ESSAYS ON ADVANCE SELLING OF NEW TO-BE-RELEASED PRODUCTS

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by

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The undersigned, appointed by the Dean of the Graduate School, have examined the dissertation entitled:

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ABSTRACT

This dissertation comprises three essays on the same topic: advance selling of new to-be-released products.

The first essay studies the retailer's optimal strategy in a two-period model where the demand uncertainty comes from both the market size and the distribution of consumers' valuations. I find that there are three types of advance selling strategies: advance selling at a deep discount, advance selling at a moderate discount and no advance selling. I also characterize the conditions under which the retailer adopts advance selling and perform comparative statics analysis.

The second essay studies the retailer's optimal advance selling strategy in a model with the presence of experienced consumers. We divide consumers into two groups, experienced and inexperienced. Pre-orders from experienced consumers lead to a more precise forecast of future demand by the firm. We show that the firm will always adopt advance selling and that the optimal pre-order price may or may not be at a discount to the regular selling price.

The third essay investigates advance selling at a price premium. I show that advance selling at a price premium always yields more profit for the retailer compared with advance selling at the regular selling price. In addition, I analyze conditions under which the retailer is more likely to implement advance selling at a price premium instead of a price discount. Sensitivity analysis is also presented to show how the retailer's optimal advance selling price premium and optimal total profit are affected by some important parameters in the model.

Chapter 1

Introduction

This dissertation focuses on the study of advance selling for a retailer[1] before he releases a new product. Due to the increased competition and rapid product replacements, advance selling has been a very popular strategy for the retailer to forecast the future demand and manage the inventory. When the retailer adopts advance selling, he takes pre-orders which are guaranteed to be delivered on the release date. Since consumers cannot try the product before its release, they are uncertain about their valuations for this product when they place pre-orders. To induce consumers to pre-order, advance selling is usually carried out with a discount. Examples in practice show that advance selling is widely used in many categories.

- In 2006, FamilyVideo.com began taking pre-orders for Gears of War for Xbox 360 at $49.99 (MSRP $59.99).
- In 2007, Amazon offered a 49% discount to induce consumers to pre-order the

[1]The retailer will be referred to as "he" hereinafter, whereas a consumer will be referred to as "she"

book "Harry Potter and Deathly Hallows".

- In 2008, Nintendo Wii offered a $20 discount to consumers who pre-ordered Wii Fit.

- In 2009, Windows 7 Home Premium Upgrade and Professional Upgrade were available for consumers in the U.S. to pre-order at $49.99 (MSRP $119.99) and $99.99 (MSRP $199.99), respectively.

- Before releasing iPhone 3GS in 2009 and iPhone 4 in 2010, Apple allowed consumers to pre-order the new generation at $199 for the 16GB version and at $299 for the 32GB version.

- On March 12, 2010, Apple started advance selling iPad to the U.S. customers.

Consumers pre-order a new product because of two main considerations. First, pre-orders are guaranteed with prompt delivery on the release date. The guaranteed fulfillments of pre-orders seem to be very attractive to consumers with high valuations. They do not need to wait to buy this product in the regular selling season and face the risk of stock out if they love this product. Second, pre-orders usually come with a discount to compensate for consumers' valuation uncertainty. With a pre-order discount, consumers who will not buy in the regular selling season might be induced to pre-order.

As to the retailer, there are several major benefits associated with advance selling discussed in the literature. First, it may help the retailer to reduce demand uncertainty (Chen and Parlar 2005 and Prasad, Stecke, and Zhao 2011). Second, it can provide the retailer with opportunities to better forecast the future demand (Tang, Rajaram, Alptekinoğlu, and Ou 2004 and Zhao and Stecke 2010). Third, it utilizes consumers' uncertainty of valuations and carry out price discrimination (Chu and Zhang 2011 and Nocke, Peitz, and Rosar 2011).

Four important features of the models in this dissertation make it distinctive. In contrast to the literature on advance selling, which commonly assumes that the retailer offers price discount to consumers, this dissertation examines the profitability of advance selling with a price premium (Chapter 4). Also, it captures an important phenomena of advance selling, that is, the retailer cannot get the accurate distribution of consumers' valuations before the regular selling season. Thus, learning by the firm in this dissertation is not only on the consumer pool but also on the consumer valuation distribution (Chapter 2, 3 and 4). Third, I present a model with experienced consumers and study the role played by them (Chapter 3). Advance selling of new generation of serial products is studied in this model setup. Last, since stock-out probability affects consumers purchase decisions in the advance selling season and these decisions in turn affect the stock-out probability in the regular selling season, I model the stock-out probability through endogenous determination (Chapter 2, 3 and 4). However, most papers in the literature model it as exogenously given.

The first essay, "Advance Selling of New Products", focuses on advance selling of a completely new product. I study a two-period dynamic model in which consumers are uncertain about their valuations in the first period (advance selling season). The retailer does not know the mean of consumer valuation distribution in addition to the market size. As a result, the demand in the second period (regular selling season) is uncertain. To better match supply with demand, the retailer can implement advance selling strategy to learn the consumer valuation distribution, with which he is able to forecast the future demand. Based on this model setup, I show that the retailer may or may not implement advance selling before the product is released. Also, I provide the conditions under which the retailer should sell in advance and what should be the optimal advance selling price. Furthermore, the impacts of some parameters on the retailer's optimal advance selling strategy are examined in this essay.

The second essay, "Advance Selling in the Presence of Experienced Consumers", introduces experienced consumers into the model and studies the role they play. The key feature of this model is that consumers are classified into two groups: experienced and inexperienced. Experienced consumers know their valuations in the advance selling season, while inexperienced consumers learn their valuations only in the regular selling season. The presence of experienced consumers yields new insights. Specifically, pre-orders from experienced consumers lead to a more precise forecast of future demand by the firm. We show that the firm will always adopt advance selling and that the optimal pre-order price may or may not be at a discount to the regular selling price.

The third essay, "Advance Selling with Price Premium", extends the analysis of the second essay to study whether the retailer can improve his profit through advance selling at a price premium. It was motivated by the observations that some products were sold out either in the advance selling season or shortly after the release, which implies that retailers might be able to improve their total profits by advance selling at a premium. I show that the retailer will implement advance selling either at a discount or at a premium. Furthermore, I study the conditions in which a retailer is more likely to sell in advance at a premium rather than a price discount.

Chapter 2

Advance selling of new products

2.1 Introduction

Advance selling is a sale strategy by a retailer which allows consumers to submit pre-orders before the release of a new to-be-released product. It is often implemented when the retailer faces demand uncertainty and needs to decide how much to produce before the regular selling season. Since consumers are uncertain about their valuations for this product in advance of the regular selling season, advance selling is usually carried out with a discount to induce consumers to pre-order, guaranteeing that pre-orders will be fulfilled promptly after release. For example, Amazon allows consumers to pre-order books and music CDs which will be released soon. Bestbuy offers different levels of discounts to consumers who pre-order new video games. With remarkable developments in the Internet and information technology, advance selling is widely used in many product categories, such as books, CDs, video games, smart phones, software, fashion products, and travel services.

There are three major benefits associated with advance selling. First, it helps the retailer to reduce the demand uncertainty because he can capture some of the market demand in advance through pre-orders. Second, it provides the retailer with opportunities to better forecast the future demand. In particular, pre-order information may work as a signal for the retailer to update the forecast of market demand. Third, it helps the retailer to utilize consumers' uncertainty of valuations and increase the overall demand. In the regular selling season, consumers with valuations below the selling price will not make purchases. However, they may be attracted to pre-order at a discount because they do not know their own valuations in the advance selling season.

This paper studies the optimal advance selling strategy of completely new products (for serial products, it focuses on the first generation). The motivation for the present study comes from two observations. First, some retailers either adjust the pre-order prices in the advance selling season or refund early adopters in the regular selling season.[1] Since the products are completely new, it seems to be very difficult for retailers to actually capture the consumers' valuation information in advance. Second, we do not observe advance selling for all new products. Table 2.1 reports release history for some well-known products. Pre-orders were only available for some of these products, with or without discounts.

After considering the retailer's uncertainty of consumer valuation information, it is very interesting to ask, for a new product, when should a retailer implement

[1]For example, Apple adopted advance selling without a discount for its first generation of iPhone in 2007, but later it gave \$100 rebates to early adopters after the release. Amazon started taking pre-order for Nokia N900 at \$649 in September 2009, but later it dropped the pre-price to \$589.

advance selling? What should be the optimal advance selling price? How does the retailer's optimal choice change with some important parameters in the model, such as salvage value, profit margin in the regular selling season, uncertainty of market size, and some consumer characteristics?

Table 2.1: Release history for several products

Product	Release date	Pre-order availability	Discount
Harry Potter Book 1	Sep. 1, 1998	No	N/A
iPhone	Jun. 29, 2007	No	N/A
iPod Touch 1st	Sep. 5, 2007	No	N/A
Amazon Kindle 1	Nov. 19, 2007	No	N/A
PlayStation 1	Sep. 9, 1995	Yes	No discount
Nokia N900	Nov.11,2009	Yes	No discount
iPad 1	Apr.3, 2010	Yes	No discount
Motorola Xoom Wi-Fi	Mar.23, 2011	Yes	No discount
Gears of War	Nov.7,2006	Yes	\$10 off
Nintendo Wii Fit	May 21, 2008	Yes	\$20 off

I consider a two-period dynamic model. The first period is the advance selling season, and the second period is the regular selling season. Consumers in the model are heterogenous in their valuations, which are assumed to follow a normal distribution. Consumers do not know their own valuations in the advance selling season. When pre-orders are available, consumers make purchases in advance by comparing the expected payoffs from pre-orders and not. If they decide to wait, consumers with valuations above the regular selling price will make purchases in the regular selling season. However, they will face a risk of not being able to get the product. With regard to the retailer, he is uncertain about the market size, and he does not know the mean of consumer valuation distribution because this product is completely new to the market. To reduce the uncertainty caused by these two factors, the retailer decides on adopting advance selling or not after considering consumers' decision-making process. If yes, he chooses the advance selling price at the same time and makes the quantity decision at the end of the advance selling season.

I find that there are three types of advance selling strategies for the retailer: advance selling at a deep discount, advance selling at a moderate discount and no advance selling. In addition, I show that the retailer will implement advance selling if and only if the marginal cost is below the threshold on it. Numerical tests are also presented to show how these parameters in the model impact the retailer's optimal advance selling strategy.

This paper contains several contributions to the literature on advance selling.

- Most of the studies on advance selling assume the consumer valuation distribution is known to the retailer. They only consider the the demand uncertainty from the randomness of the market size. However, this paper includes the uncertainty of the consumer valuation distribution into the model and studies how it affects the retailer's optimal advance selling strategy.

- Rather than build up a correlation between the demands in these two periods and forecast the second-period demand with the realized first-period demand, this paper studies the retailer's active learning of the consumer valuation distribution or the market size, with which he updates the forecast of the future demand.

- Most of the studies on advance selling take the stock-out probability as exogenously given. This paper is the first paper to examine the endogenous stock-out probabilities under different scenarios. Specifically, it corrects the formula for the stock-out probability in Prasad, Stecke, and Zhao (2011), and studies it according to three learning scenarios for the retailer.

2.2 Literature Review

The research on advance selling can be classified into two strands. The first strand focuses on advance selling under limited capacity, with applications to service industry (Xie and Shugan (2001), Shugan and Xie (2004), and Möller and Waternabe (2010)). The second strand focuses on advance selling without capacity constraints, with applications to the manufacturing industry. Since the manufacturer/retailer also needs to decide the quantity for a specific product, this line of research studies the quantity decision in addition to the price decision. Below I expand the literature review of the second strand by dividing it into papers dealing with (i) advance selling from manufacturers to retailers and (ii) advance selling from firms (manufacturers) and retailers to consumers.

Most papers in the second strand deal with advance selling from manufacturers to retailers. For example, Cachon (2004) examines inventory risk under three types of contract offered by a supplier. Taylor (2006) studies the manufacturer's sale-timing decision, and Boyaci and Özer (2010) characterize the optimal advance selling price and optimal stopping policy for a manufacturer.

The literature that is closest to the present study is on advance selling from firms (manufacturers) and retailers to consumers without capacity constraints. Weng and Parlar (1999) are the first to develop a model in which pre-orders are offered with a discount to attract consumers. Tang, Rajaram, Alptekinoğlu, and Ou (2004) extend the model by Weng and Parlar (1999) and examine the benefits of advance selling. McCardle, Rajaram, and Tang (2004) present a duopoly model and focus on competition between two firms. Chen and Parlar (2005) introduce two different models and solve for the optimal advance selling discount and optimal quantity. In these four papers discussed above, consumers are modeled to be non-strategic.

Other papers assume strategic consumers and incorporates consumers' decision-making process into retailers' consideration. Optimal advance selling strategies are examined in different settings. For example, Zhao and Stecke (2010) classify consumers into two groups according to whether they are loss averse. Prasad, Stecke, and Zhao (2011) divide consumers into two groups, informed consumers and uninformed consumers, based on the accessibility to the pre-order information. Chu and Zhang (2011) allow the firm to control the release of information about the product at pre-order.

In the existing literature, the demand uncertainty in the regular selling season only comes from the uncertainty of the market size. These papers ignore that the retailer might have problems to get the exact distribution function of consumers' valuations before he releases a complectly new product. In practice, before he releases a new product to the market, the retailer may be able to know that this product is favored by some consumers in the market. However, it is very difficult for him to know how much consumers love this product on average. In this paper, I consider a model in which the demand uncertainty not only comes from the randomness of the market size but also from the uncertainty of consumer valuation distribution.

The rest of the paper is organized as follows. Section 2.3 introduces the model. Section 2.4 presents optimal solutions to the Newsvendor Problem when there is no advance selling. Section 2.5 studies the retailer's advance selling strategies and provides the conditions under which the retailer will implement advance selling. Section 2.6 performs comparative statics analysis. Section 2.7 concludes the paper.

2.3 Model Setup

Consider a retailer who sells a new product to a set of consumers over two periods. The first period is the advance selling season and the second period is the regular selling season. Each consumer in the market wants to purchase at most one unit of the specific product, either in the first period or in the second period. Orders submitted in the first period at the advance selling price are guaranteed to be fulfilled after the product release. With regard to the orders submitted in the second period, there is a risk that this product will be out of stock. The cost for the retailer to implement advance selling is k (sufficiently small), which captures the labor, technology, advertising and other costs. Table 2.2 lists the notation in this paper.

2.3.1 Retailer

The retailer produces the product at marginal cost c and charges price p during the regular selling season. At the end of the regular selling season, the retailer gets salvage value s for each unsold unit. I assume $s < c < p$, which ensures that the retailer makes positive profit and avoids infinite stock. Also, as described in detail in Subsection

Table 2.2: Notation A

Parameters/Variables concerning a retailer	
c	marginal cost
s	salvage value
p	price in the regular selling season
k	adoption cost of advance selling
π	retailer's expected profit from the regular selling season
Π	retailer's total expected profit (includes pre-orders)
Parameters/Variables concerning consumers and market	
D_1, D_2	demands in the first and second periods
η	stock-out probability
$V \sim \mathrm{N}\left(\mu, \sigma^2\right)$	consumer valuation distribution, with realized value v
$\mu \in \{\mu_H, \mu_L\}$	two-point distribution, $\mathrm{Prob}(\mu_H) = \gamma$ and $\mathrm{Prob}(\mu_L) = 1 - \gamma$
$M_i \sim \mathrm{LN}\left(\nu_i, \tau_i^2\right)$	market size distribution, mean $m_i = \exp\left\{\nu_i + \tau_i^2/2\right\}$
Decision variables	
q	quantity produced for the regular selling season
Q	total quantity produced (includes pre-orders)
x	advance selling price
Distribution and density functions	
$F(\cdot)$	cdf of $\mathrm{N}\left(\mu, \sigma^2\right)$, $F(y) = \Phi\left(\frac{y-\mu}{\sigma}\right)$
$f(\cdot)$	density function of $\mathrm{N}\left(\mu, \sigma^2\right)$, $f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(y-\mu)^2}{2\sigma^2}\right\}$
$G(\cdot)$	cdf of $\mathrm{LN}\left(\nu, \tau^2\right)$, $G(y) = \Phi\left(\frac{\ln y - \nu}{\tau}\right)$
$g(\cdot)$	density function of $\mathrm{LN}\left(\nu, \tau^2\right)$, $g(y) = \frac{1}{y\sqrt{2\pi\tau^2}} \exp\left\{-\frac{(\ln y-\nu)^2}{2\tau^2}\right\}$

2.3.2, the market size is a random variable, and the consumer valuation distribution is unknown to the retailer. Because of these two factors, the retailer faces uncertain demand.

At the beginning of the advance selling season the retailer makes a decision on the advance selling price x; also, he announces the regular selling price p to the market. In the model, I assume $x \leq p$, i.e, pre-order discount is offered to induce consumers to order in advance. After pre-order is available, all consumers are allowed to submit pre-orders at price x which will be fulfilled by the retailer in the regular selling season.

