Middle and upper class**
Diana	HMS	41%	2%	57%	100%	-	-
Juan	SMS	9%	3%	88%	23%	77%	-
Pedro	SJS	-	-	100%	-	-	100%

Students

After selecting these teachers, I observed some of their algebra lessons in different classrooms to identify differences and similarities. In addition, I sought to get a sense of variations in the classroom environments. For instance, I observed Diana's and Juan's lessons in three different classrooms, and Pedro's lessons in each of the two eighth grades at SJS. I selected the groups with the highest percentage of black students in Diana's and Juan's classrooms. I randomly opted for one of the two eighth grades at SJS because of the similarities in their racial compositions. After this process, I observed one lesson in each selected classroom looking for students who could wear cameras to assist with data collection (see explanation of this method below) as well as to focus my observations. I sought students from diverse racial and social backgrounds to discuss with the teachers about expectations and beliefs about these students. I selected:

1. Three black and one mestizo students at HMS. Three females and one male.
2. One black, two mulatto, and one mestizo students at SMS. All were females.

3. Four mestizo students at SJS. Three males and one female.

Table 3.4 shows the profile of the participating students. It is important to mention that although I interviewed the students I did not analyze these data. I carefully reviewed the videos filmed by the students searching for particular interactions and class episodes particular relevant for the study. Although the classroom camera captured such episodes, the sunglasses cameras worn by the students provided me with a better sense of these episodes taking place during the participating teachers' instruction. I also selected video clips taken from the students' cameras to debrief the teachers in order to procure their thoughts about selected class episodes.

Table 3.4

The participating students' profile

School	Name	Age	Race	Gender	SES
HMS	Luis	14	Black	Male	Low
	María	14	Mestizo	Female	Low
	Carmen	16	Black	Female	Low
	Juana	17	Black	Female	Low
SMS	Lucía	14	Black	Female	Low
	Rosa	13	Mulatto	Female	Low
	Ana	13	Mestizo	Female	Working Class
	Katherine	13	Mulatto	Female	Working Class

(continued)

Table 3.4

The participating students' profile (continued)

School	Name	Age	Race	Gender	SES
SJS	Lucas	13	Mestizo	Male	Middle
	Luisa	13	Mestizo	Female	Upper Middle
	Marcos	13	Mestizo	Male	Middle
	Jesús	13	Mestizo	Male	Upper Middle

Data Collection Methods

I used five data sources. As I describe in the following paragraphs, the data sources were chosen to enable persistent observation and significant engagement in the classrooms over the course of four weeks. Interviews, debriefings, filming, and participant observation enabled me a deeper examination of phenomena as they took place during the teaching of algebra at the three schools.

Interviews

I used interviews in this study for several purposes. First, I interviewed the teachers to explore their general expectations about their current students, as well as their ideas about students' performance, learning, and ability to learn algebra. I sought to elicit how these ideas were related to students' social backgrounds and the schools' neighborhoods. Second, during the interviews I scrutinized the different ways in which teachers constructed categories to classify the students and the different ways of using these categories to talk about particular learners in their classrooms. Finally, the interviews allowed me to explore

the participating teachers' perceptions of the school and classroom environments and how these elements were related to their expectations of students' ability to learn algebra. I used semi-structured interviews to openly explore topics that emerged in the course of the conversations. I conducted one interview with each teacher at the beginning of the data collection process (See Appendix B).

Participant Observation

The use of participant observation as a method of data collection in this study had a two-fold purpose. First, participant observation allowed me to directly observe the instructional strategies used by the teachers to teach algebra, their use of textbooks and other curriculum materials, and their classroom organization to either foster or inhibit students' engagement in the lessons. Being immersed during eight sessions in each classroom enabled me to gain a better understanding of teachers' routines, discourses, and students' reactions to teachers' practices. Although some researchers consider that participant observation requires considerable time in the field, Hatch (2002) argues that this is not a requirement when the researcher has a narrow focus, specific questions, and interests, as in this study.

Second, observations of cultural and social groups for a few weeks and even days can enable researchers to gain an intuitive understanding of practices and interactions as they take place in the natural settings (Hatch, 2002). In this regard, I gained a better sense not only of the classroom environment in relation to the interactions between and among teachers and students but also of the school climate in general. As part of my observations of the school environment, I spent time in the teachers' lounges and talked with some of them to capture their general perceptions of the school and the students. Moreover, I informally talked with

different students during class recesses about their perceptions of the school environment and attended a parent meeting. I walked the schools and had access to different spaces such as libraries, restrooms, cafeterias, among others, in order to record both, the practices and interactions taking place and the buildings' conditions. In short, participant observation enabled me full immersion in the schools during the data collection. I used field note sheets to record my impressions, thoughts, and feelings during the eight times I visited each school and observed the lessons (See Appendix C).

Video Recording

As a way to collect detailed information of the classroom environments and in order to capture patterns of interactions that might be missed from my observations, I filmed the classes of the participating teachers twice a week during four weeks using a tripod-mounted camera. Because of differences in the schools' organization, the length of the lessons varied across classrooms. For instance, Diana's lessons took place twice per week in periods of 60 and 95 minutes. Juan's lessons also took place twice per week in periods of 100 and 110 minutes respectively. Finally, Pedro's lessons occurred three times per week in periods of 45, 90 and 45 minutes. In addition, I provided sunglass cameras to four students during each lesson in order to capture the teacher-student and student-student interaction (see Figure 3.1). The clips from the students' and the classroom cameras offered rich information that I used to help the teachers elucidate their thinking about specific moments from the lessons during individual debriefings.

Figure 3.1 Student wearing a sunglass camera

Debriefings

The main goal of the debriefings was to explore the teachers' ideas and explanations for their students' mathematics activity during instruction. In addition, the debriefing allow ed the teachers to explain their goals for the lessons and expectations for particular students. Based on the research goals and the theoretical perspective, I selected specific moments of the lessons and created video clips that I used to debrief the teachers. I explored the goals they set for the lessons, the purposes of specific activities and instructional strategies they introduced, and the reasons they posed particular questions to students or the responses they gave to students. To elaborate on these video clips, I took into account different criteria such as the interactions captured by the students wearing the sunglass cameras; particular practices introduced by the teachers to help students learn the algebraic content, and so forth. I debriefed Diana and Juan in three different occasions. Due to the difficulties in getting access to SJS, I debriefed Pedro twice.

Mathematics Classroom Artifacts

Teachers' curriculum materials provide important information about expectations and learning goals, and in this sense, constitute an important source for analysis. I collected the teachers' curriculum materials such as textbooks and the teachers' classroom and year-plans to explore their learning goals and the characteristics of the algebraic content to be taught to the learners. For instance, by collecting these curriculum materials, I sought to get a sense of (a) the learning goals that teachers set up for their students; (b) the levels of thinking that the teachers expected to promote in their students as reflected in tasks and worksheets; and (c) the deepness and complexity of the mathematics content that the teachers proposed to their students. In this regard, the analysis of curriculum materials contributed to uncover the participating teachers' assumptions about the learning of algebra. Table 3.5 summarizes the amount of data collected in the study.

Table 3.5

Summary of Data Collected

Instruments and Sources of Data	Diana	Juan	Pedro	Total
Interviews (Teachers).	1	1	1	3
Interviews (Students).	4	5	4	13
Debriefings.	3	3	2	8
Curriculum materials and school policies documents.	5	6	4	15
Lesson plans.	2	2	0	4
Field notes.	8	8	8	24
Videos (Classrooms)	8	8	8	24
Videos (Students)	21	27	18	66

Data Analysis Procedures

I used an interpretative analysis perspective to make sense of the participating teachers' expectations and practices. According to Hatch (2002), interpretative analysis helps the researcher make inferences of the observed facts, attach meanings to them, and draw conclusions from them. The process of conducting an interpretative analysis comprises different stages. First, the transcriptions of a small piece of the data sources are read to look for potential categories, that is, potential themes that might arise. Second, as the categories emerge, the process continues going over the data and pulling them from those categories. Third, it is needed to explore the links between the categories and finally, the relations are used to build a theoretical model that allows the researchers to understand and describe the phenomenon under study. Hallberg (2005) argues that the result of the analysis is "the researcher's interpretative understanding, rather than the researcher's explanation, of how the participant creates his or her understanding and meaning of reality" (p. 146). Drawing on these assumptions, the data analysis in this study comprised the phases described in Table 3.6

Table 3.6

Phases of data analysis

Phase	Activity	Results
Line-by-line coding.	Identification of phrases, words, and paragraphs that were expressions of ideas related to the initial proposed categories.	Initial codes refined. Emerging codes.
Episode-by-episode coding.	Identification of patterns in the videos related to the initial proposed categories and codes found in the phase 1.	Initial codes refined. Emerging codes
Focused coding.	Selective and more conceptual coding of transcripts and videos.	Themes associated to categories.
Conceptual coding.	Characterization of categories and their relations.	Model representing the relationships among categories.

In the following paragraphs, I describe in detail each phase.

Phases

Phase 1. I transcribed the interviews and debriefings in Spanish, the original language of the data. I selected one of the teachers and started the screening process of the transcripts by closely reading these narratives. Using different color markers, I underlined phrases, words, and paragraphs that were expressions of ideas related to the initial themes I proposed (see Table 3.7). I also wrote labels in the margins of the pages regarding these themes and others that began emerging from the transcripts. For instance, I found that the first teacher I initially coded introduced different criteria to distinguish the students between two categories

of performance, "good" and "bad." In particular, the teacher used the location of the students in the classroom to explain their performance, as illustrated in the following narrative:

> Usually, the students who want to pay attention and contribute to the class sit in the front of the classroom. The students, who not want to pay attention, sit in the back. That is always a common feature (Interview, August 20th)

The teacher persistently associated the students who sat in the front of the classroom as "good" students and the students who sat in the back as "bad" students. I labeled these teacher's allusions as "comparing students" although initially I did not make any distinction among the criteria the teacher used to describe the students. Other labels included teachers' descriptions of students and their social environment; teachers' explanations of their students' attitudes and behaviors; teachers' ideas of algebra, its teaching and learning; and so forth.

I repeated this process with the transcripts of the two remaining teachers. As an important part of the data analysis process, I constantly compared emerging codes, words, and regularities within the data of each teacher and across the data of all of them. Using this constant comparative method (Charmaz, 2006) enabled me to both identify commonalities and differences in themes and codes and to strongly ground the analysis in the data (Hallberg, 2006).

Table 3.7

Initial categories of data analysis

Teachers' expectations	Teachers' instructional practices
Narratives about: • Students' ability to learn algebra; • Students' performance in algebra; • Students' behaviors and attitudes in the classroom. • Students' school success and failure.	• Characteristics of instruction. • Discourses about the teaching and learning of algebra. • Discourses that support the teachers' instruction. • Characteristics of the algebraic content as presented by the teachers.

Phase 2. I repeated the process in phase 1 with the teachers' videos. I uploaded the videos to the Nvivo 10 software and started searching for patterns in the teachers' instructional strategies as stated in the initial category. I selected one video for each teacher to conduct this phase of the data analysis. In particular, I sought key class episodes or events that illustrated important patterns associated with the initial categories. A key event or class episode was defined as situations in which "the researcher assumes intuitively that the event chosen has the potential to make explicit a theoretical 'loading'. A key event is key in that it brings to awareness latent, intuitive judgments the analyst has already made about salient patterns in the data" (Straehler-Pohl, Fernández, Gellert, & Figueiras, 2014, p. 183).

I also sought new themes. I used my field notes to focus on particular aspects and ideas I highlighted during the data collection process that would help me to identify emerging themes regarding teachers' instructional routines, specific characteristics of the algebraic content taught, and so forth. For instance, during the data collection process, I noticed that the teachers displayed different interactions with learners that seemed to be based on their

own constructions about "good" and "bad" students. I consigned this idea in my field notes and, when watching the videos, I paid close attention to this fact that resulted in creating two new codes that I called "teacher and student interactions" and "student participation." The finding of these emerging themes led me to carry out a revision of the literature review to further refine my analysis. Research approaching issues about the interactions between teacher and student and student-student in classroom across different social contexts was therefore included in this study. The new codes that emerged in the previous phase were organized in a new category called classroom climate, as shown in Table 3.8.

Table 3.8

Final categories of analysis

Teachers' expectations	**Teachers' instructional practices**	**Classroom climate**
• Students' ability to learn algebra; • Students' performance in algebra; • Students' behaviors in the classroom; and • School success and failure.	• Characteristics of Instruction. • Discourses about the teaching and learning of algebra. • Discourses that support the teachers' instruction. • Characteristics of the algebraic content as presented by the teachers.	• Student behavior management. • Student positioning. • Discursive interactions.

Again, the use of the constant comparative method made possible to distinguish patterns, regularities and differences within and between the teachers' data. At the end of this process, I organized the codes that resulted from phase 1 and 2 into a coding dictionary in the Nvivo software, and uploaded all data to start a new coding process.

Phase 3. In this phase, I coded all videos and transcripts in a reiterative process (Bernard, 2006) that led me to deeply cover the fundamental elements in the data. I emphasized my search in findings the links among themes and categories, and in this sense, the process of coding was more selective and conceptual. To do so, I used queries to help me to synthesize and organize the data into conceptual categories.

The significance of the codes was given by both the research questions and the theoretical perspective that guided the analysis. It this phase of the data analysis, I started elaborating memos about the themes and categories. Although I had previously written memos during the data collection, in this phase I deepened into the initial memos and produced more refined and conceptual descriptions of the categories. The memo-writing activity was critical in underpinning the identification and elaboration of the conceptual categories. Writing memos helped me establish connections among themes and relate the themes to broader issues and conceptual developments in the field of mathematics education. Table 3.9 shows the characterization of the three categories and their themes.

Phase 4. The ultimate purpose of this study was to identify the forms wherein teacher expectations of poor and black students ability to learn algebra influenced their teaching practices. In order to illuminate such influences, I sought to relate the categories previously found in the final phase of the analysis. Comparing across the three cases, I explored each category to identify its properties and characteristics. Based in the theoretical perspective assumed in this study as well as in research in the field addressing issues of power and mathematics education, I looked for possible relationships between categories to build a model that would allow me to interpret the ways wherein the teachers' expectations were

related to students' social backgrounds and translated into instructional decisions. I finished this process when all sources of data had been meticulously covered and the categories had been saturated. In other words, the process finished when all the regularities and patterns found across the three cases were integrated in a model.

It is important to mention the role of the theoretical perspective assumed in this study in the data analysis process. I used a dialectical approach that allowed me to move back and forth between the data and the theory. In one hand, the theoretical perspective enabled me to focus my search on particular aspects of the participating teachers' narratives and practices. I sought to uncover teachers' beliefs about black and poor students and the characteristics of their strategies to help students build meaning for algebraic objects and procedures. On the other hand, along the data analysis process and as a result of it, I refined some conceptual elements to gain deeper insights about specific emerging themes. For instance, I introduced notions–such as agency–as the result of emerging codes related to teacher and student interactions. I also refined and specified the notion of hegemonic representations to describe the ways in which teachers explained and justified students' attitudes and behaviors and the discursive strategies displayed to describe and portray them. In this sense, the theoretical perspective guided my analysis and at the same time, the emerging codes and themes exhorted me to refine and better develop concepts and ideas as proposed in the theoretical perspective.

In the following chapter, I discuss the findings and present this model.

Table 3.9

Categories and Themes

Categories		Themes	
Teachers' Expectations	Frameworks expressing hegemonic ideological representations of black and poor students that anticipate, explain, and justify their school performance. Discursive strategies and communicative styles used to talk about black and poor students' failure.	**Students' social environment**	Social environment as source of teachers' justifications of students' abilities to learn algebra.
		Student social background	Class, race, and ethnicity as sources of teachers' justifications of students' ability to learn algebra.
		Students' attitudes and behaviors	Class, race, and ethnicity as sources of teachers' explanations of the students' attitudes and behaviors in class.
		Individual students' attitudes and behaviors	Class, race, and ethnicity as sources of teachers' explanations of individual students' attitudes and behaviors in class
		Role Models	Class, race, and ethnicity as sources of teachers' representations of good and bad students.
Teaching Practices	Instructional strategies implemented by the teachers to build meaning of algebraic objects and procedures. Practices implemented by the teachers to assess the students' learning process of algebra.	**Meaning of Algebra**	Teachers' instructional and discursive practices aimed to help students build meaning for algebraic concepts and procedures. Teachers' ways of defining and characterizing algebraic concepts.
		Assessment Practices	Strategies to assess student learning of algebra. Goals of assessment practices.

(continued)

Table 3.9

Categories and themes (continued)

Categories		Themes	
Classroom Climate	Teacher and students' interactions and relationships during instruction that either hinder or foster the learning of algebra	**Teacher and students' interactions**	Discursive interactions to support the students' learning of algebra
		Student participation	Construction of a safe learning environment.

CHAPER 4. Findings and Discussion

In this chapter, I present the findings of this study in three sections. In the first section, I address the first research question and discuss Colombian's Colombian teachers' expectations about eighth graders' ability to learn algebra related to the students' racial and social backgrounds. I present findings in relation to the second question (i.e., How are these teacher expectations expressed in their teaching practices?) in the second section, "Teaching Practices," and the third section, "Classroom Climate."

Teachers' Expectations

In this section, I present the findings related to teachers' expectations for each teacher. I coded the data into four major categories outlined in Table 4.1 to capture the teachers' expectations about their students.

Table 4.1

Teachers' expectations category and themes

Category		Themes	
Teachers' Expectations	Frameworks expressing hegemonic ideological representations of minority and poor students that anticipate, explain, and justify their school performance.	**Students' social environment**	Social environment as source of teachers' justifications of students' abilities to learn algebra.
		Student social background	Class, race, and ethnicity as sources of teachers' justifications of students' ability to learn algebra.
	Discursive strategies and communicative styles used to talk about minority and poor students' failure.	**Students' attitudes and behaviors**	Class, race, and ethnicity as sources of teachers' explanations of the students' attitudes and behaviors in class.

(continued)

Table 4.1

Teachers' expectations category and themes (continued)

Category	Themes	
Teachers' Expectations	**Individual students' attitudes and behaviors**	Class, race, and ethnicity as sources of teachers' explanations of individual students' attitudes and behaviors in class
	Role Models	Class, race, and ethnicity as sources of teachers' representations of good and bad students.
	Assessment Practices	Strategies to assess student learning of algebra. Goals of assessment practices.

Diana's Expectations of the Students' Ability to Learn Algebra

Diana began teaching geometry and algebra to seventh and eighth graders at HMS three years ago. HMS is situated in a community that faces extreme poverty, overcrowding, unemployment, and violence. When I asked Diana to describe her students, she replied:

> Some of them are completely interested in nothing and they do nothing. They do not care about education (Interview, August 20).

The first image that comes to Diana's mind is a negative representation that depicts "some" learners who do not appreciate education, lack motivation and interest, and possess low aspirations. Diana also communicated that her students lack desire to build a better future and do not display the necessary effort needed to achieve it:

> I have always taught in eighth grades and sometimes in seventh grade since I came here. And they are children that, I mean, they do not care about studying. I feel they

> are not envisioning very well that in order to get a good future they need to study. So, you have to push them up a lot (Interview, August 20).

Diana links the lack of interest in education to the characteristics of the social and economic environment surrounding the students. According to her, the atmosphere of violence and poverty that the inhabitants of the distrito experience nurtures the students' aspirations about their social and professional status. Violence and poverty contribute to the students' expectations about their futures and consequently, she depicts boys as aspiring to be gangsters and girls as desiring to become mothers:

> They think: "I do not need to study a lot because for that [becoming a gangster], I won't need it." The girls get pregnant pretty soon. So, they just drop school. I think they do not see education as an alternative to improve their lives (Interview, August 20).

An additional representation of the students held by Diana relates to the apparent lack of mainstream social values such as hard work. The students do not persevere enough in order to achieve the goal of being academically successful. This deficit view explains their low performance and difficulties in learning algebra:

> They are not aware. They believe things are simple, easy. They do not believe in what we are showing them, because there are difficulties and to reach the goals [of graduating and going to college] they need to work hard. I think they do not believe

in hard work and reality crashes in front of them when they get low grades in the Saber 11 test[7] (Debriefing, September 3).

Interestingly, these narratives reflect a key feature of Diana's discursive strategies. Diana attributes lacking values and dispositions as a matter of individual choices. The students consciously decide to drop out of school, to become gangsters, to be teenage mothers, and therefore, their lack of motivation to study and low aspirations are the result of their willingness and personal decisions. Diana does not refer to unchallenging learning environments and social and school practices of exclusion that disadvantage students by unequally distributing knowledge and opportunities. Instead, Diana expands on narratives that depict students as accountable for not only their own school failure but also for the debacle of their futures; in this sense, she diminishes her responsibility for the students' mathematics outcomes. Blaming the students enables Diana to convey a negative representation of them and a positive or at least neutral self-representation in the explanations of the students' low achievement in algebra.

Successful and Unsuccessful Students

Diana sharply classifies the students into "good" and "bad" students. She depicts "good" students as:

> industrious students that have a well-defined core family. They know where they are going and what they want. They care about doing everything you teach them; they comply with homework and schoolwork; I mean, they are the most responsible students. Their tests are very good, they study for exams. Homework is done

[7] The Saber 11 is a test the students take in their last year of schooling. Colombian universities require specific scores in this test to accept students in their programs.

throughout. They are not like the others who merely write a few sentences to comply with the tasks. They participate in class, contribute to class (Debriefing, September 3).

Diana's representation of a "good" student begins with family. The family determines if the student will possess values and dispositions needed to succeed in school and in life. Clearly, "good" students possess values such as hard work, effort, appreciation for education, and responsibility, among others. In addition, Diana depicts a learner who does not speak without the teacher's consent and that remains quiet in class. They respond to the teacher and make contributions in class, as Diana explains:

> Very quiet. That is a general behavior of good students. I am not just thinking about the [good] students of 8-6[8] but all the students I [currently] teach. And all of them are very quiet, well behaved. They pay attention to the class all the time. They are engaged doing schoolwork. They participate and contribute to the class (Interview, August 20).

From this narrative, docility and compliance arise as features to characterize the students' academic performance. A "good" student in Diana's classroom surrenders autonomy and self-expression to obey and follow her orders and comply with her desires. The narrative conveys a sense of submission and passivity associated with successful learning. In contrast, Diana characterizes students with high chances of failing her class as those who do not care or those that do not have the ability:

[8] This is a common form of distinguishing groups at schools. 8-6 means the sixth group of eighth graders at HMS, or a specific class.

[They] do not really care about doing homework; others do not have the ability to do homework and they do not care (Debriefing, September 17).

Diana also noted that students choose to sit in locations in the classroom that convey if they are good or bad students. Diana argues that:

Usually, the students who want to pay attention and contribute to the class sit in the front of the classroom. The students, who not want to pay attention, sit in the back. That is always a common feature. For instance, when I attend a class, I always sit in the front (Interview, August 20).

Diana believes "good" and "bad" students locate in different places in the classroom and she makes a connection to where she sits when she attends a class as a student.

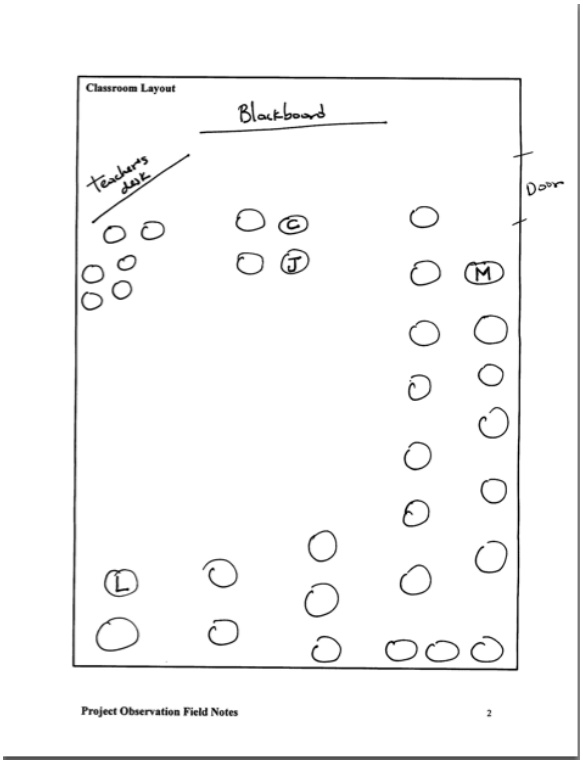

Figure 4.1. Diana's classroom layout during one observation

This division results in a subtle geography of inclusion and exclusion in Diana's classroom by segregating the students in different sectors as shown in Figure 4.1[9]. Diana's classroom layout contributes to both the marginalization of the majority of the learners and their academic disadvantage by hindering their opportunities to access Diana's mathematics instruction. For instance, I started noticing that the "good" students in the group–according to Diana–always sat near Diana's desk and in the first rows independently of the classroom in which they were assigned[10]. I recorded this situation in my field notes:

> There is a spatial gap between this group and the rest of the students. They are far away from their classmates and always work together. They barely talk with other students out of this small group and their interactions mainly consist in responding to simple requests such as lending a pencil or a notebook. During instruction, Diana makes frequent visual contact with them and directs her questions to this group. She seems to wait for a response coming from a student of this group. When Diana sits on her desk, she chats with the students and responds to the questions they have. There is a special and close interaction between Diana and this group (Field notes, September 2).

As may be noticed, there are three different groups in the classroom. The first one is the group of "good" students who sit near Diana's desk. This group includes 6 mestizo students. The second group of students sits between this group and the "bad" students who comprise the majority of the class. The group of "good" students is clearly separated from the

[9] The circles represent the students' location. The letters L (black), J (black), C (black), and M (mestizo) represent the students who wore the sunglasses cameras in Diana's classroom.

[10] HMS has more groups of students than available classrooms. Every day the students are placed in different rooms that sometimes may include the school library.

rest of the learners in the classroom. I asked Diana to describe the groups based on their location. She told me:

> In that space [in the front of the classroom and close to her desk] are the students that comply with homework and solve exercises. They do pay attention to class. I mean, they are always attending to class and solving exercises. They always comply with what I ask them to do (Debriefing, September 3).

In contrast, Diana elaborated on the "bad" students who particularly sit in the back of the classroom:

> Those guys over there [the students in the back of the classroom] come to the school to do other things [different from studying]. Learning is not a goal for them. They come here to do other stuff. They come to have fun and hang out. That is like throwing off the things they live outside on the streets, and unfortunately, one as a teacher does not see every student but all of them in general, so one learns to isolate [the good students from the bad students]. So, ok, they do not want [to learn], they do not want [to learn], so I go on with those who want (Debriefing, September 3).

It is worth noticing the form in which Diana creates and justifies two different mechanisms of exclusion. The first one relates to the spatial segregation of the classroom that secures access to her tutoring, advising, and caring to a small group of students while excluding the bulk of students from these privileges. The second mechanism of exclusion relates to her denying teaching to the majority of learners in the group. Diana overtly recognizes her decision of ignoring the "bad" students and taking into account just the needs of the small group of students who, in her opinion, want to learn. Thus, Diana's

representations–that depict the majority of the students as incapable of learning by appealing to deficit perspectives–enable her to validate school practices of exclusion at school. By blaming the students for their lack of interest in learning, Diana justifies her decision of excluding the majority of the students form her instruction and concentrating her teaching exclusively on the "good" ones. In other words, making the students responsible for their lack of interest and dispositions is used to justify her denying of teaching and attention to the majority of the students.