At the end of the advance selling period the retailer gets the number of pre-orders, denoted by D_1, from which he might be able to learn consumer valuation distribution. Let D_2 be the random demand in the regular selling season. Informed by the pre-orders D_1, the retailer must decide how much to produce: $Q = D_1 + q$, where D_1 fulfills the pre-orders immediately after the release and quantity q satisfies the demand during the regular selling season.

2.3.2 Consumers

Consumers are risk-neutral. Each consumer has an idiosyncratic valuation, i.e., the maximum amount of money she would like to pay for this product. Since the product is new to the market and unavailable before its release date, it is assumed that consumers are uncertain about their own valuations in the advance selling season. The consumer valuation of this product V follows normal distribution with mean μ and variance σ^2, i.e., $V \sim \mathrm{N}(\mu, \sigma^2)$. Consumers know the distribution from the beginning of the advance selling season while the retailer lacks information of it. Retailer's uncertainty is modeled by assuming that μ follows a two-point distribution: $\text{Prob}(\mu_H) = \gamma$ and $\text{Prob}(\mu_L) = 1 - \gamma$, where $\mu_L < \mu_H$ and $\gamma \in (0, 1)$. In the regular selling season, each consumer realizes her valuation as v and she purchases when $v \geq p$.

The size of the consumer market M_i is a random variable. It is assumed that the distribution of M_i is common knowledge and it follows lognormal distribution $\mathrm{LN}(\nu_i, \tau_i^2)$. Let m_i denote the expected value (mean) of M_i, $m_i = \exp\{\nu_i + \tau_i^2/2\}$.

During the advance selling season, consumers are uncertain about their valuations and thus make decisions on whether to purchase by comparing the expected payoffs from pre-orders and regular season purchases. If a consumer pre-orders, she pays discount price x and is guaranteed to get the product right after it is released. If not, she waits until the regular selling season and makes a purchase when the realized valuation v is no less than the regular selling price p, but she might face a stock out.

2.4 No Advance Selling

First, consider the benchmark case when the retailer does not implement the advance selling strategy. There is only one period, that is, the regular selling season. Before the regular selling season starts, the retailer has to decide how much to produce at a given price p. Then he produces the quantity at a marginal cost c, charges price p during the selling season, and gets a salvage value s for each unsold unit at the end of the selling season. As mentioned in the previous section, the market size $M_i \sim \mathrm{LN}(\nu_i, \tau_i^2)$ and it is a common information. Furthermore, consumer valuation follows a Normal distribution, $V \sim \mathrm{N}(\mu, \sigma^2)$, where μ follows a two- point distribution and takes values μ_H or μ_L. The key point here is that the retailer does not know the exact value of μ but consumers do.

Since there is no advance selling season in the benchmark case, $D_1 = 0$. The regular selling season demand

$$D_2 = M_i \operatorname{Prob}(v > p) = \begin{cases} M_i \overline{F}_H(p), & \text{with probability } \gamma, \\ M_i \overline{F}_L(p), & \text{with probability 1-}\gamma. \end{cases}$$

The retailer has to decide the optimal quantity to maximize his total expected profit when random variable D_2 is as described above. That is, he solves

$$\max_{Q \geq 0} E_{D_2} \left[p \min \{Q, D_2\} + s(Q - D_2)^+ - cQ \right]. \tag{2.1}$$

This problem is known as the Newsvendor Problem because the retailer has to make the quantity decision before observing the demand in the regular selling season. It is important to point out that the random demand in the regular selling season D_2 does not follow a Lognormal distribution as M_i does. Gallego (1995) gives a closed form formula which maximizes the total expected profit considering the worst possible distribution of D_2 when the mean and variance are given. Fortunately, it provides a very good approximation to the solution of the Newsvendor problem under D_2.

Let μ_0 and σ_0^2 denote the mean and the variance of the random demand D_2 in the regular selling season when there is no advance selling. Then,

$$\begin{aligned} \mu_0 &= \mathrm{E}[D_2] = \gamma \, \mathrm{E}\left[M_i \overline{F}_H(p)\right] + (1-\gamma)\, \mathrm{E}\left[M_i \overline{F}_L(p)\right] \\ &= \left(\gamma \overline{F}_H(p) + (1-\gamma)\overline{F}_L(p)\right) \mathrm{E}[M_i] \\ &= \left(\gamma \overline{F}_H(p) + (1-\gamma)\overline{F}_L(p)\right) m_i, \\ \sigma_0^2 &= Var[D_2] = E[(D_2)^2] - (E[D_2])^2, \end{aligned}$$

where

$$
\begin{aligned}
\mathrm{E}[(D_2)^2] &= \gamma \, \mathrm{E}\left[(M_i \overline{F}_H(p))^2\right] + (1-\gamma) \, \mathrm{E}\left[(M_i \overline{F}_L(p))^2\right] \\
&= \left(\gamma \overline{F}_H^2(p) + (1-\gamma) \overline{F}_L^2(p)\right) \mathrm{E}[(M_i)^2] \\
&= \left(\gamma \overline{F}_H^2(p) + (1-\gamma) \overline{F}_L^2(p)\right) m_i^2 \exp\{\tau_i^2\}.
\end{aligned}
$$

Let Q^0 denote the optimal quantity for this Newsvendor Problem and Π^0 be the optimal total expected profit. Following Gallego (1995), given μ_0 and σ_0, the solution to the Newsvendor Problem with an unknown distribution of random demand D_2 is

$$
Q^0 = \mu_0 + \frac{\sigma_0}{2} \left(\sqrt{\frac{p-c}{c-s}} - \sqrt{\frac{c-s}{p-c}} \right). \tag{2.2}
$$

Thus,

$$
\Pi^0 = \gamma \Pi_H(Q_0) + (1-\gamma) \Pi_L(Q_0), \tag{2.3}
$$

where $\Pi_H(Q)$ and $\Pi_L(Q)$ represent the total expected profits with quantity Q for $\mu = \mu_H$ and $\mu = \mu_L$, respectively. The explicit expression for Π^0 (derived in Appendix) is

$$
\Pi^0 = (p-c) Q^0 - (p-s) \left(\gamma A_H + (1-\gamma) A_L \right), \tag{2.4}
$$

where

$$
\begin{aligned}
A_j &= Q^0 \Phi(T_j) - \overline{F}_j(p) m_i \Phi(T_j - \tau_i), \\
T_j &= \frac{\ln Q^0 - \left(\nu_i + \ln \overline{F}_j(p)\right)}{\tau_i}, j = H, L.
\end{aligned}
$$

2.5 Advance Selling

The goal of this section is to study the optimal advance selling strategy for the retailer. First, I derive the endogenous stock-out probabilities in different scenarios and examine consumers' optimal purchasing decisions. Then I study the retailer's learning from pre-orders, present the retailer's total expected profit function and solve for the optimal advance selling price. In Subsection 2.5.5, advance selling is compared with the benchmark case and the conditions under which advance selling is superior to no advance selling are characterized.

2.5.1 Stock-out probability

Before studying consumers' optimal purchasing decisions, we need to solve for the stock-out probability η. The stock-out probability is the fraction of excess demand in the second period over the total second-period demand. It captures the probability of any consumer who wants to purchase the product in the regular selling season but is unable to get it, i.e.,

$$\eta = E\left[\left(\frac{D_2 - q^*}{D_2}\right)^+\right], \tag{2.5}$$

where q^* is the optimal quantity for the random demand in the second period D_2. For the lognormal distribution $D_2 \sim \mathrm{LN}\left(\nu, \tau^2\right)$ the optimal production quantity is given by

$$q^* = \exp\{\nu + \tau z_\beta\} \tag{2.6}$$

and

$$\pi(q^*) = (p - s)\left(1 - \Phi(\tau - z_\beta)\right)\exp\left\{\nu + \frac{\tau^2}{2}\right\}, \tag{2.7}$$

where z_β is the β-th percentile of the standard normal distribution, $z_\beta \equiv \Phi^{-1}(\beta)$. See Appendix for derivations of (2.6) and (2.7).

In addition, the stock-out probability for $D_2 \sim \mathrm{LN}\,(\nu, \tau^2)$ is

$$\eta = \int_{q^*}^{+\infty} \frac{D_2 - q^*}{D_2} g(D_2)\,\mathrm{d}D_2,$$

where $g(\cdot)$ is the density function of $\mathrm{LN}\,(\nu, \tau^2)$ and q^* is given in (2.6). The explicit expression for the stock-out probability is obtained in Appendix,

$$\eta = 1 - \beta - \exp\left\{\tau z_\beta + \frac{\tau^2}{2}\right\}(1 - \Phi(z_\beta + \tau)). \tag{2.8}$$

From the expression of stock-out probability (see (2.5)), it is easy to see that the quantity decision made by the retailer in the second period affects η directly. Because the retailer's learning is very important for him to make quantity decision (Subsection 2.5.3), with regard to the stock-out probability, the following three learning scenarios are considered.

(i) If the retailer learns from pre-orders that $\mu = \mu_L$,

$$D_2 = M_i \overline{F}_L(p) \sim \mathrm{LN}\left(\nu_i + \ln \overline{F}_L(p), \tau_i^2\right).$$

Following (2.6) and (2.7), the optimal order quantity q_L^* and the resulting expected profit π_L are

$$q_L^* = \exp\{\nu_i + \tau_i z_\beta\}\overline{F}_L(p)$$

and

$$\pi_L = (p - s)\,(1 - \Phi(\tau_i - z_\beta))\,m_i \overline{F}_L(p),$$

where $\beta \equiv (p-c)/(p-s)$ and z_β is the β-th percentile of the standard normal distribution, i.e., $z_\beta \equiv \Phi^{-1}(\beta)$. Following (2.8), the stock-out probability η_1 is

$$\eta_1 = 1 - \beta - \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}(1 - \Phi(z_\beta + \tau_i)).$$

(ii) If the retailer learns from pre-orders that $\mu = \mu_H$,

$$D_2 = M_i \overline{F}_H(p) \sim \text{LN}\left(\nu_i + \ln \overline{F}_H(p), \tau_i^2\right).$$

The optimal order quantity q_H^* and the resulting expected profit π_H are

$$q_H^* = \exp\{\nu_i + \tau_i z_\beta\}\overline{F}_H(p)$$

and

$$\pi_H = (p-s)\left(1 - \Phi(\tau_i - z_\beta)\right) m_i \overline{F}_H(p).$$

The stock-out probability η_2 is

$$\eta_2 = 1 - \beta - \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}(1 - \Phi(z_\beta + \tau_i)).$$

Since $\eta_1 = \eta_2$, let η^* denote the stock-out probability when the retailer can infer the value of μ from pre-orders, $\eta^* = \eta_1 = \eta_2$. So,

$$\eta^* = 1 - \beta - \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}(1 - \Phi(z_\beta + \tau_i)). \tag{2.9}$$

(iii) If the retailer does not learn from pre-orders the value of μ, the optimal order quantity Q^0 and the resulting expected profit Π^0 are given in Section 2.4 as (2.2) and (2.4). When $\mu = \mu_L$,

$$\eta_L = E\left[\left(\frac{D_2 - Q^0}{D_2}\right)^+\right],$$

where $D_2 = M_i\overline{F}_L(p)$. The explicit expression for η_L (derived in Appendix) is

$$\eta_L = 1 - \Phi\left(\frac{\ln Q^0 - \nu_L}{\tau_i}\right) - Q^0 \exp\left\{\frac{\tau_i^2}{2} - \nu_L\right\}\left(1 - \Phi\left(\frac{\ln Q^0 - \nu_L + \tau_i^2}{\tau_i}\right)\right),$$

where $\nu_L = \nu_i + \ln \overline{F}_L(p)$. When $\mu = \mu_H$,

$$\eta_H = E\left[\left(\frac{D_2 - Q^0}{D_2}\right)^+\right],$$

where $D_2 = M_i\overline{F}_H(p)$. The explicit expression for η_H (derived in Appendix) is

$$\eta_H = 1 - \Phi\left(\frac{\ln Q^0 - \nu_H}{\tau_i}\right) - Q^0 \exp\left\{\frac{\tau_i^2}{2} - \nu_H\right\}\left(1 - \Phi\left(\frac{\ln Q^0 - \nu_H + \tau_i^2}{\tau_i}\right)\right), \tag{2.10}$$

where $\nu_H = \nu_i + \ln \overline{F}_H(p)$. (2.10) is used later in (2.17).

Lemma 1 (Optimal quantity)**.** *When there is no advance selling, the optimal quantity Q^0 satisfies $q_L^* < Q^0 < q_H^*$, where q_H^* and q_L^* denote the optimal second-period quantities when $\mu = \mu_H$ and $\mu = \mu_L$, respectively.*

With the results in Lemma 1, the lemma below shows the relationship between η_L, η^* and η_H.

Lemma 2 (Stock-out probability)**.** $\eta_L < \eta^* < \eta_H$ *holds.*

The intuition is straightforward. First, when $\mu = \mu_L$, η_L corresponds to the output Q^0 when the retailer does not know the value of μ, while η^* corresponds to the output q_L^* when the retailer learns it. From Lemma 1, $q_L^* < Q^0$, which implies η_L corresponds to a higher output amount compared to η^*. Thus, the stock-out probability is lower, $\eta_L < \eta^*$. Following the same logic, $\eta^* < \eta_H$ holds when $\mu = \mu_H$.

2.5.2 Consumers' optimal purchasing decisions

As described is Subsection 2.3.2, each consumer purchases at most one unit of the product and she makes the decision to pre-order or wait by comparing the expected payoffs.

When a consumer pre-orders in the advance selling season, it is easy to see that her expected payoff is

$$\mu - x.$$

When she does not pre-order and waits until the regular selling season, with probability η this product is out of stock and she gets a payoff of zero; with probability $1-\eta$ this product is in stock and she purchases if $v \geq p$, which yields $\int_p^{+\infty}(v-p)f(v)\,\mathrm{d}v$ to her. Thus, her expected payoff is

$$(1-\eta)\int_p^{+\infty}(v-p)f(v)\,\mathrm{d}v,$$

where $f(\cdot)$ is the density function of $\mathrm{N}\left(\mu, \sigma^2\right)$.

When $\mu = \mu_L$, the consumer pre-orders if and only if

$$\mu_L - x \geq (1-\eta)\int_p^{+\infty}(v-p)f_L(v)\,\mathrm{d}v;$$

when $\mu = \mu_H$, the consumer pre-orders if and only if

$$\mu_H - x \geq (1-\eta) \int_p^{+\infty} (v-p) f_H(v) \, \mathrm{d}v.$$

Let

$$x_L \equiv \mu_L - (1-\eta) \int_p^{+\infty} (v-p) f_L(v) \, \mathrm{d}v \tag{2.11}$$

and

$$x_H \equiv \mu_H - (1-\eta) \int_p^{+\infty} (v-p) f_H(v) \, \mathrm{d}v \tag{2.12}$$

denote the threshold values for $\mu = \mu_L$ and $\mu = \mu_H$, respectively. Here $f_L(\cdot)$ is the density function of $\mathrm{N}(\mu_L, \sigma^2)$ and $f_H(\cdot)$ is the density function of $\mathrm{N}(\mu_H, \sigma^2)$. Therefore, when $\mu = \mu_L$, the consumer pre-orders if and only if $x \leq x_L$; when $\mu = \mu_H$, the consumer pre-orders if and only if $x \leq x_H$.

It is important to point out that η in (2.11) and (2.12) does not need to be the same.[2] They are to be determined endogenously in the model. In particular, η can be either η_L or η^* in (2.11), depending on whether the retailer knows the value of μ. Similarly, η can be either η_H or η^* in (2.12).

Lemma 3 below shows that the threshold value x_L is always less than x_H no matter which value η takes in (2.11) and (2.12).

Lemma 3 (x_L and x_H)**.** *The threshold values x_L and x_H always satisfy that $x_L < x_H$.*

According to Lemma 3, a deeper advance selling discount is needed to induce all consumers to pre-order if the valuation expectation is μ_L. This is because low type consumers are willing to pay less compared to the high type. Since $x_L < x_H$ always holds, there are three regions associated with consumers' purchasing behaviors.

[2] The expressions for x_L and x_H in terms of the exact value of η are given in Subsection 2.5.4.

- Region A: $x \leq x_L$. All consumers pre-order.

- Region B: $x_L < x \leq x_H$. All consumers pre-order if $\mu = \mu_H$; all wait until the second period if $\mu = \mu_L$.

- Region C: $x > x_H$. All consumers wait until the second period.

2.5.3 Retailer's learning from pre-orders

Since the retailer is uncertain about the consumer valuation distribution, he implements advance selling at a fixed cost k and takes pre-orders from consumers at a discount price x. With the information on pre-orders obtained during the advance selling season, the retailer learns the second-period demand D_2 immediately, or he tries to infer μ, and then uses it to forecast the future demand.

When advance selling price x is in region A, all consumers in the market pre-order. The retailer captures this information and learns that the demand in the second period is $D_2 = 0$. Thus, he produces $Q = D_1$. Let $\Pi^A(x)$ be the total expected profit in region A:

$$\Pi^A(x) = E\left[(x-c)D_1\right] - k = m_i(x-c) - k. \tag{2.13}$$

When advance selling price x is in region B, the retailer learns μ based on the number of pre-orders D_1 at the end of advance selling season. If $D_1 = 0$, the retailer infers that $\mu = \mu_L$. Thus, he produces $Q = D_1 + q_L^*$, where q_L^* is the optimal quantity for the random demand of the second period,

$$D_2 = M_i \overline{F}_L(p) \sim \text{LN}\left(\nu_i + \ln \overline{F}_L(p), \tau_i^2\right).$$

Following the solution to the Newsvendor Problem under the lognormal distribution,

$$q_L^* = \exp\{\nu_i + \tau_i z_\beta\}\overline{F}_L(p),$$

yielding

$$\pi_L = (p - s)\,(1 - \Phi(\tau_i - z_\beta))\, m_i \overline{F}_L(p)$$

to the retailer. If $D_1 \neq 0$, then the retailer infers that $\mu = \mu_H$ and concludes $D_2 = 0$. Thus, he produces $Q = D_1$. Let $\Pi^B(x)$ denote the total expected profit in region B:

$$\Pi^B(x) = \gamma m_i(x - c) + (1 - \gamma)\pi_L - k. \tag{2.14}$$

When advance selling price x is in region C, all consumers will wait, $D_1 = 0$. The retailer can not learn μ in the advance selling season. In this case, the retailer faces the same situation as that under no advance selling. He produces $Q = Q^0$, which yields a total expected profit

$$\Pi^C(x) = \Pi^0 - k, \tag{2.15}$$

where Q^0 and Π^0 are given in (2.2) and (2.4), respectively.

It is easy to see that the retailer's learning is imperfect. When x is in region A or B, the retailer either learns that $D_2 = 0$, or learns μ and forecasts the future demand with it. When x is in region C, all consumers wait till the second period, but the retailer can not learn μ to forecast the demand.

In addition, from the analysis above, the retailer's total expected profit $\Pi(x)$ as a function of advance selling price x is

$$\Pi(x) = \begin{cases} m_i(x-c) - k, & x \le x_L, \\ \gamma m_i(x-c) + (1-\gamma)\pi_L - k, & x_L < x \le x_H, \\ \Pi^0 - k, & x > x_H. \end{cases}$$

As introduced in Section 2.3, the adoption cost of advance selling k is a small fixed number. In the following analysis, I assume $\Pi(0) \gg k$.

2.5.4 Optimal advance selling price

For the rest of the analysis, it is assumed that $x_L < x_H \le p$.[3] From the discussion of stock-out probability in Subsection 2.5.1 and retailer's learning in Subsection 2.5.3, it follows that $\eta = \eta^*$ in (2.11) and $\eta = \eta_H$ in (2.12). Thus, the threshold values expressed in (2.11) and (2.12) can be written out as

$$x_L \equiv \mu_L - (1-\eta^*) \int_p^{+\infty} (v-p) f_L(v)\, \mathrm{d}v \tag{2.16}$$

and

$$x_H \equiv \mu_H - (1-\eta_H) \int_p^{+\infty} (v-p) f_H(v)\, \mathrm{d}v, \tag{2.17}$$

where η^* and η_H are expressed by (2.9) and (2.10), respectively.

Before solving for the optimal advance selling price x^*, I present two important features of the profit function. First, $\Pi(x)$ increases in x in both region A and B (see (2.13) and (2.14)). It is important to point out that it does not imply $\Pi^A(x_L) <$

[3]There are three relationships between x_L, x_H and p: $x_L < x_H \le p$, $x_L < p < x_H$ and $p \le x_L < x_H$.

$\Pi^B(x_L)$.[4] A jump down at x_L can occur if $\Pi^A(x_L) > \Pi^B(x_L)$.

Second, $\Pi(x)$ remains constant at the value $\Pi^0 - k$ in region C. There could be a jump down at x_H if and only if $\Pi^B(x_H) < \Pi^0 - k$; otherwise there is a jump up. Extensive numerical examples indicate that both jump down and jump up can happen at both x_L and x_H.

Because the profit function can jump down/up at both x_L and x_H, it is easy to get that x_L, x_H and any $x \in (x_H, p]$ could be the optimal advance selling price (see Figure 2.1).

Proposition 1 (Optimal advance selling price)**.** *If the retailer implements advance selling, the optimal advance selling price x^* can be either x_L, x_H, or any $x \in (x_H, p]$.*

Proposition 5 suggests two values and a region for an optimal advance selling price. Moving in the direction $x_L \to x_H \to x \in (x_H, p]$, the pre-order price increases but the expected sales decrease. In detail, $\Pi^A(x_L)$ corresponds to the profit of low price-high sales, $\Pi^C(x)$ corresponds to the profit of high price-low sales, while $\Pi^B(x_H)$ shows a mixed profit from middle price high sales and high price low sales. These tradeoffs imply x^* can be either x_L, x_H, or any $x \in (x_H, p]$. However, it is important to note that if the optimal total expected profit is $\Pi^0 - k$, i.e., $x^* \in (x_H, p]$, the retailer will not implement advance selling.

To better demonstrate the pricing decision, Figure 2.1 shows three numerical examples in which the optimal advance selling price occurs in each of the three regions. In all three cases, $p = 200$, $c = 100$,$s = 80$, $\tau_i = 0.65$, $\gamma = 0.5$, $m_i = 200000$ and $k = 3000$.

Example 1. *It is constructed with $\sigma = 80$, $\mu_L = 180$, $\mu_H = 200$. In this example, the*

[4] Since $\Pi_B(\cdot)$ is defined on $(x_L, x_H]$, $\Pi^B(x_L)$ is the limiting value.

endogenous values are $\eta^* = 0.04$, η_H*=0.17*, $x_L = 158.10$, $x_H = 173.59$*; the optimal advance selling price* $x^* = x_L$*. This example is illustrated in Figure 2.1(a).*

Example 2. *It is constructed with* $\sigma = 100$, $\mu_L = 115$, $\mu_H = 180$*. In this example, the endogenous values are* $\eta^* = 0.04$, η_H*=0.24*, $x_L = 104.49$, $x_H = 156.53$*; the optimal advance selling price* $x^* = x_H$*. This example is illustrated in Figure 2.1(b).*

Example 3. *It is constructed with* $\sigma = 100$, $\mu_L = 115$, $\mu_H = 130$*. In this example, the endogenous values are* $\eta^* = 0.04$, η_H*=0.17*, $x_L = 104.49$, $x_H = 118.14$*; the optimal advance selling price* $x^* \in (x_H, p]$*. This example is illustrated in Figure 2.1(c).*

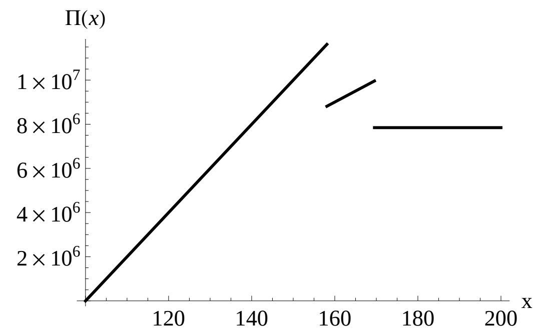
2.5.5 Advance selling vs. no advance selling

For comparison purposes, let $\Pi^* = \Pi(x^*)$ denote the corresponding total expected profit at the optimal advance selling price x^*. The condition for the retailer to implement advance selling is $\Pi^* \geq \Pi^0$. With advance selling, it is obvious from (2.13) and (2.14) that both $\Pi^A(x)$ and $\Pi^B(x)$ increase with advance selling price x. Thus, the retailer will adopt advance selling if and only if

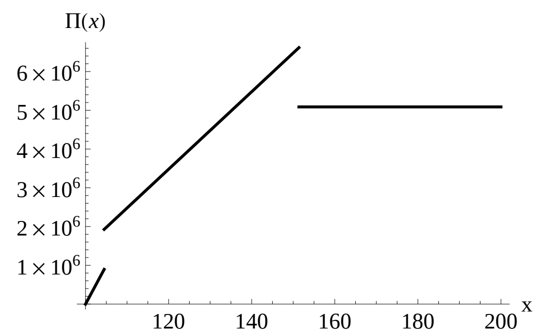
$$\Pi^* = max\{m_i(x_L - c) - k; \gamma m_i(x_H - c) + (1-\gamma)\pi_L - k; \Pi^0 - k\} \geq \Pi^0. \qquad (2.18)$$

Define the boundaries of the marginal cost for the retailer to produce the products,

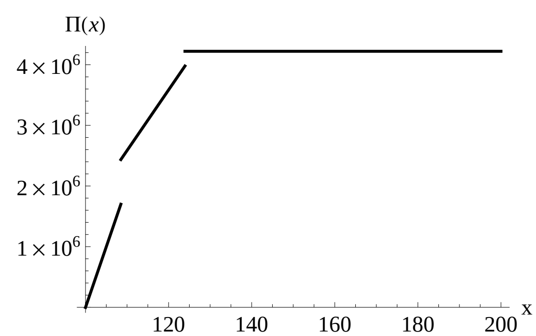
$$c_1 = \mu_L - (1-\eta^*) \int_p^{+\infty} (v-p) f_L(v)\, \mathrm{d}v - \frac{\Pi^0 + k}{m_i},$$
$$c_2 = \mu_H - (1-\eta_H) \int_p^{+\infty} (v-p) f_H(v)\, \mathrm{d}v - \frac{\Pi^0 + k - (1-\gamma)\pi_L}{\gamma m_i}.$$



(a) $x^* = x_L$



(b) $x^* = x_H$



(c) x^* in region C

Figure 2.1: Optimal advance selling price

For the retailer, c_1 and c_2 play a very important role to make the advance selling decision. Specifically, if $c < c_1$, it will be optimal for the retailer to implement advance selling at x_L and induce all consumers to pre-order compared to no advance selling. If $c < c_2$, it will be optimal for the retailer to implement advance selling at x_H compared to no advance selling. Proposition 2 shows the condition under which advance selling is superior to no advance selling.

Proposition 2. *The retailer should implement advance selling if and only if* $c \leq max\{c_1, c_2\}$; x^* *is either* x_L *or* x_H.

It implies that there is a threshold value for the marginal cost. Only with a marginal cost below the threshold value, the retailer can benefit from implementing advance selling. Moreover, the retailer is less likely to implement advance selling as c becomes larger. Proposition 2 can be expressed in a different way. It is equivalent to say that the retailer should sell in advance if and only if

$$\mu_L \geq (1-\eta^*) \int_p^{+\infty} (v-p) f_L(v) \, \mathrm{d}v + \frac{\Pi^0 + k}{m_i} + c \tag{2.19}$$

or

$$\mu_H \geq (1-\eta_H) \int_p^{+\infty} (v-p) f_H(v) \, \mathrm{d}v + \frac{\Pi^0 + k - (1-\gamma)\pi_L}{\gamma m_i} + c. \tag{2.20}$$

Equations (2.19) and (2.20) define threshold values for μ_L and μ_H, respectively.

Proposition 4 indicates that (i) advance selling is not always the best choice for the retailer; (ii) if the retailer chooses advance selling, the optimal choice could be either x_L or x_H, where $x_H \leq p$. As a result, to maximize his total expected profit, a retailer always has three strategies to choose from: no advance selling, advance selling at x_L, advance selling at x_H.

Proposition 3. *Before the release of a new product, the retailer has three advance selling strategies to choose from: no advance selling, advance selling with a deep* *discount ($x = x_L$), advance selling with a moderate discount ($x = x_H$).*[5]

2.6 Numerical Analysis

As described in Proposition 3, to obtain the highest total expected profit, a retailer chooses from three strategies: no advance selling, advance selling at x_L, and advance selling at x_H. It is interesting to examine how the retailer's advance selling strategy changes with some important parameters of the model.

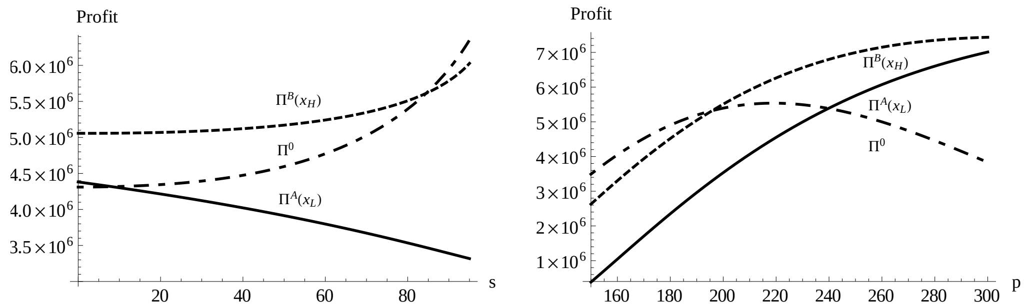
2.6.1 Retailer-related parameters

In this subsection, I consider the impact of some parameters concerning the retailer, salvage value s and profit margin $p - c$, on the retailer's advance selling decision.

Both numerical tests are constructed with the following initial values: $p = 200, c = 100, s = 80, m_i = 200000, k = 3000, \tau_i = 1.0, \sigma = 120, \gamma = 0.5, \mu_L = 140, \mu_H = 170$.

First, to examine how the retailer's decision on advance selling price changes with s, I vary s from 0 to 95 and keep the other parameters fixed. With these values, it shows in Figure 2.2(a) that $\Pi^A(x_L)$ is dominated by the other two strategies when s is high. On one hand, a higher s helps the retailer to better satisfy the market need and get higher profit from salvaged products. On the other hand, with the increase of s, consumers are willing to pay less to pre-order because the probability of stock out decreases. As a result, $\Pi^A(x_L)$ decreases. Therefore, as s increases, the retailer is more likely to choose no advance selling or advance selling with a moderate discount, i.e., $x = x_H$.

[5]when $x_H = p$, no discount will be offered.



(a) Profits change with s (b) Profits change with p

Figure 2.2: Impact of retailer related parameters on advance selling decision

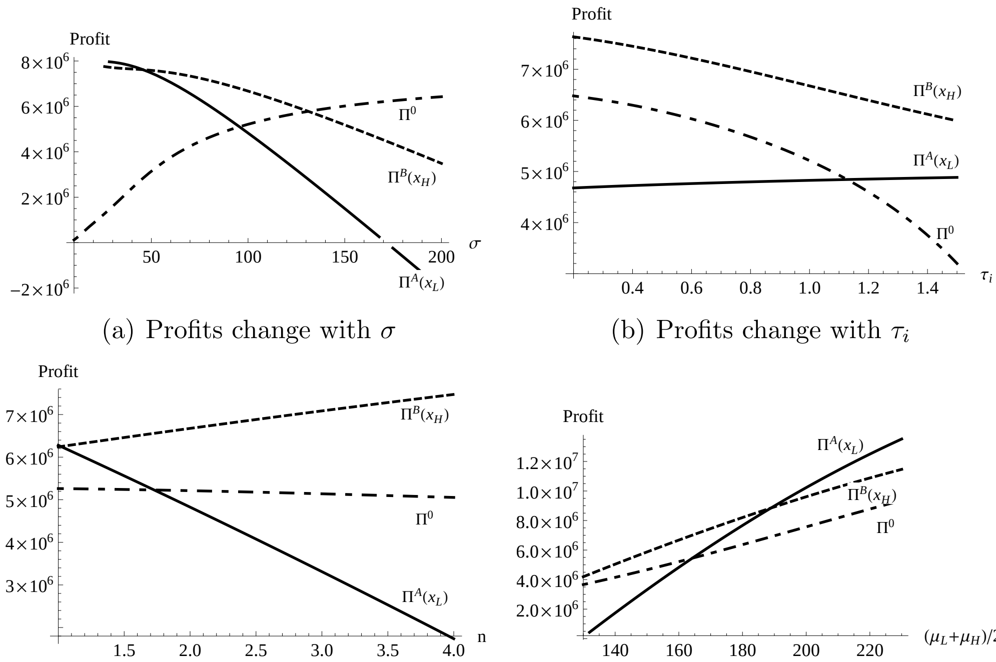
Next, I consider that profit margin $p - c$ increases. By varying p from 150 to 300, Figure 2.2(b) illustrates that Π^0 is dominated by the other two strategies when p is high. As p increases, the consumer is willing to pay a higher price to pre-order because the expected payoff of waiting decreases with p; as a result, $\Pi^A(x_L)$ and $\Pi^B(x_H)$ increase. However, the number of consumers who will purchase in the regular selling season decreases with p. As p increases, the gain from a higher price margin is less than the loss from lost buyers (Π^0 decreases). Therefore, as profit margin $p - c$ increases, the retailer is more likely to choose advance selling at either x_L or x_H to induce consumers to pre-order.