The closeness between Diana and the good students in the classroom is not just physical but also symbolic. Diana positions herself as a good student and, thus, the small group of brilliant learners in the classroom is *close* to the principles she believes in and values. In this regard, Diana believes that her main goal when teaching the students at HMS is:

> To give to the students positive messages about life. More than teaching algebra my goal is to show them a good example and to inculcate values, those types of things. That is more valuable for them (Debriefing, September 3).

Thus, the spatial segregation of the classroom seems to be literal and symbolic between two groups of people who do not share values and dispositions, and that in fact have nothing in common. Diana believes that:

> The boys and girls [that are "good" students] are very dedicated, so dedicated that I say: "Jeez, they are in the wrong place!" Because they do not fit within this environment and with their classmates. They do not fit in this milieu, and I wish they

were in another place where they could take advantage of education (Interview, August 20).

Diana also identifies the habits and characteristics of bad students as an illness that can be transmitted to others pupils. Bad students are depicted as carriers of a virus that can contaminate the good students and even spoil them. So, to some extent, they need to be separated:

> [Good students] have all the characteristics I told you before. A general [characteristic] of these little persons is that they do not allow the others to infect them. They are not troublemakers and they are always paying attention to what I am dictating. They contribute to the class. They do not allow the others to contaminate them (Debriefing, September 3).

Interestingly, there are not any Black students among the group that sits near Diana's desk although they constitute about the half of the classroom population. However, I noticed one black student, Juana, who seems to fulfill the criteria to be a "good" student. She always sat in the front of the classroom although a little far from Diana's desk. Juana, who wore one of the sunglass cameras, was very quiet. She paid attention and sometimes raised her hand to solve exercises on the whiteboard and contributed to the class discussions. As a result, I asked Diana about Juana:

L: Is Juana a good student?

D: Not really.

L: Why? I am asking you this because I have noticed that she is very dedicated and contributes to the class. She frequently raises her hand and goes to the whiteboard. She is very quiet. She is kind of a good student.

D: Yeah, she is very quiet. That is good. But, I mean, she needs to work harder.

L: What do you mean? Has she got low grades?

D: She has got good grades.

L: But she is not a good student?

D: I mean, she tries to pay attention. She concentrates in the class for short periods, but she needs to work harder (Debriefing, September 3).

Despite Juana's contributions to the lessons, her positive attitudes in class, and her good grades, Diana positions her as lacking work ethic. Although there are not explicit allusions to the students' race, class, or ethnicity in the representations of "good" and "bad" students and in Diana's justifications, her narratives reflect negative images of poor and minority students and reveal her low expectations for these student populations.

Diana's Expectations of the Students Ability to Learn Algebra

Diana communicated frustration that her students did not have strong mathematics knowledge when they entered her classroom. Moreover, they did not understand her multiple explanations. Although she repeats explanations, gives students many opportunities to obtain good grades, and provides individual tutoring when needed,

There are a lot [of students] that do not learn at the same pace [of my teaching] because they do not have strong prior mathematics knowledge; and one explains in so

many different ways and they do not understand; they are just spaced out (Debriefing, September 17).

Diana conveys that students' low-level mathematics skills and incapability to learn are presented as factors that undermine Diana's ability to effectively teach. Diana tries to sequence her teaching to what she considers the students' levels of understanding. For instance, she holds:

> I start from the simplest [topics] and scale up to the most complex for the students to understand. I try to go from the simplest to the more complex (Interview, August 20).

Yet, when Diana examines the results of tests, she determines she must focus on making it easier. She explains:

> Yeah, I use the [results of] tests to, like, for instance, when I grade [the test], I see [the students' failure] and I said myself: "God, I have to find an easier way [to teach them] to see if I can harvest something" (Debriefing, September 3).

In this statement, Diana introduces a metaphor that identifies the students as infertile land in which fruits do not grow and teaching the pupils at HMS is similar to plowing in barren soil. To make the algebra easier, Diana introduces familiar contexts and objects for the students to model such concepts. She explains:

> So, I have always tried to explain to them that algebra is not something [that comes from] out of the world. I tell them, "Look, algebra is something that you see every day." I say, for instance, "Let us group all desks and count them. Then, we just make a set with the desks and come up with an algebraic expression" (Debriefing, September 3).

These narratives reinforce the low expectations elaborated and conveyed by Diana. In her views, the students are incapable of learning complex concepts by exploring challenging tasks. Diana reduces the complexity by making the meaning of objects and procedures dependable of real life, contrived contexts. However, in reality, these practices may mislead the students' understanding of the algebraic objects and hinder the development of students' algebraic reasoning.

I asked Diana about possible explanations for the students' failure in learning algebra despite the easiness and repetition of her instructional methods. She responded:

> There are a lot of explanations. One, they are so ill prepared that when they look at the whiteboard, they feel lost. No matter how hard you try to explain to them, they do not catch on. Another thing is they do not care. They do not see the utility [of the topic, so they say], "What is that for?" Another, they come to the school to goof around, to have fun, so [they say] "I will hang out and chat with my mates about what I did yesterday, before yesterday, last night, and I will have fun." But anything related to doing homework or thinking, [they say], "No way, not for that route." So, there are tons of things that explain that (Debriefing, September 17).

Again, Diana reveals that the responsibility for the students' failure is ascribed to them. Diana's explanations for the students' low performance mainly rely on both the weak prior mathematics knowledge that hinders their comprehension of the algebraic knowledge and the students' lack of values and dispositions. Additional blame could be attributed to teachers from earlier grade levels; however, Diana never made that claim. Furthermore, she did not tie a lack of success to her instruction.

In summary, Diana's low expectations are expressed in two different ways. First, her use of adjectives such as "simplest" and "easier" characterize the instructional methods she implements. Second, low expectations are also expressed in her representations of the students as "slow," "spaced out," and "the worst" of the school. The adjectives and expressions she uses to describe her students and her teaching communicate a deficit view in which she anticipates failure.

Her low expectations also capture a particular representation of the role of school in the students' lives. Diana depicts HMS as a symbolic island in the middle of a harmful neighborhood in which the students are isolated from the dangers of streets and the hardships of their lives. The school becomes a safe place for the socialization and amusement of the students, as Diana affirms:

> They mainly come to the school to pass the time with their mates and to have fun. They do not do homework or schoolwork. I mean, they do little [work] here and assigning them homework is a waste of time. Anyhow they are safe here and far away from streets (Interview, August 20).

In this perspective, the school is not a site for the students to learn, to develop thinking abilities, and to transform the social world. This representation of school frees Diana from assuming responsibility of the students' learning. If the school is a place for socializing and protection, then the priority is not about developing meaningful teaching practices that influence student learning.

Students' Social and Cultural Background as Sources of Explanations for their Failure at School

Three different themes (i.e., students' racial, social, and ethnic backgrounds) emerge from Diana's narratives to explain the students' failure in learning algebra. I discuss each of these themes below.

Students' Cultural Background

Diana seems to deny any influence of the students' cultural backgrounds in their mathematics performance, as stated in the following:

> L: Do you believe that [cultural difference] is troublesome for the guys' mathematics performance?
>
> D: No, I do not. I think the problem is *what has brought them here* [to the city]. I mean, violence and forced displacement. They may have had to flee from the guerilla and all these kind of stuff. That is what makes them different little persons when learning. But I do not believe that [they learn differently] is because they come from Buenaventura or Tumaco[11]. It is because of the *reasons* they came here [to Cali] (Debriefing, September 3).

According to this narrative, differences are tied to the students' experiences of violence and displacement rather than to cultural explanations or justifications associated to race or ethnicity. Diana also believes that differences in performance may be explained by differences in the speed to learn instead of the students' ethnic backgrounds, as she affirms:

> I do not think so [that black students need other ways of mathematics instruction]. I think all of us, I mean some of us learn faster or easier than others, but I do think that

[11] Buenaventura and Tumaco are the most important ports on the Colombian Pacific coast and largely inhabited by black people.

it is not related to culture. Learning has nothing to do with culture (Debriefing, September 3).

In the same line of reasoning, Diana considers that mathematics performance is not affected or influenced by the students' race. Although Diana did not identify any "good" black students in the group I observed, she holds that:

I have brilliant black students. That [race] does not have anything to do [with the students' performance] (Debriefing, September 3).

The previous narratives could lead us to conclude that, in Diana's views, the students' cultural and racial backgrounds do not influence their mathematics performance at school. However, others narratives do convey a contradictory, negative image about black students. The following excerpt expresses such contradiction:

L: Do you think black students in your classrooms need additional support to learn mathematics?

D: And why just for learning mathematics? I think it is not just to learn mathematics; they need support to learn anything. They do not like to read and reading is fundamental for [learning] any subject matter at school; they do not like to read, so they are not going to understand the different concepts or basis they are given in the different classes. And they do not have the culture of [complying with] homework.

L: What type of support do you think they would need?

D: I think they would need psychological and occupational therapy.

L: Why?

D: Because they could take their problems out. That is something teachers cannot do; we do not have time for that, we are not trained for that. We are trained for teaching, so we do not know how to face the difficulties these students are having– displacement, drugs, gangs–all this kind of stuff. Sometimes you just do not know how to deal with them, so you ask yourself, "God! What else should I do? What else can I do?" Besides we do not have time for that (Debriefing, September 17).

In this excerpt, Diana clearly expresses a negative representation of black students by associating them with undesirable activities such as drugs and gangs. Her narrative reveals the inferior social and cultural status in which Diana positions this student population and that depicts them as lacking the dispositions needed to perform well at school. They are also different and divergent from expected and accepted behaviors. That is clearly implied in her suggestion for providing occupational therapy to treat the variety of conditions they seem to carry. It is important to notice that, on the one hand, Diana displays narratives that convey a message of racial egalitarianism. If differences in mathematics performance exist among students, they are due to personal characteristics rather than to racial and ethnic differences. On the other hand, her narratives seem to be framed within hegemonic and biased representations of black people that depict them from a deficiency perspective (e.g., "they do not like to read," they do not have the culture of homework"). In consequence, the chances of academic success of black students in Diana's opinion are low.

Students' Family Compositions

According to Diana, the forms in which families are constituted as well as their social environment play a fundamental role in students' mathematics performance. For instance,

high achievers originate from homes that include a father, mother, and siblings and a "good" family environment. Diana describes:

> L: How would you explain that despite the [social] environment you describe, there are good students in this school?
>
> D: I mean, I explain that based on their families [composition]. That part is fundamental to me. I mean, ["good" students] have a very solid core family, super good bases and values. The ["good"] students have dad and mom; they live with and have dad and mom; they live with siblings, and have a good family environment (Debriefing, September 3).

In contrast, low achievers come from what she defines as dysfunctional or broken families that are essentially depicted as deviant from the traditional social model. As may be noticed in the following narrative, the absence of a "normal" family at home is used to explain what she considers irregular students' attitudes and behaviors:

> They have dysfunctional [family] environments. They have very, very diverse families. Usually some of them do not have dads or moms; their grandparents have reared them. Some of them do have dads, but not moms. Others have moms and siblings from different dads. So, they are families with tons of troubles and then, that environment leads to, makes them different (Debriefing, September 3).

Diana also blames the parents for their lack of commitment with their children's education. She believes that these parents do not provide sufficient supervision and support with the students' work from HMS, and therefore they have little chance to perform well:

They learn through examples and they need guidance all the time. So, let's say one is trying to guide them here in the school and they go back home and they are loose, they do not have somebody who takes care of them, who supports them and clarifies their doubts; so, they remain that way (Debriefing, August 27).

Thus, a hegemonic representation of family emerges in Diana's narratives as a criterion to anticipate the future academic success of an individual. In addition, Diana also negatively depicts the parents who, in her words, do not provide positive guidance for their children. The "good" example she represents for the students at school dissipates at home where the students find their broken families. Thus, particular hegemonic representations of family in general, and low-income families in particular, shape Diana's expectations that she communicates to her students.

Students' Social Environment

Diana justifies the students' behaviors and attitudes towards education by appealing to features of the social context in which they live. In other words, the social environment of the *distrito* embodies a series of anti-values that determine the students' chances to successfully learn algebra. This idea emerges from the following narrative:

L: How would you explain the students' attitudes? I mean, you said the students do not value education; they do not see the importance of education for their futures. Why do you think these students have this attitude?

D: Well, I believe it is the [social] environment, isn't it? I mean, the *distrito.*

L: Could you explain a little bit more your response?

> D: I think their family environments, the society in which they are living [influences their attitudes]. They are surrounded by gangs, hit man offices, drug dealers, and other similar stuff, so their ambitions are "I want to be a gangster" (Debriefing, August 27).

The *distrito* embeds features and characteristics that make it a place different from other areas of Cali. Its environment makes the students "special," and in this regard, Diana establishes a demarking line between students out and inside the *distrito:*

> She [another teacher] has always told me that the students, the group of students here is different, I mean, the environment they have here is different, the conditions they have here in the *distrito*, and it is not possible to do the same work here that could be done in another school [out of the *distrito*] (Debriefing, September 3).

Clearly, the students' attitudes towards education are grounded in and nurtured by the social environment of the *distrito.* As I described, the social environment of the *distrito* to which Diana is referring comprises the highest levels of poverty, violence, and presence of black population in Cali. These are the environment characteristics that seem to have a significant impact in the students' ways of being. Although Diana does not explicitly mention such features, these are widespread images commonly associated to the people who live in the *distrito*.

A revealing feature emerging from this narrative is Diana's allusion to the restrictions that working with students of the *distrito* imposes upon her teaching. If we assume that poverty, violence, and blackness are the main characteristics of the *distrito,* what Diana implies is that these impede effective teaching. That the *distrito*'s social environment is the

main source to explain the students' lack of motivation to study as well as their undervaluation of education becomes clear in the following narrative:

> L: Why do you think that idea is so widespread?
>
> D: Because the context they live in? People have generalized in that way. I mean the areas inhabited by vulnerable people, that have low employment opportunities; then, you relate [the areas] with people who do not want to study and do not want to reach something better or they cannot reach something else, I do not know (Debriefing, September 3).

It is worth noticing that in these narratives, Diana opts for politically correct and neutral terms such as "vulnerable people" to describe inhabitants of the *distrito.* She avoids using overt racist language and uses terms such as lazy, low skilled, spaced out, and careless. Introducing these terms serves Diana in the purpose of talking and representing the black and poor students at HMS without dealing with racial and class implications. Her narratives also appeal to dominant representations of the *distrito* that depict its inhabitants as ignorant, dangerous, lazy, and violent criminals to justify her low expectations about the students' opportunities to learn algebra. In the last part of the excerpt, it becomes evident the associations Diana makes between what she calls "vulnerable people" and their lack of attributes, values, and dispositions to success. The entire responsibility of the current life conditions of "vulnerable people" relies on their own attitudes towards education, lack of ambitions, and laziness rather than on the unequal social structures of society that marginalize them. Thus, blaming the victim arises as a discursive strategy that allows Diana to deny any racist and discriminatory attitude against her students and their parents.

Diana's expectations may have been influenced by statements made by other teachers at HMS as evidenced by the following quote:

> The director of the group[12] told me that, two years ago, they [the school administrators] grouped the worst students of the school in what is today 8-6. So, the bad [students] are in 8-6. Since there, they have met each other, they know each other and have formed small groups according to their interests. There are some little islands there and some of them are interested in nothing. Others, that may be interested, do not want to leave their group of mates so they just keep doing nothing (Interview, August 20).

Furthermore, Diana's narratives of the students at HMS are the result of prejudiced and hegemonic images about the *distrito* and its inhabitants is clear from the following statement:

> I know from my colleagues that, they say that the *distrito* has a special environment, it is a different society; they [the students] bring a lot of problems from home; particularly Emma who has worked here for twenty-three years [told me that] (Debriefing, September 3).

Evidently, Diana's representations of the students are not the result of first-hand experiences in the *distrito*, but have been nurtured by the stories that circulate within the institutional level and that seem to be passed from teacher to teacher. Diana barely leaves the school to meet the parents or neighbors or to learn about the school community. She does not

[12] Within Colombian schools, each group of student has a teacher in charge who is called the director of the group. This teacher provides information to the parents about the students' performance and behaviors and serves as a link between them and the school administrators.

need to walk on the barrio streets because she is one of the few teachers who drives a car to school. In this regard, besides dominant and widespread images of the *distrito* usually conveyed through the local media, her representations of the neighborhood and the students come from broader narratives that seem to have been naturalized at the school.

Summary of Diana's Expectations of the Students' Ability to Learn Algebra

Diana appeals to the students' social and cultural backgrounds as sources of explanations for their attitudes and behaviors as previously discussed. In her narratives, ethnic, class, and racial backgrounds as well as family environments and compositions are presented as factors that explain and justify the students' lack of values and dispositions needed for school success. In the case of the students who have immigrated to Cali due to different situations and that comprise an important number of the HMS population are represented as "the others," the foreigners whose cultures and values are strange to the culture of people in Cali. Diana associates his "otherness" status with deviant behaviors and helps her explain student failure at school in general and in learning algebra in particular.

Deficit views are the main sources of Diana's explanations for the students' failure in learning algebra. Her representations of the students–that convey their lack of values and dispositions to success at school–are supported and nurtured by hegemonic representations about the *distrito* and its inhabitants than associate them with laziness, violence, ignorance, irresponsibility, and low aspirations. The class, racial, and ethnic composition of the *distrito* enable Diana to justify her low expectations about the students' performance in algebra. Nevertheless, her avoidance of racial, ethnic, and class terms is replaced by discursive strategies aimed to convey a negative representation of the students and their parents while

preserving a positive representation for her and her colleagues. Thus, blaming the students for their failure is the main discursive strategy that allows Diana to divert her responsibility in the students' low performance towards them and their parents. In addition, the introduction of "neutral terms" such as "vulnerable people" enables her to express racist and classist opinions in subtle and implicit ways. It is clear then, that Diana holds low expectations for this particular group mainly constituted by black and poor students and that such expectations are strongly tied to the social background of the students.

Juan's Expectations of his Students' Ability to Learn Algebra

Juan began teaching algebra to eighth graders at SMS three years ago. SMS is situated in a working-class neighborhood in a traditional *comuna* inhabited by the social, political and economic elites of the city. In general terms, Juan holds a positive representation of his eighth graders. When asked to describe the group, his narrative was:

> They are doing better [now] than before. I have seen better grades. There are just like three or four students who have very deficient mental schemas (Interview, August 27).

Juan initially explained the differences in attitudes implicit in this excerpt by pointing out the form wherein the group was set up. It was the result of a last minute decision made by the school administrators. Due to the size of the eighth grades, some students were randomly selected and separated from their original groups to organize a new one. This situation was a source of disagreement, outrage, and protests among the students, affecting, in Juan's opinion, their motivation and desire to attend the school. In this regard, Juan told me:

> Currently I see they are different, more docile; at the beginning of the school year they looked annoyed because they were taken out from their original groups (Interview, August 27).

Although Juan's general description of the students conveys a positive image, he is emphatic in distinguishing industrious and good students from the troublesome and low achievers:

> Some guys, at least the guys I know and whose parents have approached me and told me about their situations at home, come from dysfunctional families; they have tons of problems at home and in the neighborhoods. Some of them have brothers who are gangsters and, believe me, that has an effect on their behaviors (Interview, August 27).

Juan also describes this particular group of students as lacking ambitions and expressing low aspirations. They are not interested in education and, as a result, the students consider the school as a place to socialize and pass the time as evidenced by the following statement:

> They do not have ambitions. Their goals are to draw off along life, to have fun, and to hang out. I have asked them, "Why do you come here?" [and they say] "to have fun" (Debriefing, September 4).

Juan conveyed a positive representation of the majority of the students, and one that has improved over the school year. Juan describes the few students who do not fall within this representation as coming from dysfunctional families, lacking ambitions about their futures, and not displaying interest about education. It is worth noticing Juan's discursive

strategy of holding the students and their parents accountable for their lack of motivation and low aspirations. He blames the students for their own failure and in this sense, he evades his responsibility in the students' academic failure, as may be noticed in the following narrative:

> The students' performance is not exclusively our responsibility. [It is the responsibility of] the parents, the guys themselves and their lack of motivation, the hardships at home, and the neighborhoods (Debriefing, September 18).

Although Juan's representations of the students are positive, he points out a few students who lack the values and dispositions for learning algebra. Juan uses this deficit frame to distinguish good and bad students and anticipate their mathematics performance.

Successful and Unsuccessful Students

Juan describes "good" students in the following narrative:

> [A good student is one] who never takes her look off of me. She always looks at the whiteboard. Look, there is a special case in 8-2. Valentina Giraldo is a girl that partners with a small group of students, like six or seven. They are always chatting and chatting. But when I glance at her to check what she is doing, Valentina is always looking at me. She never takes her look off of the whiteboard or me. She is a good student (Interview, August 27).

For Juan, this is an important characteristic of "good" students. He is reiterative in underscoring this type of attentiveness as a distinctive feature of the students who would succeed in learning algebra. Juan seems to identify a cognitive act–learning mathematics–with a physical one–looking at–and rises it to a condition *sine qua non* learning hardly would

take place. “Good” students also like algebra and are responsible, as Juan holds in the following narrative:

> [Good students] like algebra; I can see they like math; I can see they are interested. And these students set their look on you and do not take it off. They pay attention and are attentive of the things they are doing. They comply with the tasks I give them (Interview, August 27).

In addition to these characteristics, the location in the classroom is an indicator of the students’ performance according to Juan’s representation of the students. “Good” students usually sit in the front of the classroom. In contrast to Diana’s representations, they do not need to sit close to the teacher’s desk but near to the whiteboard. This position would guarantee steady attention, “better vision” of the teacher’s explanation, and more frequent interactions with him. Moreover, the students who are not interested in learning algebra and come to the school to socialize are easily identifiable: They sit in the back. When describing a student who has been struggling with learning algebra, Juan explains:

> I have tried to help her. I have told her “Come here, sit in the front; here, I can help you. Over there, in the back, you get distracted.” She gets distracted in the back (Debriefing, September 18).

These excerpts reflect a subtle meaning of teaching as showing facts. Juan needs the entire students’ attention because his teaching mainly consists of exhibiting procedures and concepts on the whiteboard while learning algebra results from looking fixedly at him and “seeing” his explanations.

In Juan's classroom, the "best" students partner in a group that sits on the right side and in the front of the classroom. I recorded my impressions of the group in the following excerpt:

> Juan describes the students in this group as the best of his class. They always sit together in the front and on the right of the classroom. They are not as quiet as other students in the class but pay close attention when Juan is explaining on the whiteboard. They frequently raise their hands either to pose questions or to ask permission to solve problems on the whiteboard. The main talking of the class happens between Juan and this group. Actually, in order to maintain a close interaction with the students, Juan always positions his body in such a way that he is able to easily direct his look to the group and control the students' actions. By doing this, Juan impedes the access of other students to his explanation on the whiteboard, particularly to the group of two black and one indigenous students who sit right on the opposite side of the classroom (Field notes, August 22).

Figure 4.2[13] shows the distribution of the students in Juan's classroom. The two black and the only indigenous student of the group sat on the upper left side of the classroom. The students that Juan considered to be the best of the group sat on the opposite side.

[13] The letters L (black), A (mestizo), R (Mulatto) and K (Mulatto) stand for the students who wore the sunglass cameras. The letters B and I represent the other black and the sole indigenous students in Juan's classroom.

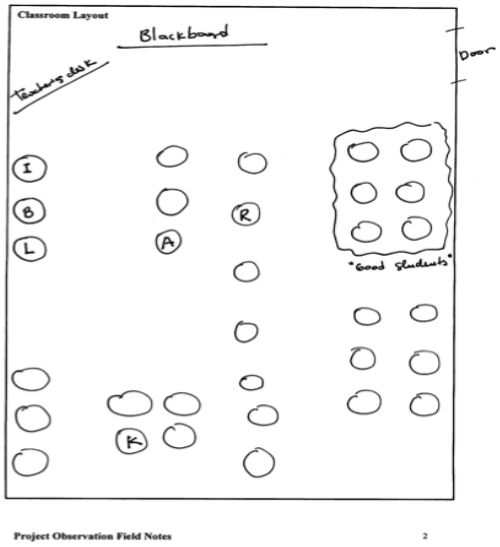

Figure 4.2. Juan's classroom layout

It is interesting to observe how a simple body gesture generates a geography of exclusion in Juan's classroom. The students who sit on the left side of the classroom and on the first rows are marginalized from his teaching and interactions. This is because Juan's body position privileges communication with some students–the best–and enables them access to his explanations while denying it to others. Figure 4.3, a picture taken from one the students' sunglass camera illustrates Juan's body position and the student's difficulties for accessing his explanations. Although Juan is aware of the situation, he minimizes the situation and just blames the students for sitting there:

> I have told them many, many times to move on to other places in the classroom so they can better see the whiteboard (Debriefing, September 18).

Figure 4.3. Juan's physical positioning in the classroom

I observed a close interaction between the group of six students on the right and Juan. In occasions, Juan exclusively directed his explanations to this group while the rest of the students were completely disengaged. I recorded several class episodes in which this close interaction between Juan and this group was noticeable:

> Juan is explaining the procedure to divide polynomials. The classroom is noisy. From the back of the classroom, I can barely hear Juan's voice. Some students are texting. Others are listening to music in their cellphones. Others are just chatting. Despite the evident level of the students' disengagement, Juan keeps talking. He is exclusively interacting with a small group of students. When he asks, they respond. The students are the only ones copying and paying attention. Juan's explanations are just for them. He does not care about the disengagement of the rest of the group (Field notes, August 30).

The processes of exclusion operate in different ways in Diana's and Juan's classrooms. Whereas in the former the spatial disposition of the students is source of exclusion, in the latter a body gesture denies access to mathematics instruction and interactions to a group of students. Interestingly, in both cases the teachers are aware of how their actions ostracize the students.

In contrast to Diana's representation of successful and unsuccessful students, Juan includes a third representation: The majority of the students who are neither the best nor the worst. They make the minimal effort in class to learn, are irresolute, and intermittent in their willingness to comply with schoolwork and homework. However, according to his narratives, these students have a chance to pass the class although with significant help from him and others. Juan classifies his students as "good," "regular," or "bad."

Although Juan's representations of the students may appear to be devoid of racial and class references, this is not the case. In fact, the students' social background is an important source to anticipate their ability to learn algebra.

Students' Social and Cultural Background as Sources of Explanations for their Failure at School

Juan's narratives reflect his beliefs that the students' family composition and social background are critical in anticipating their school performance and possibilities of success. In contrast to Diana, Juan overtly attributes differences in algebraic learning to differences in the students' race, class, and culture.

Students' Cultural Background

Juan depicts students from culturally diverse communities, such as indigenous and Blacks, as lacking the cognitive abilities to learn algebra in particular, and mathematics in general. When I asked him about the difficulties that the indigenous and black students in his classroom were experiencing, Juan responded:

> L: How do you think the cultural differences impact the students' learning of algebra?
>
> J: First, the absence in those places of a type of education that foster the development of mathematics thinking. Second, there is more interest for other types of activities than for cognitive activities (Debriefing, September 4).