2.6.2 Consumer and market-related parameters

In this subsection, I consider how the retailer's advance selling decision is affected by some parameters concerning consumers and the market, such as standard deviation of consumer valuations σ, demand uncertainty τ_i, the difference between the expected consumer valuations $\mu_H - \mu_L$ and the expectation of μ, $(\mu_L + \mu_H)/2$.

All these numerical tests are constructed with the following initial values: $p =$

$200, c = 100, s = 80, m_i = 200000, k = 3000, \tau_i = 1.0, \sigma = 100, \gamma = 0.5, \mu_L = 140, \mu_H = 180$.



(a) Profits change with σ (b) Profits change with τ_i

(c) Profits change with $\mu_H - \mu_L$ (d) Profits change with $\frac{\mu_H+\mu_L}{2}$

Figure 2.3: Impact of consumer/market related parameters on advance selling decision

First, I vary σ from 10 to 200 to examine how a retailer's decision on advance selling price changes with σ. Figure 2.3(a) shows that no advance selling dominates the other two strategies when σ is large. A high σ implies that there is a large number of high valuation consumers in the market, so it is more profitable when all high valuation consumers purchase in the regular selling season at p by not offering pre-orders. Therefore, as σ increases, the retailer is more likely to choose no advance selling.

Second, τ_i is varied from 0.2 to 1.5 to see how the optimal profits in each region are affected. As we can see from Figure 2.3(b), τ_i affects the optimal profits in an opposite way as s. $\Pi^A(x_L)$ increases with τ_i, while both $\Pi^B(x_H)$ and Π^0 decrease. Therefore, as τ_i increases, no advance selling will gradually become dominated by the other two strategies. The retailer is more likely to implement advance selling at either x_L or x_H to induce consumers to pre-order.

Next, I am going to examine the impact of $\mu_H - \mu_L$ on the retailer's decision on advance selling price. Denote $\mu_H = 160 + 20n$, and $\mu_L = 160 - 20n$. By moving n from 1 to 4, Figure 2.3(c) shows that $\Pi^B(x_H)$ is greater than the other two when $\mu_H - \mu_L$ is high. This is because x_H increases to be close to p as $\mu_H - \mu_L$ increases, while x_L decreases to be close to c. So setting the pre-order price at x_H will yield the most profit to the retailer because it attracts all consumers to pre-order at a price close to p with probability γ. Therefore, as $\mu_H - \mu_L$ increases, the retailer should consider implementing advance selling with a moderate discount, i.e., $x = x_H$.

Last, to examine how a retailer's decision on advance selling price is affected by the expectation of μ, I fix $\mu_H - \mu_L$ at 40 and other parameters at their initial values. By moving $(\mu_H + \mu_L)/2$ from 130 to 230, it appears in Figure 2.3(d) that when $(\mu_H + \mu_L)/2$ is large, Π^0 yields the lowest profit. This is because both x_L and x_H increase as close as p when $(\mu_H + \mu_L)/2$ increases. So it is profitable for the retailer to implement advance selling at either x_L ($\mu_H - \mu_L$ is small) and x_H ($\mu_H - \mu_L$ is large) to induce all consumers pre-order. Therefore, as $(\mu_H + \mu_L)/2$ increases, the retailer should consider implementing advance selling, either with a deep discount or with a moderate discount.

2.7 Conclusion

This paper studies a retailer's advance selling strategies before he releases a new product. Consumers are strategic and their valuations of this new product are unknown in the advance selling season. When pre-orders are available, they make purchases in advance by comparing the payoffs from pre-orders and waiting. With regard to the retailer, he faces uncertain demand in the regular selling season, which is because of the randomness of the market size and the uncertainty of the consumer valuation distribution. To reduce the uncertainty of demand and thus improve the total expected profit, the retailer decides whether to adopt advance selling after considering consumers' decision-making process. If yes, he determines the advance selling price immediately and makes the quantity decision at the end of advance selling season.

The main results of this paper are summarized below.

- It is not always optimal for the retailer to implement advance selling. There exists a threshold on the marginal cost, above which the retailer will not accept pre-orders.

- There are three types of advance selling strategies for a retailer: no advance selling, advance selling with a deep discount and advance selling with a moderate discount.

- The retailer learns from pre-orders if he implements advance selling, but the learning is limited.

- Numerical tests show how the retailer's advance selling decision changes with some important parameters of the model. For example, as σ increases, the retailer favors no advance selling; as $\mu_H - \mu_L$ increases, he prefers advance selling at x_H; as $(\mu_H + \mu_L)/2$ increases, he would like to implement advance selling at x_L.

For future research, several issues are worthy of investigation. First, it would be interesting to introduce competition in the model and study the optimal advance selling decisions of the retailers. Under competition, I expect that a retailer is more likely to implement advance selling to win consumers.

Second, studying price premium in an advance selling model could be another direction. Consider a new generation of some series product, the current data base for the old generations can help the retailer to update his forecast of consumer valuation distribution. For a warmly welcomed product, it might be beneficial to charge a price premium because of a possible high stock-out probability.

Last, it would be interesting to study a dynamic model in which consumers arrive in the advance selling season at different times and they can update their valuations based on prior pre-orders. Because of the development of the Internet, consumers who noticed the availability of pre-orders usually gather together online to share information and discuss the new product. The popularity of the product could work as a signal for consumers to update their valuations.

Chapter 3

Advance selling in the presence of experienced consumers

3.1 Introduction

Advance selling occurs when firms and retailers offer consumers the opportunity to order the product or service in advance of the regular selling season. Remarkable developments in the Internet and information technology have made advance selling an economically efficient strategy in many product categories. Examples include new books, movies and CDs, software, electronic games, smart phones, travel services and vacation packages.

There are several major advantages of advance selling. First, advance selling reduces uncertainty for both the firm and the buyer, because advance orders are pre-committed. In situations when the firm needs to decide how much to produce (procure) prior to the regular selling season, advance orders reduce demand uncertainty. For the buyer, an advance order guarantees delivery of the product in the regular selling season, possibly at a discount to the retail price. Second, orders from advance selling may provide valuable information for the firm to better forecast the future demand. In particular, the firm may be able to update its forecast of the size of consumer pool and the distribution of consumers' valuations. Finally, advance selling may increase the overall demand. Indeed, when a consumer pre-orders the product, she commits to purchase it. In the absence of advance selling the same consumer will not purchase the product if she learns her valuation is low.

The motivation for the present study is based on two observations. One is that many pre-orders are from consumers who have previous experience with the product or its earlier versions. The other is that some products were not made available for pre-orders when they were first introduced, but pre-orders became possible for later versions. These observations point to an important role played by experienced consumers in advance selling.

Table 3.1 reports the product release history of several well-known products. These products are also widely cited as examples of advance selling. In the first four examples there were no pre-orders for the first one or two versions, and pre-orders were offered for later versions, some with discount and some without.

While inexperienced consumers learn more about their valuations of the product when it becomes available, experienced consumers are likely to have a good idea about their valuations of the product in advance. Therefore, experienced consumers have less incentives to wait until the regular selling season. It follows that when there are experienced consumers, advance selling is more likely to be utilized by consumers. In addition, pre-orders from experienced consumers are more informative

Table 3.1: Release history and pre-order availability for several products

Product	Version	Release date	Pre-order availability/Discount
Amazon Kindle	Kindle	Nov. 19, 2007	No
	Kindle 2	Feb. 23, 2009	Yes/No discount
	Kindle 3	Aug. 27, 2010	Yes/No discount
Harry Potter	Book 1	Sep. 1, 1998	No
	Book 2	Jun. 2, 1999	No
	Book 3	Sep. 8, 1999	Yes/40% off
	Book 4	Jul. 8, 2000	Yes/40% off
	Book 5	Jun. 21, 2003	Yes/40% off
	Book 6	Jul. 16, 2005	Yes/40% off
	Book 7	Jul. 21, 2007	Yes/49% off
iPhone	iPhone	Jun. 29, 2007	No
	iPhone 3G	Jul. 11, 2008	No
	iPhone 3GS	Jun. 19, 2009	Yes/No discount
	iPhone 4	Jun. 24, 2010	Yes/No discount
iPod Touch	iPod Touch 1st	Sep. 5, 2007	No
	iPod Touch 2nd	Sep. 9, 2008	No
	iPod Touch 3rd	Sep. 9, 2009	Yes/No discount
	iPod Touch 4th	Sep. 8, 2010	Yes/No discount
PlayStation	PlayStation 1	Sep. 9, 1995	Yes/No discount
	PlayStation 2	Oct. 26, 2000	Yes/No discount
	PlayStation 3	Nov. 17, 2006	Yes/No discount
Nintendo Wii	Wii Fit	May 21, 2008	Yes/$20 off
	Wii Fit Plus	Oct. 4, 2009	Yes/$10 off

than those from inexperienced consumers. One can thus conclude that the presence of experienced consumers makes the first two of the aforementioned advantages of advance selling more pronounced.

A number of papers in the literature have emphasized some or all of the three advantages of advance selling (see the literature review in Section 3.2), but none have modeled experienced consumers and the role they play in advance selling. This paper is the first study of advance selling with both experienced consumers and inexperienced consumers.

The model has two periods. The first is the advance selling season and the second is the regular selling season. In the first period, the firm chooses whether to make its product available for pre-orders, and if so, the level of discount from the retail price. There are two groups of consumers – experienced and inexperienced. Experienced consumers know their valuations of the product from the outset, while inexperienced consumers learn their valuations only in the second period. All consumers decide whether to pre-order the product (if this option is available) or wait until the regular selling season, in which they will face a probability of not being able to get the product (the stock-out probability). At the conclusion of the first period, the firm must choose its production quantity, which has to be at least the size of pre-orders. The product is delivered at the end of the second period.

Consumers are heterogeneous in their valuations, which are assumed to follow a normal distribution. The firm does not know the mean of this distribution. The group size of experienced consumers is fixed and known to the firm. However, the firm is uncertain about the number of inexperienced consumers.

In the second period the firm faces the Newsvendor Problem by analogy with the situation faced by a newsvendor who must decide how many copies of the day's paper to stock on a newsstand before observing demand, knowing that unsold copies will become worthless by the end of the day. If the produced quantity is greater than the realized demand, the firm must dispose of the remaining units at a loss (due to the salvage value being below the marginal cost). If the produced quantity is lower than the realized demand, the firm forgoes some profit.[1]

The main research questions are the following. Will the firm adopt the advance selling strategy? If so, will an advance selling discount be offered? How do experienced and inexperienced consumers behave in the advance selling season? What can the firm learn from pre-orders? How much should the firm produce? How are the answers to (some of) these questions affected by parameters of the model, such as the salvage value and the composition of experienced/inexperienced consumers in the population?

The main results are summarized below.

- The firm always adopts advance selling. Advance selling may be at a discount and may be not.

- Experienced consumers never wait until the regular selling season. Inexperienced consumers sometimes pre-order, sometimes wait until the regular selling season. When the pre-order discount is deep, inexperienced consumers pre-order. When the discount is moderate, inexperienced consumers pre-order if the mean of the distribution from which their valuations are drawn is high, and wait if otherwise.

- The firm learns from pre-orders, which softens the Newsvendor Problem. It learns whether there are any consumers who have chosen to wait until the regular selling season. If nobody waits, the firm only needs to fill all pre-orders.

[1] If there is no uncertainty about the second-period demand or if the salvage value equals the marginal cost, then the Newsvendor Problem disappears.

In the case when some consumers wait, the firm learns the mean of consumers' valuations. However, the uncertainty about the number of inexperienced consumers remains.

- The sensitivity analysis in regard to changes in some parameters of the model yields several interesting results, some intuitive and some counterintuitive. For example, as the salvage value decreases, the firm's expected profit may decrease (intuitive), but may also increase (counterintuitive). Likewise, as the proportion of experienced consumers decreases, the firm's expected profit may decrease (intuitive), but may also increase (counterintuitive).

This paper contains several contributions to the literature on advance selling. First, as mentioned earlier, this is the first paper to study advance selling in a model with experienced consumers. We believe the model captures an important aspect of the advance selling phenomena. Second, learning by the firm in this model is not only on the consumer pool but also on the distribution of consumers' valuations of the product. Finally, the stock-out probability that consumers face when they wait until the regular selling season is endogenously determined in our model. In the literature, the stock-out probability has been modeled as exogenously given. We think that the correct way to model the stock-out probability is through endogenous determination, since this probability affects consumers' choices in the advance selling season and these choices in turn affect the stock-out probability in the regular selling season.[2]

After the literature review (Section 3.2), the rest of the paper is organized as follows. In Section 3.3 we introduce the model. Section 3.4 is devoted to equilibrium analysis. In Section 3.5 sensitivity analysis results are presented. In Section 3.6 we

[2]See footnote 9 for further elaboration on the stock-out probability.

consider two extensions. Concluding remarks are provided in Section 3.7. Proofs of all lemmas and propositions, as well as derivations for some expressions and claims, are relegated to Appendix.

3.2 Literature Review

Several strands of the literature have studied advance selling. One deals with advance selling from manufacturers to retailers, e.g. Cachon (2004) and Taylor (2006). Another is on advance selling from firms and retailers to consumers under limited capacity, with applications to the airline and hotel industries (Xie and Shugan, 2001, Liu and van Ryzin, 2008, Boyaci and Özer, 2010). The literature that is closest to the present study is on advance selling from firms and retailers to consumers without capacity constraints.

Our review below focuses on the third strand.[3] Two modeling approaches have been adopted by researchers. In the first approach consumers are non-strategic in their decisions on whether to pre-order the product. In the second approach consumers are strategic.

Papers that model consumers as non-strategic include Weng and Parlar (1999), Tang, Rajaram, Alptekinoğlu, and Ou (2004), McCardle, Rajaram, and Tang (2004), and Chen and Parlar (2005). In all of these papers the fraction of consumers who place advance orders is an exogenously given decreasing function of the advance selling price.[4] In Tang, Rajaram, Alptekinoğlu, and Ou (2004) and McCardle, Rajaram, and

[3]The main difference between the second and the third strands is that in situations of limited capacity firms mainly choose prices, while without capacity constraints firms choose their production quantities as well as prices.

[4]In Chen and Parlar (2005) an alternative model is considered, in which the probability that

Tang (2004) there are two brands belonging to rivalry firms. Advance selling by a firm attracts customers of the other brand. The former paper examines the decision on advance selling by a single firm, while the latter focuses on competition between two firms in adopting the advance selling strategy.

Several papers have treated consumers as strategic. Strategic consumers compare the options of ordering in advance and of waiting until the regular selling season. Zhao and Stecke (2010) classify consumers according to whether they are loss averse. A loss averse consumer is more averse to a negative surplus (when the realized valuation is below the advance selling price) than is attracted to the equivalent positive surplus. Prasad, Stecke, and Zhao (2011) divide consumers into two groups. The informed group consists of consumers who know about the option to buy in advance, while the uninformed group is not aware of this option. Chu and Zhang (2011) allow the firm to control the release of information about the product at pre-order.

The common issues present in the literature are (i) the Newsvendor Problem, and (ii) learning and updating by the firm.[5] The Newsvendor Problem arises because the firm, facing uncertain demand, has to choose its production quantity prior to the regular selling season. Obviously, learning from pre-orders benefits the firm because it helps to better forecast the demand in the regular selling season. Both issues are also central in our paper. Because we assume that the mean of the distribution of consumers' valuations is unknown to the retailer, learning in our model is not only on the consumer pool, but also on the distribution of consumers' valuations.

Like Zhao and Stecke (2010), Prasad, Stecke, and Zhao (2011), and Chu and

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each consumer orders in advance is a beta-distributed random variable.

[5] An exception is Chu and Zhang (2011) in which the Newsvendor Problem is (implicitly) assumed away because the salvage value equals the marginal cost.

Zhang (2011), consumers in our model are strategic. The key difference between our paper and existent literature is the introduction of experienced consumers into the model. As stated before, experienced consumers make the strategy of advance selling more attractive to the firm.

3.3 Model Setup

Consider a firm or a retailer who sells a product over two periods.[6] The first period is the advance selling season and the second period is the regular selling season. Any consumer who pre-orders in the first period is guaranteed delivery of the product in the second period. Those who do not pre-order can buy in the regular selling season, but there is a risk that the product will be out of stock. There are two types of consumers – experienced and inexperienced. Experienced consumers know their valuations (i.e., their willingness to pay for the product) from the outset, whereas inexperienced consumers learn their valuations only in the second period. Each consumer is willing to buy at most one unit of the product.

The number of experienced consumers is m_e. The number of inexperienced consumers, M_i, is a random variable; the distribution of M_i is lognormal $\mathrm{LN}\left(\nu_i, \tau_i^2\right)$ with the mean $m_i = \exp\left\{\nu_i + \tau_i^2/2\right\}$.[7] Both m_e and the distribution of M_i are common knowledge.

Consumers' valuations of the product are normally distributed with mean μ and variance σ^2 (i.e., $v \sim \mathrm{N}\left(\mu, \sigma^2\right)$). While all consumers know the distribution from

[6]From now on we shall speak of the firm as the retailer.

[7]We use a lognormal distribution to avoid negative realizations of the number of inexperienced consumers.

Table 3.2: Notation B

Parameters	
c	marginal cost
s	salvage value
p	price in the regular selling season
m_e	number of experienced consumers
Random variables	
$v \sim \mathrm{N}\left(\mu, \sigma^2\right)$	distribution of consumers' valuations
$\mu \in \{\mu_H, \mu_L\}$	two-point mass distribution, $\mathrm{Prob}(\mu_H) = \gamma$ and $\mathrm{Prob}(\mu_L) = 1 - \gamma$
$M_i \sim \mathrm{LN}\left(\nu_i, \tau_i^2\right)$	number of inexperienced consumers, mean $m_i = \exp\left\{\nu_i + \frac{\tau_i^2}{2}\right\}$
Decision variables	
q	quantity produced for the regular selling season
Q	total quantity produced (includes pre-orders)
x	advance selling price
Distribution and density functions	
$F(\cdot)$	cdf of $\mathrm{N}\left(\mu, \sigma^2\right)$, $F(y) = \Phi\left(\frac{y-\mu}{\sigma}\right)$
$\overline{F}(y)$	$= 1 - F(y)$
$f(\cdot)$	density function of $\mathrm{N}\left(\mu, \sigma^2\right)$, $f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{(y-\mu)^2}{2\sigma^2}\right\}$
$G(\cdot)$	cdf of $\mathrm{LN}\left(\nu, \tau^2\right)$, $G(y) = \Phi\left(\frac{\ln y - \nu}{\tau}\right)$
$g(\cdot)$	density function of $\mathrm{LN}\left(\nu, \tau^2\right)$, $g(y) = \frac{1}{y\sqrt{2\pi\tau^2}} \exp\left\{-\frac{(\ln y - \nu)^2}{2\tau^2}\right\}$
Other notation	
D_1, D_2	demands in the advance and regular selling seasons
π	retailer's expected profit from the regular selling season
Π	retailer's total expected profit (includes pre-orders)
η	stock-out probability
β	$= \frac{p-c}{p-s}$

which their valuations are drawn, the retailer does not. We model the retailer's uncertainty by assuming that μ is high, μ_H, with probability γ and low, μ_L, with probability $1 - \gamma$.

The marginal production cost is c and the price during the regular selling season is p. For each unsold unit of the product at the end of the regular selling season the retailer gets its salvage value s. We assume $s < c < p$.

The retailer decides on advance selling price $x \leq p$ in the beginning of the advance selling season.[8] After the conclusion of the advance selling season the retailer must decide how much to produce. Let D_1 denote the number of consumers who buy in the advance selling season. Then the retailer's quantity choice is $Q = D_1 + q$, where D_1 fulfills the pre-orders. Quantity q satisfies the (stochastic) demand during the regular selling season, denoted by D_2.

Table 3.2 lists the notation introduced above and also some of the notation introduced later. Figure 3.1 displays the timeline of the model. In the beginning of the first period, all consumers learn μ and all experienced consumers learn their valuations. During the first period, the retailer announces advance selling price x, then each consumer decides whether to pre-order. At the end of the first period, the retailer observes the number of pre-orders D_1, updates his forecast of the second-period demand D_2 and chooses production quantity Q. During the second period, all inexperienced consumers learn their valuations and those consumers who did not pre-order then decide whether to purchase the product at price p. The product is delivered at the end of the second period.

[8] We assume in this paper that $x \leq p$ so that to focus on pre-order discounting.

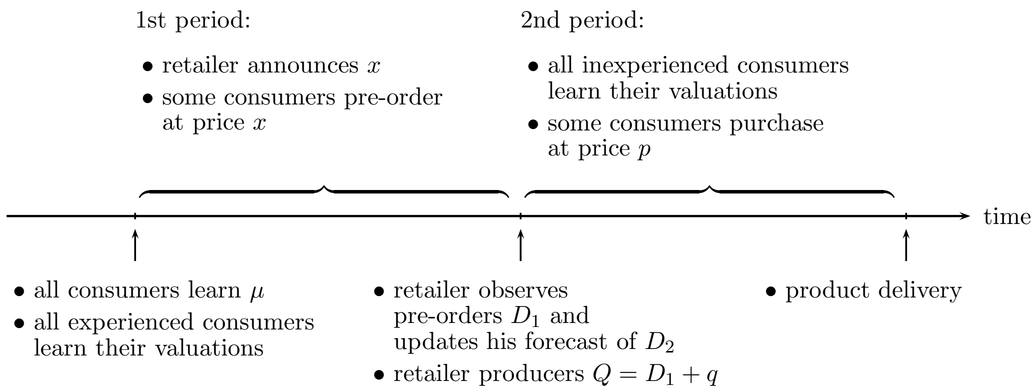


Figure 3.1: Timeline of the model

3.4 Equilibrium Analysis

Our goal in this section is to find the optimal advance selling price. To do this, we first derive consumers' optimal responses to any advance selling price, and how the retailer learns from the pre-orders and chooses his output. We then determine the endogenous stock-out probability and present the retailer's expected profit function.

3.4.1 Consumers' optimal purchasing decisions

Since experienced consumers know their valuations from the outset, they never wait until the regular selling season. Experienced consumers with valuations above x pre-order the product and pay discounted price $x \leq p$.

Inexperienced consumers do not know their valuations in the advance selling season. An inexperienced consumer has two options. The first is to pre-order and pay

x. In this case the consumer's expected payoff is

$$\mu - x.$$

The other option is to wait until the regular selling season. The consumer learns her valuation v and purchases the product (provided it is in stock) if $v \geq p$. Her expected payoff is

$$(1-\eta)\int_p^{+\infty}(v-p)f(v)\,\mathrm{d}v,$$

where η is the stock-out probability and $f(\cdot)$ is the density function of $\mathrm{N}(\mu, \sigma^2)$, from which the consumer's valuation is drawn. The stock-out probability is the probability that the consumer will not be able to get the product when she actually wants to purchase it.

Thus, inexperienced consumers pre-order if and only if

$$\mu - x \geq (1-\eta)\int_p^{+\infty}(v-p)f(v)\,\mathrm{d}v,$$

or, equivalently,

$$x \leq \mu - (1-\eta)\int_p^{+\infty}(v-p)f(v)\,\mathrm{d}v.$$

Let

$$x_L \equiv \mu_L - (1-\eta)\int_p^{+\infty}(v-p)f_L(v)\,\mathrm{d}v \tag{3.1}$$

and

$$x_H \equiv \mu_H - (1-\eta)\int_p^{+\infty}(v-p)f_H(v)\,\mathrm{d}v \tag{3.2}$$

denote the threshold values for $\mu = \mu_L$ and $\mu = \mu_H$, respectively. Here $f_L(\cdot)$ is the density function of $\mathrm{N}(\mu_L, \sigma^2)$ and $f_H(\cdot)$ is the density function of $\mathrm{N}(\mu_H, \sigma^2)$. In Subsection 3.4.4 we show that the endogenously determined η is the same in (3.1) and (3.2). The explicit expressions for x_L and x_H (derived in Appendix) are

$$x_L = \mu_L - (1-\eta)\left((\mu_L - p)\overline{F}_L(p) + \sigma^2 f_L(p)\right)$$

and

$$x_H = \mu_H - (1-\eta)\left((\mu_H - p)\overline{F}_H(p) + \sigma^2 f_H(p)\right),$$

where $F_L(\cdot)$ and $F_H(\cdot)$ are the cumulative distribution functions of $\mathrm{N}(\mu_L, \sigma^2)$ and $\mathrm{N}(\mu_H, \sigma^2)$, respectively, and $\overline{F}(\cdot) = 1 - F(\cdot)$.

Lemma 4 (Properties of x_L and x_H)**.** *The threshold values $x_L(\eta, \sigma)$ and $x_H(\eta, \sigma)$ possess the following properties:*

(i) *$x_L(\eta, \sigma) < x_H(\eta, \sigma)$ for all η and σ;*

(ii) *$\partial x_L/\partial\sigma < 0$ and $\partial x_H/\partial\sigma < 0$;*

(iii) *$\partial x_L/\partial\eta > 0$ and $\partial x_H/\partial\eta > 0$.*

Since $x_L < x_H$ always holds, we consider the following three regions for advance selling price x.

- Region A: $x \le x_L$. All inexperienced consumers pre-order.
- Region B: $x_L < x \le x_H$. Inexperienced consumers pre-order if $\mu = \mu_H$.
- Region C: $x > x_H$. All inexperienced consumers wait until the second period.

Properties (ii) and (iii) will be useful for our sensitivity analysis in Section 3.5.

3.4.2 Learning by the retailer from pre-orders

When advance selling price x is in region A, experienced consumers with valuations above x and all inexperienced consumers pre-order. No one will wait until the regular selling season. As the retailer's forecast of the second-period demand is $D_2 = 0$, he produces $Q = D_1$.

When x is in region B, the retailer learns μ through observing D_1. If $D_1 = m_e\overline{F}_L(x)$, then the retailer infers that $\mu = \mu_L$. The retailer produces $Q = D_1 + q$, where q satisfies the second-period demand that comprises of inexperienced consumers with valuations above p,

$$D_2 = M_i \operatorname{Prob}(v > p) = M_i\overline{F}_L(p).$$

Because $M_i \sim \mathrm{LN}\,(\nu_i, \tau_i^2)$, it is straightforward to show that

$$D_2 \sim \mathrm{LN}\left(\nu_i + \ln \overline{F}_L(p), \tau_i^2\right).$$

If $D_1 \neq m_e\overline{F}_L(x)$, the retailer infers $\mu = \mu_H$ and $D_2 = 0$, hence produces $Q = D_1$. (Like in region A, no one waits until the regular selling season – experienced consumers with valuations above x and all inexperienced consumers pre-order.)

In region C the retailer also learns μ. If $D_1 = m_e\overline{F}_L(x)$, then the retailer infers that $\mu = \mu_L$. The retailer produces $Q = D_1 + q$, where q is for the second-period demand

$$D_2 = M_i\overline{F}_L(p) \sim \mathrm{LN}\left(\nu_i + \ln \overline{F}_L(p), \tau_i^2\right).$$

If $D_1 = m_e\overline{F}_H(x)$, the retailer knows $\mu = \mu_H$. The retailer produces $Q = D_1 + q$, where q is for the second-period demand

$$D_2 = M_i \overline{F}_H(p) \sim \mathrm{LN}\left(\nu_i + \ln \overline{F}_H(p), \tau_i^2\right).$$

3.4.3 Optimal value of q

In regard to the retailer's choice of q, it remains to find the optimal q when D_2 follows the distribution $\mathrm{LN}\left(\nu_i + \ln \overline{F}_L(p), \tau_i^2\right)$ and when it follows $\mathrm{LN}\left(\nu_i + \ln \overline{F}_H(p), \tau_i^2\right)$.

For any D_2, if q units are produced, then $\min\{q, D_2\}$ units are sold and $(q-D_2)^+ = \max\{q - D_2, 0\}$ are salvaged. The retailer's expected profit from the second period, denoted by π, is

$$\pi(q) = p\,\mathrm{E}\left[\min\{q, D_2\}\right] + s\,\mathrm{E}\left[(q - D_2)^+\right] - cq. \tag{3.3}$$

The problem of maximizing (3.3) is the Newsvendor Problem, well-known in the operations management literature. Using the fact that $\min\{q, D_2\} = D_2 - (D_2 - q)^+$, we can rewrite the retailer's expected profit as

$$\pi(q) = \mathrm{E}[D_2](p - c) - \mathrm{E}\left[(D_2 - q)^+\right](p - c) - \mathrm{E}\left[(q - D_2)^+\right](c - s).$$

The optimal value of q, therefore, minimizes the expected underage and overage cost

$$\mathrm{E}\left[(D_2 - q)^+\right](p - c) + \mathrm{E}\left[(q - D_2)^+\right](c - s).$$

The first-order condition is

$$\mathrm{Prob}(D_2 \le q^*) = \beta,$$

where

$$\beta \equiv \frac{p-c}{p-s}.$$

It is clear that q^* selected this way increases in β and therefore increases in the per unit underage cost $p-c$ and decreases in the per unit overage cost $c-s$.

As shown in essay "Advance selling of new product", for the lognormal distribution $D_2 \sim \mathrm{LN}\left(\nu, \tau^2\right)$ the optimal production quantity is given by

$$q^* = \exp\{\nu + \tau z_\beta\} \tag{3.4}$$

and

$$\pi(q^*) = (p-s)\left(1 - \Phi(\tau - z_\beta)\right)\exp\left\{\nu + \frac{\tau^2}{2}\right\}, \tag{3.5}$$

where z_β is the β-th percentile of the standard normal distribution, $z_\beta \equiv \Phi^{-1}(\beta)$.

Applying (3.4) and (3.5) to $D_2 \sim \mathrm{LN}\left(\nu_i + \ln \overline{F}_L(p), \tau_i^2\right)$, the optimal q, denoted by q_L^*, and the resulting expected profit π_L are

$$q_L^* = \exp\{\nu_i + \tau_i z_\beta\}\overline{F}_L(p)$$

and

$$\pi_L = (p-s)\left(1 - \Phi(\tau_i - z_\beta)\right) m_i \overline{F}_L(p).$$

Similarly, under $D_2 \sim \mathrm{LN}\left(\nu_i + \ln \overline{F}_H(p), \tau_i^2\right)$ the optimal q, denoted by q_H^*, and the resulting expected profit π_H are

$$q_H^* = \exp\{\nu_i + \tau_i z_\beta\}\overline{F}_H(p)$$

and

$$\pi_H = (p - s)\left(1 - \Phi(\tau_i - z_\beta)\right) m_i \overline{F}_H(p).$$

3.4.4 Stock-out probability

Given D_2, the (conditional) probability of any consumer who wants to purchase the product in the regular selling season but is unable to get it is the fraction of excess demand,

$$\text{Prob(stock-out}|D_2) = \left(\frac{D_2 - q^*}{D_2}\right)^+ = \begin{cases} 0, & D_2 \leq q^*, \\ \frac{D_2 - q^*}{D_2}, & D_2 > q^*. \end{cases}$$

Hence, the stock-out probability is the expected value of this expression over the distribution of D_2,

$$\eta = \mathrm{E}\left[\left(\frac{D_2 - q^*}{D_2}\right)^+\right]. \tag{3.6}$$

For $D_2 \sim \mathrm{LN}\left(\nu, \tau^2\right)$,

$$\eta = \int_{q^*}^{+\infty} \frac{D_2 - q^*}{D_2} g(D_2)\,\mathrm{d}D_2,$$

where $g(\cdot)$ is the density function of $\mathrm{LN}\left(\nu, \tau^2\right)$ and q^* is given in (3.4). The explicit expression for the stock-out probability is

$$\eta = 1 - \beta - \exp\left\{\tau z_\beta + \frac{\tau^2}{2}\right\}\left(1 - \Phi(z_\beta + \tau)\right). \tag{3.7}$$

Note that the expression in (3.7) is independent of ν. Accordingly, the same η results from $D_2 \sim \mathrm{LN}\left(\nu_i + \ln \overline{F}_L(p), \tau_i^2\right)$ and $D_2 \sim \mathrm{LN}\left(\nu_i + \ln \overline{F}_H(p), \tau_i^2\right)$. This finding is presented in Lemma 5.

Lemma 5 (Stock-out probability η)**.** *The stock-out probability under the second-*

period demand $D_2 \sim \mathrm{LN}\left(\nu_i + \ln \overline{F}_L(p), \tau_i^2\right)$ *is equal to that under* $D_2 \sim \mathrm{LN}\left(\nu_i + \ln \overline{F}_H(p), \tau_i^2\right)$ *and is given by*

$$\eta = 1 - \beta - \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}(1 - \Phi(z_\beta + \tau_i)).$$

It is important to note that the stock-out probability in our model is endogenously determined, because q^* is the optimal choice. It follows that $\eta < 1 - \beta$, which is expected, as

$$\begin{aligned} \eta = \mathrm{E}\left[\left(\frac{D_2 - q^*}{D_2}\right)^+\right] &< \mathrm{E}\left[\mathbb{1}(D_2 > q^*)\right] = \mathrm{Prob}(D_2 > q^*) \\ &= 1 - \mathrm{Prob}(D_2 < q^*) = 1 - \beta, \end{aligned}$$

where $\mathbb{1}(D_2 > q^*)$ is the indicator of the corresponding event, and

$$\left(\frac{D_2 - q^*}{D_2}\right)^+ \leq \mathbb{1}(D_2 > q^*)$$

for all D_2, with strict inequality for $D_2 > q^*$.[9]

Lemma 6 (Properties of η)**.** *The stock-out probability* $\eta = \eta(\beta, \tau_i)$ *possesses the following properties:*

(i) $\partial\eta/\partial\tau_i > 0$*,* $\eta(\beta, 0) = 0$*, and* $\lim_{\tau_i \to +\infty} \eta(\beta, \tau_i) = 1 - \beta$*;*

(ii) $\partial\eta/\partial\beta < 0$*,* $\eta(0, \tau_i) = 1$*, and* $\eta(1, \tau_i) = 0$*.*

[9] In most papers in the literature consumers take the risk of stock out as exogenously given. An exception is Prasad, Stecke, and Zhao (2011). In their study the "stocking out probability" (defined on page 5) has the same meaning as η in our model: a consumer who waited until the regular selling season will not be able to get the product with probability η. However, instead of (3.6), they incorrectly use the expression $\eta = \mathrm{Prob}(D_2 > q^*)$, which yields $\eta = 1 - \beta$ (page 7).