While emphasizing minority students' difficulties to acquire complex knowledge, Juan underscore their abilities to perform tasks and activities that apparently require low mental demands and involvement such as elaborating handcrafted artifacts and dancing. Racial and ethnic minority students and their communities are also depicted as deeply interested in bodily-related activities such as, for instance, dancing and sports, that do not require learning mathematics. This ideological position that overestimates thought upon action and rationality over feelings is embedded in Juan's narratives as he builds particular representations of black and indigenous students and their communities. Juan's narratives state that:

> There may be little interest to develop [mathematics thinking] in those communities. They might think "our educational project is not aimed to get students ready for college but to prepare them to farm and become farmers." Or they might think "our educational project is oriented towards developing handcrafted abilities" and then,

> learning mathematics is not a goal for them. Or they might think "we have the biotype for becoming athletes". I think [cultural differences] might affect [the students' performance]. [They affect] the majority of students although there are some exceptions (Debriefing, September 4).

This deficit frame enables Juan to negatively represent indigenous and black communities by underscoring the absence of values and dispositions highly valued in the "mestizo world." According to Juan, these communities do not value particular forms of rationality–such as the mathematics rationality–, situations that lessen their chances of school success. In addition, cultural differences are not considered as positive and important parts of the country and city cultural identities. Instead, the students' cultural differences are presented in terms of deficiency and incapability in the context of an ideological view that expresses the dominance of a particular form of approaching and representing the social world. In such dominant views, there is not room for the recognition and valuation of cultural diversity.

When describing minority students, Juan appeals to a discursive strategy consisting in lessening the negative impact of his narratives by introducing "exceptions to the rule." Thus, black people are depicted as having natural abilities for sports *but* there are some exceptions. Juan has met some Blacks and indigenous who are more interested in studying than, for instance, in dancing and farming. In this regard, the discursive strategy seems to subtlety convey the message that "the exception confirms the rule." The following narrative exemplifies this strategy:

> Cultural differences might have an impact [in the low performance of students]. That does not mean that all Blacks behave in the same way. I met black classmates [at the university] and they were supposed to be deeply dedicated to the revelry but they were brilliant guys. They did well at the university. They did not necessarily like the revelry and dancing. I also met an indigenous guy at the university. Jair was his name, [and he was] very, very brilliant. So, I would not say that culture is determining for all of them, no. But it could be affecting the majority of them (Debriefing, September 18).

Students' Family Composition

The students' family compositions and environments are used by Juan to explain their low performance at school. As in Diana's narratives, Juan also seems to hold a traditional view of the family whose absence might explain the students' academic failure. He also attributes the students' lack of motivation to study to the absence of the parents at home, as stated in the following narrative:

> J: You know, the father is not at home, the father was killed, the father is in jail. So, these types of situations [such as] broken families [influence students' performance].
>
> L: How does coming from dysfunctional families influence the students' performance?
>
> J: First, the students cannot concentrate because of the problems they have at home. They pay little attention. They cannot hold their attention for long periods during the class. They need distractions, they need to forget their problems, so they come to [school to] hang out (Interview, August 27).

Juan's representation of the students' family conveys negative images of the parents who are depicted from a deficit perspective. The narratives portray them as absent in the students' lives, careless about the education of their children, and lacking the knowledge to support their children:

> I insist, familiar problems. Sometimes they come here and have fun. But when they go back home, they go back to their struggling lives where mom and dad are not there for them. And when one of them is at home, [the parent] is very aggressive. They do not understand the stage of development of their children (Interview, August 27).

In addition, the students' family compositions and environments arise in Juan's narratives as impediments for his teaching. The lack of parents' commitment as well as their absence in the students' lives generates obstacles in Juan's effort to effectively teach algebra. He reported:

> The guys are really, really alone at home. We do not have the parents' support to make our work. There are exceptions, but most of the time we do not have parents' support. The students are very, very alone, so the street is the best companion, the street buddies (Debriefing, September 4).

In summary, Juan's narratives convey a deficit view of the students' families. More than the compositions of the families, their environment constitutes an important source of justifications of the students' academic attitudes and dispositions as well as their low performance at school. Moreover, Juan draws upon deficient views of the students' families to justify his difficulties in effectively teaching algebra. In addition, he blames the parents for their children' academic failure. It is worth noticing the assumptions that Juan makes about

the parents. For example, he assumes they are not at home and does not attribute these absences to economic reasons or the urgency to obtain money to meet the families' financial needs. Rather, Juan displays a discursive strategy that enables him to avoid his and the school responsibility in the students' failure.

Students' Class Background

Juan' links poverty to students' performance in school. Juan believes that poverty significantly influences not only the students' motivation to study but also their chances for academic success. According to Juan:

> Poverty is a factor that determines what the guys do here [at school] (Debriefing, September 4).

Moreover, poverty is associated with the students' low aspirations. Their economic status constitutes an impediment for the students to value education, and then, poverty shapes the ways in which they envision and plan their future paths. Juan explained:

> Poverty might contribute to the lack of motivation to come to study; the [poor] students do not come with the desire to say "I'm going [to school] to learn because I need this knowledge to move forward, because I'm going to finish my secondary studies, because one day I'm going to go to the university and I envision myself as a professional" (Debriefing, September 4).

In addition, and not surprisingly, poverty also justifies the parents' lack of commitment with their children's education:

> It is the lack of [economic] resources, time. Poor parents have less time [to dedicate to their children]. They are never there for their children. They never are there when

they have doubts, “Mom, Dad, why don’t you explain this to me?” (Debriefing, September 18)

Juan’s narratives about poverty convey an image of deficiency and a social deficit frame to explain the students’ academic failure. Poverty is identified with underestimation of education, low aspirations, and apathy. Poor students lack the values and dispositions required to perform well at school.

L: So, you are saying that poverty is a factor determining what the students do here?

J: Yes, indeed. In most cases, it is atypical to find a boy with huge economic limitations and doing well at school. But I have seen a few and very well behaved (Debriefing, September 4).

In short, Juan’s narratives depict poor, racial and ethnic minority students as incapable of learning and academic success. These narratives are framed in deficit views that provide explanations and justification of the *why* and *how* of the students’ failure at school. The students’ performance is not the result of school practices that might contribute to marginalization and disadvantage at school but a matter of the students, families, and communities’ social and cultural characteristics. Juan appeals to two different discursive strategies to depict the students’ attitudes and dispositions. The first one consists in blaming the students and their families for school failure. The second discursive strategy consists in “confirming the rule” by presenting some exceptions to Juan’s general assumptions about poor, black, and indigenous students.

In contrast to Diana’s classroom in which black and low-income students make the majority of the student population, in Juan’s classroom there are just two black students, one

mulatto, one indigenous, and eight students come from low-income families. He expresses his low expectations about these students in different forms as I show in the following section.

Juan's Expectations of the Students' Ability to Learn Algebra

The references to expectations of the students' ability to learn algebra in Juan's narratives are scarce. He believes that:

> All students in my class have the same chances to get good grades. They just need to pay attention. As I told you before, there are a few [students] that have difficulties; they do not know the basics and have a story of low achievement. Those are the hard ones. And I do not have the time to sit with each one and explain again and again. I cannot say, "Come and sit here and tell me what didn't you learn in the previous years." However, with help, they can also get good grades (Interview, August 27).

However, there are not Blacks in the group Juan describes as the best students in his classroom. Actually, when I asked him about the worst students in his class, he told me:

> You gave the sunglass cameras to some of them. Just one girl out of the four is a regular student (Interview, August 27).

Juan's narrative refers to Ana. She is the only mestizo among the participating students wearing the sunglass cameras. In order to illustrate Juan's expectations of the minority and poor students' ability to learn algebra, I present the case of Lucía, a 13-year-old girl. Lucía is one of the two black students in the classroom who lives in poverty. Her double marginalized status by race and class makes her case interesting and sheds light on the dynamics of exclusion in Juan's mathematics classroom.

Lucía

Lucia lives with her grandparents, one cousin, her mother, and four siblings in a three-bedroom house located in one of the poorest barrios of the *comuna*. Lucía shares one of the bedrooms with her mother and two siblings. Her family came to Cali fleeing from the violence in Buenaventura. Lucía's mother works as a housekeeper in one of the wealthy neighborhoods of the *comuna*. Her father recently showed up at Lucía's home after abandoning them two years ago. During the time of my observations, he was trying to convince Lucia's mother to allow him to move in again. In Lucía's home the money from her mother's work and her grandpa's pension is not enough to make ends meet. Despite this fact, Lucía describes the economic situation of her family as "normal" because they have what they need. Lucía plans to attend the public university in the city. Her cousin studies there and helps Lucía with her homework. He encourages her to do well at school to continue her post-secondary studies.

Lucía has always studied at SMS. She is bussed from the hill to the school every day. In the classroom, she is the object of jokes and derisions by her classmates because of her race and her particular voice tone that is characteristic of black people from the Pacific coast. I observed several occasions in which her classmates scornfully imitated her accent to ridicule her. Moreover, they gave her a nickname, *Chocó*[14], and would call her by using it rather than by using her name. During a class, I observed one episode that I recorded in my field notes:

[14] Chocó is the Colombian's department with the highest percentage of black population**.** A Colombian department is the equivalent to a state in the U.S.

> The students are completing seatwork assigned by the teacher. Suddenly, Lucía gets up from her desk and approaches Damian asking him for an eraser. Damian lends the eraser accompanying his action with a "joke": "Chocó, you are very annoying. Go to your desk and let me alone." The class explodes in laughter while Lucía returns to her desk with evident signs of anger. The teacher does not intervene (Field notes, August 21).

I never saw Lucía protesting or talking back. She just quietly sits at her desk and continues to engage with her assigned tasks. When I asked Juan to describe Lucía's performance, he told me:

> Well, Lucía. This is the first year I teach her. She has tons of problems. I have never met her parents, I do not even know if she has, whom she lives with. She does not know the basic math facts. I have told her, "Lucía, you need to memorize the time tables." But she does not (Debriefing, September 4).

Juan's narrative about Lucía is aligned with his representations of poor and minority students that depict them as incapable of academic success. Allusions to the absence of the family at school, its lack of support, and its composition are explicitly used by Juan to depict Lucía. Juan's first narrative depicts a negative representation of Lucía and her family and a deficit view that frames her low levels of mathematics knowledge. Juan's negative representation of Lucía's family is confirmed in the following narrative:

> I have never met her parents. They are not interested in her education. They have never approached me in the parents meetings to know how she is doing in class. There may be a lot of problems in her home (Debriefing, September 4).

Juan also thinks that, even though Lucía sits in the front of the classroom, she gets easily distracted and, therefore, she does not focus on him, which is required for learning. In fact, Juan is surprised when she gets good grades on tests:

> You know, sometimes she, one day she surprised me. She got five (the highest grade) on a test, so, it is possible [that she can get good grades] (Debriefing, September 4).

Because of these representations, Juan does not see many options to help Lucía improve her learning process:

> L: Why Lucía is not a good student? I mean, she sits in the front of the classroom and pays attention to your explanations.
>
> J: I do not think she pays attention. She is very absent-minded, indeed.
>
> L: But I have seen her paying attention to the class. Let me show you some video clips from her sunglasses camera. *(After watching the video clips)*. I think she is looking at you and copying, do not you think?
>
> J: See? You are giving me the reason. She does not set her look on me. She is copying when I am explaining, so she is missing my explanations and that explicates why she has so many voids in algebra. She does not pay attention.
>
> L: So, what would she need to do to improve her performance in your class?
>
> J: She needs somebody who helps her with her difficulties. I do not have time for individual tutoring (Debriefing, September 18).

Juan's expectations about Lucía are also expressed in his interactions with her during the lessons. Juan barely directs his questions or explanations towards Lucía or the other two

students who sit close to her. In fact, Juan ignores her when she raises her hand to contribute to the class. I registered one of these class episodes in my field notes:

> Juan has given the students an exercise and told them he would grade the first five students who solve it. Lucía is the third one in handing her notebook out to Juan. After Juan has collected not five but ten notebooks, he asked for one student to solve the exercise on the whiteboard. Lucía jumps up to the whiteboard to solve the problem but Juan ignores her. He selects Lorena, one of the students he considers the best. Lucía just goes back to her desk (Field notes, August 28).

Juan's representations of Lucía are aligned to the narratives he has built for minority and poor students, and in this sense, her options to successfully learn algebra are scarce. In fact, as his narratives convey, Juan will not help Lucía improve her learning of algebra; he assigns that responsibility to other people. Lucía's low performance is the result of her family composition and environment, and her lack of motivation and attentiveness.

Juan's low expectations about Lucía became clear during our last debriefing. The final grades of the second school trimester had just been revealed and I wanted to know Lucía's grades. After finishing our conversation, I asked Juan to share with me this information. I recorded in my field notes the episode that took place:

> Juan invites me to look for the director of Lucía's group to know her grades in the school trimester. The director of the group handed the list of final grades of the students out to Juan and me. The students' names in the list were organized from the highest GPA to the lowest. Lucía obtained the highest GPA in the trimester. Her name was the first on the list. And Juan's reaction was one of bewilderment and

> disappointment. He asked the teacher to recheck to be sure that there were not mistakes. He checked Lucía's last name several times and compared her grades with the grades on his own list. Juan kept saying that there would be a mistake in the grades. He looked very disappointed and confused. Then he looked for the "best" students in the class to find that two of them had got lower grades than Lucía and the other two had not passed one subject, so they were below the middle of the list. Juan could not believe that Lucía, a poor and black student, had had such excellent grades (Field notes, September 18).

The case of Lucía illustrates how issues of power take place in the mathematics classroom. Based on hegemonic ideas about poor and black students, Juan built representations for these students that translate into low expectations. Juan's reaction to Lucía's high grades illustrates the way in which Juan's low expectations are strongly linked to the students' class, ethnic, and racial background.

Summary of Juan's Expectations of the Students' Ability to Learn Algebra

Juan appeals to deficit frames to explain and justify the low expectations and the academic failure of the poor, racial, and ethnic minority students in his classroom. His representations of the students–that convey deficiency and incapacity–are supported and nurtured by ideological discourses of racial and ethnic minority students. Thus, these students are depicted as lacking the values and dispositions to succeed at school.

The students' social backgrounds constitute key elements in anticipating their chances of successfully learning algebra as well as in explaining their attitudes and behaviors at school. Thus, Juan associates poverty and cultural diversity with deficiencies and lack of

dispositions and attitudes that end up shaping the students' experiences at school. In contrast to Diana's discursive strategies, Juan overtly appeals to racial and social characteristics of the students to describe their mathematics performance and chances of academic success. Besides the discursive strategy of blaming the students for their academic failure, Juan repeatedly introduces exceptions to the statements he makes about the students. Both strategies are aimed to communicate a positive self-representation of Juan. It is clear then, that Juan holds low expectations for the minority and poor students in his class and that such expectations are strongly tied to the social background of the students.

Pedro's Expectations of the Students' Ability to Learn Algebra

Pedro is an experienced teacher. He has been teaching high school mathematics for 13 years at SJS and started teaching eighth grade algebra four years ago. SJS is located in one of the wealthiest *comunas* in the city. In general terms, Pedro holds a highly positive representation of the students at SJS. The students are depicted as motivated to learn algebra in particular, and mathematics in general. Such motivation is expressed in their high level of engagement in the algebra class and their willingness to comply with the schoolwork and homework. Pedro's narrative describing the students was:

> These students love algebra. They like mathematics a lot. Most of them do very well in algebra. Although I have just applied a few tests, I have seen them working, asking questions, and solving exercises in their notebooks. They always want to solve problems on the whiteboard (Interview, September 5).

Hard work, motivation, responsibility, and engagement seem to be the main characteristics of Pedro's eighth graders. In addition to these qualities, the significant number

of students who select mathematics-related careers is another indicator of the high degree of motivation for learning mathematics and the students' excellent performance in this subject. Pedro explains that:

> The students in this school choose mathematics-related careers at the university. 70% of our graduates choose engineering, business, sciences, and so forth. Just a few select humanities (Interview, September 5).

Pedro's explanations for the students' attitudes towards learning algebra and their high performance are grounded in school practices that foster the teaching and learning of mathematics. According to Pedro, the successful implementation of these practices has contributed to a positive identity of the students as high mathematics achievers, as well as social recognition that the school is one of the leaders in the teaching of mathematics in the city. According to Pedro's narrative:

> The school has been implementing an instructional approach based on the developments of constructivism. From preschool through sixth grade, the teachers mainly use mathematical games and manipulatives that allow the students to explore the mathematics concepts. A group of teachers have also written the textbooks that we use in the different school levels. So, the students like mathematics a lot (Interview, September 5).

In addition, Pedro believes that the school's engagement in national and international mathematical competitions has allowed the students to gain mathematical experiences and to challenge themselves by battling with other students in the city and the country. In this regard, Pedro told me:

> Since the first weeks of the school year, the students start asking me "when are the math competitions this year? I want to participate!" So, see? A lot of students want to participate in the Mathematical Olympics. That has lifted the math level of the students (Interview, September 5).

Interestingly, there are not references to the students' social background as sources of explanations for their attitudes and academic performance in Pedro's narratives. As I discuss in the following section, in Pedro's view the success of the students depends on personal dispositions and attitudes as well as their previous performance rather than on social and racial factors.

Successful and Unsuccessful Students

When distinguishing between "good" and "bad" students, Pedro's narratives focus on the characteristics of outstanding students and the need to support the learners who have difficulties. He barely refers to "bad" students and when he does, Pedro alludes to possible weaknesses in the prior mathematics knowledge as the main source of difficulties for these students. Pedro describes "good" students in the following way:

> They are guys that pose questions about the topics you are teaching, participate, and catch on what one is explaining. They have a clear idea of what one is doing in class. So, these guys show you [interest], they constantly call on you "Teacher, is this [exercise] right?" so you see what they are doing (Debriefing, September 21).

There are not differences between the representations of "good" students among the three participating teachers. In Pedro's opinion, attentiveness, interest, participation, and engagement are the main characteristics of successful students. Bad students do not

necessarily lack these attributes. Pedro believes that their low performance is the result of a deficient mathematics background that hinders their chances to understand algebra. However, these students can improve with the teachers' help. As a matter of fact, Pedro does not use the adjective "bad" to describe the low achievers in his class but always refers to them as "students that need support." Pedro depicts these students as follows:

> I have seen like three or four students that could fail. They did not get good grades in the first test I gave them. Their grades were really low. And sometimes when I check their work in class, they are kind of lost. Their homework is kind of incomplete. They may have some problems with the prior mathematics knowledge that is interfering with the things we are doing. However, it is still early to say: "They are not going to pass." I want to talk with the parents. And I want to wait until the end of the school trimester to see what else I can do for them (Debriefing, September 21).

Clearly, Pedro's narratives represent "bad" students as capable of improving their learning over time. Their low performance is not the result of social or domestic deficiencies but might be the result of weak and inadequate prior mathematics understanding. The students' failure is not associated to out-school factors but to the practices that the teachers implement during their instruction. In this sense, Pedro assigns the responsibility for the students' failure to the teachers and at the same time he assumes responsibility for helping them overcome their mathematical weaknesses.

Although Pedro is careful in describing the "bad" students, he profusely characterizes what he calls "outstanding students." According to him:

> Talented student is, for instance, the guy that when you look at him, when you give him a challenging exercise, he solves it. A talented student does not necessarily get the highest grades. There are very good students that always get 10 in their tests, but they are not talented. Their grades show that they dedicate time to study and are committed. But they do not have that spark needed in mathematics. There are students who are disorganized, their tests might be messy, but they pass all the tests. And I always introduce a challenging problem in my tests, a problem that requires more effort and knowledge to be solved. A talented student can solve it, because that is a problem out of the normal; it requires a higher step [to be solved it]. Those are talented students and we need to cultivate them (Debriefing, September 21).

Again, the criteria exposed by Pedro to describe the levels of the students' mathematics achievement underline features associated to their mathematics knowledge and talent rather than to social characteristics. In contrast to Diana and Juan, Pedro's criteria do not include references to the students' location in the classroom or to their family composition. The students in Pedro's classroom sit on organized lines and each student has his\her own desk marked with the name. On some occasions, Pedro asks the students to work in small groups to solve a mathematics task. Figure 4.4[15] shows the classroom layout.

[15] The letters C and J represents the location of two students who wore the sunglass cameras.

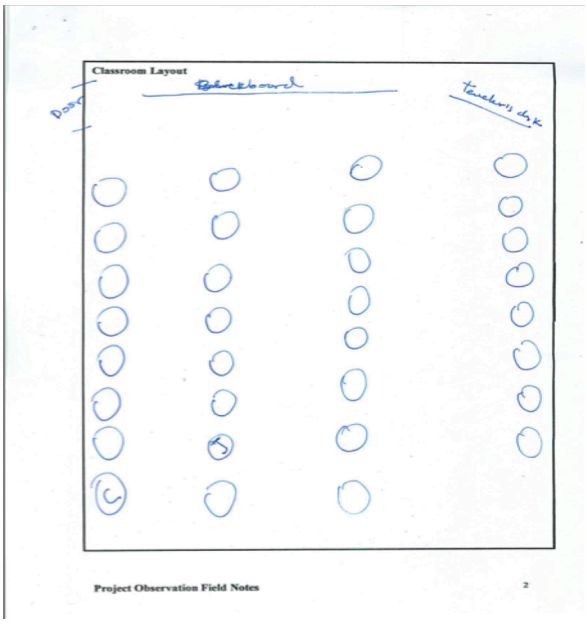

Figure 4.4. Pedro's classroom layout

As I discuss in the following section, Pedro's expectations are aligned to the previous representations of the students.

Pedro's Expectations of the Students' Ability to Learn Algebra

Accordingly with Pedro's representation of the students, he holds high expectations for his eighth graders' performance in algebra. Pedro thinks that:

> All students in my class have the same chances to pass. They like algebra a lot. I think they enjoy the class and want to learn. There are a few students that might have some problems, but [their future performance] depends on you, as the teacher. If they failed is because there was a lack of teacher's effort (Interview, September 5).

The narrative reveals an important feature. Pedro considers that the students must have multiple opportunities to pass the course and that it is the teachers' responsibility to

focus on the low achievers to help them improve. Assuming responsibility for the students' learning is a recurrent feature in his narrative as may be noticed in the story he told me:

> P: Last year, there were three seventh grades. This year there are just two eighth grades because twenty-eight students did not pass. That is a big problem. I cannot believe that you have twenty-eight bad students in a group. I think some problems must have taken place last year.
>
> L: What type of problems do you think?
>
> P: The [instructional] methods, the [instructional] methods, because in a group of students there are two or three who learn faster than the rest. But if you go just with the fastest, you are going to spoil the rest. I guess that is what happened last year (Interview, September 5).

Pedro's commitment with the students' learning is also reflected in the different extra-school activities he implements to help the students with their mathematics doubts. For instance, he told me:

> I started paying attention to four students who have failed the tests. I looked at their notebooks and I see their difficulties, they do not catch on. So, I know I have to dedicate more time to these guys. So I convened them the last Saturday. We meet at 8 a.m. and most of them came. Just one did not come and I am going to call home to find why he did not come (Interview, September 5).

Interestingly, Pedro's expectations changed when I asked him about the students in the poor public school in which he works in the afternoons. Although he maintains his view about the teachers' responsibility in student learning, his expectations lessen in relation to

these students, evidencing, in this way, the forms in which such expectations are linked to the learners' social background.

Students' Social Background as a Source of Explanations of Pedro's Expectations

I asked Pedro to compare the students in the two schools. He told me:

> The guys there [at the public school] do not have expectations. They go to school just to get ready to get a job, so they go to pass the time and get a certificate. But the guys here have high expectations. They want to get high quality education to go to a good university (Debriefing, September 21).

In this case, Pedro's narratives draw upon dominant ideologies that represent poor students as having low aspirations for the future. The students' lack of interest in education is reflected in their apathy towards schoolwork and their disinterest in going to college:

> What I would say. [In the public school] there are some students that assist for earning a certificate rather than for learning. [They think] "education is free, I am forced to attend, I have to comply with [it]." Currently, I'm teaching in two ninth grades there and when I talk with my students, I know where they are going: nowhere. If you asked them, "What do you want to do in the future?", [you would know that] most of them do not have expectations (Debriefing, September 21).

As may be noticed in the following narrative, Pedro uses the discursive strategy of introducing the exception to the rule. He recognizes some isolated examples in which his poor students are outstanding performers in mathematics:

P: There have been some students in the public school who have gone to the university and pursued doctoral degrees. One out of a thousand, a few, but they have. My question is could it be possible that the teachers are spoiling the students?

L: Why do you think so?

P: It is sad, Luz. The [Colombian] public education is sad. What happens is that the teachers in the public school say, "What a bunch of dummies, they do nothing." They despise the guys. And the principal does nothing (Debriefing, September 21).

Pedro's representations of the students are aligned to the images that Diana and Juan have elaborated for racial and class marginalized learners. However, Pedro also recognizes the role of the school nature and environment on the students' mathematics performance. In his opinion, teachers' practices matter as well.

The nature of the school seems to have an important role in the attitudes Pedro assumes regarding the students' chances to successfully learn algebra. For instance, he underscores the role of the parents at SJS in pressuring the teachers and the school for good academic results:

You cannot say to the parents, "Your child did bad on the Saber 11 test," because it is the entire school's responsibility. The parent is investing in his child's education. It is an investment and you have to show results (Interview, September 5).

In contrast to poor parents, the parents at SJS recognize the importance of education. These parents consider education as an investment or in other words, as a process that will give value to their children in the future ensuring the maintenance of the social positions and privileges they have already reached. In this regard, they are willing to pay a considerable

amount of money on a monthly basis for high quality education that is only possible in a private school.

In short, Pedro holds two different types of expectations for the students clearly linked to their social background. In the previous narrative it is worth noticing the presence of features of the social deficit frame also used by Diana and Juan to explain and justify the marginalized students' performance in algebra. Pedro's lack of references to the social background of the students at SJS may be due to the homogeneous social and racial compositions of the group and their belonging to a particular class that, in ideological terms, fulfill the requirements to be successful at school, as he told me:

> Here [at SJS] the students have all the resources they need [to study]. They cannot complain about lacking something because they have everything, whereas in the public school, they have nothing (Interview, September 5).

In this regard, and insofar as it is not possible to appeal to social and cultural deficits to explain the students' failure, both the school and the teachers are accountable for the students' learning.

Summary of Pedro's Expectations of the Students' Ability to Learn Algebra

Pedro's expectations for the SJS students are aligned to dominant ideologies about middle and upper middle class students' performance at school. These ideologies position these students as capable of both learning and school success. They are depicted as motivated, attentive, dedicated, and committed to high social aspirations. They and their families demand excellent education from the school to the extent that the students need to gain access to good universities. In this regard, Pedro feels accountable for the students'

performance at school. In fact, both teachers and school need to guarantee the resources and environments to foster students' learning.

When describing his expectations for poor and working-class students who attend public schools, Pedro appeals to social deficit frames to explain their school failure as Juan and Diana do. He also introduces the discursive strategy of blaming the victim conveying in this way a negative representation of the students. However, this strategy is ameliorated by his consideration about the role of the teachers in either fostering or hindering the students' learning.

Interestingly, Pedro's narratives shed light on the role of institutional contexts in framing the expectations of the students' performance. The public-private nature of the schools as well as their organizations are underlined as factors that shape the ways wherein anticipations about the students' academic success are elaborated and justified. This is an important feature that I analyze later.

Comparing the Participating Teachers' Expectations

The analysis of the narratives and representations of the participating teachers sheds light on the ways wherein the students' social backgrounds permeate and shape expectations. Hegemonic ideologies about poor and racial and ethnic minority students manifest in the teachers' narratives and nurture the expectations they have elaborated about their pupils. Low-income and black and indigenous students were represented as more likely to fail in their process of learning algebra whereas middle and upper middle and mestizo students were depicted as having higher chances to succeed. In this regard, the findings draw attention to the nature and sources of the teachers' representations about school failure and success in

their relation to social categories such as race and class and coincide with the results of other studies that have approached these issues (Lim, 2008; Zevenbergen, 2003).