Combining the results of Lemma 4(iii) and Lemma 6 we can conclude that the threshold values x_L and x_H increase in τ_i and decrease in β.

3.4.5 The retailer's expected profit

We can now write the retailer's expected total profit Π as a function of advance selling price x. The part of the retailer's expected profit that comes from experienced consumers equals

$$m_e \left(\gamma \overline{F}_H(x) + (1-\gamma)\overline{F}_L(x) \right) (x-c),$$

as experienced consumers never wait until the regular selling season and only those with valuations above x (fraction $\overline{F}_H(x)$ in the case $\mu = \mu_H$ and fraction $\overline{F}_L(x)$ in the case $\mu = \mu_L$) purchase the product in the advance selling season.

The purchasing behavior of inexperienced consumers depends on the region that x belongs to. If $x \le x_L$ (region A), then all inexperienced consumers pre-order. Hence,

$$\Pi^A(x) = m_e \left(\gamma \overline{F}_H(x) + (1-\gamma)\overline{F}_L(x) \right) (x-c) + m_i(x-c). \tag{3.8}$$

Next, consider $x_L < x \le x_H$ (region B). In the case $\mu = \mu_H$ all inexperienced pre-order, yielding $m_i(x-c)$ to the retailer. In the case $\mu = \mu_L$ all inexperienced consumers wait until the second period, yielding π_L (calculated in Section 3.4.3) to the retailer. Hence,

$$\Pi^B(x) = m_e \left(\gamma \overline{F}_H(x) + (1-\gamma)\overline{F}_L(x) \right) (x-c) + \gamma m_i(x-c) + (1-\gamma)\pi_L. \tag{3.9}$$

Finally, if $x > x_H$ (region C), then all inexperienced consumers wait until the second period. Since the retailer learns μ in the first period, his expected payoff from inexperienced consumers is $\gamma\pi_H + (1-\gamma)\pi_L$. Hence,

$$\Pi^C(x) = m_e\left(\gamma\overline{F}_H(x) + (1-\gamma)\overline{F}_L(x)\right)(x-c) + \gamma\pi_H + (1-\gamma)\pi_L. \tag{3.10}$$

The retailer's total expected profit as a function of advance selling price x is, therefore,

$$\Pi(x) = \begin{cases} \Pi^A(x), & x \le x_L, \\ \Pi^B(x), & x_L < x \le x_H, \\ \Pi^C(x), & x > x_H. \end{cases}$$

3.4.6 Advance selling vs. no advance selling

Before moving onto the optimal advance selling price for the retailer, we explore next whether there always is an incentive for the retailer to implement the advance selling strategy.

Without advance selling, the retailer sells in the regular selling season at price p. Let $\Pi_H(Q)$ and $\Pi_L(Q)$ denote the expected profits as functions of production quantity Q for $\mu = \mu_H$ and $\mu = \mu_L$, respectively. Let Q_0 denote the optimal quantity. Then, without advance selling, the retailer's maximum expected profit is

$$\Pi^0 = \gamma\Pi_H(Q_0) + (1-\gamma)\Pi_L(Q_0).$$

We will show that advance selling at price p leads to a higher expected profit than no advance selling, that is, $\Pi(p) > \Pi^0$. It follows that the retailer's expected profit under advance selling (at the optimal price which may or may not be equal to p) must be greater than that under no advance selling.

Advance selling brings two benefits for the retailer: learning the true value of μ before choosing production quantity and receiving precommitted orders. We show next that the first benefit alone improves the retailer's expected profit over that under no advance selling. Let Q_H and Q_L denote the optimal quantities for $\mu = \mu_H$ and $\mu = \mu_L$, respectively. It follows that $\Pi_H(Q_H) \geq \Pi_H(Q_0)$ and $\Pi_L(Q_L) \geq \Pi_L(Q_0)$ with at least one in strict inequality. Accordingly,

$$\gamma\Pi_H(Q_H) + (1-\gamma)\Pi_L(Q_L) > \Pi^0.$$

This inequality indicates that knowing the true value of μ before choosing Q is superior to choosing Q without knowing the true value of μ. Since $\Pi(p)$ incorporates the benefit from possible pre-orders, we must have

$$\Pi(p) \geq \gamma\Pi_H(Q_H) + (1-\gamma)\Pi_L(Q_L).$$

Hence, $\Pi(p) > \Pi^0$.

Therefore, we have the following proposition.[10]

Proposition 4 (Advance selling vs. no advance selling)**.** *Advance selling is always superior to no advance selling for the retailer.*

[10]We do not consider the cost of adopting advance selling in this paper. Proposition 4 holds as long as the adoption cost is lower than $\Pi(p) - \Pi^0$.

3.4.7 Optimal advance selling price

For the rest of our analysis, we will assume that $c < x_L < x_H < p$ holds.[11] Furthermore, we make the following simplifying assumption.

Assumption 1. *The function*

$$\left(\gamma\overline{F}_H(x) + (1-\gamma)\overline{F}_L(x)\right)(x-c)$$

increases in x *on* $[c, p]$.

This assumption implies that the expected profit from experienced consumers,

$$m_e\left(\gamma\overline{F}_H(x) + (1-\gamma)\overline{F}_L(x)\right)(x-c),$$

is an increasing function of x for all $x \in [c, p]$. It follows that, as far as experienced consumers are concerned, the retailer has no incentives to offer an advance selling discount. Accordingly, if discounting for pre-orders is offered it must due to the presence of inexperienced consumers.

It is easy to see that under Assumption 1 the retailer's expected profit $\Pi(x)$ increases in x in each of the three regions A, B, and C. This does not imply, however, that $\Pi(x)$ increases in x on $[c, p]$, as $\Pi(x)$ can jump down at $x = x_L$ and/or at $x = x_H$. A jump down at x_L occurs if and only if $\Pi^A(x_L) > \Pi^B(x_L)$.[12] That is,

$$(1-\gamma)m_i(x_L - c) > (1-\gamma)\pi_L,$$

[11]There are six possible relationships between c, p, x_L, and x_H: $c < p < x_L < x_H$, $c < x_L < p < x_H$, $c < x_L < x_H < p$, $x_L < c < p < x_H$, $x_L < c < x_H < p$, and $x_L < x_H < c < p$.

[12]Since $\Pi_B(\cdot)$ is defined on $(x_L, x_H]$, $\Pi^B(x_L)$ is the limiting value.

or equivalently,

$$x_L - c > (p - s)\,(1 - \Phi(\tau_i - z_\beta))\,\overline{F}_L(p).$$

Similarly, a jump down at x_H occurs if and only if $\Pi^B(x_H) > \Pi^C(x_H)$.[13] That is,

$$\gamma m_i(x_H - c) > \gamma \pi_H,$$

or equivalently,

$$x_H - c > (p - s)\,(1 - \Phi(\tau_i - z_\beta))\,\overline{F}_H(p).$$

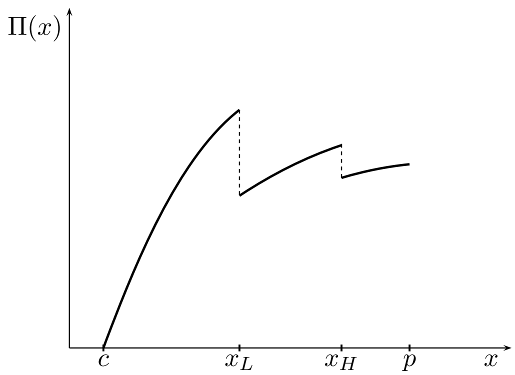
Hence, we have the following four patterns for $\Pi(x)$ (see Figure 3.2).

- Pattern 1: $\Pi(x)$ jumps down at both x_L and x_H.
- Pattern 2: $\Pi(x)$ jumps up at both x_L and x_H.
- Pattern 3: $\Pi(x)$ jumps up at x_L, jumps down at x_H.
- Pattern 4: $\Pi(x)$ jumps down at x_L, jumps up at x_H.

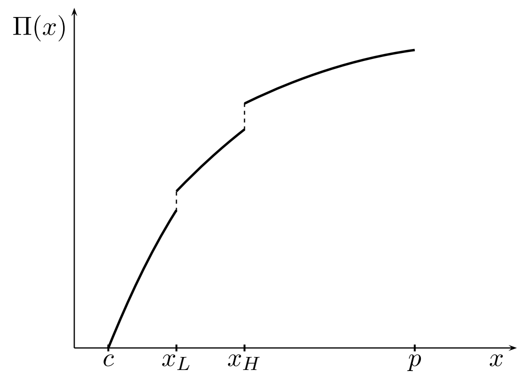
Regarding the optimal advance selling price x^*, under pattern 1 it is either x_L, x_H, or p; under pattern 2 it is p; under pattern 3, it is either x_H or p; under pattern 4, it is either x_L or p. Our extensive numerical simulations indicate that all four patterns may arise and under each pattern the optimal advance selling price can take any of the possible values mentioned above. It follows that the retailer's optimal advance selling price can be any one of the three values: x_L, x_H, or p.

Proposition 5 (Optimal advance selling price)**.** *Under Assumption 1, the optimal advance selling price x^* is either x_L, x_H, or p.*

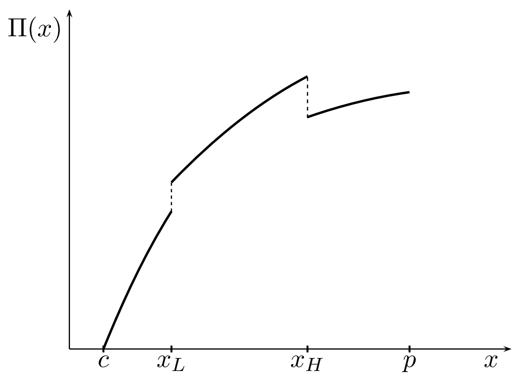
[13] Since $\Pi_C(\cdot)$ is defined on $(x_H, p]$, $\Pi^C(x_H)$ is the limiting value.



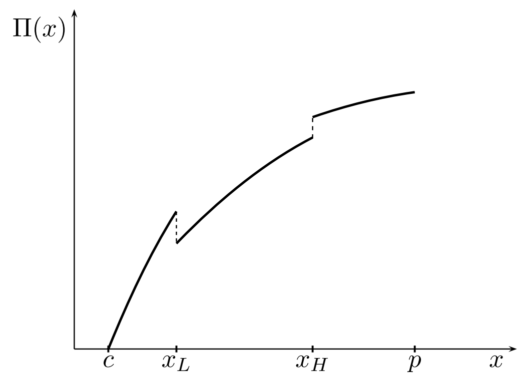
(a) Pattern 1



(b) Pattern 2



(c) Pattern 3



(d) Pattern 4

Figure 3.2: The retailer's expected profit as a function of x

The three likely optimal advance selling prices reflect two different tradeoffs for the retailer: between low price-high sales and high price-low sales, and between low price-low expected overage and underage cost and high price-high expected overage and underage cost. Under all three prices, experienced consumers only buy in the advance selling season. Hence, the tradeoff for the retailer from this group of consumers is only between a lower price and therefore higher expected sales and a higher price and lower expected sales.

For the inexperienced group of consumers, let us first focus on the comparison between x_L and p. At x_L all inexperienced consumers pre-order while at p none pre-order and only those with realized values above p will buy in the regular selling season. Hence, both tradeoffs described above are present here. First, x_L corresponds to higher sales and a lower price, while p corresponds to much lower sales and a higher price. Second, x_L means zero overage and underage cost, while p leads to a positive expected overage and underage cost.

Adding the intermediate price x_H to the mix, the same two tradeoffs are involved between each pair of these prices. For example, moving from x_L to x_H, the expected sales decrease while the expected overage and underage cost increases, both are due to the fact that there is a positive probability that at x_H all inexperienced consumers will wait until the regular selling season. Comparing all three possible advance selling prices, we conclude that, as the advance selling price changes from x_L to x_H to p, the expected sales decrease and the expected overage and underage cost increases.

3.5 Sensitivity Analysis

In this section we consider how the retailer's optimal advance selling price x^* and the expected profit $\Pi(x^*)$ are affected by some important parameters of the model. Let $\Pi^* \equiv \Pi(x^*)$. Intuitively, we should expect that a decrease in the salvage value s or an increase in demand uncertainty τ_i would result in lower Π^*. Surprisingly, we find that Π^* might actually increase (Subsection 3.5.1).

We are also interested in how consumer characteristics – the relative number of experienced consumers (measured by parameter α introduced below) and valuation uncertainty of inexperienced consumers (captured by σ) – affect the retailer. Will a decrease in α and/or an increase in σ lead to lower Π^*? As we show in Subsections 3.5.2 and 3.5.3, both lower and higher Π^* are possible.

In all of our sensitivity analysis in this section, we focus on small changes in the parameter values so that the retailer's optimal choice stays the same in that it does not jump to one of the other two points. Although we work with τ_i and σ, the same results apply to small changes in τ_i^2 and σ^2.

In Table 3.3 all of the directional changes in the parameters (the first row) are chosen to "hurt" the retailer on an intuitive basis. Therefore, all cases in which Π^* increases ($\Pi^* \uparrow$) represent counterintuitive results.

3.5.1 Sensitivity analysis – s and τ_i

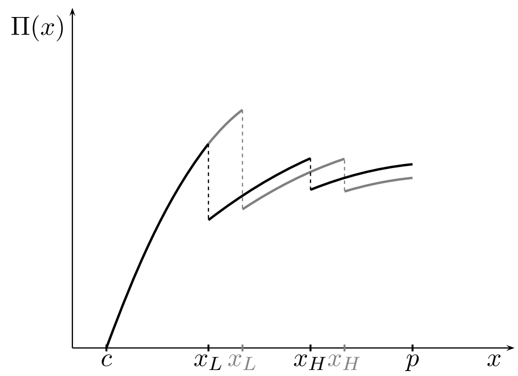
In this subsection we consider how a decrease in the salvage value s and an increase in demand uncertainty τ_i affects the retailer.

We first focus on s. Suppose s decreases. Then $\beta = (p-c)/(p-s)$ decreases. By Lemma 6(ii), η increases. An increase in η positively affects x_L and x_H. As to π_L and π_H, they go down as s decreases. Indeed, applying the Envelope Theorem to (3.3) yields

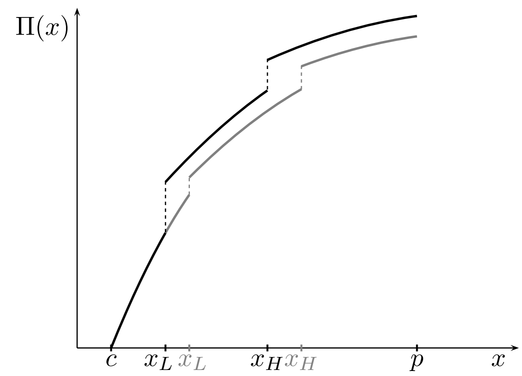
$$\left.\frac{\partial \pi}{\partial s}\right|_{q=q^*} = \mathrm{E}\left[(q^* - D_2)^+\right] > 0.$$

Geometrically, a decrease in s shifts the thresholds x_L and x_H to the right and the curves $\Pi^B(x)$ and $\Pi^C(x)$ defined in (3.9) and (3.10) down.

Consider, for example, pattern 1 and suppose $x^* = x_L$. In Figure 3.3(a) the black and gray curves represent, respectively, $\Pi(x)$ before and after a decrease in s. It is



(a) Pattern 1



(b) Pattern 2

Figure 3.3: The effects of a decrease in s (increase in τ_i) on $\Pi(x)$

easy to see that both x^* and Π^* increase. If $x^* = x_H$, then a decrease in s leads to higher x^*; Π^* can increase or decrease. If $x^* = p$, then a decrease in s leads to lower Π^*.

Consider next pattern 2. The optimal advance selling price is p. As can be seen from Figure 3.3(b), a decrease in s leads to a lower Π^*. Similar reasoning applies to patterns 3 and 4. It is important to point out that the same results on the directional changes of x^* and Π^* hold across all four patterns as long as the same optimal choice is obtained. The second column of Table 3.3 reports the sensitivity results on x^* and Π^* in terms of s.