Two frameworks were identified as the main sources nurturing these representations. The frameworks serve the teachers in the purpose of justifying and explaining such representations as well. In addition, two discursive strategies emerged from the teachers' narratives as a way to convey their representations of the students. The discursive strategies were introduced to organize and structure biased conversations about poor and minority students without giving a negative impression to the listener. Thus, the strategies were aimed to present a negative representation of the students and their families and a positive representation of the teachers. Frameworks and discursive strategies are vital components of hegemonic ideologies (Apple, 1990; Bonilla-Silva, 2010) that contribute to naturalize the failure of marginalized students and the success of their wealthier and mestizo peers. In the following paragraphs, I describe both the frameworks and discursive strategies in relation to the teachers' expectations.

Frameworks

Cultural and class deficit frameworks emerged from the teachers' narratives as the dominant sources for the representations of their students. Consistently, poor and racial and ethnic minority students and their families were depicted as holding a set of values that hinder their chances of school success. Hard work, responsibility, appreciation of education and ambition, were proposed as fundamental values that teachers did not recognize in their students and that helped them explain the failure in the learning of algebra. Likewise, the

teachers represented low-income and black and indigenous parents as lacking commitment with the children education, ignorant, and absent in their children's lives.

A *cultural deficit framework* embodies dominant images of cultural minority pupils. This cultural racism (Bonilla-Silva, 2010) or ethnicism (Van Dijk, 2004) depicts black and indigenous students as culturally inferiors due to their lack of appreciation for certain ways of being and acting highly valued in the Western world. Racial and ethnic minority students were represented as possessing other forms of rationality unaligned to Western ways of thinking that favor thought over action, mind over body, and literacy over orality, as manifested in Juan's and Diana's narratives. According to the cultural deficit framework, black and indigenous students appreciate other forms of knowledge and living that, although important, do not guarantee academic success, and in this sense, are worthless.

In particular, the dominant representation of mathematics in the Western part of the world seems to leave minority students without many opportunities to learn this subject. In broadly accepted characterizations of mathematics, "abstract is valued over concrete, formal over informal, objective over subjective, justification over discovery, rationality over intuition, reason over emotion, general over particular, theory over practice, the work of the brain over the work of the hand, and so on" (Ernest, 1991, p. 259). Bishop (2001) also remarks rationalism, objectivity, control, progress, openness and mystery as the main values associated with Western mathematics. According to hegemonic ideologies, these are the values that black and indigenous students either lack or disregard and therefore, it is possible to anticipate their failure in learning mathematics. The chances of academic success seem to lessen when the subject to be learned is algebra. If black and indigenous students mainly

value orality over literacy, the learning of a written symbolic system such as algebra (Bosch, 1994), arises as problematic if not impossible.

A *class deficit framework* comprises dominant stereotypes of poor students. This framework depicts economically disadvantaged learners as anti-work, anti-school, anti-family and therefore, anti-success (McNamee & Miller, 2004). The participating teachers privilege certain values and norms such as hard work, ambition, perseverance, family harmony, and individual merit that usually correspond to middle-class values and dispositions (Rist, 1970; Zevenbergen & Niesche, 2000) and that poor students are less likely to possess (Lim, 2008). In this sense, the teachers use values, dispositions, and attitudes that most closely reflect the culture of the middle class (Giroux, 1981) and that when compared with the values, dispositions, and attitudes of poor students, reinforces the stereotypes and negative images about this student population.

The participating teachers consistently used the cultural and class deficit frameworks to explain and justify their expectations about the students' learning of algebra. There were not differences about the negative representations of and the low expectations about poor and black students among the teachers. The noticeable silence about poverty and race in Pedro's narratives might be explained by the fact that the students in SJS are closer to the representations held by the teachers about successful students. They are middle class and mestizos, and therefore their values are aligned to the attitudes and behaviors required for academic success. The presence in the teachers' narratives of cultural and class deficit views aligned to hegemonic ideologies about black, indigenous, and poor people reflects the ways

wherein student racial and class backgrounds permeate teacher expectations. In this sense, the teachers hold lower expectations for poor, black, and indigenous students.

Discursive Strategies

Across the three social contexts, the participating teachers used two discursive strategies to talk about poor and racial and ethnic minority students without appearing as racist or biased. The first discursive strategy consists in assigning responsibility to the students for their own academic failure. The parents were portrayed as accountable for this failure as well. The discursive strategy more frequently emerged in the narratives of the teachers of the poorest students. They often appealed to blaming the students for the lack of attitudes and dispositions needed to succeed in school. The discursive strategy was also present in Pedro's narratives although combined with the assumption that the teachers are also responsible for the students' failure. In contrast to Diana and Juan, Pedro recognizes that teachers may have an effect on student learning, a possibility that is completely absent in the narratives of the other two teachers. In fact, both teachers blame the students and parents for the failure in their teaching methods and, in this regard, they free themselves from any participation in such failure.

The second discursive strategy consists in lessening the impact of a biased statement about a particular student population by presenting a counter example. The teachers minimize the impact of stereotypical images of poor and minority students by presenting an exception to this representation. In this sense, implicitly, the message conveyed is that this exception seems to confirm the stereotype. This discursive strategy was mainly found in Juan's and Pedro's narratives. They would state a negative and biased representation of, for

instance, poor students and promptly would appeal to a specific story of a poor friend or student who does not fall into the general statement they made. Interestingly, the discursive strategy, beyond avoiding a negative representation of Pedro and Juan, allows them to divert the responsibility of the students' failure from the teaching practices. The success of a few black and poor students provides evidence that the causes of failure must be found in individual attitudes and dispositions.

Van Dijk (2004) spotlights the role of discourses not only in the communication and reproduction of discrimination but also in the formation of racist beliefs. In fact, discriminatory discourses as expressed in the narratives of the participating teachers are, according to this researcher, forms of discriminatory practices. However, the teachers introduce the previous discourse strategies to avoid a negative impression as racist or discriminators. Van Dijk (1992) calls these strategies as *denials* and are primarily aimed to transfer to others the responsibility for the current state of racial and class matters. In particular, van Dijk argues that denials play a fundamental sociopolitical function in society to the extent that they challenge the existence of racism and classism and therefore there is no need of intervention to attack these social issues. They just do not exist. In the case of the participating teachers, blaming the students for their failure and highlighting the few successful minority students would hinder the discriminatory role of school and teaching practices and challenge the need of transforming such practices.

The emerging frameworks and discursive strategies assist the teachers in the organization of certain aspects of the school dynamics. In this particular case, these elements, as manifestation of hegemonic ideologies, enable the teachers to present themselves as

neutral participants in the configuration of poor and minority students' low mathematics performance and academic failure. Apple (1990) underlines this role of hegemonic ideologies as saturating the forms in which naturalization of discrimination occurs in society.

It is worth noticing that these findings are aligned with the result of other studies that have pointed out the close relationship between teacher expectations and student social background (Boaler, Altendorff, & Kent, 2011; Frankestein, 1995; Hoadley, 2007; Lim, 2008; Ogbu, 1988; Reyes & Stanic, 1988). They confirm that teachers' expectations about the students' ability to learn mathematics embody and enact comprehensions of the social world, individuals, and social practices that transcend school.

Teaching Practices

The second section of this chapter addresses the research question: *How are these teacher expectations expressed in their teaching practices?* In order to investigate this question, I analyzed the nature of the teachers' practices that focused on helping the students build meaning for algebra. I approach instructional and discursive practices as implemented in the classroom and the consequences of such practices for the meaningful understanding of algebra by the students. I coded the data into two major themes outlined in Table 4.2 to capture the teachers' practices for teaching algebra.

Table 4.2

Teaching practices category and themes

Categories		Themes	
Teaching Practices	Instructional strategies implemented by the teachers to build meaning of algebraic objects and procedures. Practices implemented by the teachers to assess the students' learning process of algebra.	**Meaning of Algebra**	Teachers' instructional and discursive practices aimed to help students build meaning for algebraic concepts and procedures. Teachers' ways of defining and characterizing algebraic concepts.
		Assessment Practices	Strategies to assess student learning of algebra. Goals of assessment practices.

Diana's Teaching Practices

Two of Diana's practices interact to shape the meanings Diana's students develop in her classroom. The first one relates to her selection of real world contexts as sources to help the students build meaning for algebraic objects and procedures. The second practice is discursive and relates to the language employed to define and characterize objects and procedures. In the following paragraphs I introduce these practices and discuss their impact on the students' learning of algebra.

Making Sense of Algebraic Objects and Procedures

The first characteristic of Diana's teaching practice relates to the contexts she chooses in order to signify objects and procedures. Real and familiar contexts are important sources for fostering students' opportunities to build meaning of mathematics objects and to use them

as tools for exploring and understanding the social world. However, contexts must be appropriated to foster the understanding of the mathematics concepts to be taught and to produce the desired effects on students' learning. Contexts must be carefully selected by the teacher for mathematical understanding to take place.

In Diana's algebra lessons, real world contexts are not used to design meaningful tasks to support the students' construction of algebraic meaning and the development of the students' algebraic thinking. Instead, Diana constantly appeals to situations that are apparently close to students' experiences and interests in order to help them learn rote procedures such as mastering calculations with algebraic symbols. Real and familiar contexts are introduced as mnemonic devices to carry out arithmetic algorithms with integers and fractions.

This practice is noticeable in her efforts to provide meaning for the calculation of integers with different signs. Diana systematically brings in either "to owe" or "to own" situations to help the students make sense of additive situations in the set of the integers. She explains that the negative numbers stand for money and objects they owe and the positive ones for what they own or pay for. During a class episode, the students were practicing like terms reduction. One of the exercises involved simplifying $5x - 10x$. After asking the students for the response and getting no answer, she explains:

> Remember that positive numbers are things that we own and negative numbers are things that we owe. So, if I own five and I owe 10, do I owe [something]? Do I own [something]? How much [do I own or owe]? I owe 5, and then I write the $\boldsymbol{x}$. So, the answer is minus five $\boldsymbol{x}$ (Lesson, September 16).

Although using these types of contexts is a widely-accepted strategy found in textbooks and employed by teachers to model additive situations with integers, it may be a source of confusion for students when it involves an algebraic expression, as in this class episode. Even though the context "to owe-to own" makes sense for the calculation between the quantities, the conceptual issue arises when she introduces such context to transform algebraic expressions ignoring what they represent. As a matter of fact, Diana overlooks the mathematical relationship depicted by this expression. Diana disregards the relationship between the coefficients and the variable and introduces a context that does not make sense for carrying out the subtraction of the algebraic terms. The calculation of integers is brought to the front while the variable seems to be an accessory that appears after the operation has been performed. Further, while students were practicing the simplification of like terms, she explains that the result of $-0.4c + 0.4c$ is 0 because:

> You owe zero point four and pay zero point four, so you get nothing (Lesson, September 16).

Besides ignoring the mathematical relationship depicted by the algebraic expression, the context would hardly help students make sense of the operation, especially because there are not cents in Colombia currency. Diana's students would not find this type of situation in their daily lives and therefore, the context is neither mathematically relevant nor familiar to the students. Hence, the context Diana has introduced works as a mnemonic device and as such, whose main function is to help the students remember how to deal with positive and negative numbers regardless of the numerical systems in which they belong. Rather than building meaning for operations upon the numerical systems properties, Diana introduces a

device that would help the students remember how to carry out such operations, although it does take out the mathematics characteristics of the algebraic procedures. In this sense, conceptual understanding is replaced with rote procedures whose meaning arises from not mathematically relevant contexts.

Diana also uses real life and familiar contexts to help students build meaning for algebraic objects. This is the case for the notion of variable. In an effort to help her students understand this notion, Diana introduces contexts that she thinks will appeal to students. Yet, they could hinder the mathematics meanings of this core algebraic object. For example, Diana uses a set of hearts to represent the notion of variable as shown in the following class episode:

(Diana draws a set of five hearts on the blackboard)

D: Here we have a set, and because we are always in love, we have little hearts. This is a set of what?

S: (*Students answering in chorus*) Hearts!

D: (*She writes "hearts" on the blackboard*) How many hearts do we see here?

S: Five!

D: We have five hearts, don't we? (*She writes "five hearts" on the blackboard*). However, writing "five hearts" takes forever, so we can express it with the number five and a little heart that it is the symbol (*She writes the number five and draws a heart*). But, what [would happen] if instead of a heart, we had a butterfly? It would take too much time to write "butterfly," so we can replace the heart with a letter. Which letter?

S: the [letter] ***h***!

S: The [letter] ***m***!

D: Ok, let us write the [letter] ***m***. What is the meaning of that [letter] ***m***?

S: Heart!

D: Right, heart (*She writes the word "heart" on the top of its drawing*). Look, here we have started algebra. It is as simple as this. The letters are going to replace unknown and known objects, things that we may know or not. Let us observe that after having a set of five hearts, we have got "five ***m***", where the letter ***m*** is representing an object called "heart" (Lesson, August 22).

As may be inferred from Diana's utterance in the first line, the context is selected under the premise that it may attract the students' attention due to Diana's belief that the students may be in love. She selects a context that responds to her perceptions of students' interests. However, taken into account the mathematical definition of variable, the context is mathematically irrelevant to introduce such a notion. Rather than a quantity that varies in a numerical range[16], the variable is introduced as a letter that stands for a material object. In this particular case, the status of the variable is that of a letter that depicts an object. The variable is not characterized as a mathematics object that makes sense in the context of mathematics theories but it is objectified as a letter that can be easily manipulated. The letter representing the variable is an abbreviation or *label* (Philipp, 1992) whose introduction avoids the students writing long sentences. In addition, the mathematical relationship represented by the algebraic expression 5***m*** is completely ignored. The process through

[16] I am considering variables in the field of real numbers.

which the expression is built does not focus on the multiplicative relation between the coefficient and the variable but rather on an economy of words and as a result, conceptual understating is sacrificed.

Diana implements a process of objectification in her teaching consisting in turning algebraic objects into objects of the sensorial experience. As a systematic practice taken place in Diana's classroom, this objectification process fulfills a two-fold purpose, as may be noticed in the following class episode:

> *(Diana is explaining to the class the procedure carried by one of her students to simplify the algebraic expression* $7a - 9b + 6a - 4b$).
>
> Let us remember what we did last class. We explained like terms. Like terms are those that have the same letter and the same exponent. This one has the letter ***a*** *(pointing to* $\mathbf{7a}$*),* and that one also has the letter ***a*** (*pointing to* $\mathbf{6a}$), so I can put them together and get one answer. Seven plus six [equals] thirteen accompanied with the letter ***a***. So, this little piece here (*pointing to* $\mathbf{13a}$) is right. Now, I have minus nine ***b***, and I have another one that also has the [letter] ***b*** that is minus four ***b***. Ok. Remember that the sign minus is like money I owe. So, we owe nine *(pointing to the number -9*) and let us make sense of it [-9$\boldsymbol{b}-4\boldsymbol{b}$], [let us say] nine balloons minus four balloons, I owe nine balloons and four balloons, how many balloons do I owe? Thirteen; and that is the final answer. The answer to this exercise is thirteen ***a*** minus thirteen ***b*** (Lesson, September 9).

The first purpose relates to the need of providing a context that would help students understand the algebraic expression. In order to make sense of the notion of variable, Diana

replaces it with a non-mathematical object, a balloon, and, in this regard, the variable *is* the object represented by a letter instead of a mathematical entity. The second purpose relates to the need of helping the students operate with algebraic expressions. Turning the variable into an object provides Diana with a tool to explain to the students a process for simplying like terms. The final purpose of reifying the variable is to ease the students' challenges in dealing with symbolic manipulation, and hence, it is also clear that Diana's emphasis is on teaching rote procedures with algebraic symbols rather than building conceptual understanding of the algebraic objects. This purpose is clear in the following class episode that took place in the classroom:

> (*Diana asks the students to simplify the algebraic expression* $6a^{x+1} + 8a^{x+1}$*. After one of the students writes* $14a^{x+1}$*, Diana explains*:)
>
> Six and eight [equals] fourteen and we repeat this part here (*pointing to* $\boldsymbol{a}^{x+1}$). Don't be worried if there is an ugly doodle [here] (*pointing to* $\boldsymbol{a}^{x+1}$). If this one (*pointing to the variable in* $6\boldsymbol{a}^{x+1}$) is similar to that one (*pointing to the variable in* $8\boldsymbol{a}^{x+1}$), we write the same [variable]. There is no need to worry [about the variable] (Lesson, September 5).

In short, the process of objectification proceeds by turning the variable into a non-mathematical object represented by a letter, and treating the letter as an object. Clearly, the variable turns into a "thing" that accompanies the coefficient without both representing any mathematical entity and involving a mathematical relationship. The objectification of the notion of variable is a practice used by Diana to make the manipulation of the algebraic objects and their meaning accessible to the students' comprehension.

The Algebraic Language and the Construction of Algebraic Objects

Diana's second practice relates to the language she uses to define and depict algebraic objects. Diana's language appeals to a process of building comprehension of the algebraic objects upon their visual features. To the extent that the algebraic objects have been approached as material entities deprived of their mathematics properties, Diana relies on visual features to describe and define them. For instance, she often refers to the variable as the letter that accompanies the number in a term. In a class episode in which she was explaining the degree of an algebraic expression, she tells the students:

> Let us remember that an exponent is the little number on the top of either the letter or the coefficient (Lesson, August 26).

The language used to characterize the algebraic objects is not grounded in mathematics theories but rather is elaborated to help students remember the object by a visual characteristic. In this sense, the object is devoid of mathematical meaning. Diana's use of informal language to portray algebraic objects is also noticeable. The following class episode shows the way in which Diana's language is used to characterize algebraic objects:

> *(Diana is simplifying an algebraic expression that includes adding* $\boldsymbol{m}^2 + 3\boldsymbol{m}^2$. *One student does not understand why she says "one* $\boldsymbol{m}$ *to the second power if there is not a number one in* $\boldsymbol{m}^2$. *Her explanation goes on as follows).*
>
> D: *(Lifting up the marker she holds in her hand),* how many markers do I have here?
>
> J: One
>
> D: And do you see a [number] one in front of the marker?
>
> J: No, I don't.

D: How many Juan Gómez *(the student's name)* are there in this classroom?

J: [there is] just one, me!

D: And do you see a [number] one in front of you?

J: No

D: So, that is the reason $\boldsymbol{m}^2$ does not have a [number] one in front of it. We do not write it (Lesson, September 2).

The justification for the absence of the number one as a coefficient in algebraic terms does not rely on mathematical reasons and explanations; instead, it involves drawing upon the objectification of the algebraic objects to appeal to visual features as if they had a material existence. In the absence of mathematical language to construct an accurate explanation for the mathematical phenomenon, Diana asks the student to "see" the number one as if it were an object of the material world accessible through the senses and treats it as such. It is worth noticing the context in which the class episode takes place. Diana is responding to a student's request for clarifying the meaning of an algebraic term, and in this regard, the episode reflects Diana's attempt to make the expression comprehensible for the student. The way in which Diana faces the student's question conveys the message of incapacity of mathematical understanding by the student and the need of appealing to informal language and extra mathematical contexts. In this regard, the construction of mathematical meaning for an algebraic object by the student is denied by the teacher's practice.

In summary, the algebraic knowledge taught by Diana acquires a particular form comprising two characteristics. First, the algebraic knowledge is not the result of systematic

examination of properties through, for instance, the exploration of patterns, the solution of word problems, or the modeling of a set of problems. Instead, the algebraic knowledge is the product of memorization of definitions and repetition of rote procedures grounded in a variety of daily contexts that accomplish a mnemonic function. Second, algebraic objects, such as variable, are treated as material objects rather than mathematics entities that must be characterized in the context of mathematical theories. In fact, the characterization of algebraic objects is not situated in mathematical concept and meaning, which are absent in Diana's lessons. They are described by their visual appearance using a language that leaves aside their mathematical attributes. As a result of these practices, the algebraic knowledge in Diana's classroom has been deprived from its mathematical meaning, its conceptual foundation, and reduced to non-sense rules to calculate with algebraic expressions. I argue that these practices are mediated by the teacher's beliefs that the group of students is not able to learn algebra, so she needs to reduce the level of complexity to make it accessible to the students.

Understanding Diana's Stance towards her Presentation of the Algebraic Knowledge

The practices I have described are directed towards motivating the students not by posing challenging and interesting problems to build meaning but to attract and keep their attention by using contexts that may be appealing to them. These practices are rooted in Diana's beliefs about the students' difficulties to learn algebra. This is worth noticing in Diana's narratives about her students' difficulties:

L: What are the most challenging situations you face when teaching algebra?

D: Teaching the negative numbers. It is [a topic] really, really, really hard [for the students]. And when one brings in the negative numbers, they feel, I feel they are not ready, they are never ready.

L: How do you deal with this situation?

D: I try to do it in a simple way, I mean, I try to make it easier for the students to catch on. And I try really hard that they understand it.

L: How do you do it?

D: I try to make it clear; when they do not understand, I want them to ask me and we will repeat and repeat until, I will repeat it in different ways, in the simplest way. I try to use a very understandable vocabulary to facilitate their comprehension. And I also look at [the work of] every individual student to make sure that they understand (Interview, August 20).

Diana's use of informal language is intentional and aimed to help the students understand a topic she believes is difficult for them. Diana assigns to the students the entire responsibility for their lack of understanding and selects an approach to deal with their difficulties that sacrificed the quality of the algebraic knowledge to be learned. She opts for practices that deprive the algebraic knowledge from its cognitive and mathematical complexity such as repeating procedures, using "a very understandable vocabulary," and avoiding the challenges of its learning.

Reflecting about the criteria Diana used to select the examples of the set of hearts when introducing the notion of variable, she again appeals to the students' difficulties in learning this concept, as may be noticed in the following narrative:

L: Tell me about the criteria you used to select the examples for that class. What were your goals?

D: The goal [of the activity] was that they began to relate themselves with what algebra is because that was the first time we would talk about numbers and letters, because we had seen just numbers. I wanted them to understand that there are letters, and that those letters can be added; I mean, had I started with numbers and letters, that would have been something horrible for them. They have a lot of problems understanding the notion of variable, you know. For instance, it is really difficult for them to understand why I can add $2\boldsymbol{a}$ with $\sqrt{3}\boldsymbol{a}$. It is really difficult for them to understand that the variable $\boldsymbol{a}$ can turn into any object, any object. So, it is really hard for them. The last year there was a group that I explained in different ways and, my God! There were children that did not understand. So, this year I said: "I am going to make it in a slow, slow form", but you know, trying to make sure that that little piece [understanding the notion of variable], they are understanding it (Debriefing, September 3).

One could think that her references to students' difficulties encompass the general population of eighth graders or, in other words, that learning the notion of variable is a complex endeavor for all students. However, in her narratives, the students she refers to are the students of HMS, as may be inferred from her allusions to the group of students she taught the previous school year. In this sense, the narrative about the students' difficulties for learning the notion of variable is grounded in Diana's general expectation of their ability to learn algebra. Her repetitive and persistent use of words to describe her instructional

strategies, such as “slow,” “repeat,” “clear,” “easy,” “simply,” and “understandable,” as well as her words to describe the students’ process of learning, such as “hard,” “difficult,” “unready to learn,” and “horrible,” express the way in which her expectations of the students’ ability to learn algebra permeate and shape her practices to help students build algebraic meaning and develop algebraic thinking. The form acquired by the algebraic knowledge in Diana’s class is not casual, but rather the result of a series of practices mediated by her beliefs about the students’ ability to learn algebra. These practices are aimed to facilitate the learning process by reducing the quality of the algebraic knowledge.

Diana’s Assessment Practices

According to Diana, her main goal of assessment is to ensure that the students have learned the concepts and procedures previously taught. The following narrative describes her assessment method:

> Well, I try to pose exercises similar to those we have solved in class and in the worksheets (Interview, August 20).

The similarity between the exercises solved in class and those posed on the tests and quizzes reveals Diana’s association between learning and memorization. It seems that for Diana, the purpose of assessing students’ learning is to determine if they remember the procedures that Diana has introduced.

Diana implements several practices to assess her students’ learning of algebra. For examples she administers quizzes, examines homework every day, and asks the students to solve exercises on the whiteboard. The last two practices consume a considerable amount of the class time. Regarding the practice of examining homework, Diana takes the time to check

the task of each student in the classroom. She would walk across the classroom and stop by each student's desk to ensure that he/she did the homework. I asked her to explain the importance she assigned to this practice:

> They are not aware of the importance of homework. The [main goal of] homework is learning; they come to the school to learn. However, if I did not grade the homework, they would not complete it, the homework would be insignificant for them. So, I use the homework to help them acquire pre-concepts and to give them a grade. They have to complete homework. Otherwise, they will get a low grade (Interview, August 20).

According to this narrative, the goal of assigning homework every day is two-fold. First, the homework is assigned to help the students learn mathematics by exploring new topics or developing skills through drills. What Diana calls "pre-concepts" are the topics she is going to introduce in the following lesson, and in this sense, the homework mainly consists of looking for the definition of concepts or the steps of a given algebraic procedure. Sometimes, Diana also assigns a homework worksheet with a large number of exercises to be solved. Second, the homework also fulfills a normative role to the extent that it is used to raise awareness about the value of education in students' lives. Homework is a means to create consciousness among the students about the importance of learning. However, because students lack responsibility and awareness about the value of doing homework, they need to be "motivated" by grading it. This view becomes clear in the following narrative:

> When I taught at the university I did not do this, but when I started teaching here the students did not do homework. I asked, "Who did the homework?" and just one, two, three students raised their hands, the usual students. So, I decided to grade homework.

> I began giving them a five (the highest grade) per every five homeworks completed in the notebook. So, they started doing the homework although some of them do not do it anyway (Debriefing, September 17).

In this sense, the formative function of assessment is lost and Diana recognizes this fact:

> I started checking if each exercise was right, but the process took me forever. So, what I do now is, I give them a mark [when they do the homework] and I put the exercises on the whiteboard and in this way they can check their mistakes. I say "it is important for me that you at least tried to do the homework no matter if is right or wrong," so I value their effort (Debriefing, September 3).

The students comply with the homework Diana gives them because it is a way to get better grades. The students understand this fact as demonstrated by the following class episode from my field notes:

> Diana starts the class checking the homework of each student. María, one of the students who wears a sunglasses camera, suddenly jumps up from her desk and approaches one of the "good" students. She sits near to the student and starts copying the homework while watching Diana's steps to avoid being caught. When finished, María quickly goes back to her desk just in time to get her homework checked. Diana examines María's notebook and gives her a five. María compares her grade with the grade of the student whose homework she copied and bursts out laughing. Diana gave a lower grade to the student who had originally completed the homework (Field notes, September 16).

In fact, María is not the only student cheating. This is wide-spread practice among the students who share their notebooks to get better grades. When I showed Diana this episode as recorded in María's camera, she told me:

> The goal of the homework is not reached by those students, I mean, the goal of acquiring the pre-concepts. I will keep grading the homework because that means appreciating the hard work that some of them are doing. I know that some students are cheating; I have seen that. And I know who are the students who lend their notebooks. But I am not able to give them a low grade because those are the good students, the students who comply with. So, I just write a note saying "do not lend your notebook." However, many, many times, I do not know they are cheating. However, if just a few do homework and are aware, I'm doing something for them (Debriefing, September 17).

Diana also frequently calls on each student to the whiteboard to solve exercises. Sometimes she spends the entire class time doing this. She tells the students:

> I give three to the student who comes to the whiteboard no matter what she does. The student who not comes to the whiteboard gets one (the lowest grade) (Lesson, September 5).

Clearly, this assessment practice is aimed to neither provide feedback about the effectiveness of Diana's teaching practices nor improve the students' learning. What Diana has set up is a fiction in which she pretends to assess the students' learning and the students pretend that they are, in fact, learning. However, this does not actually happen. The fiction around the process of assessment becomes clear in the following narrative:

D: I do not care about giving them high grades when they come to the whiteboard or when I check their homework. Despite this help, a lot of students fail. Look at this girl *(showing me her grades)*. Her final grade in the trimester was 2.7. She got high grades in doing homework and solving exercises on the whiteboard. But she failed the quizzes, so despite the facilities I give them, they fail because they are incapable or they do not care. The students who pass are the students who care. If I did not do what I do, the majority of the students would fail. So, I need to help them pass.

L: What do you mean? I do not understand.

D: I need to help them pass because just 10% of the students can fail mathematics. It is a school policy. So, I need them to pass. Can you imagine what would happen if I did not do this? Most of them would fail (Debriefing, September 17).

Two different elements shape Diana's assessment practices. First, her representations of the students according to which they are incapable of learning and disinterested. Despite the multiple opportunities the students have to pass the course and the easiness of her assessment practices, they either fail or get low grades. In this regard, the self-fulfilling prophecy takes place in the form of the students' low grades reinforcing not only Diana's representation and low expectations about the students' ability to learn algebra. Second, an institutional policy contributes to shape the assessment practices Diana implements. The restrictive policy imposes a limit in the number of students who can fail mathematics and therefore, teachers are accountable not for students' learning but for ensuring they will "pass." Assessment is not formative. It is not aimed to provide feedback of the state of the students' learning. Instead, Diana's assessment practices are intended to help the students

pass no matter whether learning is taking place or not. Moreover, the students receive grades for getting involved in non-mathematics activities as the following class episode reveals:

> The class ended up early as usual. Diana uses a few minutes to recall the students that there is an easy way to improve their grades: Those who want to raise their final mathematics grade can do it by recycling. They must bring plastic bottles and bags, broken glass, and cardboard to the school to contribute to the school recycling project. Then, she will give them a grade depending on the amount of recycling they bring. The students approach her to know more about this opportunity. (Field notes, August 22).

Juan's Teaching Practices

Two practices interact to shape the form and meaning acquired by the algebraic knowledge as taught in Juan's classroom. The first practice consists in using arithmetic procedures to carry out algebraic operations and to help the students make sense of them. The second practice is discursive and primarily consists on describing algebraic procedures and objects by using an informal language that is closer to the language used by students in their daily lives. I analyze these practices in the following paragraphs.

Making Sense of Algebraic Objects and Procedures

In contrast to Diana's practices, references to real and familiar contexts as sources of meaning for algebraic objects and procedures are absent in Juan's instruction. In the classes I observed, Juan mainly focused on the presentation and explanation of rules to operate with algebraic expressions. The teacher provided the students with step-by-step techniques aimed to facilitate the mastering of procedures to add, multiply, subtract and divide algebraic

expressions. Juan heavily relies on the techniques as stated by several textbooks he uses as guides for his instruction, and in this sense, the main sources to build meaning for algebraic objects and procedures are the textbooks rules.

The teacher systematically implements this practice of providing the students with guidelines to deal with algebraic symbols, as illustrated in the following class episode. After dictating the rules to expand algebraic expressions, Juan proceeds to exemplify the technique to do so by solving the following exercise on the whiteboard:

$$-\{15a^2 + ab^2 - [3ab - 27a^2 + (-7a^2 + 15ab + 2ab^2) - a^2]\}$$

> First, we start removing the parentheses because they are inside of the other two [types of brackets]. And, this is really important, do not forget to take into account the sign in front of the brackets. If you would not do it, you will get a wrong answer. So, because the sign [in front of the parentheses] is plus, all the terms keep the same sign. (*Juan proceeds removing the parentheses*). Now, we continue removing the square brackets. Look! In this case, there is a minus [sign] in front of the box bracket, so you have to change the signs of the enclosed terms. If they are positive [terms], write them negative. If they are negative [terms], write them positive. (*Juan proceeds removing the square brackets and changing the signs of the terms*). We have now the last brackets, the braces. Again, it has a minus [sign] in front of it, so we have to change the signs of all the enclosed terms (Lesson, August 14).

Juan closely follows the steps as proposed in the textbook and is careful in associating his moves when expanding the algebraic expression with every step he has previously dictated to the students. In this process, it is worth noticing the lack of mathematics

explanations in Juan's discourse that support the steps involved in the procedure. Juan does not ground mathematically the process involved in expanding algebraic expressions. Instead, changing signs is presented as an additional step in the whole process of expanding the algebraic expression and, in this regard, Juan introduces the rule as a rote device to help the students successfully carry out the procedure in the future.

Later on in the class episode and after removing the brackets, Juan reminds the students how to reduce like terms:

> Now, we need to reduce like terms. Let us take the positive [terms] in one side and the negative [terms] in the other side, provided that they are alike. (*Juan continues circling the terms whose variable is* a^2 *that are all negative and organizes them in a column as is shown in figure 4. He carries on with the procedure by adding the numbers in a process that resembles the addition of natural numbers. He, then, finds the sum of these numbers, writes the variable next to the sum, and finally writes the minus sign getting* $-50a^2$ *as the final answer* (Lesson, August 16).

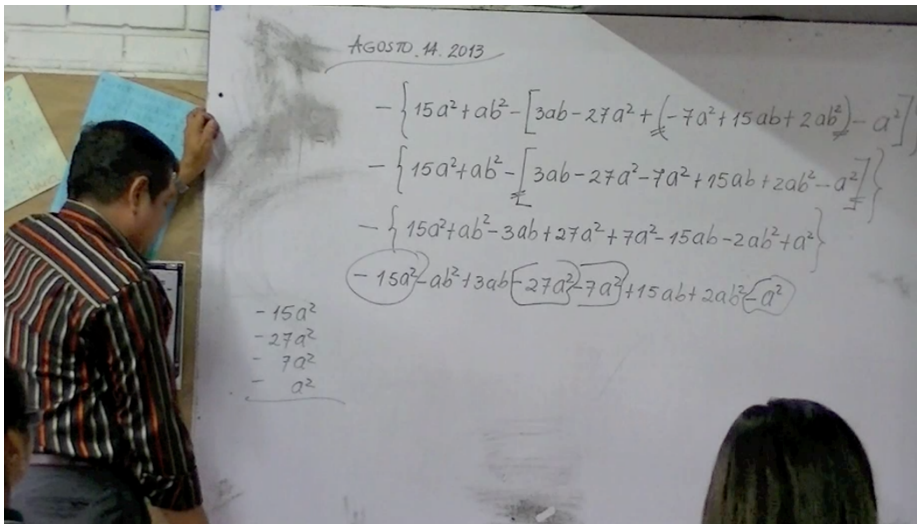

Figure 4.5. Vertical addition of algebraic expressions

Juan has turned an algebraic procedure into an arithmetic one. In order to calculate with negative algebraic expressions, he brings up a procedure that resembles the algorithm to add natural numbers but that it is not mathematically accurate in the context of the whole number set. The practice is aimed to build understanding of the algebraic procedure upon the students' previous arithmetic experiences acting as an instructional aid for allowing the learners to deal with the manipulation of algebraic symbols. This is clear in a class episode that took place later.

> *(After dictating the steps to multiply polynomials, Juan goes on exemplifying the procedure).*
>
> Let us multiply $(1/4\,m - 2/5\,n)$ by $(2/3\,m + 1/4\,n)$. Let us try this: let us write [the terms] in vertical form to make it easier. It may be easier for you to make it in this way (Lesson, August 20).

In the process of teaching the procedure, Juan privileges the vertical representation of multiplication used in arithmetic over the horizontal one. As he explains to the students, the goal of this change is to make the procedure accessible to the learners' comprehension by avoiding them facing the complexity of applying mathematics properties. Juan tries to take advantage of the students' arithmetic experiences to build the new algebraic knowledge. And although building knowledge upon students' previous mathematics experiences is a desirable teaching practice, the problem arises from the method employed by Juan to do so; this is because both the conceptual understanding of the procedure as well as its mathematical accurateness may disappear in the process of easing the students' learning. During the time I observed Juan's classes, his instruction about multiplying and adding algebraic expressions

as described above did not evolve towards algebraic procedures but exclusively relied on the arithmetic operations. The transition from the arithmetic procedures towards algebraic ones did not take place.

Trying to facilitate the students' understanding of the algebraic procedures, Juan also introduces some features to avoid the use of—what would be for him—complex mathematics properties. For instance, Juan does not use the distributive property to explain the multiplication of polynomials. Instead, he introduces a gesture to replace the use of this property that consists in linking each term of the first factor in the multiplication with each term of the second factor by using an arrow, as shown in Figure 4.6. By using these practices, Juan attempts to facilitate his students' learning, although in the process conceptual understanding and mathematical accurateness are sacrificed.

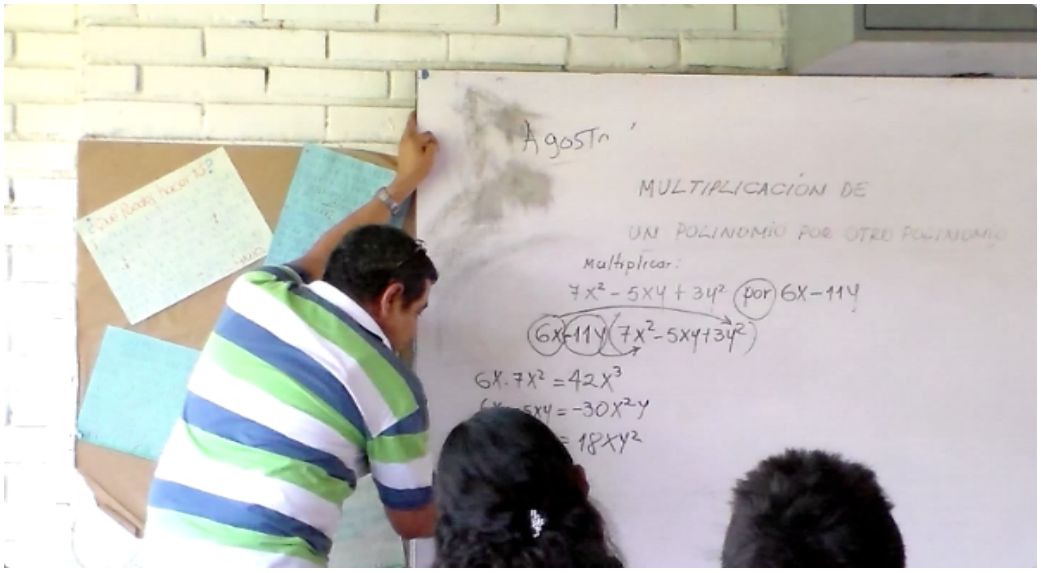

Figure 4.6. Juan's gestures

In short, the main sources of meaning for algebraic procedures and objects in Juan' class are the rules and step-by-step techniques as stated in the textbooks he uses. During the time I observed his classes, Juan did not use instructional methods that are widely accepted as useful to build meaning for algebraic objects and procedures, such as word problems, tasks involving generalizations, and modeling. His teaching mainly relies on the mastering of syntactic rules to carry out operations with algebraic expressions. In addition, Juan attempts to help the students build conceptual understanding upon their previous arithmetic experiences. Although building algebraic upon arithmetic knowledge is an approach that has been recommended in the process of learning algebra at school, Juan's method completely diverts and detracts its strength. This is because the building of such understanding does not proceed by exploring and problematizing properties and characteristics of the arithmetic objects and procedures but rather by assimilating these procedures in an algebraic form that is mathematically misleading. In this process, mathematical discourses that support the algebraic procedures are absent and replaced by gestures that function as mnemonic devices. As the result of these practices, algebra is primarily a set of rules barely supported by mathematics explanations. Further, algebraic objects and procedures are deprived of their mathematics meaning.

The Algebraic Language and the Construction of Algebraic Procedures

In Juan' discourses the absence of rigorous and precise mathematics language to describe and characterize algebraic objects and procedures is notable. Juan barely supports the process of building meaning upon the theoretical developments of mathematics but rather

he combines the presentation of rules as stated in the textbook and informal language close to his students' experiences. The following episode evidences this practice:

> *(One student has come to the whiteboard and is multiplying two polynomials that include rational numbers in their fractional expression. After the student ends, he interprets the process she followed to solve the exercise)*. Ok. So, Lorena started multiplying each member of the first expression with each member of the second expression *(using the gesture previously explained)*. Now, remember how we multiply fractions: the [number] on the top with the [number] in the top and the [number] on the bottom with the [number] on the bottom. Then, you have to add the exponents of the variables, as she did (Lesson, August 30).

Instead of using the mathematics names of numerator and denominator to refer the numbers in a fraction, Juan decides to use a visual feature to explain a rote procedure: The position of the numbers in the fraction. In another class episode, Juan also appeals to informal language to explain the algorithm to divide algebraic expressions. In this case, he presents the common algorithm that the textbook has turned in a series of steps to follow and again, introduces informal language to explain the steps:

> The rule says that you first need to organize the terms in descending order. If we had a missed term, you have to replace it by writing 0 and the variable to the missed. Then, you need to find the term to multiply the divisor to get the dividend. I have already explained how to do so. Now, you multiply [the divisor and the quotient] and write down the result changing the signs of each term. Right? Then, you just copy

down the following part of the divisor. Remember, it is like when you cut-and-paste, cut-and-paste a text in the computer (Lesson, August 23).

In the episode, Juan introduces a familiar expression from the informal language to help the students remember the algebraic procedure. The expression "cut-and-paste" makes complete sense to the students to the extent that Spring Middle has a vocational program in computers, so it is a phrase and a practice the learners are familiar with. These practices are related to Juan's beliefs about his students' ability to learn algebra, as I argue in the following section.

Understanding Juan's Stance towards his Presentation of the Algebraic Knowledge

The practices that Juan uses to help his students learn algebra and make sense of its objects and procedures are mainly aimed to facilitate the learning process. Such practices are mediated by Juan' beliefs about his students' abilities to learn algebra, as he states during the initial interview:

L: What are the biggest challenges you face when teaching algebra to eighth graders?

J: Teaching [to operate with] integers. The integers are the first obstacle that we [the teachers] have with the students in the teaching of algebra. Once it is overpassed… Look, look, I know I am advancing really, really slow. I know I should go more forward in my teaching based on my lesson plan, but these are slow students, some of them. I cannot go faster because if I did so, I would have to repeat and repeat again and again topics I had already taught (Interview, August 27).

As in the case of Diana, the students' difficulties to learn are faced by slowing the pace of instruction rather than searching for and implementing meaningful alternative instructional strategies. Juan utilizes a slow instructional approach to teach mathematics to what he considers slow students. His emphasis on repetition of rote procedures may respond to his perceptions that it is hard for the group of students to learn certain mathematics topics. Based on these perceptions, it may be easier for Juan to focus on the presentation of rules to deal with algebraic symbols and operations rather than on developing conceptual understanding and algebraic thinking.

In relation with his systematic use of informal language to depict algebraic algorithms, Juan justifies this practice in the following narrative:

> When I was in the normal school, they taught me that teaching comprises three phases; one of these is motivation. So, sometimes I bring in jokes [to motivate the students] and believe me, I have been changing my language, because sometimes I am very…But the [school] coordinator tells me "do not worry about it [the language], that is what better works with the students". Because, sometimes I think, "Ok. If I say numerator [multiplied] by numerator, and denominator [multiplied] by denominator", that is a technical language and they [the students] are not going to understand me. And if a mathematician heard me explaining "the [number] on the top with the [number] in the top and the [number] on the bottom with the [number] on the bottom," she would say: "Are you mad? Why are you doing that? That is not the right way to say so." However, the students, I explain [algebra] in my own way and that works pretty well, they remember [the procedures] easier" (Debriefing, September 4).

Juan's narrative reveals several facts. First, he believes he should use strategies (e.g., telling jokes, singing recent musical hits, and narrating funny stories) to motivate students and keep their interest in class. On some occasions, when Juan feels he is losing the students' attention, he would bring up these strategies to capture the students' interest and regain the control of the classroom. In this regard, the motivation to learn a particular algebraic concept does not reside in mathematical needs. Second, his use of informal language to explain algebra is conscious and intentional. It is not a matter of a weak foundation in his mathematical knowledge but rather the result of his efforts to make the algebraic knowledge accessible to a group of students he considers incapable of learning challenging algebraic knowledge. Although Juan recognizes that his language is not accurate and mathematically rigorous, he systematically uses it in his algebraic instruction. Finally, introducing informal language to explain algebra serves a two-fold purpose. On the one hand, it is aimed to facilitate the students' learning. And on the other hand, informal language functions as a mnemonic device to help the students remember algebraic procedures. Hence, Juan's expectations and practices to build meaning for algebraic objects and procedures are closely related.

Juan's Assessment Practices

Juan systematically implements two assessment practices. The first one is administering quizzes. The second one is what he calls "the first five." This assessment practice consists of asking the students to solve an exercise in the notebook and then collecting the work of the first five students who find the answer. The latter practice took

place in every class I observed and several times in the same lesson. Juan explains the main goals of administering quizzes to the students:

> A quiz is a short test. I do not warn the student when I am going to give them a test. I want to know what they recall. Besides looking for the students' prior weaknesses, I also assess the students' attentions, their degree of attention towards the class. When I give them a test, I get grades and at the same time, I know where they are (Interview, August 27).

The narrative seems to convey the idea that the implementation of this practice fulfills a formative purpose to the extent that it is aimed to provide feedback about the student learning process. From the narrative it is also clear the strong association between attention and learning. It seems that for learning to take place, the students must pay attention in the sense that Juan assigns to this expression: fitting the students' look on him and looking at the whiteboard. Besides apparently formative purposes, the assessment practices in Juan's class also are used to help the students pass the course. After getting the final grades of the school trimester, Juan realized that a large number of students had failed. In order to help them, he gave them a quiz consisting in finding the answer of a single exercise previously solved in class. Juan explains that:

> Look, in the last class, well, I said: "This is the quiz of the opportunities." What I meant was: "I do not want you to fail, but I won't gift you a good grade" (Debriefing, September 18).

Thus, the formative purpose of the assessment practice turned into a mechanism to help the students improve their final grades no matter whether learning was taking place.

Because of this, Juan recognizes that although the students cheat on the quizzes, it is not a problem for him, because the goal of the quiz is to help the students pass:

> I know that sometimes they cheat. But even though they cheat, I think: "OK, at least they are doing something." At least one is giving opportunities to the students to improve their grades and to practice for the final test (Debriefing, September 18).

As in Diana case's, the fictional character of assessment becomes more evident in "the first five" practice. I asked Juan to describe the purpose of this practice:

> First, it motivates the competence among them. It is like a tournament. They want to get a mark in their notebooks, it is important for them. Second, I know that the students who have got marks have interest. Even though some of them copy and cheat, at least that is class participation. I know that some students make mistakes and that they haven't developed skill drills. But I give them good grades because for me that is participation in class, and I also know that their test grades are worst, so it is a way to help them (Debriefing, September 18).

From this narrative the fictional character of the two assessment practices becomes clear. Rather than using assessment to improve learning and teaching, Juan's assessment practices are aimed to help the students move along the schooling process. It is also evident that Juan values student characteristics such as speed, ability to memorize, and competition.

Juan's focus on helping the students to pass is not the result of school impositions or policies. The school administration encourages the teachers to provide multiple opportunities for the students to foster their learning process. The teachers should design different tasks

and activities throughout the school year to enable the students to overcome the learning difficulties they face. This school philosophy is stated as follows:

> Each teacher must plan continuous activities to monitor the students' learning process and to determine the students' progress (SMS Assessment System, 2011).

Juan's assessment practices might reflect low expectations related to the students' ability to learn. The emphasis on memorization and speed seems to reveal that Juan considers his students are incapable of learning complex content in a process that involves time and active engagement. The easiness of the questions and their characteristics indicates that in Juan's views, the students lack the abilities to solve challenging problems and tasks and therefore they need that type of help to pass. He for instance would pose exercises similar to those previously solved by him on the whiteboard. Juan would never introduce a word problem in his tests to challenge students' thinking. Juan does not focus on the student learning process but on increasing their chances to pass by watering down the algebraic content. In this sense, Juan affirms:

> I want my students to pass algebra. Although I think the students are capable of learning, many students have failed in the past. Sometimes I ask: "Would it be that my quizzes are difficult?" And sometimes the students tell me: "Teacher, give us a test, do not put it off, let us do it now," so I know they are ready, they have grasped the content. But sometimes I am concerned that in two months they will have forgotten what they have learned today. So, the better [I give them quizzes] the best [grades]. (Interview, August 27).

Pedro's Teaching Practices

Three practices interact to shape the characteristics acquired by the algebraic knowledge in Pedro's lessons. The first practice consists in building meaning for algebraic objects and procedures by introducing different instructional strategies such as posing exercises, word problems, and generalization tasks. The second practice relates to the use of different systems of representation (Duval, 1999) to depict and cope with algebraic objects. The third practice is discursive and mainly consists in appealing to the precision of algebraic language to describe and talk about algebraic objects and procedures.

Making Sense of Algebraic Objects and Procedures

The main sources used by Pedro in his instruction to help the students make sense of algebraic objects and procedures are exercises, word problems, and generalization tasks. Pedro systematically appeals to these types of instructional strategies to start his classes giving the students the opportunity to explore and determine properties of algebraic objects and procedures. The following class episode illustrates this practice.

> P: Today, we are going to start using variables. Write down this problem. Take your time solving it. *(Pedro dictates the problem and at the same time, writes it on the blackboard)*. Write down three consecutive numbers and add them up. Repeat this process with five different sets of three consecutive numbers.
>
> S: Teacher, could the numbers be forty-one, forty-two, and forty-three?
>
> P: Any three consecutive numbers. Just be sure that the numbers are consecutive, ok? I can, for instance, pick six, and which is the consecutive [number]?
>
> S: (Answering in chorus) seven!

T: Seven, and eight. So then, add them up. Observe all your responses and write down what you observe. Do you see a pattern? Do you see a rule or a regularity? What relationships between the three consecutive numbers and its sum do you observe? How would you describe such relationships? Take your time. Write down your answers and we will go back to them later (Lesson, September 6).

Pedro uses this generalization task to introduce the notion of variable. During the lesson, the teacher gave the students enough time to explore the activity and allowed the students to come to the whiteboard and present and discuss their answers. Pedro did not provide a definition of the notion of variable. Instead, he spent the class time to help the students figure out the relationship between the variables involved in the task, as the following class episode shows:

P: What do you observe? What rule did you find? What pattern could you deduce?

S: Me, me! There are three numbers, right?

P: aha.

S: and always the number in the middle multiplied by three equals the sum.

P: Ok, ok! Everyone pay attention to what he is saying. He came up with something. Could you repeat it, please?

S: The sum is the result of multiplying the number in the middle by three.

P: do you all agree with him? Go to the whiteboard and show us what you did. *(The student goes to the whiteboard and writes his examples)*. Observe, he has chosen big numbers and small numbers. So, does the size of the numbers matter?

S: No, it does not.

P: It seems the size of the numbers does not matter. It appears that the rule holds for any triplet of consecutive numbers you choose. Is this true? (Lesson, September 6).

It is clear that Pedro is not searching for a right answer. His efforts are directed towards helping the students clarify their thinking by focusing on the relationship between the variables of the task. Pedro also engages the entire group of students in the discussion of the student's answer. The meaning of the algebraic object is therefore, the result of a group activity grounded in the exploration and deduction of mathematical relationships. The context provided by the task contributes to both the comprehension of the notion of variable and the mathematical signification of such algebraic concept.

An additional practice that Pedro implements in his instruction to promote the construction of meaning is to introduce different representational systems to depict the algebraic objects, as the following class episode demonstrates:

P: So we have two different ways to express the relationship we have found. García has already written the rule in natural language. But we cannot calculate with that language. I mean, we could but it is inefficient. So, the symbolic language of algebra is powerful. So, we are going to express the rule by using symbols (Lesson, September 5).

The need of introducing the algebraic language is explained in the power of its symbolism and the facilities it offers to deal with mathematical relationships. In other class episodes, Pedro would propose different activities to translate from one representational system to another one. For instance, he constantly asks the students to write in natural language the algebraic expressions they find when solving problems and exercises. Pedro

would also pose tasks to the students in which they need to translate from the natural language to the algebraic one, always in the context of working on solving different types of problems, as in the following class episode:

> *(Pedro divides the whiteboard into two columns. In the first one, he writes the following expression in natural language: A number increased by five. The double of a number. A number decreased by ten. And so forth)*
>
> P: Ok, guys. I have written in this column several expressions. I want you to find the algebraic expression for each one. Check in your notebooks if you have doubts and we will solve them later in the whiteboard (Lesson, September 23).

Pedro would not only use this practice for translating algebraic language into natural language but also for the representation of numbers. When calculating with numbers Pedro would ask the students, for instance, to find the decimal expression of a fraction and operate with it and vice versa.

The Algebraic Language and the Construction of Algebraic Objects and Procedures

Pedro uses mathematics language to describe algebraic objects and procedures and requires his students to do so, as shown in the following class episode:

> P: What do we need to do to find the decimal expression of a fraction? Do you, guys, remember [the procedure]?
>
> G: Divide the numbers?

P: No, mister Gómez, I do not know what numbers you are talking about. We need to divide the numerator by the denominator and in that way you would get the decimal expression (Lesson, August 29).

Pedro constantly encourages his students to call the objects using formal mathematical terminology, as in this episode. When he describes the properties of an object, Pedro uses the proper words to name them. There is no attempt to avoid the students facing the complexity of using mathematics language to describe processes and objects. Instead, Pedro stimulates and motivates the use of precise mathematics language in his lessons.

Pedro also appeals to mathematics properties to justify procedures and to characterize objects. In the following episode, the class is reviewing the process to solve equations with one unknown quantity.

(Pedro is comparing the solutions of the equation $-33 - 6x = 25$ *as solved by two different students)*.

P: We have talked about two mathematics properties useful to solve equations. Let us observe that Mister Suarez omitted these properties [to solve the equation] and just wrote the results in each step. And the answer is right but we need to display the procedure. In contrast, Mister Gonzalez used the mathematics properties. Let us see. Because he had minus thirty-three, he needed to get rid of it, so which property did he use?

S: The additive inverse property?

P: That is right! The additive inverse, so he wrote plus thirty-three on both sides of the equation. Why did he do this?

S: Because he used the additive property of equality.

P: Ok. So, now, this thirty-three and the other one are eliminated. What is the result of this sum?

S: Zero

P: Yeah, zero. On the right side of the equation, the sum is fifty-eight, isn't it? And on the left side, he got minus six x. So, we need to leave the unknown on the left side. How did he proceed? He applied the multiplicative inverse property and got rid of minus six. So he cancelled minus six on the left side and divided fifty-eight by minus six on the right side. The value of the unknown is minus fifty-eight sixths. He just forgot to simplify the fraction (Lesson, August 29).