What is the effect of an increase in τ_i on x^* and Π^*? By Lemma 6(i), η increases. As a result, x_L and x_H increase. Clearly, an increase in τ_i negatively affects π_L and π_H. It follows that an increase in τ_i affects x^* and Π^* in similar ways as a decrease in s (See the third column of Table 3.3).

The intuition for the above results in regard to a change in the salvage value s is as follows (similar for an increase in τ_i). When s becomes lower, the per unit

Table 3.3: Sensitivity analysis results

optimal choice x^* \ parameter change	$s \downarrow$	$\tau_i \uparrow$	$\alpha \downarrow$	$\sigma \uparrow$
x_L	$x^* \uparrow$ $\Pi^* \uparrow$	$x^* \uparrow$ $\Pi^* \uparrow$	x^* unchanged $\Pi^* \uparrow$	$x^* \downarrow$ $\Pi^* \downarrow$
x_H	$x^* \uparrow$ $\Pi^* \uparrow\downarrow$	$x^* \uparrow$ $\Pi^* \uparrow\downarrow$	x^* unchanged $\Pi^* \uparrow\downarrow$	$x^* \downarrow$ $\Pi^* \uparrow\downarrow$
p	x^* unchanged $\Pi^* \downarrow$	x^* unchanged $\Pi^* \downarrow$	x^* unchanged $\Pi^* \downarrow$	x^* unchanged $\Pi^* \uparrow\downarrow$

overage cost becomes higher and therefore the retailer reduces his output to avoid too much unsold product at the end of the regular selling season. This raises the stock-out probability η for consumers who wait until the regular selling season. As a result, inexperienced consumers are willing to pay a higher advance selling price (i.e., higher x_L and x_H) to secure delivery of the product. Consider the case in which the retailer's optimal advance selling price $x^* = x_L$. Since the retailer's expected profit from the group of experienced consumers is an increasing function of x (Assumption 1), it becomes higher. Obviously, the retailer's expected profit from the group of inexperienced consumers rises as well since all of them pre-order at a higher price. Hence, the total expected profit of the retailer becomes greater. In this case, we obtain the counterintuitive result that the retailer can benefit from a decrease in s.

The increased stock-out probability does bite the retailer if the optimal advance selling price $x^* = p$. In this case, the retailer's expected profit from experienced consumers remains the same as there is no change for this group both in price and in the number of buyers. However, the retailer's expected profit from the group of inexperienced consumers, who all wait until the regular selling season, becomes lower due to the increased per unit overage cost. It follows that the retailer is hurt by a decrease in s.

Finally, consider $x^* = x_H$. With probability γ the retailer will benefit from a decrease in s (similar to the case $x^* = x_L$) and with probability $1 - \gamma$ the retailer will be hurt by a decrease in s (similar to the case $x^* = p$). As a result, depending on these probabilities and the respective gain and loss, both directions of change are possible for the retailer's total expected profit.

3.5.2 Sensitivity analysis – α

Let $m = m_e + m_i$ denote the total expected number of (experienced and inexperienced) consumers, and let α denote the proportion of experienced consumers in the market. Thus, $m_e = \alpha m$ and $m_i = (1 - \alpha)m$. In this subsection we consider how a decrease in α affects the retailer.

Note that η, x_L and x_H are independent of α. Hence, x^* stays the same. To see how the retailer's expected profit is affected, we substitute $m_e = \alpha m$ and $m_i = (1 - \alpha)m$ into the expressions (3.8) through (3.10), remembering that π_L and π_H depend on m_i. In Appendix we calculate the derivatives of $\Pi^A(x_L)$, $\Pi^B(x_H)$, and $\Pi^C(p)$ with respect to α, and show that the first derivative is negative, the second can be positive or negative, and the third is positive. Therefore, if $x^* = x_L$ then a decrease in α leads to higher Π^*, and if $x^* = p$ – to lower Π^*. If $x^* = x_H$, Π^* can increase or decrease.

The fourth column of Table 3.3 reports the results of our sensitivity analysis with respect to α. The intuition is straightforward. A decrease in α changes the composition of experienced and inexperienced consumers in the total consumer population by decreasing the group size of experienced consumers and increasing the group size of inexperienced consumers. Such a change does not alter the incentive for the retailer to produce to satisfy the demand in the regular selling season and does not affect each individual consumer's incentive to pre-order in the advance selling season. That is, η, x_L and x_H all remain unchanged. It does, however, affect the retailer's expected profit. In the case in which the retailer's optimal advance selling price $x^* = x_L$, no one buys in the regular selling season. The size of pre-orders placed by experienced consumers decreases while the size of pre-orders placed by inexperienced consumers increases. The latter change is greater than the former, because only a fraction of experienced consumers purchase the product. Hence, the retailer's expected profit increases.

If the optimal advance selling price $x^* = p$, the opposite occurs. In this case all experienced consumers whose valuations are above p pre-order but only a fraction of those inexperienced consumers whose valuations are above p get to buy the product in the regular selling season due to the positive stock-out probability. This leads to a net loss in the expected sales for the retailer and therefore to a lower expected profit.

In the case in which the optimal advance selling price $x^* = x_H$, the retailer's expected profit rises with probability γ (corresponding to all inexperienced consumers pre-ordering like in the case $x^* = x_L$) and falls with probability $1 - \gamma$ (corresponding to all inexperienced consumers waiting to buy in the regular selling season like in the case $x^* = p$). As a result, depending on these probabilities and the respective gain and loss, both directions of change are possible for the retailer's total expected profit.

3.5.3 Sensitivity analysis – σ

Parameter σ captures variation in consumer valuations. For inexperienced consumers an increase in σ also means they become more uncertain about their valuations.

By Lemma 4(ii), both x_L and x_H decrease as σ increases. In Appendix we show that whenever $x^* = x_L$, an increase in σ leads to lower expected profit for the retailer. If $x^* = x_H$ or p, Π^* can increase or decrease (we constructed numerical examples that show that both directions are possible). The fifth column of Table 3.3 reports the results.

The intuition is as follows. When σ increases, inexperienced consumers become less certain about their valuations of the product. As a result, they require better incentives than before to be willing to pre-order in the advance selling season (i.e., lower x_L and x_H). In the case in which the optimal advance selling price $x^* = x_L$, all consumers who buy pre-order, now at a lower price. By Assumption 1, the retailer's expected profit from experienced consumers becomes lower. The retailer's expected profit from inexperienced consumers falls as well since all of them pre-order at a lower price. Hence, the total expected profit of the retailer becomes smaller.

Consider now the case in which the optimal advance selling price $x^* = p$. As σ increases the valuation distribution functions become more dispersed in that more consumers have valuations farther away from the mean value than before. As a result, if $p > \mu$ then more consumers have valuations above p and if $p < \mu$ then less consumers have valuations above p. Suppose $p > \mu_H$, which also implies $p > \mu_L$. The retailer gets more pre-orders from experienced consumers in the advance selling season and a larger demand from inexperienced consumers in the regular selling season, both yielding greater profit. On the other hand, for $p < \mu_L$ (hence, $p < \mu_H$), the opposite occurs and the retailer's total expected profit decreases. Obviously, either an increase or a decrease in the retailer's expected profit is possible if $\mu_L < p < \mu_H$. Thus, in the case $x^* = p$ the directional change in the retailer's expected profit depends on the value of p in relation to μ_L and μ_H, all of which are exogenously given in our model.

Finally, consider the case in which the optimal advance selling price $x^* = x_H$. The retailer's total expected profit falls with probability γ (corresponding to all inexperienced consumers pre-ordering like in the case $x^* = x_L$) and falls or rises with probability $1 - \gamma$ (corresponding to all inexperienced consumers waiting to buy in the regular selling season like in the case $x^* = p$). As a result, depending on these probabilities (γ and $1-\gamma$) and the respective gain and loss, both directions of change are possible for the retailer's total expected profit.

3.6 Extensions

In this section, the equilibrium analysis in Section 3.4 is extended in two directions. We first relax the assumption that $c < x_L < x_H < p$ so as to explore the possibility that the retailer sets an advance selling price that is below cost. Then, we discuss the retailer's optimal pricing strategy without Assumption 1.

3.6.1 Advance selling below cost ($x < c$)

Based on our analysis in Section 3.4, only if the assumption that $c < x_L < x_H < p$ is relaxed can it become possible that the optimal advance selling price x is less than c. Accordingly, we examine the possibility of $x^* < c$ under the following two scenarios: $x_L < c < x_H < p$ and $x_L < x_H < c < p$. Following the same arguments as those presented in Section 3.4, we have that the optimal advance selling price in these two scenarios must take one of the three values: x_L, x_H, and p.

Can x_L be the optimal advance selling price? Since

$$\Pi^A(x_L) = m_e \left(\gamma \overline{F}_H(x_L) + (1-\gamma)\overline{F}_L(x_L)\right)(x_L - c) + m_i(x_L - c) < 0,$$

setting $x = x_L$ would lead to a negative expected profit under either scenario. It is therefore inferior to setting the advance selling price at p, which implies a positive expected profit. Thus, the optimal advance selling price can never be x_L in either of the above two scenarios.

In the scenario $x_L < c < x_H < p$, setting $x = x_H$ does not imply pricing under cost. So the question becomes, can x_H be the optimal advance selling price in the scenario $x_L < x_H < c < p$? We have

$$\begin{aligned}\Pi^B(x_H) &= m_e \left(\gamma \overline{F}_H(x_H) + (1-\gamma)\overline{F}_L(x)\right)(x_H - c) + \gamma m_i(x_H - c) + (1-\gamma)\pi_L \\ &< m_e \left(\gamma \overline{F}_H(p) + (1-\gamma)\overline{F}_L(p)\right)(p - c) + \gamma \pi_H + (1-\gamma)\pi_L = \Pi^C(p).\end{aligned}$$

Thus, the optimal advance selling price cannot be x_H.

In conclusion, we have shown above that the retailer's optimal advance selling price is never below the unit production cost c.

3.6.2 Interior optimum

Our study of the retailer's optimal advance selling price and the subsequent sensitivity analysis were based on the simplifying Assumption 1 that $\left(\gamma \overline{F}_H(x) + (1-\gamma)\overline{F}_L(x)\right)(x-$

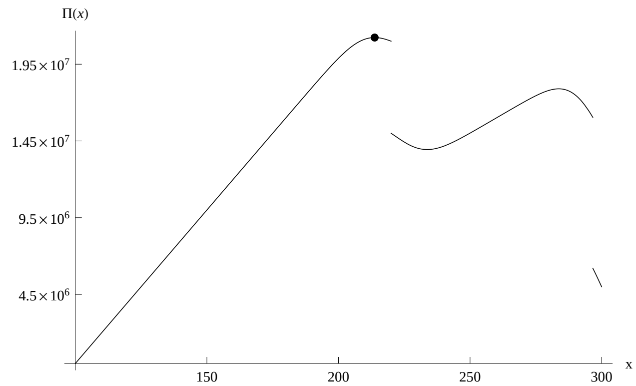
c) increases in x on $[c, p]$. In this subsection, we wish to point out that our main results continue to hold if Assumption 1 is not maintained. In particular, the following numerical examples demonstrate that the retailer's optimal advance selling price can occur in all three relevant regions, although it is in general an interior optimum in the respective region (Figure 3.4).

In these examples, we present three different interior optimal choices by the retailer that are located in the three respective regions. In all three cases, $c = 100$, $m = 200000$, $\alpha = 0.5$, and $\gamma = 0.5$.

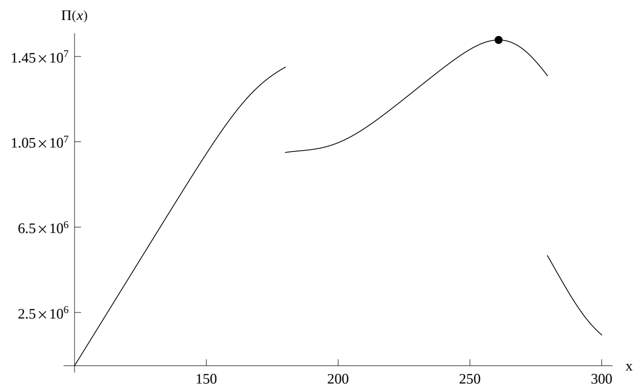
Example 4. *We use $p = 300$, $s = 45$, $\sigma = 10$, $\mu_L = 220$, $\mu_H = 300$, and $\tau_i = 4.5$. In this example, the endogenous values are $\eta = 0.16$, $x_L = 220$, and $x_H = 296.66$; the optimal advance selling price $x^* = 213.75$ is located in region A. This example is illustrated in Figure 3.4(a).*

Example 5. *We use $p = 300$, $s = 60$, $\sigma = 15$, $\mu_L = 180$, $\mu_H = 280$, and $\tau_i = 1$. In this example, the endogenous values are $\eta = 0.06$, $x_L = 180$, and $x_H = 279.40$; the optimal advance selling price $x^* = 260.88$ is located in region B. This example is illustrated in Figure 3.4(b).*

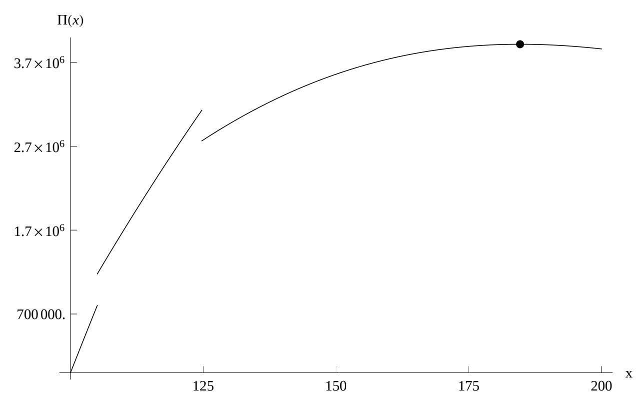
Example 6. *We use $p = 200$, $s = 55$, $\sigma = 100$, $\mu_L = 115$, $\mu_H = 140$, and $\tau_i = 0.65$. In this example, the endogenous values are $\eta = 0.10$, $x_L = 105.05$, and $x_H = 124.74$; the optimal advance selling price $x^* = 184.65$ is located in region C. This example is illustrated in Figure 3.4(c).*



(a) $c < x^* < x_L$



(b) $x_L < x^* < x_H$



(c) $x_H < x^* < p$

Figure 3.4: Interior optimum

3.7 Concluding Remarks

This paper has studied advance selling when the firm faces inexperienced consumers as well as a group of experienced consumers who have prior experience with an earlier version of the product. We find that it is always in the firm's best interest to adopt advance selling. The optimal pre-order price may or may not be at a discount to the regular selling price.

A number of issues are worthy of further investigation. One issue is the possibility of a price premium for pre-orders, which has not been analyzed in the present paper. As experienced consumers know their valuations of the product, some of them may be willing to pay a price premium so as to avoid the possibility of stock-out in the regular selling season. However, with a price premium for pre-orders, some experienced consumers will choose to wait until the regular selling season. As a result, both learning by the firm and the calculation of the stock-out probability will be much different and more involved.

Another issue concerns the assumption that the distribution of valuations is the same for experienced and inexperienced consumers. One straightforward generalization of our model is to assume that the distribution of valuations for experienced consumers is a rightward shift of that for inexperienced consumers (i.e., experienced consumers value the product more than inexperienced consumers on average). We expect many of the results in our paper to continue to hold in this extension. An alternative might be to assume a more general level of correlation between the two distribution functions. Much remains to be explored about this case.

Chapter 4

Advance selling with price premium

4.1 Introduction

Advance selling is a selling strategy which allows consumers to pre-order a product and guarantees prompt delivery after release. Consumers can also place orders in the regular selling season, however, they might not be able to get the product immediately because a stock out might happen. The iPhone 4S by Apple Inc. is a very good example. From Oct. 7th, 2011, Apple started taking pre-orders for the iPhone 4S in the United States. Consumers who pre-ordered got their iPhones right after the release. However, some consumers who decided to buy after Oct. 14th, 2011 (release date) were informed "backordered" because the demand in a short time after the release is too huge.

To consumers, there are two main reasons for them to order in advance, guaranteed prompt delivery of pre-orders and a price discount (if there is a discount). To the retailer, advance selling is a tool to reduce the demand uncertainty and therefore improve his profit. With rapid development in the Internet and information technology, advance selling has gained popularity in a wide range of product categories, especially for books, music CDs, video games, smart phones, software, fashion products, and travel services.