When comparing the solution processes, Pedro privileges the procedure that embodies the precision in using mathematics properties. He recognizes the correctness of the first student's solution but also remarks its lack of alignment to the mathematical steps involved in that solution. Pedro sends a clear message to his students in this episode. Beyond getting the right answer, what is most important is to use mathematics properties to deal with the solution process directing students' attention towards the mathematics involved in the solution process.

The later reflection does not imply that informal language is not present in Pedro's instruction. Sometimes, he introduces mnemonic devices to help the students remember procedures without providing mathematical support for these operations. He also introduces informal language to describe algebraic objects. However, these episodes rarely took place in Pedro's classroom during the time I observed his classes.

Understanding Pedro's Stance towards his Presentation of Algebraic Knowledge

Pedro's practices are mediated by his awareness of the future mathematical needs of his students. As middle and upper middle class students, they are college-bound. They will continue their post-secondary studies in top Colombian universities in mainly mathematics-related careers and Pedro is aware of this. So, his efforts are aimed to prepare the students for facing the challenging mathematics knowledge they will find in their undergraduate programs, as he stated in the interview.

> P: The eighth grade algebra is key [for the students] in the university mainly in the first semester. One day, one former student came and told me "Teacher, please help me. I did not do well with the eighth grade algebra. There were some things that they did not teach me and I am struggling with this class now." So, I spent two hours explaining it to him again. You know, if you teach well eighth grade algebra, man! The boy is going to graduate with good [mathematical] bases (Interview, September 5).

The pressure for preparing students influences Pedro's practices that configure and shape the algebraic knowledge in his classroom. This pressure is also exerted by several features, such as parent and administrator expectations related to student performance on national standardized tests that guarantee students' access to top Colombian universities. Moreover, the well-known status of the school as a top performer on national and international mathematics competitions obliges Pedro to strive for getting good learning results. In short, the high expectations of parents and school administrators translate into high expectations of Pedro who feels responsible for his students' learning.

Pedro's Assessment Practices

Pedro's assessment practices are regulated by the school policy. According to the SJS assessment system, the teachers have to administer three tests during each school trimester and they are scheduled since the beginning of the school year. Pedro informs both the parents and students about the test dates and associated topics to be assessed. Pedro explains that:

> I cannot come into the classroom and tell the students: "Today I am going to give you a test." That is not fair, the students need to study; they need to get ready ahead. Besides, the school administration does not allow the teachers to do so (Interview, September 5).

The first test took place during the time of my observations. The test lasted the entire class period. Pedro instructed the students to write their names and read carefully each question. He finally reminded the students to take enough time to think about the questions and write the answers. In the following class, the students received their grades and Pedro provided feedback about the test. Pedro and the students worked together to find, explain, and clarify the students' mistakes on the test. He called on some students to approach the whiteboard to show and explain their answers. The students who failed the test were required to attend small group or individual tutoring during the next Saturday. I participated in one of these activities and recorded my impressions:

> Pedro cited four students who failed the first test of the trimester and three of them showed up for the Saturday session. There are three other students who want to attend because they have not deeply understood the topic. Pedro asks the students to work individually on a worksheet. While the students are working, he walks across

> the classroom and spends time with each student posing and answering questions, checking problems, and so on. The session lasts from 8 a.m. to 12 p.m. (Field notes, September 14).
>
> Extra class tutoring was an important activity for Pedro. He explained:
>
> That is an opportunity to help the students. I am able to focus on each student and know what difficulties she or he is having. In class you cannot do that because you have thirty students. If they do not improve and fail, the parents cannot tell me I did not do something to help the students (Debriefing, September 21)

Thus, this practice is not only aimed to satisfy a formative goal but also to respond to parents' concerns that may arise in the future. In this sense, the teachers do not individually decide about the assessment practices but they respond to an institutional policy aimed to provide enough opportunities for all students to overcome their difficulties and to get good grades. For instance, the grading scale in SJS goes from one to ten. During my observations, there was a contentious debate among the teachers regarding the minimum grade needed to pass a course. Two years ago, the school administrators decided to set the minimum grade at 6.5. But, according to Pedro, some students exerted minimum effort to get this grade to pass. Some teachers wanted to increase the minimum grade to 7 to force the students to study more. There was a concern about weakening the learning process and as a result, the students' performance on standardized tests. Pedro described his viewpoint:

> The school needs to raise the academic exigencies. And I agree with that. I think the students need to get ready for the university. The students have many opportunities to pass, you know, tests, class participation, auto evaluation, and so forth. And when

they fail or have problems, they can ask for individual tutoring in extra classes (Debriefing, September 21).

Pedro's narrative expresses the ways in which expectations are linked to institutional requirements and dynamics. In this case, the school's prestige and the students' college-bound trajectories influence the assessment policies and practices. As a result, the teachers need to make sure that the students gain both knowledge and abilities to be successful at a university and to maintain high mathematics performance. The school administrators' and parents' high expectations translate into the teachers' high expectations and make them accountable for the students' learning. Students need to not only get good grades, but also learn algebra to face university exigencies.

Comparing Teaching Practices across Three Different Social Contexts

Based on the previous discussion, it is possible to affirm that the practices implemented by teachers are aligned to and express their expectations in relation to the students' ability to learn algebra. This statement holds true for the three schools analyzed in this study. As I have shown, Diana's and Juan's low expectations translate into particular practices of teaching and assessment which, in short, scale down the quality of the algebraic knowledge and reduce the students' chances to successfully access post-secondary education. In contrast, Pedro's high expectations give rise to teaching and assessment practices aimed to prepare the students for the challenges they will face in a university setting. Thus, the differences in mathematics experiences to which the students are exposed are strongly linked to differences in teachers' expectations that, in turn, are nurtured by the students' class, racial, and ethnic backgrounds. In this regard, this finding coincides with results from other

studies that have shed light on the relationships between student background and their mathematical experiences at school (Hoadley, 2007; Zevenbergen, 2000).

Although my findings emphasize a pronounced relationship between teacher expectations and teaching practices, it is important to underline that this relationship is not unidirectional as it could be expected. Indeed, I argue that this relationship is dialectical insofar as expectations nurture and shape teaching practices while the results of teaching practices as evidenced in course grades and standardized test scores confirm teachers' expectations. I discuss these aspects in the following paragraphs.

Teaching Practices as Expressions of Teacher Expectations

The teaching practices implemented by the participating teachers were aligned to their expectations of the students' ability to learn. Although the participating teachers implemented teacher-centered mathematics instruction, there were differences in their practices that ended up providing unequal access to high quality algebraic knowledge. The teaching practices were aimed to satisfy the levels of the students' abilities as perceived by the teachers. In this regard, depending on the teachers' representations of their students, diverse and particular instructional practices took place in the mathematics classrooms shaping the quality of the algebraic knowledge and determining the nature of the cognitive abilities to be developed.

The teaching practices implemented in the economically disadvantaged schools led the students to experience a shallow and sometimes inaccurate algebraic knowledge. Drawing upon Bernstein's theoretical developments, Hoadley (2007) differentiates between two types of knowledge: context-dependent and context-independent knowledge. For

instance, in this study, the practice of tying the addition of negative and positive numbers to contexts of owing-owning is a good example of context-dependent meanings. It is expected that mathematics teachers provide opportunities for students to move from this type of knowledge towards more specialized, context-independent forms of knowledge as found in mathematics textbooks. However, this is not the case in the low-income schools in this study. The introduction of context-dependent meanings for algebraic operations in the ways in which Diana did in this study expresses her low expectations and contributes to the marginalization of the students by narrowing down the students' possibilities of at least carrying on their higher education.

A second practice implemented in the schools attended by poor and working class students consisted in using informal mathematical and everyday language to describe and define algebraic objects and procedures. The practice also responds to the teachers' low expectations and is aimed to facilitate the process of learning of a complex subject matter through the introduction of expressions and words familiar to students. Radford (2003) has highlighted the importance of gestures in the learning of algebra. However, in this case, the gestures as introduced by Juan are aimed to avoid the use of formal algebraic properties and language. In addition, appealing to visual characteristics of the algebraic objects to define them compensated the absence of formal and precise mathematical language in the classroom. The teachers also brought into the classrooms everyday expressions–such as cut-and-paste–to explain and describe algebraic procedures privileging, in this way, context-depending meanings over more mathematically formal and precise significations. Thus, in

the process of apparently facilitating the learning process, algebraic meaning was blurred and its construction by the students was hindered.

In contrast, the middle and upper middle class students in the study were exposed to teaching practices that enabled them to explore algebraic concepts in more mathematically relevant contexts and in deeper ways. The students, represented by the teacher as capable of learning and expected to develop high and complex processes of mathematics thinking, were more likely to experience classroom activities in which formal mathematics language and processes were used to build meaning for algebraic concepts and procedures. As a matter of fact, real world problems were absent in Pedro's instructions at least during the time of my observations. Word problems were mathematics bound and some of them were similar to problems usually found in mathematics textbooks. Although they were usually introduced as starting points for the students to explore concepts and procedures, the teacher quickly moved the learners towards focusing on the mathematics relations embedded in the problems by engaging the students in the use of different semiotic systems to represent and deal with the situation. Clearly, the practices were intended to help the students deepen their understanding of the complexity of the algebraic knowledge.

Several researchers have shed light on the ways wherein different students are exposed to different types of mathematics knowledge. In her study of the role of school mathematics in the reproduction of social inequalities in South Africa, Hoadley (2007) found that students in different social-class contexts are introduced to different forms of knowledge. In particular, Hoadley affirms that "context-dependent meanings and everyday knowledge are privileged in the working-class contexts, and context-independent meanings and school

knowledge predominate in the middle class schooling context" (p. 682). The processes of thinking that are expected to be developed by the students also vary across social context (Oakes, 2005). Mathematics teachers of economically marginalized students tend to reinforce the development of low abilities of thinking such as procedural skills and memorization whereas middle class students are required to perform complex thinking such as generalizing and solving problems. The findings of the present study seem to corroborate these phenomena.

Relationship between Teacher Expectations and their Teaching Practices

The teachers' assessment practices as implemented in the mathematics classroom played an important role in reinforcing their expectations. The enacted assessment practices pursued different goals that ranged from helping the students pass the course to identifying strengths and weaknesses.

The strategies developed for assessing the state of the algebraic learning of the economically disadvantaged students were primarily aimed to help them obtain better grades in order to pass the course. There was a strong emphasis on helping the students improve their grades rather than providing them feedback about their learning or determining the impact of instructional practices. The students were given multiple and sometimes unplanned quizzes, tests, and extra-mathematics activities whose final purposes were to increase individual GPAs. The students and teachers engaged in a sort of game in which the former pretended to be learning and the latter feigned to be measuring that learning. In this sense, the final grades did not reflect the real state of the students' algebraic learning but the success of the game in which they were involved. In spite of the easiness of the instruments used to

assess the students' learning, the teachers complained about the low grades and the difficulties for them to obtain better grades, and then, this situation confirms the teachers' low expectations. In the participating teachers' views, low mathematics performance is not the result of poor teaching practices that lessen the quality of the algebraic knowledge and restrict the students' opportunities to learn meaningful content. According to the teachers, the students' failure originates in their incapability and lack of dispositions for learning complex algebraic content despite the low-demanding nature of the teachers' instruction.

In the case of wealthy students, the assessment practices were aimed to monitor their progress and difficulties. The results of the tests were used to design and implement supporting plans for low achievers to overcome their weaknesses. A noticeable characteristic is the strict organization of the assessment practices that involved providing both parents and students with enough information about dates of assessments and topics to be assessed making the process public. In this case, the students' mathematics outcomes also confirm Pedro's expectations of the students' ability to learn algebra.

Interestingly, the assessment practices were mediated by institutional exigencies and policies that are particularly clear in the poorest and wealthiest schools. In both cases, the teachers were pressured to fulfill determined conditions such as a fixed percentage of student failure or respond to parents' expectations. Although I do not approach the role of institutional expectations in shaping and influencing teacher expectations and their practices, it arises as a fundamental feature in need of future study. Understanding the ways in which schools configure and depict student expectations would help to comprehend its role in contributing to the unequal distribution of knowledge and opportunities to learn.

The Algebraic Knowledge Taught by the Teachers

As a consequence of the differences in teaching and assessment practices, the meaning of the algebraic knowledge taught by the teachers also differs. First, the poor and working class students have access to an algebraic knowledge that has been deprived of its power to model mathematics and social phenomena. By watering down the potentialities of algebraic knowledge, the students are left without a powerful tool of thinking and a key instrument for both their future access to more complex forms of mathematics knowledge and thought and their access to post-secondary education. Second, in the efforts of easing the process of learning algebra, the teachers introduce contexts and definitions that might mislead the students in their construction of mathematically accurate meaning of algebraic concepts and objects. For instance, defining algebraic and mathematics objects such as variable and exponent by appealing to their visual features hinder the mathematics relations they embed and represent and lead the students to build wrong significations. Finally, the algebraic knowledge is reduced to a series of rules and abstract procedures irrelevant to the students' lives. Rather than a potent tool for solving problems and for making sense of the social world, algebra becomes a meaningless symbolic game useless out of school.

The wealthier students are more likely to build meaning for algebraic objects and procedures in ways more aligned to those mathematically accepted in the field. Interestingly, Pedro's teaching practices expose the students to mathematics experiences that researchers have pointed out as important for developing algebraic thinking such as solving problems (Rojano, 1996), generalizing patterns (Mason, 1999), and using different semiotic systems to represent situations of change and variation (Radford, 2006). In this sense, their chances to

access the algebraic knowledge needed to succeed in the school systems are higher than those of their poor peers. For instance, Pedro frequently focuses on asking the students to translate from the natural language to the algebraic symbolism and vice versa. He would ask the students to express in natural language an algebraic expression or, given and algebraic expression to represent it using natural language. This ability to represent mathematical entities in different semiotic systems is considered fundamental for the mathematics comprehension of objects. Because of their nature, mathematics objects need to be depicted by using different representational systems or registers (Duval, 1999) in order to be accessible and usable by individuals. The same mathematics object can be depicted in several representational systems and the comprehension of the objects involves the act of mathematically seeing the same object in different representational systems. Researchers have highlighted the central role of this ability in the processes of making sense of mathematical objects (Duval, 1999; Kieran, 2007). Kieran (2007) argues that "the opportunity to coordinate objects and actions within two different representations, such as the graphical and the letter symbolic, is considered by many to be essential in creating meaning in algebra" (p.710). Duval (2006) sheds light on the complexity for the students of "seeing" the same object as represented in what he calls different semiotic systems, as for instance, recognizing the same function either in a tabular form or in an algebraic representation. The process demands from the student the ability to recognize the object into two different registers "whose contents have very often nothing in common" (p 112). In this sense, Pedro engages his students in a complex process of understanding and building meaning for algebraic objects that is absent in Juan's and Diana's instruction.

It is in this sense that the role of teaching practices as source of inequality and marginalization can be understood. Because of the critical role of algebra in the Colombian mathematics curriculum and its role as a gateway to higher mathematics, poor and working-class students who are exposed to the type of instructional practices I have described are more likely to fail at school. Their opportunities to attain higher education and ensure better jobs significantly decrease and in this sense, their marginalization is reinforced and maintained.

Classroom Climate

As part of the teachers' effort in creating the appropriate conditions for the students' learning, I analyzed the interactions and relationships they set up to foster the development of algebraic thinking. I identified the two major themes outlined in Table 4.3 to capture the teachers and students' interactions during instruction.

Table 4.3

Classroom climate category and themes

Categories		Themes	
Classroom Climate	Teacher and students' interactions and relationships during instruction that either hinder or foster the learning of algebra	**Teacher and students' interactions**	Discursive interactions to support the students' learning of algebra
		Student participation	Construction of a safe learning environment.

Diana's Classroom Climate

Several factors shape the classroom climate during Diana's instruction. First, the characteristics of the questions Diana poses to her students as well as her goals in raising such questions determine the forms in which the students are positioned as doers and learners of mathematics. A second factor relates to the absence of social norms to regulate the students' participation and interactions during instruction. I discuss these factors in the following paragraphs.

Diana and the Students' Interactions

Diana's instruction is teacher-centered. Diana usually begins her lessons either checking student homework responses or introducing a new algebraic concept and procedure. When Diana introduces a new algebraic object or procedure, she defines it, then provides multiple examples, and dedicates the remaining class time to student practice to reinforce the learning of rote algebraic procedures. Subsequent to the introduction of the algebraic concept, Diana spends one or two classes to the solution of a large number of exercises aimed to master the manipulation of algebraic expressions. In order to do so, Diana calls one student to the whiteboard to check that student's work and progress and then repeats this process with the next student. In this sense, her instruction focuses on the mastering of rules to operate algebraic expressions, the development of procedural skills, and the memorization of facts. The roles of the students in Diana's class consist of copying her dictations, answering her simple-response questions, and completing seatwork. She controlls the lesson flow by using interactions as in the following class episode:

D: Can anybody give me an example of an algebraic expression with just one term?

S: X to the second power.

D: X to the second power and what else?

S: Seven y to the second power

D: Seven y to the second power, good. This is an algebraic expression with one term. Can anybody give me another one?

(The students talk at the same time, it is noisy and hard to listen to a sole response)

D: Mr. Silva says "nine x to the third power y to the second power" and let us write a minus in the front *(Diana writes the algebraic expression on the whiteboard)*. It only has one term and that is why it is a monomial. Copy that example in your notebooks (Lesson, August 26).

The Initiation-Reply-Evaluation or IRE (Mehan, 1978) is the privileged mode of interaction in Diana's classroom. She usually poses a question directed to elicit a mathematical characteristic of an algebraic object or procedure, as in the class episode above. Next, Diana assesses the correctness of the answer. If the answer is right, she will go on explaining the topic or pose a new question. Otherwise, Diana will lead the student towards the right answer by offering clues until he or she reaches the response Diana expects. The following class episode illustrates this last type of interaction:

(Diana calls on the students to determine what type of polynomial is a given algebraic expression and to identify their components. This is homework assigned the previous class)

D: What type of algebraic expression is this? (*pointing to* $3/2\,x^5y^2$). [Is it a] monomial, binomial, trinomial, or polynomial?

S: Trinomial

D: Trinomial? Does it have three terms?

S: [It has] one!

D: When it has one [term], how is it called?

S: Monomial?

D: Monomial. Good. Remember, one term is separated from another one by using what?

S: A sign?

D: Yes, by writing a sign (Lesson, September 2).

In this case, Diana asks the students about mathematics facts that have been previously taught such as polynomials classification. The questions are not aimed to bring about the discussion of new knowledge but to recall mathematics features of the algebraic expressions. In a different type of interaction, Diana poses questions about the result of algebraic or arithmetic operations, as illustrated in the following class episode:

(Diana is explaining how to add like terms. She draws two sets. In the first one there are five letters "p" and in the second one Diana writes 3p, 2p, and 5p)

D: *(Directing the question to the group)*, How many [letters p] do I have here?

S: *(Answering in chorus)*, five ***p***'s!

D: *(pointing to each p while counting),* one, two, three, four, five p's. *(Diana writes "5p" under the first set).* And how many [letters p] do I have here?

S: *(students chatting indistinctively, hard to listen to)*

D: *(underlining each term while saying)* Two ***p*** plus three ***p*** plus five ***p***. How many ***p*** do I have?

S: Ten ***p***.

D: Ten ***p***, good. Can I add these ***p***?

S: Yes!

D: What is the sum?

S: Fifteen!

D: Fifteen ***p***. I can add [the terms] because this one *(pointing to 5p)* has the same letter as this one *(pointing to 10p)* (Lesson, September 2).

An additional feature of these types of interactions is that they are neither intended to explore the students' thinking, build upon prior knowledge, or deepen understandings about the algebraic objects and procedures. Moreover, there is no attempt to engage the group in meaningful discussions about different mathematical problems and situations that may arise in the course of the lessons. I never observed Diana confront the students' responses or ask them to clarify and explain their thoughts when solving an exercise on the whiteboard. During the time of my observations Diana never invited the students to discuss or analyze a particular student's response. Instead, Diana engaged the students in one-response type of questioning aimed to regulate the lesson flow. In part, the explanation of this practice may rely on the fact that Diana never introduces challenging problems or at least problems that require the students both to explore the properties of algebraic objects and to search for more than one solution path. In this sense, the nature of the teacher-students' interactions are

mediated by the quality of the questions and problems Diana poses to the students, as the following class episode reveals:

> *(Diana has previously found the general formula of the perimeter of a rectangle and expressed it in algebraic language. The expression 2z+ 2w is written on the whiteboard)*
>
> D: Let us solve this problem. What is the linear measure of a fence needed to enclose a rectangular yard whose length is 15 m and its width is 7 m?
>
> S: *(Answering in chorus just after Diana announces the problem)* 44!
>
> D: What is the length of fence needed?
>
> S: 44!
>
> D: That was fast! Let us check whether [the answer] is right. What is the width?
>
> S: 15
>
> D: and the length?
>
> S: 7
>
> D: *(Plugging the numerical values of the variables into the formula)* And we need to calculate the perimeter of the rectangle. That would be twice the width plus twice the length. The [letter] z is 15 and the [letter] w is 7. So, the answer is 44. You are right! (Lesson, August 22).

This is one of the two word problems Diana posed during the eight classes I observed. As may be noticed, Diana is surprised by the facility and speed displayed by the students in solving the problem. She expected a different solution path more focused on plugging the numbers into the algebraic expression and carrying out the operations involved, as she did.

However, the nature of the problem allowed the students to quickly solve it without appealing to written calculations.

A different goal of Diana's type of questioning is related to managing the group discipline. Questioning is designed to control the students' behaviors and to take their attention back to Diana's explanations. In fact, this goal seems to be prevalent in the teacher-student interactions insofar as the episodes in which Diana calls on students who interrupt the class or misbehave during her instruction take place more frequently. The following class episode exemplifies this type of interaction:

(Juan, a student who sits in the back of the classroom is laughing aloud and interrupting the class)

D: Juan! What is the coefficient of this term?

J: Minus nine? *(The students laugh loudly "celebrating" Silva's mistake)*

D: No, it is not minus nine.

S: *(Answering in chorus)* it is nine!

D: Juan, see? It is nine not minus nine. Pay attention (Lesson, August 26).

On some occasions, Diana combined this practice with giving a bad grade when the students do not respond correctly to her questions, as demonstrated in the following class episode:

(Alexandra sits in the back of the classroom. She is texting on her phone while Diana tries to start the class)

D: Ok, let us get started. Alexandra!

A: aaaaaah!

D: What is the sign of this term?

A: minus?

D: Ok. Because it was plus but you said minus I will give you a one (the lowest grade).

A: Come on, teacher! It is not fair! (Lesson, September 2).

This is not an isolated episode. Diana frequently called on the students who did not have their attention on her and graded them for the purpose of maintaining control of the classroom discipline. The students were, then, academically punished for their behavior in the classroom. Although this practice often took place, its frequency reveals its failure. A lack of classroom and behavior management and the resulting student disengagement were distinctive features of Diana's classroom.

The students' disengagement was obvious in the actions that students displayed in class such as persistently chatting, texting on the phone, listening to music, or just taking a nap on their desks. It was also evident through their withdrawal from class participation. Despite Diana's insistence in asking the students to contribute, they rarely did unless she threatened them with lower grades. I recorded in my field notes one of the episodes in which the students' disengagement was evident:

> The students are completely disengaged. Diana attempts to call their attention by raising her voice and hitting the whiteboard with the eraser but nothing seems to work. The students just keep chatting and laughing. I even can listen to the music from the cellphone of one student sat near to me. After a few minutes, Diana stops

trying to call the students' attention and just stands up in front of the classroom. I can see the anger and disappointment in her face. (Field notes, September 5).

The students also arrived late to class or skipped Diana's class altogether. Several times I observed Diana leaving the classroom to look for students who were hiding so that they did not have to attend her class. In summary, the teacher-students interactions in Diana's classroom can be framed in an Initiation-Reply-Evaluation or IRE model (Mehan, 1978) that responds to a twofold purpose. On the one hand, the IRE model of interaction enables Diana to control the lesson flow by posing one-answer and easy questions. In this regard, she searches for correct procedures and responses rather than for scaffolding and fostering the students' mathematics thinking. On the other hand, the IRE model allows Diana to manage the classroom discipline controlling the students' behaviors through the use of questions and scores. Along with these practices, the mode in which Diana organizes her daily instruction contributes to a classroom climate that hinders the learning process and prevents the students to build positive identities as doers of mathematics, as I discuss in the following section.

Student Participation

Besides the students' disengagement and their lack of motivation to participate in Diana's class, the absence of social norms to regulate the teacher-students and student-student interactions is evident in her classroom. Norms for supporting and encouraging the explanation and justification of the students' thinking, respecting others' interpretations, and attempting to make sense of the explanations given by others were absent in Diana's classroom. This was not a safe classroom environment that stimulated active student participation and the development of a community of learners that would foster algebraic

thinking. Unfortunately, students who wanted to participate were the object of derision and aggression. For example, the following class episode took place during one lesson:

(Diana is calling on each student to come to the whiteboard and solve one exercise from a worksheet she has previously assigned. She calls on Luis who wears one of the sunglass cameras. Luis comes to the whiteboard and starts solving the exercise. While doing this, a group of students starts verbally attacking him).

S: He does not know!

S: Donkey!

S: If the glasses are clouding you over, take it off!

S: Teacher, send him back to his desk. He does not know.

S: Mr. Bean!

S: He does not know!

D: Come on, everyone has the opportunity to solve an exercise. *(Luis finishes the exercise)*. Good job!

S: Wow! He used his brain! I said, "He is my buddy."

D: He did it well (Lesson, September 5).

In a similar episode, Diana calls on a student who is chatting. The student's answer gives rise to jokes and laughing as may be noticed in the excerpt above:

D: Pinzon!

P: aaaaaah!

D: what are the variables in this term?

P: x to the third power, y to the second power.

D: Very well!

S: Wow! She thought!

S: She used her brain!

D: And who said she could not do it?

Both episodes provide evidence of negative social norms in Diana's classroom that prevent students from safely and productively participating in the lesson. The students are verbally attacked and ridiculed by their classmates if they appear to comply with Diana's expectations. It is also worth noticing that Diana does not attempt to prevent the students' attacks. Rather than directing her efforts to build social norms for productive student interaction, she timidly addresses the micro-aggressions or avoids them altogether. Her practices likely facilitate fear and hinder motivation for students to participate. As a result, students rarely pose questions during Diana's lessons and they rarely raise their hands to ask Diana for further clarifications. Posing questions in front of their peers will likely result in peer mockery and derision, as happens in the following class episode:

D: Let us reduce the like terms in this polynomial. We have a [term with] x to the second power. Where is another one with x to the second power?

S: seven ***x*** to the second power.

D: So we have nine ***x*** to the second power. *(Carmen raises her hand and poses a question)*

C: Why? I cannot figure out where that nine comes from.

S: aaaah! Donkey! Stupid!

S: Can I not ask?

D: (*Answering to Carmen*) Because they are like terms, so you can add them up (Lesson, September 5).

The social norms as set up in Diana's classroom contributes to the establishment of a learning environment in which many students are rude and disrespectful of one another. Therefore, the environment is not conductive for students to share, negotiate, and build mathematics knowledge. During one of our debriefings, I called Diana's attention to the level of student engagement and the persistent attacks to the learners who wanted to participate:

L: How would you explain the students' disengagement during the lessons?

D: I do not know. I mean they do not care.

L: Sometimes, I have seen you kind of uncomfortable with the students' behavior in class. Let me show you this video clip *(I show her the class episode in which she is fruitlessly trying to call the students' attention).*

D: Yes, I can see. I do not know. I try to be respectful with them. I mean, I have not been rude with them. So, I do not feel afraid.

L: What do you mean?

D: I know that I do not react when they misbehave. They just piss me off. My children also [piss me off] and I get mad at them and I yell at them. But here, I do not react.

L: Do you feel afraid? Why would you feel afraid?

D: I do not feel afraid of the students because I am respectful with them. I am very careful with the words I use when I admonish them. I do not know. I would need to think about it. Sometimes I said, "I need to take it easy because this going to be my

job until I retire, so I do not need to be stressed out for now" (Debriefing, September 17).

Although I insisted in my question about the sources of her fears and the situations or persons that could threaten her at school, I did not get a straightforward answer from Diana. Her representations of the students as gang members and drug dealers might contribute to her fears and her apprehensiveness to deal with the classroom discipline beyond assigning a bad grade. Diana may think that the students would potentially assail her, as demonstrated in the following exchange:

(Diana has refused to grade homework. She opted for giving the students a quiz based on the homework, a situation that provoked irate reactions of the students).

D: Everyone get ready for the quiz.

S: No, teacher!

D: Go ahead and write the first question.

S: See? And then you ask why the tires of your car were punched! (Lesson, September 19)

She may feel threatened by the students, as she states in this conversation. As far as she does not react against their aggressions and provocations, she can be sure that nothing "bad" is going to happen. For her, being respectful means to ignore the students' attacks of heir classmates and their bad behaviors during instruction.

In summary, Diana's classroom climate does not constitute a trusting environment that fosters the development of students' algebraic thinking. Diana's classroom is not a place where the students can expose their thinking, respectfully argue with others, consider and

analyze others' ways of thinking, and interact to collectively build knowledge. Conversely, Diana's classroom is a place in which meaningful learning of algebra does not occur.

Several factors seem to influence the nature of the classroom climate. First, Diana positions the students as incapable of engaging in challenging processes of solving problems. They are neither encouraged to openly express their thinking nor stimulated to assume responsibility for their own thinking. The type of instructional interactions in which the students are just passive spectators of Diana's performance and whose participation is limited to answer low-level questions is a clear indication of how the students are positioned. This feature of the classroom climate is strongly related to the absence of social norms that enable the students to respectfully interact with their peers in the purpose of collectively building algebraic knowledge. This is a second factor that seems to shape the relationships and interactions in Diana's classroom. As a matter of fact, Wood (1994) underscores the critical role of teachers in fostering the maintenance of a safe community of learning by establishing productive social norms. In his words, "the teacher plays a major role in determining and guiding the development of these norms and in providing the means of support for maintaining them" (p. 151). Diana seems to renounce this role and limits herself to exerting partial control of the classroom social norms by threatening the students with low grades. Finally, the representations she has built about the students in HMS seem to influence both her positioning in the classroom and her decision of refusing to react against the students' aggressions and misbehaviors.

Juan's Classroom Climate

Juan's instruction is teacher-centered. Similar to Diana, Juan controlled the lesson flow by posing multiple questions. During the time I observed his classes, Juan's instruction mainly consisted in explaining procedures to carry out operations with algebraic expressions. First, he would dictate the steps involved in the procedure and illustrate it with multiple examples. Following, Juan would pose multiple exercises similar to those solved by him. Finally, he would spend the rest of the class supervising seatwork and grading the students' tasks. Juan tried to cover as many cases as possible when explaining a particular procedure. For instance, when teaching how to multiply algebraic expressions, Juan would first introduce the multiplication of a monomial with a monomial, then with a binomial, and so forth. Then, he would focus on introducing exercises with different types of numbers to show how to proceed in each case. Thus, for instance, he would begin with expressions involving integers and then, rational numbers in fraction and decimal expressions. In this sense, Juan's instruction focused on the development of abilities to operate with algebraic expressions. The role of the students in Juan's classroom consisted of paying attention to his explanations, answering his questions, and solving the exercises he proposed.

Juan and the Students' Interactions

The following class episode exemplifies the type of interactions in Juan's classroom:

(Juan is explaining the procedure to divide polynomials)

J: What does four divided by two equal?

S: Two

J: What does $\boldsymbol{x}$ to the second power divided by $\boldsymbol{x}$ equal?

S: $\boldsymbol{x}$

J: And what does $\boldsymbol{x}$ divided by $\boldsymbol{x}$ equal?

S: $\boldsymbol{x}$ to the zero power

J: And what does $\boldsymbol{x}$ to the zero power equal?

S: One?

J: Yes, that is correct! $\boldsymbol{x}$ to the zero power equals one (Lesson, August 28).

In this class episode, Juan engages with the students in a very well-known interaction. It consists of posing questions with one answer, waiting for the right answer to surface, and continuing with the next question. In this type of interaction, Juan often asks the student to find the sum, product, or subtraction of two or more integers with the corresponding mathematical sign of the answer. In this sense, Juan mainly raises questions that just require the students to develop rote procedures. The following class episode illustrates another type of interaction:

(Juan is explaining how to expand algebraic expressions using a particular example. He has removed the braces and obtained the algebraic expression:

$-15a^2 - ab^2 + 3ab - 27a^2 - 7a^2 + 15ab + 2ab^2 - a^2$*)*

J: So, now, we have removed all the grouping symbols. What do we need to do next?

G: Subtract?

J: What did you say?

D: We need to separate positive from negative terms.

J: Why we need to do so?

D: I do not know.

J: Why we need to do so?

S: We need to do it to reduce like terms.

J: Good! We need to do it to reduce like terms (Lesson, August 16).

In this class episode, Juan inquires about the next step in a given procedure and asks the students to explain the reasons to carry out such a procedure. This is a common type of interaction in Juan's classroom. Its frequency may be explained by the fact that his teaching centers on the presentation of rules to carry out algebraic operations, and then, he inquiries for the steps in a given procedure. During algebra instruction, Juan also introduces the following type of interaction:

J: Ok, guys. Let us revise the procedure to find out whether we made a mistake. Six times minus 2 equals minus twelve, right or wrong?

S: Right!

J: Ok. $\boldsymbol{x}$ to the second power multiplied by $\boldsymbol{x}$ equals $\boldsymbol{x}$ to the third power, right?

S: Yes!

(The conversation between Juan and the students continues)(Lesson, August 23).

In this class episode, Juan engages the students in a conversation aimed to determine the correctness of an answer. He asks the students to check whether or not a given result of an algorithm has been accurately carried out. Usually, it is Juan who has solved a particular exercise and then engages the students in this type of interaction.

In summary, Juan employs three different strategies to establish instructional interactions with his students. Besides asking them to assess the correctness of a procedure, Juan also requests that the students find the next step in a given procedure, and to obtain the

result of an operation. These interactions enable Juan to control the flow of the lesson by posing questions that just require students to complete a basic operation or to assess it.

The role of the students in Juan's classroom mainly consists of examining the correctness of the procedure. Rather than having an active participation in the process of solving exercises, Juan relegates the students to the position of passive spectators. The students do not need to explain their thoughts, assess other students' responses, or engage in meaningful mathematics conversations. For instance, when the students come to the whiteboard to solve exercises, Juan does not ask the students to explain their solution paths. Instead, he interprets for the class the procedure that is written, as in the following class episode:

> *(Sandra is finding the product of* $3a^2$ *and* $5a^3 + 3a^2 - 2a + 8$ *on the whiteboard)*
>
> J: Let us see what Sandra is doing. She is finding the first product of $3a^2$by $5a^3$. She sets that product aside to make it easier and says "five times three equals fifteen" and a to the second power multiplied by a to the third power equals a to the fifth power. Very good! *(Sandra stands up near to the whiteboard while Juan is explaining her procedure. Sandra continues solving the exercise).* So, Sandra found that five a to the third power multiplied by three a to the second power is fifteen a to the fifth power, three a to the second power multiplied by three a to the second power is nine a to the fourth power, very well. And then she got three a to the second power multiplied by minus two a is six a to the third power. Good. And she finally got twenty-four a to the second power. Good job, Sandra! (Lesson, August 16).

In addition to these interactions, Juan uses the IRE model to control the students' behavior during his instruction, as the following class episode illustrates:

(Juan is explaining how to multiply polynomials. Salazar is chatting and laughing aloud. Juan has warned Salazar several times but he keeps interrupting Juan's explanation).

J: Salazar! What does six times seven equal, Salazar?

S: Forty-two.

J: Forty-two. And [what does] ***x*** to the third power multiplied by ***x*** to the second power [equal], Salazar?

S: ***x*** to the fifth power?

J: Are you answering or asking? (Lesson, August 16).

Juan often tries to take back the students' attention by using this strategy. However, the level of disengagement in his class is noticeable. The students are usually texting with their cellphones or just chatting. In addition to calling on the students who are misbehaving, Juan also uses different strategies to retake the attention such as moving the students to different places in the classroom, raising his voice, whistling, and hitting the whiteboard with the eraser. Although Juan does not use the grade as a threat to gain the students' attention; however, he employs a similar strategy:

J: Let us start the division. Let us start! *(Hitting the whiteboard with the eraser).* Let us start this division and pay attention because after it, I am going to give you a quiz. Guys! Attention to the whiteboard! What are you doing? (Lesson, September 4).

The episode reflects the ways in which Juan turns an assessment practice into a tool to control the students' behaviors. However, despite his efforts and strategies, the students easily disengage. Although Juan expresses preoccupation about the students' disengagement and their misbehavior in class, he responds to these attitudes by concentrating his instruction on the group of good students, as I discuss below.

Student Participation

In one of our debriefings, Juan expressed his preoccupations about the students' behaviors and the effects of the students' disengagement in the learning process:

> J: What really bothers me is not having the guys' attention. That stresses me out a lot.
>
> L: Which strategies have you used to keep the students' focused on the lesson?
>
> J: I have talked with them; I say, "Come here, Buddy. What is wrong with you?," and I move them to a new place in the classroom so they cannot chat with their mates. But sometimes it does not work. If I have to make jokes and tickles I would do it, but I want their attention (Debriefing, September 4).

He also recognizes that some students bully their peers when they come to the whiteboard:

> J: They are teasers, you know. Sometimes they say nicknames to their mates and that is not a good environment. I have not been able to create an environment in which that does not happen. The last year, I could make it with the eighth grade and the students would ask and come to the whiteboard. But I have not been able to do it with this group.
>
> L: Why has it been difficult for you to do it with this group?

J: I told you before the way in which these students were grouped. I guess they are not motivated to study (Debriefing, September 18).

This perceived lack of motivation has led Juan to focus his instructional interactions on the group of "good" students. As I discussed before, Juan positions his body in the classroom in such a way that he can direct the attention to this group. The questions are mainly addressed to the students in this group and Juan usually asks them to solve the exercises he poses, as the following class episode reveals:

(Juan writes an exercise in the whiteboard)

J: Who thinks is able to solve this exercise? Anybody? Sandra, do you want to solve this exercise?

S: Ok.

J: Ok, Sandra is going to solve it (Lesson, August 16).

Juan then proceeds to closely interact with Sandra while she solves the exercise on the whiteboard and finally, he would explain to the group what she did, as I illustrated above. I recorded the close interaction between Juan and the "good" students in my field notes:

Juan is explaining the steps to divide polynomials. He writes two polynomials on the whiteboard to demonstrate the process. Although he asks the students to read each step as stated in the textbook, Juan concentrates his interactions on the group of "good" students. He asks them to read the steps and to check the correctness of the procedure he produces. The students also pose questions and lively contribute to the conversation. The rest of the group is completely disengaged. They are chatting and texting on their cellphones. The classroom is noisy, and I barely can hear the

> conversations between Juan and the six students. However, Juan ignores the situation and keeps interacting with the "good" students. When he finishes the explanation, he asks the class if there are any questions. No one answers. Juan proceeds to ask Sandra to solve an exercise on the whiteboard. I can hear a student protesting, "Always Sandra, always Sandra." I do not know if Juan heard the student and ignored him or just did not hear him all at (Field notes, August 30).
>
> I asked Juan about his close interaction with this group. His narrative was:
>
> I always count on Sandra and Lorena. I mean, that group of students is excellent; they are both outstanding. They are the best. So, I always count on them. You can see they pay attention, they do not take their look off of me, they pose questions. So, I count on them (Debriefing, September 4).

Rather than creating a safe context in which all the students can participate and discuss their thinking, Juan opts for ignoring the group of students he considers to lack motivation. During my observations, interacting with these students was the common pattern of interactions. I rarely saw a different student coming to the whiteboard to solve an exercise; when that happened, as in the case of Lucía, Juan would ignore such a student and rely on Sandra and Lorena, his "best" students.

In summary, Juan's classroom climate does not offer the majority of students a meaningful environment to expose their thinking, respectfully argue with others, consider and analyze others' opinions and ways of thinking, and interact to collectively build knowledge. Juan privileges close instructional interactions with a reduced group of students

who respond to his representations of "good" students. The rest of the class is positioned as passive receptors and spectators of Juan's explanations.

The type of interactions with the reduced group of "good" students, particularly with Lorena and Sandra, influences Juan's classroom climate. Juan excludes the majority of the students from his mathematics instruction reducing their opportunities to learn meaningful algebra. The types of instructional interactions in which the students are just passive spectators of Juan's explanations and whose participation is limited to answer low-level questions demonstrates how Juan positions the students. This positioning is related to the expectations Juan holds of his students. The cultural and class deficit frames that nurture the representations of poor and racial minority students have enabled Juan to classify the students into two well-differentiated categories of performance. Based on this categorization, Juan establishes a series of interactions that privilege the learning process of some students and denies this right to others. A small group of students is positioned as active doers of algebra while the rest is placed in an inactive role that is contested by some learners either protesting or just disengaging from the lesson. Thus, the classroom climate created by Juan responds to the expectations he elaborated about his students.

Pedro's Classroom Climate

Pedro's instruction is mainly teacher-centered. He controls the lesson flow by posing questions to the students and organizing different instructional activities. Pedro usually started his classes by posing a word problem or exercise. He would give the students time to explore the problem individually and then, would call on them to compare and analyze their answers. Pedro finally would focus on the mathematical characteristics of the concept or

procedure and provide formal mathematical definitions. Although Pedro controls the lesson flow, in contrast to Diana and Juan's classrooms, he tries to purposely engage the students in the construction of meaning for algebraic objects, as I discuss below.

Pedro and the Students' Interactions

Pedro uses the IRE model to interact with his students, as the following class episode illustrates:

(Pedro is reviewing with the students their responses to a test about properties of real numbers and operations).

P: What does five halves plus seven halves equal, Kelly?

K: twelve halves

P: Twelve halves. Is that sum right?

S: Yes

P: But what is twelve halves equal?

S: Six

P: Right, six (Lesson, September 13).

He asks the students to solve rote procedures such as adding two fractions and simplifying the sum. Pedro also engages the students in the following type of interaction:

(Pedro is reviewing with the students how to solve equations with one unknown. He asks the students to individually solve the equation $-6 + x/8 = -5$*)*

P: What would be the first step to solve this equation?

S: Me, teacher! *(A student raising his hand)*.

P: Wait a minute. You have already participated. Let us allow another student who has not participated, yet. Molina, what would be the first step to solve this equation?

M: We need to eliminate the minus six.

P: How do we do that, Lopez?

L: We have to add positive six to both sides of the equation.

P: So, minus six plus six plus one-eighth $\boldsymbol{x}$ equals minus five plus six. Ok. So I am adding six on both sides. What property are we applying here?

S: Additive inverse!

P: Additive inverse, very well. So, what is next? (Lesson, September 13)

In this class episode, Pedro asks students to identify the following step in the process to find the value of the unknown quantity. Following the typical IRE model, Pedro poses a question and assesses the answers. Besides these types of interactions in which Pedro controls the lesson flow, he also involves the students in a different kind of conversation, as the following class episode reveals:

(Pedro is calling on the students to solve equations)

P: Felipe, you have not participated today. Come to the whiteboard and show your work. *(The student solves the equation* $5 + x/11 = 12$*.)*

P: Felipe, could you explain what you did?

F: I added minus five to both sides of the equation and then I got one-eleventh of $\boldsymbol{x}$ equals 7. Then I multiplied by eleven to eliminate the eleven in the denominator. So, I cancelled the elevens here and then I multiplied seven times eleven. That is.

P: How did you get that seven on the right side?

F: Which one? This? *(Pointing to the number seven that resulted from subtracting twelve and five)*

P: Yeah, that one.

F: This is the difference between twelve and minus five. And then I multiplied 7 times eleven and got seventy-seven that is the value of $\boldsymbol{x}$.

P: Felipe says that seventy-seven is the value of the unknown. Who got a different answer?

S: It is right, I proved it!

P: You did it! Great. Come to the whiteboard and show your work (Lesson, September 13).

In this class episode, Pedro does not provide the answer, but invites the students to explain their solution paths. Pedro closely observes the students who are constantly participating and encourages the learners who seem to be disengaged using phrases such as "I have not heard your voice today" or "I do not remember you writing." In this way, Pedro constantly motivates all students to engage in the class activity. The following class episode illustrates a similar type of interaction:

P: Say false or true: one-half belongs to the set of the positive integers. David, could you respond to this question?

D: False

P: False. Why do you think it is false?

D: Because if I divided the numerator into the denominator, I would get a periodic decimal.

P: So, David says that it is false because one-half is a periodic decimal. Who agrees? Who disagrees?

S: I agree it is false.

P: Ok. We agree it is false, right? And what about his arguments? How can we decide whether or not one-half is a periodic decimal?

S: Dividing one into two?

P: Let us do the division *(Pedro divides one into two on the whiteboard)*.

S: Teacher, if a number is a decimal it is not an integer. One-half is a decimal, so it is not an integer.

P: Right, that is a better argument, isn't it? However, David says that one-half is a periodic decimal, so I ask you, is this *(pointing to 0.5)* a periodic decimal?

S: No, it is not.

D: Oh! Yeah! But my answer was right.

P: Yes, the answer was right but your argument was wrong, so we need to pay attention to how we support our responses (Lesson, September 6).

In this class episode, Pedro engages the class in discussing a student's answer. He does not correct the student's mistake but rather opens the discussion and invites the students to explore better arguments. Pedro tries to help the student builds upon his mistake to understand a mathematical concept with the help of his peers. This is a frequent type of interaction in Pedro's classroom. He often gives the students the opportunity to express and explain their thoughts and to value different responses, as the following class episode shows:

(A student is solving an exercise on the whiteboard. He is trying to find a rule to express the sum of three consecutive integers).

P: After adding the three consecutives numbers, what did you notice?

S: I found that the sum is always the number in the middle multiplied by three.

P: He is using the word "always." Does it hold true for your answers, guys? Who got a different answer? Let us see. What is your answer? *(Pointing to one student)*

S: The sum of the three numbers is a multiple of three.

P: You *(pointing to an another student),* what did you find?

S: Three is the divisor of the sum.

P: You, what is your answer?

S: The number in the middle can be multiplied by three and the result of the multiplication equals the sum of the three consecutive numbers.

P: Great! See, guys? That is one the advantages of the natural language. There are multiple forms of expressing our answers! (Lesson, September 20).

The class episode also reveals the form in which Pedro values the responses of all the students. It is worth noticing that although Pedro controls the lesson flow, he assigns the students the responsibility to carry out the proposed activity. He does not attempt to impose a particular way of approaching the problem but allows the students to build meaning of the task by engaging them in its solution. In this sense, the student individually and the group collectively are responsible for the construction of algebraic meaning.

It is also important to mention that, during the time I spent in the classroom, Pedro never used direct questions to control the students' behaviors. There were few episodes in

which Pedro reprimanded a student because he or she was either chatting or distracted with a cellphone. The class was usually engaged in the different activities proposed by Pedro. This environment was not fortuitous but the result of a conscious effort of building a safe community of learning, as I discuss in the following section.

Student Participation

Besides the type of instructional interactions, the class episodes reveal an environment in which the students expose their thoughts and share their work. They actively engage in discussion without attacking or disrespecting their classmates. Pedro tries to create a classroom climate in which the students can safely participate and support each other. He explains that:

> These students are just teenagers. They need guidance. For me it is really important that all of them participate. So they need to learn that there are rules: respecting others' opinions, listening to their peers, respecting the turns to talk. At the beginning of the school year I come to an agreement with them about these rules. I call them "the non-negotiable minima." These are the basic norms of coexistence. You can see them posted on the wall (Debriefing, September 21).

The agreement consists of a set of the rules to interact and behave in the classroom during mathematics instruction. They include raising the hand before talking, listening when another person is talking, avoiding making jokes or laughing when a student makes a mistake, and actively participating in class, among others. Thus, the rules are explicit and the students must follow them. The following class episode shows Pedro's reaction when one of these rules is broken:

(A student is explaining his answer to an exercise. He provides multiple details and some students starts laughing)

P: Guys, stop it. This is a big problem. And it has always taken place in this school. It always happens. A common denominator is the derision. Sometimes when somebody participates, the students make fun of him. And what happens? Well, the student never participates in class again. Remember, we need to learn to respect our classmates. Everyone has the right to talk and you have the obligation to listen to that person. In my class everyone has that right. If somebody comes to the whiteboard and makes a mistake, nothing happens. We come to the school to learn. Nobody knows everything. Even none of the students who are repeating the school year know everything. So, let us try to avoid ridiculing our classmates. Otherwise we are doing nothing here at school as persons. First of all, the respect! Remember the non-negotiable minima (Lesson, September 20).

The rules are not intended to be punitive but rather to guarantee the conditions in which the students can safely participate. This class episode reveals Pedro's effort in creating an environment of mutual respect and trust for one another's thinking. Pedro assures that all the students' ideas will be respected regardless of their correctness.

Pedro is also careful in engaging all the students. There is not any attempt to privilege interactions with particular groups of students in the classroom. He uses different strategies to ensure that in fact all the students participate in the discussions and that their ideas are heard. Thus, for instance, and besides using phrases such as "I have not heard your voice

today" or "I do not remember you writing," Pedro calls on a student of each line. So, his goal of including each student in the instructional activities is clear.

Pedro positions the students as active doers and producers of algebraic knowledge. Not only do all of them have the right to participate in the collective construction of knowledge but they also have an obligation to do so. Pedro tries to elevate the motivation for participating by valuing student work, questions, and answers, as the following class episode illustrates:

> *(Pedro is helping the students find clues in the word problems to decide what mathematical operation to include in an algebraic expression. He writes a double column table and on one side writes the operation and on the other column the words associated with that operation)*
>
> S: Teacher, when the problem includes the word "rate," does it mean division?
>
> P: That is an excellent question, Carolina! Please repeat your question for your classmates to listen to and we can discuss it (Lesson, September 23).

In short, although Pedro introduces an IRE model for interacting with his students, other forms of interaction are implemented. These different ways are aimed to provide the students the opportunity to engage in meaningful conversations and discussions about mathematics problems and exercises. Pedro makes a conscious effort to create the conditions in which the students can express their ideas and discuss them with others and respectfully assess other student responses. The students are positioned as having the right to express their ideas to be heard but, at the same time, they have the obligation to participate and respect. The high expectations held by Pedro help facilitate a learning environment in which

the students are actively engaged in their learning process. They are invited to take responsibility for the ideas they expose. This positive positioning contributes to create an environment that plays a major role in the students' learning and outcomes.

Comparing the Three Participating Teachers' Classroom Environments.

Although the teachers in the study implemented the IRE model of interactions, the previous analysis reveals fundamental differences in the classroom climate across the three sociocultural contexts. The findings draw attention to the different goals for using particular patterns of interactions in the classrooms as well as the organization of the students' participation in the learning process. The variations in the classroom climate reflect the different ways in which power operates at the micro-level of the classroom through the actions used by the teachers to position the students during instruction. Accordingly, the patterns of the students' behaviors indicate the different forms in which they contest and challenge such positioning. As I discuss below, the classroom climate responds to the expectations the participating teachers hold about their students.

Expectations and Positioning

As different researchers have argued (e.g., Darling-Hammond, 2004), the patterns of teacher and student interactions differ in relation to the sociocultural contexts in which they occur. Although the IRE model emerged in each classroom, it was particularly persistent in the poorest schools. The interactions in HMS and SMS were less academically engaging as well as less focused on fostering the development of higher order thinking. The nature of the questions posed by the teachers emphasized the correctness and completeness of an answer rather than in the construction of meaning. This indicates the form wherein their low

expectations translate into ways of interacting in the classroom that position the students as incapable of developing complex processes of mathematical thinking such as reasoning, solving problems, and using formal mathematics language to communicate their ideas. The interactions were also less oriented towards fostering learning and more focused on directing and assessing the students' thought in a process that Turner et al. (2002) have called "nonscaffolded forms of instructional discourse" (p. 91) and that Stein and Lane (1996) associated to low level cognitive demands. According to these authors, the implementation of such forms of interactions in which the teachers control the students' thinking, limit not only their opportunities to learn but also their autonomy. In this sense, the students are positioned as passive recipes of fixed algebraic content and inactive participants in the construction of knowledge.

In contrast, the wealthier students were engaged in forms of interactions that positioned them as capable of developing complex processes of algebraic thinking such as finding patterns and regularities and using different semiotic systems to represent algebraic objects. The interactions were mainly aimed to help the students deepen the characteristics of the algebraic objects by asking them to discuss different answers and ways of solving particular problems. The algebraic language was presented as a useful tool to deal with complex operations rather than as a monolithic set of rules to carry out rote procedures. In this regard, the students in the three schools experienced, in deeply different ways, the meaning, and use of the algebraic knowledge.

In addition, the poor and black students in this study were less likely to engage in the collective construction of algebraic knowledge. There were fewer opportunities for them to

expose their ideas, to respectfully discuss and argue with their peers, and to elaborate arguments to defend their thinking. These students did not have the opportunity to experience alternative forms of building algebraic knowledge because the teachers presented it as a final, pre-elaborated knowledge that the pupils had to learn. The absence of social norms to interact ended up giving way to a learning environment in which the students assumed hostile attitudes towards their peers and impeded the collective construction of new knowledge. In contrast, the wealthier students were motivated to elicit and explain their thinking and respectfully debate others students' opinions. In this regard, negotiation arises as a fundamental factor that reflects both the teacher efforts in building meaning with the students and to help them attain higher levels of competence (Turner et al., 2002).

Interestingly, the different patterns of interactions influence not only the learning of algebra but also convey a sense of students' places in the social world. As a matter of fact, the interactions as mediated by the teachers' expectations seem to develop particular attitudes and identities. It is claimed that the particular features of the interactions reveal the role of schools in maintaining the social class structure and locus of power by encouraging abilities of leadership and attitudes of self-government in middle and upper middle class students while promoting stances of observance, obedience, and submission in poor, working-class, and minority students (Nassir & Hand, 2006; Oakes, 2005). The students in this study were not only learning particular forms of algebraic knowledge, but also, and mainly, they were learning to *be* (Boaler & Greeno, 2000) and to interiorize discourses and representations of who they are and their future roles in society. Poor and black students were learning to

surround their agency and obey while their wealthier peers were learning leadership and autonomy.

It is in this way that power operates at the micro level of teacher and student interactions in the mathematics classroom. On the one hand, the students experience the learning of algebraic knowledge in deeply different ways. Poor and black students have access to a set of rules to carry out rote procedures. They have fewer opportunities to experience the power of the algebraic symbolism in the construction of new knowledge not only about mathematics but also of the social world. The nature of the interactions in their classrooms limits the options to develop higher order thinking and in consequence, erodes the chances to enter to higher education. The economic and social marginalization of the students is then, reproduced. The interactions in the poorest schools prevent the students from developing the abilities and competences needed to successfully participate in the democratic life (Pinnow & Chval, 2014). Skovsmose (1990) argues that democratic competence "is a socially developed characteristic of the competence which people to be ruled must possess so they can be able to judge the acts of the people in charge" (p. 122). Democratic competence implies, among other aspects, the ability to discuss; to understand other's people stances; and elaborate arguments to support or challenge such stances. Based on the findings, the poor and black students are less likely to develop this democratic competence and in this sense, the political marginalization is also reproduced. This is how instructional interactions in the micro-level of the classroom contribute to the reproduction of inequality and the marginalization of particular groups of students.

CHAPTER 5. Conclusions, Significance, and Implications

In this chapter, I discuss the findings, implications, and limitations of this study. I first contextualize the findings within the broader body of research that has addressed issues of power and mathematics education. I propose a model representing the relationships among teacher expectations, teaching practices, and classroom climate. I locate the model within the context of the theoretical developments in the field of mathematics education. Subsequently, I discuss the significance, limitations, and implications of my findings for research approaching issues of power and mathematics education, particularly for the Colombian research community.

Understanding Power and Mathematics Education

This study drew upon recent work in the field of mathematics education that addresses issues of power from a critical sociopolitical perspective (Apple, 1990; Oakes, 2005; Popkewitz, 1988; Valero, 2012). The paradigm that supports this type of research contends traditional approaches that introduce deficit views of marginalized students to seek explanations for their mathematics failure. Instead, this study was grounded on the premise that, in order to understand such failure, it is fundamental to investigate and focus on the school practices that unequally distribute knowledge, dispositions, and opportunities to learn (Flores, 2007; Weber, Radu, Mueller, Powell, & Maher, 2010). Beyond trying to bridge the achievement gap between minority and mainstream students, the main purpose of research conducted within the theoretical perspective assumed in this study is to uncover the structure and mechanisms that install and perpetuate inequality at school (Martin, 2009). This is a

critical step in the process of empowering students, teachers, and researchers to understand and act against injustice in society.

Teacher Expectations and Student Social background

The findings of this study draw attention towards the critical role of the hegemonic representations of poor and black students in the expectations teachers hold about the ability of these student populations to learn algebra. The findings are consistent with research that points to the significant impact of the beliefs that teachers hold about students on mathematics learning, school experiences, and outcomes (Boaler, Altendorff, & Kent, 2011; Frankestein, 1995; Hoadley, 2007; Lim, 2008; Ogbu, 1988; Reyes & Stanic, 1988). Moreover, the results confirm a dominant view in education according to which racial minority and poor students have lower ability to learn mathematics than their wealthier and mainstream peers (Zevenbergen, 2003).

The teachers in this study consistently held lower expectations for black and poor students and accordingly adjusted their teaching to such anticipations. Clearly, the teachers' expectations were supported and nurtured by ideologies related to the students' social backgrounds. Both race and class constituted the main sources of representations, ideas, and meanings that the teachers used to interpret and make sense of the students' attitudes and behaviors during mathematics instruction. Likewise, dominant ideologies about poor and black pupils helped teachers naturalize the mathematical performance of the students and present themselves as neutral participants in the configuration of their low mathematics performance. The teachers considered failure and low mathematics achievement as inherent

to the conditions of poverty and culture inferiority experienced by the students, rather than to the results of school practices that unequally distribute knowledge and dispositions.

Consistently across the three social settings, the teachers drew upon cultural and class deficit frameworks to explain and justify the low mathematics performance of black and poor students at school. The prevalent storylines were replete with negative images that depicted low-income and black students as deviant from mainstream values and dispositions needed for academic success. Cultural and class differences were seen as pathological conditions that impeded the students access to complex forms of algebraic knowledge and higher order algebraic thinking. The teachers perceived poverty and cultural differences as undesirable personal qualities that need to be overcome before any achievement could be reached. In this sense, and as Popkewitz (1988) has argued, the teachers' discourses about failure and success "do not exist independently of a complex and ongoing social world of expectations, demands, attitudes, and emotions" (p. 223). Expectations not only contain beliefs about learning and content but also, and mainly, they embody deeply rooted ideologies about individuals and the social world.

The intersection of race and class in the learning of algebra constitutes a double jeopardy for the students in this study. Black students are overrepresented in the low-income population in Cali (Rodriguez-Garavito, Alfonso, & Cavelier, 2008; Urrea, Viáfara, Ramírez, & Botero, 2007) and, in this regard, discrimination and marginalization at school doubly affect them. On one hand, black students are ostracized because of the ideologies that have naturalized representations of the black culture as inferior and its people as incapable of carrying out processes of thinking strongly associated to the dominant mathematics

rationality in the Western world. The trivialization of the black culture that associates it with the dominance of sense over thinking, handwork over intellectual work, and body over mind, translates into low expectations at school. On the other hand, the marginalization of poor students comes from dominant ideologies that present middle-class values and dispositions as "legitimate tools and goals for education while treating the cultural\communication styles of working-class people as inferior or worthless" (Lim, 2008, p. 93). According to Lim (2008), middle-class values and dispositions are depicted as indicators of academic potential and they serve the purpose of justifying either failure when the students lack them or success when the students possess them. Researchers have also argued that school, as one of the main agencies of cultural reproduction, promotes and recognizes values and dispositions of the ruling class that are foreign and unfamiliar to economically disadvantaged students (Bourdieu, 2011). Wealthy students adapt and easily respond to the dynamics of the school to the extent that they possess the cultural capital to do so, and then, school practices and discourses favor this student population (Zevenbergn & Niesche, 2008). The findings in this study coincide with this trend and show that the teachers held a set of values, such as ambition and hard work, that are presented as legitimate and whose absence indicates academic deficit and inferior status. Poor students are disadvantaged by hegemonic representations that position them as "others" in contrast to dominant assumptions of middle-class learners (Archer, 2003). Stereotypes as conveyed through the cultural and class deficit frameworks are then, ways of perpetuating the exclusion and marginalization of poor and black students. As held by Nassir and Mckinney (2013), "The very presence of these stereotypes denies students' power by

disregarding the types of capital they bring into school based on their out-of-classroom and out-of-school affiliations and knowledge set" (p. 276).

It is noticeable the ways wherein power plays out at school to position certain practices, forms of being, and knowledge as legitimate and genuine while marginalizing others (Nassir & Mckinney, 2013). The selective tradition, as mentioned by Williams (1981), acts in subtle and imperceptible ways to systematically exclude from school rationalities and epistemologies associated to poor, black and indigenous cultures while privileging values and representations of the mainstream culture of dominant groups that, in Colombia, are mainly mestizo and middle class. And although there is not agreement about which social marker, race or class, is the most important anticipator of student performance (Hoadley, 2007; Ladson-Billing & Tate, 1995; Lubienski, 2000), the findings of this study reveal the way in which they interact and overlap to maintain poor Blacks in their condition of marginalization at school.

Teacher Expectations and Teaching Practices

The findings of the study also reveal the ways wherein teacher expectations shape their teaching practices and interactions with the students. Low and high expectations resulted in different opportunities to learn for the students and then, their experiences in learning algebra were markedly different across the three social contexts. In addition, and most importantly, the students not only learned algebra. They were also either granted or denied access to the development of dispositions and abilities to participate in the social world (Secada, 1992).

As different researchers have pointed out, low expectations translate into teaching practices that, in this study, watered down the quality of the algebraic content to be taught (Anyon, 1980; Lee, Smith, & Croninger, 1997). The teachers tried to foster the learning of algebra by adjusting their instruction to the perceived abilities of the students. Both, the language and the teaching practices displayed by the teachers were aimed to facilitate the learning process of a group of students who was perceived as low achieving. Low-income students, represented as incapable of learning algebra, were exposed to teaching practices that focused on connecting algebraic content and procedures with real and familiar situations to the students. The teachers appealed to the students' experiences to help them build meaning of the new algebraic knowledge although some contexts, if not all of them, were not mathematically pertinent and robust enough to contribute to the construction of such meaning.

The system of bringing into the classroom context-dependent situations to build meaning for mathematics knowledge and operations has been found as prevalent in low-income schools (Hoadley, 2007). In this study, the process of turning algebraic objects into objects of the material world and grounding the construction of sense of algebraic procedures on arithmetic algorithms arose as common practices to facilitate the students' learning. Let us consider in detail the implications and effects of this practice. Researchers, such as Sfard and Linchevski (1994) and Harel and Kaput (1991), have shed light on the dual status of algebraic object as processes and objects. An algebraic expression such as $-4(x\ +5)-8$ can be seen as an algorithm or operation that requires finding the result–a process– or as an algebraic entity in itself–an object– that is part of a mathematical structure. The

comprehension of algebra and the development of high order levels of algebraic thinking demand, according to these researchers, to gradually overcome the operational stage to move towards the structural state in a complex process that Sfard and Lichevski call *reification* and Harel and Kaput name *entification.* Considering the case of function, in the operational stage students are able to deal with certain situations; however, according to Harel and Kaput, "it would not be sufficient to deal meaningfully with situations which involve certain operators or functions, such as the integral and differential operators" (p. 82). Therefore, teachers need to help students transition towards the structural level in which the algebraic expression is an object itself in order to foster the development of deeper understandings of algebraic knowledge and to gain access to other areas of mathematics such as calculus. By linking the meaning of algebraic objects to objects of the material world as well as to arithmetic procedures, and by focusing on the syntactic features of algebra, the teachers in this study hinder students' access to the structural level of algebra. They also deny students opportunities of developing more powerful levels of algebraic thinking and building mathematics bases for acquiring more advanced mathematics knowledge. As Sfard and Lichevski (1994) put it, in the presence of algebraic procedures and the absence of abstract objects and their unifying effects, "the students may still be able to perform these processes but their understanding will remain instrumental" (p. 221). The reification process as developed by Diana, for instance, occurs in an opposite direction as stated by Sfard and Lichevnski. Instead of creating the conditions for the students to move from an operational stage towards a structural level, she turns algebraic objects into objects of the real world by tying their meaning to real life contexts. Diana's instruction does not help students transition

towards a more specialized form of algebraic knowledge but keeps the content in an elemental, ill-defined level. The students in Diana's classroom will hardly reach the structural level and in this sense, they are more likely to fail in more advanced mathematical classes that would allow them to continue their higher education. This is how, in this study, low expectations translate into teaching practices that reinforce the marginalization of already marginalized students.

Low expectations also influenced the teacher and students' interactions. As Flores (2007) argues, racial minority and poor students are exposed to instructional interactions that pose low cognitive demands. The Initiation-Reply-Evaluation model dominated the interactions and in this sense, poor and black students in this study were positioned as passive receivers of algebraic knowledge who must surrender their agency to the teachers' authority (Boaler & Greeno, 2000). Popkewtiz (1988) argues that this type of positioning entails a political theory insofar as "the individual is denied the role of actor in the creation of history and culture. Social life is defined as fixed and unyielding to intervention" (p. 225). As I discussed in chapter four, the classroom climate played a critical role in forging democratic abilities and competencies in the poor and black students in the study.

In short, the racial minority and economically disadvantaged students in this investigation were exposed to what Popkewitz (1988) reffered to as *illusory schooling*. Although a naïve individual observing Diana's and Juan's instruction may think that a real process of teaching was taking place, in reality, they were teaching little or no algebra. The students were apparently engaged working on exercises and problems and the teachers were supposedly supervising the learning process. However, the teachers and the students were

involved in a game in which the teachers pretended to teach but they were really providing the students with multiple opportunities to pass the course regardless whether or not learning was taking place. As Popkewtiz (1988) put it,

> Illusory schooling was a response, in part, to the teachers' perceptions that the requisite dispositions for schooling were lacking in the children who came from the poor communities. The logic of schooling was that the children come from broken homes, do not have adequate discipline or the correct attitudes for schoolwork, and have few or no educational materials available to them in home; these conditions make it difficult or impossible to learn properly. To teach mathematics had less to do with learning the content than using school subjects as a vehicle to establish an orderly, busy place where children are safe, and where they can learn the "right" attitudes and behaviors that will help them when they get older (p. 232).

It could be thought that teachers deliberately decide not to teach or to lower the quality of their instruction. Nevertheless, an alternative intepretation is possible. They might be responding and acting in accordance to the dominant representations about how poor and black students learn mathematics. This is precisely the power of ideology in the lives of individuals. There is no opportunity to challenge such ideologies as social facts are presented to us as natural. Teachers are not just bad or good teachers. Perhaps they have not professionsl development opportunities that challenge dominant representations of the students, and in this sense, researchers in general and Colombian resarchers in particular need to find ways to provide the teachers with such opportunities.

Thus, the findings of the study confirm that mathematics classrooms are not neutral spaces in relation to issues of power. Based on the relationships I identified between and among categories, Figure 5.1 represents a model that identifies relationships among three components that are critical to understand the practices that reproduce marginalization and inequality in the mathematics classroom.

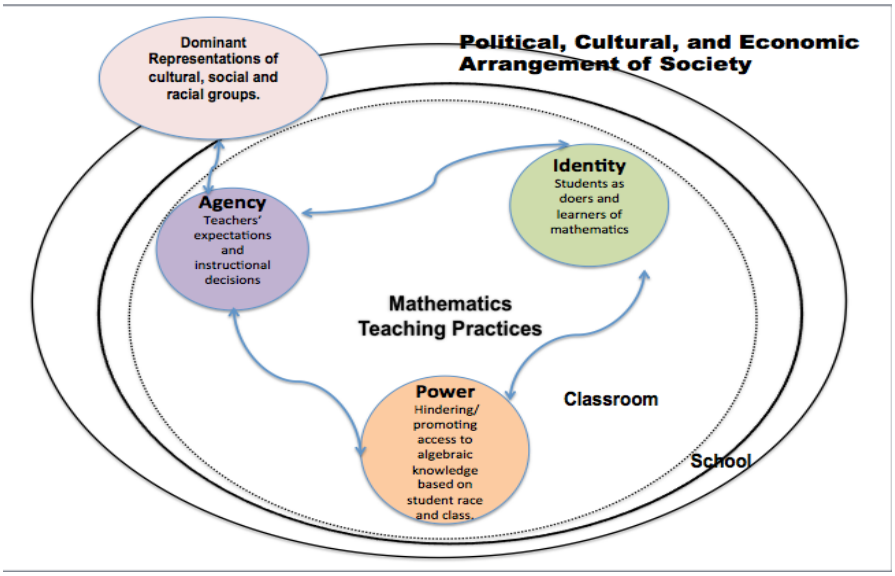

Figure 5.1 Model of critical components that influence the learning of algebra in school.

Researchers have usually emphasized the role of mathematics mediating the relationship between teachers and students (e.g., Chevallard, 1991). However, when issues of power are considered, other factors seem to mediate teacher-student interactions. The common teacher-mathematics-students triad is insufficient to explain the dynamics of teaching and learning mathematics (Valero, 2009). Figure 5.1 captures the mediating role of teacher expectations between teaching practices and the teachers and students' interactions. Teacher expectations translate into teaching practices that respond to the perceived students'

abilities to learn mathematics. Put another way, low expectations translate into low quality instruction.

However, as I discussed before, the relationship is dialectical insofar as the result of teaching practices, as measured by classroom assessment, confirms expectations. Teacher expectations also influence the configuration of the classroom climate understood as the complex net of interactions that position students and support or prevent their participation during instruction. The different patterns of interactions influence not only the learning of algebra but also convey to the students a sense of where their places are in the social world. It is important to highlight the students' agency reacting against the roles that the teachers try to impose. However, the model I propose builds exclusively upon the aspects found in the study. Students' agency needs to be studied in further investigations.

Limitations

One of the limitations of my study relates to the time spent in the classrooms. Although eight sessions provided good insights of the type of interactions occurring among teachers and students as well as the teaching practices implemented by the teachers, it was not sufficient for identifying additional teaching practices or how those practices may have changed over the course of an academic year.

An additional limitation of the study was the differences among the curricula in the three schools. The classes I observed were dedicated to build meaning for different algebraic objects. For instance, Diana and Pedro were beginning the teaching of variable and the procedures to add and subtract like terms, whereas Juan was ahead in the process. It would be interesting to observe the practices implemented by the teachers while they taught the same

algebraic content; however, this would require observations during different periods of time as the teachers did not teach the same concepts at the same time due to differences in the schools' organization.

The impossibility to conduct follow-up interviews was another limitation in the study. After the data collection process finished, I returned to the United States, a situation that impeded me from interviewing the teachers about ideas and comments that emerged during the analysis process. I missed valuable opportunities to re-interview the teachers to obtain deeper insights about their expectations and practices.

After conducting the initial interviews with the teachers, I noticed their reluctance to talk about black students and racism. When they did so, they were careful with their comments. The few times when these issues emerged during the conversations, the teachers immediately tried to smooth their narratives. I then realized that my race could be contributing to the teachers' reluctance to overtly talk about race and racism. How could they be comfortable talking about race and racism with a black interviewer? I guess this fact explains the contradictions in Diana's narratives about black students. On one hand she affirms that black students are brilliant although none of the "best" students in her class is black. On the other hand, she believes that black students need psychological attention and occupational therapy. Thus, although race and racism are taboo topics in Colombian society, I think that a mestizo interviewer would have had more details about the teachers' perceptions about black students and their ability to learn algebra.

Finally, the number of participants in the study limits the opportunity to generalize my findings. However, the in-depth examination of the three teachers' practices provides

insightful comprehensions of a social phenomenon that could lay the foundation for a larger scale study. More than trying to generalize the findings of this study, it is important to realize that they confirm and are aligned to the results of other studies conducted in different countries by several researchers. However, more research approaching issues of power and mathematics education in the Colombian educational system is needed to enrich the comprehension and improve the conditions in which poor and black students are taught and learn mathematics.

Significance and Implications of the Study

This study contributes to the field of mathematics education by providing insights into the ways wherein power operates at the classroom level to reproduce exclusion and marginalization. Although different studies have consistently shown the differences in expectations and learning opportunities that low-income and racial minority students experience in mathematics classrooms in different countries (Archambault, Janosz, & Chouinard, 2012; Dunne & Gazely, 2008; Hoadley, 2007; Zevenbergen, 2003), the present study sheds light on the forms in which teachers elaborate expectations about their students, the sources that nurture such expectations, and the impact of such expectations on their teaching practices.

The present study also contributes to the comprehension of the teaching practices used to teach algebra that might contribute to the differential achievement between racial and poor minority students and their wealthier peers. This is particularly important for two reasons. First, most of the studies addressing issues of racism and discrimination in mathematics education have approached the mathematics content in a quite general form

(e.g., Spencer, 2006). Mathematics is, sometimes, just the backdrop or context of the classroom and we are left without information about practices specific to particular mathematics knowledge, such as algebra and geometry, in low-income classrooms. In this study, I show the features that distinguish the algebraic content in low-income schools from the content in a wealthier school. In this regard, I maintain the focus on mathematics while exploring issues of power so that mathematics recovers its centrality and importance in the research process. Second, the study addresses a critical issue in need of research. The instructional practices that teachers use to teach algebra is an issue "largely unexamined" (Doerr, 2004). The present study addresses this problem in the context of the Colombian educational system where several international and national large-scale assessments have indicated students' low performance in algebra and racial and social gaps in mathematics achievement. The study was aimed to enhance the comprehension of institutionalized routines, different discourses, and different sources that determine instructional practices and discourses employed by Colombian teachers of algebra across different social contexts. It begins to examine how expectations, practices, and classroom environments differ for children of different racial and economic backgrounds.

The present study contributes to validate the use of mobile and stationary cameras in the processes of studying the teaching practices of algebra in the Colombian educational system. In particular, the use of sunglass cameras to closely capture the ways wherein minority students position themselves and the ways in which they are positioned during mathematics instructions is of great importance in this study. Further studies can contribute

to inform the strengths and limitations of using these types of devices in the understanding of privilege and marginalization in the mathematics classroom.

This study is particularly important for the Colombian community of mathematics education. Colombian researchers have not closely examined issues of power in the mathematics classroom and in particular, issues related to racism and classism. Mathematics education research in Colombia has been dominated by psychological and sociocultural approaches with the well-known consequences of relegating and hindering other perspectives that could shed light on and illuminate alternative perspectives related to the teaching and learning of mathematics. The results from this study call attention to the relationships between power and mathematics education and provide an opportunity to motivate and support the growth of a community of Colombian scholars interested in these issues.

Implications for research and teacher education arise from the findings of my study. Regarding *research*, the findings suggest the critical role of both the school organization and institutional discourses in shaping the teachers' expectations as well as the students' opportunities to learn. It seems that discourses and representations about the students are not only part of individual teachers' narratives but circulate at the institutional level. This aspect was clear in Diana's narratives about other teachers' representations of students at HMS. The form in which individual teacher and institutional expectations interact to shape black and minority students' experiences in learning algebra needs further exploration.

In addition, it is also important to inquire into the forms in which school organization influences and shapes minority students' opportunities to learn. For instance, there were clear differences between the private and public schools in relation to teachers' exigencies,

curriculum, resources, time dedicated to algebraic instruction, and so forth. More research is needed to understand whether school organization responds to particular expectations and how such organization influences the experiences of minority and poor students.

Another implication relates to the teaching practices and classroom environments implemented by black teachers to teach to black students in Colombia. As Rist (1970) showed, issues of racism and classism might emerge during mathematics instruction in classrooms in which both teachers and students are Blacks. Research in this line could shed light on the forms in which race and class interact in classrooms with black students and teachers contributing to the discussion about the weight of these two social markers in mathematics outcomes. This type of research would contribute to enhance our knowledge the ways wherein marginalization operates to the classroom level.

An important implication of the study relates to the need to design, implement, and evaluate teaching practices and environments that promote the academic and personal growth of poor and black students. Effective teaching experiences to teach algebra to marginalized students are in need of being designed, implemented, and documented. Researchers in Colombia need to address this issue as a form to erode the myth of the predestined low achievement of poor and minority students. Positive mathematics learning experiences of minority students such as the proposals developed by Gustein (2006) and the QUASAR project (Silver, Smith, & Nelson, 1995) would enhance pre and in-service teacher education.

Finally, students' acts of resistance against the dominant discourses and practices that circulate at school need to be studied. It is clear that black and poor students in this study-as in Diana's classroom-display a series of behaviors and attitudes that might represent their

opposition to the negative stereotypes that depict them. This form of agency needs to be addressed in future studies to understand how students challenge such stereotypes.

Regarding *professional development,* the results of the present study suggest the need of fostering teacher knowledge about issues of social justice and equity. Teacher educators need to find strategies to promote changes in the expectations of mathematics teachers that teach poor and black students. Efforts must be directed towards improving in-service teachers' knowledge to take advantages of cultural differences to foster the development of mathematics thinking in their students. The study raises questions about the need of providing teachers with experiences that go beyond instruction. It is important to explore alternative ways to confront teachers' ideologies about poor and black students.

Finally, regarding *pre-service teacher education*, it is critical to reorganize curricula at the university level to provide prospective mathematics teachers with learning experiences about teaching racial minority and poor students. Courses addressing issues of power and mathematics education are currently absent in the pre-service teacher education programs. The findings of this study suggest the need to address this gap in order to improve the conditions in which mathematics is taught in low-income schools in Colombia.

Concluding Remarks

The present study addressed an issue barely explored in mathematics education. Although investigators have approached issues related to teachers' beliefs of algebra, its teaching, and learning, (e.g., Agudelo, 2008; Nathan & Koedinger, 2000), research have scarcely sought for the links among expectations, practices, and opportunities to learn algebra in the case of poor and black students. The findings reveal striking features of the teaching

practices that lessen the quality of instruction and the algebraic content taught in school mainly attended by these student populations. In addition, the study contributes to the comprehension of the dialectical relationship between teachers' expectations–nurtured by hegemonic ideologies–and the practices for teaching algebra. However, it is clear that more studies approaching these issues are needed, particular in Colombia, a country with high levels of social and economic inequality. The Colombian research community needs to continue focusing on these types of issues if our interest and commitment in reaching the goal of equity in mathematics education is genuine.

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APPENDIX A. Levels of Performance and SES Criteria in the Saber 9 Test

On the Saber 9 test, students are classified into four levels of performance as described in Table 1. In addition, the schools are classified in four levels of SES based on the students' SESI index (SESI) average. The ICFES calculates the SESI using indicators of the Quality of Life Index (QLI) for four variables: parent schooling, house overcrowding, house sewer system, and the material of the house floor. The school SES is defined for each institution taking into account that at least 25% of the students belong to that SES; then, all students attending that school are located in its SES level. The lowest SES is SES 1 and the highest is SES 4.

Table 5. Description of ninth graders' levels of performance in the Saber 9 test

Level	Students' Knowledge and Abilities Associated to Each Level of Performance
Proficient	Students in this level (a) represent functions by using different semiotic representational systems and coordinate such representations; (b) establish equivalences between algebraic and numeric expressions; (c) characterize the properties of a plane figure that has been object of geometric transformations; (d) use criteria of similarity and congruency to transform and build figures; and so on.
Satisfactory	Students in this level (a) solve problems by using the properties of powers, radicals, logarithms; (b) use basic algebraic and graphic representations to model simple situations of variation; (c) recognize and apply rigid motions to 2D figures in the coordinate plane; and so on.
Basic	Students in this level (a) recognize several ways to represent a function; (b) identify some properties of 3D and 2D figures; (c) identify rigid motions in the plane; and so on.
Below-basic	Students in this level have not reached the minimum goals set up by the grade.

APPENDIX B. Teachers' Semi-structured Interview Protocol

- What are your favorite things about teaching?
- How long have you been teaching?
- Which grades have you taught?
- Why did you choose teaching as your profession?
- What are the most challenging situations that you face in your daily work as a middle school mathematics teacher?
- Which mechanisms/tools does your school offer to you to deal with these challenges?
- How would you describe the students in your mathematics classrooms?
- How would you describe a good student in your class?
- How would you describe a bad student in your class?
- How would you describe a student who is going to pass your class?
- How would you describe a student who is going to fail your class?
- How has your experience been teaching algebra to (poor/working-class/middle class) students?
- What hinders your students form learning algebra?
- What facilitates your students learning of algebra?
- How do you think the social background of students influence their learning of algebra?
- How do you think the social background of students influence your mathematics teaching practices?

APPENDIX C: Field Notes Protocol

Classroom layout			
Time	**Moment of class**	**Teacher's Strategy**	**Memo**

VITA

Luz Edith Valoyes Chávez grew up in Cali, the Colombian urban center with the highest black population percentage. She earned a Bachelor in Education with emphasis in Mathematics and Physics (1996) from the Universidad del Valle in Cali, Colombia, and a Master degree in Education with emphasis in Mathematics Education (2008) from the same university. After completing her master degree, she received a Fulbright Scholarship for Afro Colombians Leaders to pursue a doctoral degree in the United States. She earned a Ph.D. in Learning, Teaching, and Curriculum with an emphasis in Mathematics Education from the University of Missouri in Columbia (2014).

Luz has worked as a middle school mathematics teacher in low-income schools and as a teacher educator and professional development facilitator in Colombian universities. Her research interests include mathematics teaching practices and power; mathematics education and social justice; mathematics teacher agency and student identity in the mathematics classroom; and algebra teaching practices.

During her doctoral studies, Luz had the opportunity to engage in research funded by the National Science Foundation aimed to analyze the impact of professional development interventions to enhance the mathematics learning and participation of minority students. She also actively engaged in professional development and research internships.