Advance selling of new products by a Newsvendor retailer has been studied by researchers in various aspects. However, most of the studies consider advance selling at a price discount (Tang, Rajaram, Alptekinoğlu, and Ou 2004, Zhao and Stecke 2010 and Prasad, Stecke, and Zhao 2011). The dimension in advance selling price premium has been surprisingly neglected so far. In practice, there are some examples showing that retailers charge high prices in the advance selling season and cut the prices after the release. For example, in 2011, Best Buy accepted pre-orders for Motorola Xoom tablet at the price of \$1199 and then cut the price to \$599 on the release date (Rosenberg 2011). Also, for highly sought-after products, the priority to get the products seems to be very important to consumers who are either new technology lovers or loyal fans of some particular brands. As a result, these consumers might be willing to pay a premium price for a guaranteed prompt delivery.

This paper investigates the advance selling price premium for a retailer. The motivation of this paper comes from two observations. First, in practise, electronics and other new technology products are more expensive when they are first launched. Early adopters pay more to have the newest products. For example, in 2007, Apple cut down the price of iPhone (8GB model) from \$599 to \$399 just 67 days after it was launched (Hafner and Stone 2007). For the consumers who purchased it earlier, it seems that they paid an early adopter tax of \$200. In 2009, Amazon charged a pre-order price \$359 for Kindle 2 and then cut the price to \$299 after the release (Carnoy, 2009). Second, each year there are some new products sold out either in the advance selling season or shortly after the release. For example, AT&T sold out iPhone 4 on the first pre-ordering day at the regular selling price; Apple sold out iPad 2 on the opening weekend. These economic phenomenon indicate that, for some new products, the market demand in a short period after the availability exceeds the market supply, which implies retailers might be able to improve their total profits by advance selling at a price premium.

While in practice advance selling is often carried out with a price discount, it is worthwhile to explore the profitability of advance selling at a price premium. I consider a two-period dynamic model. The first period is the advance selling season, and the second period is the regular selling season. Consumers in the model are heterogenous and strategic. Consumers' valuations are drawn from a normal distribution. When pre-orders are available, consumers make decisions to pre-order or not by comparing the expected payoffs from pre-ordering and not. For consumers who decide to wait, those with valuations above the regular selling price will order in the regular selling season. However, they will face a stock-out probability. With regard to the retailer, he is uncertain about the market size, and he does not know the exact consumer valuation distribution. To reduce the demand uncertainty, the retailer decides on adopting advance selling or not. If yes, he chooses the advance selling price at the beginning of the first period and makes the quantity decision at the end of the advance selling season.

In this paper, I mainly focus on exploring the possibility of advance selling at a price premium for the retailer. Under the same model setup of the second essay, I investigate the profitability of advance selling price premium for the retailer. I show that advance selling at a price premium always yields more profit for the retailer compared with advance selling at the regular selling price. There are three types of advance selling strategies for the retailer after advance selling price premium is considered: advance selling at a deep discount, advance selling at a moderate discount and advance selling at a price premium. In addition, I study the conditions where the retailer is more likely to implement advance selling at a price premium instead of a price discount. Numerical tests are also presented to show how these parameters in the model impact the retailer's optimal advance selling price premium and optimal total profit.

The most important contribution of this paper is that it studies the profitability of advance selling at a price premium in a model with experienced consumers, and provides conditions under which the retailer is no longer willing to implement advance selling at a discount.

After the literature review, the rest of the paper is organized as follows. Section 4.3 introduces the model. Section 4.4 presents optimal advance selling strategies to the retailer after considering advance selling price premium. Section 4.5 studies the retailer's optimal advance selling price premium and provides the conditions under which the retailer will implement advance selling at a price premium. Section 4.6 performs sensitivity analysis. Section 4.7 concludes the paper.

4.2 Literature review

This paper is closely related to the literature on advance selling in manufacture industries, i.e., the retailer has unlimited capacity.

Weng and Parlar (1999) notice that the retail industries have shorter and shorter selling seasons because of fast development of products and serious competition. They present an advance selling model to predict the demand in the selling season. After pre-orders are available, a given fraction of consumers will take advantage of it and purchase the product at a price discount. Tang, Rajaram, Alptekinoğlu, and Ou (2004) assume that customers are from two different groups. One purchases from firm A which may adopt advance selling, the other purchase from other firms that do not have such a program. Their paper mostly focuses on exploring the benefits of advance selling. McCardle, Rajaram, and Tang (2004) examine advance selling in a duopoly game which involves retail competition. Under competition, they showe that both firms launch the advance selling program would be the unique equilibrium if it is optimal for one firm to adopt advance selling. Chen and Parlar (2005) introduce two models and also solve the sequentially determined decisions of optimal advance selling price and optimal production. The first model fixes the market size and assumes that the pre-order probability of each consumer increases with the advance selling discount. The second model considers an increasing market size and assumes that the purchase probability of each consumer follows a β-distribution. In these four papers, consumers are non-strategic. They follow given strategies in the market.

There are papers studying advance selling in manufacture industries with strategic consumers. Zhao and Stecke (2010) present a model in which consumers are classified into two groups, loss averse consumers and other consumers. They examine how a retailer develops the advance selling strategy to maximize his own profits. In Prasad, Stecke, and Zhao (2011), consumers are divided into two groups, informed and uninformed, depending on the accessibility to the pre-order information. Informed consumers purchase in the advance selling season while uninformed consumers purchase in the regular selling season. They build up a correlation between the numbers of informed consumers and uninformed consumers, with which the retailer forecasts the second-period demand with the realized first-period demand. Chu and Zhang (2011) consider a model in which consumers learn their initial valuations in the advance selling season. Their final valuations are influenced by many factors, one of which is how much information they know about the product. In this paper, they study the firm's optimal advance selling strategy when the firm can control the information release about the product at pre-order. In Nocke, Peitz, and Rosar (2011), consumers are heterogeneous in their expected valuations. The shock to consumers' valuations follows a binary distribution which has an expected value of zero. The monopolist seller implements advance selling at a discount to price discriminate consumers on the basis of their expected valuations.

There has been a growing interest in studying advance selling with price premium recently. Xie and Shugan (2009) show that with limited capacity and rationing, the optimal advance selling strategy might be price premium rather than price discount. Zhao and Pang (2011) study and compare three different strategies: dynamic pricing, price commitment, and pre-order price guarantee. They show that both advance selling price discount and advance selling price premium may be possible under dynamic pricing as well as price commitment. Li and Zhang (2011) study the dynamic pricing strategy for the seller. In the model, high type consumers arrive in the advance selling season while low type consumers arrive in the regular selling season. In the equilibrium, they show that the regular selling price is greater than or equal to the advance selling price.

In this paper, I examine the advance selling strategy with a price premium in a two-period model where both the retailer and consumers are strategic. The key difference between this paper and the literature on advance selling price premium is the introduction of experienced consumers into the model. Furthermore, since the retailer does not know the mean of consumer valuation distribution, learning in this model is not only on the market size, but also on the consumers' valuation distribution.

4.3 The model

Consider the situation that a monopolist retailer sells a product over two periods. The first period is the advance selling season and the second period is the regular selling season. There are two types of consumers – experienced and inexperienced, depending on the experience of the early versions of this product. Each consumer is allowed to buy at most one unit of this product. Any orders in the advance selling seasons are guaranteed to be delivered to the consumers right after the release. With regard to the orders submitted in the regular selling season, there is a risk that this product will be out of stock. Table 4.1 lists the notation in this paper.

Table 4.1: Notation C

Parameters/Variables concerning a retailer	
c	marginal cost
s	salvage value
p	price in the regular selling season
k	adoption cost of advance selling
$\hat{x}$	advance selling price premium
π	retailer's expected profit from the regular selling season
Π	retailer's total expected profit (includes pre-orders)
Parameters/Variables concerning consumers and market	
D_1	demand in the first period
D_2	demand in the second period
η	stock-out probability
$V \sim \mathrm{N}\left(\mu, \sigma^2\right)$	consumer valuation distribution, with realized value v
$\mu \in \{\mu_H, \mu_L\}$	two-point distribution, $\mathrm{Prob}(\mu_H) = \gamma$ and $\mathrm{Prob}(\mu_L) = 1-\gamma$
$M_i \sim \mathrm{LN}\left(\nu_i, \tau_i^2\right)$	market size distribution, mean $m_i = \exp\left\{\nu_i + \tau_i^2/2\right\}$
m_e	number of experienced consumers
$m = m_e + m_i$	total expected number of consumers
Decision variables	
q	quantity produced for the regular selling season
Q	total quantity produced (includes pre-orders)
x	advance selling price

4.3.1 Retailer

The retailer produces this product at a marginal cost c and receives a salvage value s for each unsold unit of the product at the end of the regular selling season. The regular selling season price p is announced at the beginning of the advance selling season. Without loss of generosity, I assume $s < c < p$.

At the beginning of the advance selling season, the retailer decides on advance selling price x and receives a certain number of pre-orders D_1. After the conclusion of the advance selling season, the retailer must decide how much to produce before the demand in the regular selling season is realized. Let D_2 denote the random demand in the regular selling season. Then the retailer's quantity choice is $Q = D_1 + q$, where D_1 fulfills the pre-orders and q is the optimal quantity for the random demand D_2.

4.3.2 Consumers

Consumers are risk-neutral and strategic. Each consumer has an idiosyncratic valuation, i.e., the maximum amount of money she would like to pay for this product. The consumer valuation of this product V follows normal distribution with mean μ and variance σ^2, i.e., $V \sim \mathrm{N}(\mu, \sigma^2)$. The mean value of this distribution is known to all consumers; however, the retailer does not know it. The retailer's uncertainty is modeled as μ is high, μ_H, with probability γ and low, μ_L, with probability $1 - \gamma$.

Depending on the experience of the early versions, consumers are divided into two groups- experienced and inexperienced. Experienced consumers know their valuations from the beginning of the advance selling season, whereas inexperienced consumers do not learn their valuations till the regular selling season. The number of experienced consumers is m_e. The number of inexperienced consumers M_i is a random variable which follows lognormal distribution $\mathrm{LN}(\nu_i, \tau_i^2)$ with mean $m_i = \exp\{\nu_i + \tau_i^2/2\}$. Both m_e and the distribution of M_i are common knowledge. Let $m = m_e + m_i$ denote the total expected number of (experienced and inexperienced) consumers, and let α denote the proportion of experienced consumers in the market. Thus, $m_e = \alpha m$ and $m_i = (1 - \alpha)m$.

During the advance selling season, consumers make decisions on whether to purchase by comparing the expected payoffs from pre-orders and regular season purchases. If a consumer pre-orders, she pays x and is guaranteed to get the product right after it is released. If not, she waits until the regular selling season and makes a purchase when the realized valuation v is greater than the regular selling price p, but this product might be out of stock.

4.4 Advance selling with price premium

The goal of this section is to study the optimal advance selling strategy for the retailer after considering advance selling at a price premium. It extends the work of the second essay in the same model setup. First, I study consumers' optimal purchasing behaviors and the retailer's learning in Subsection 4.4.1 and 4.4.2, respectively. Then I explore the stock-out probability when advance selling price premium is considered in Subsection 4.4.3. The analysis of optimal advance selling strategy is demonstrated in Subsection 4.4.4.

For the rest of the analysis, I assume that $c < x_L < x_H < p$ holds.

4.4.1 Consumers' optimal purchasing decisions

As studied in the second essay, if $x \leq p$ (we do not consider advance selling price premium in that model),

- experienced consumers will never wait till the regular selling season because of the advance selling price discount.
- inexperienced consumers,

i. when $x \leq x_L$, all pre-order;

ii. when $x_L < x \leq x_H$, all pre-order if $\mu = \mu_H$, otherwise all wait;

iii. when $x > x_H$, all wait.

It is very interesting to ask what happens if the retailer has the option to set an advance selling price above the regular selling price?

Let $\hat{x}$ denote the price, where $\hat{x} > p$. Consider the case that the retailers charges $\hat{x}$ in the advance selling season. First, let us look at experienced consumers. Since there is an advance selling price premium, some experienced will wait to buy in the regular selling season. If she pre-orders and pays $\hat{x}$, the consumer's expected payoff is

$$v - \hat{x};$$

if she waits and orders in the second period, her expected payoff is

$$(v - p)(1 - \eta),$$

where η is stock-out probability in the second period. Thus, an experienced consumers pre-order if and only if

$$v - \hat{x} \geq (v - p)(1 - \eta),$$

i.e.,

$$v \geq p + \frac{\hat{x} - p}{\eta}.$$

For those experienced consumers with

$$p < v < p + \frac{\hat{x} - p}{\eta},$$

they wait to buy in the second period. However, they will face a stock-out probability η.

Next, since x_L and x_H are below $\hat{x}$ [1], inexperienced consumers do not pre-order and always wait to make the purchase decisions in the second period. Only those inexperienced consumers with $v > p$ order in the second period. Thus, the demand in the first period is

$$D_1 = m_e\overline{F}(p + \frac{\hat{x} - p}{\eta});$$

the demand in the second period is

$$D_2 = m_e\left(\overline{F}(p) - \overline{F}(p + \frac{\hat{x} - p}{\eta})\right) + M_i\overline{F}(p),$$

which is a shifted log-normal distribution.

4.4.2 Retailer's learning

As discussed in Subsection 4.4.1, experienced consumers with $v > p$ will buy this product either in the first period or in the second period. Thus, the retailer knows that a certain number of experienced consumers will order in the second period, the number of orders is

$$m_e\left(\overline{F}(p) - \overline{F}(p + \frac{\hat{x} - p}{\eta})\right).$$

In addition, he knows that there is an uncertain number of orders $M_i\overline{F}(p)$ from inexperienced consumers. As provided in the second essay, the optimal quantity to

[1] x_L and x_H change with η. As proved in Subsection 4.4.3, η decreases when advance selling price premium is considered. According to Lemma 4 in the second essay, both x_L and x_H decrease. Thus, x_L and x_H are still below $\hat{x}$.

fulfill the random demand $M_i\overline{F}(p)$ from inexperienced consumers is

$$q^* = \exp\{\nu_i + \tau_i z_\beta\}\overline{F}(p);$$

the resulting expected profit is

$$\pi^* = (p - s)\,(1 - \Phi(\tau_i - z_\beta))\, m_i\overline{F}(p).$$

Thus, for demand in the second period, the retailer will produce

$$q = m_e(\overline{F}(p) - \overline{F}(p + \frac{\hat{x} - p}{\eta}) + q^*,$$

where $m_e(\overline{F}(p) - \overline{F}(p + \frac{\hat{x}-p}{\eta})$ intends to fulfill the orders from experienced consumers and q^* intends to fulfill the random demand from inexperienced consumers. It is important to note that for those experienced consumers who order in the second period, some of them might not be able to get this product if they arrive late.

Let the stock-out probability be η_L when $\mu = \mu_L$, and η_H when $\mu = \mu_H$. If the retailer notices that

$$D_1 = m_e\overline{F}_L(p + \frac{\hat{x} - p}{\eta_L}),$$

then he infers $\mu = \mu_L$. In this case,

- the demand in the second period is

$$D_2 = m_e\left(\overline{F}_L(p) - \overline{F}_L(p + \frac{\hat{x} - p}{\eta_L})\right) + M_i\overline{F}_L(p);$$

- the optimal quantity for the random demand $M_i\overline{F}_L(p)$, denoted by q_L^*, and the resulting expected profit π_L^* are

$$q_L^* = \exp\{\nu_i + \tau_i z_\beta\}\overline{F}_L(p)$$

and

$$\pi_L^* = (p - s)\,(1 - \Phi(\tau_i - z_\beta))\, m_i \overline{F}_L(p);$$

- the retailer produces q_L for the second period, where

$$q_L = m_e(\overline{F}_L(p) - \overline{F}_L(p + \frac{\hat{x} - p}{\eta_L}) + q_L^*;$$

If he notices the demand in the first period

$$D_1 = m_e \overline{F}_H(p + \frac{\hat{x} - p}{\eta_H}),$$

then he learns $\mu = \mu_H$. In this case,

- the demand in the second period is

$$D_2 = m_e \left(\overline{F}_H(p) - \overline{F}_H(p + \frac{\hat{x} - p}{\eta_H})\right) + M_i \overline{F}_H(p);$$

- the optimal quantity for the random demand $M_i \overline{F}_H(p)$, denoted by q_H^*, and the resulting expected profit π_H^* are

$$q_H^* = \exp\{\nu_i + \tau_i z_\beta\}\overline{F}_H(p)$$

and

$$\pi_H^* = (p - s)\left(1 - \Phi(\tau_i - z_\beta)\right) m_i \overline{F}_H(p);$$

- the retailer produces q_H for the second period, where

$$q_H = m_e(\overline{F}_H(p) - \overline{F}_H(p + \frac{\hat{x} - p}{\eta_H}) + q_H^*;$$

4.4.3 Stock-out probability

Pre-orders in the advance selling season are guaranteed to be delivered after the release according to the policy. However, for orders submitted in the regular selling season, there is a risk that this product will be out of stock, which is captured by the stock-out probability.

The stock-out probability is the ratio of excess demand in the regular selling season to the total demand in it. It shows the probability that a consumer wants to purchase this product in the second period but is unable to get it.

From the definition, the formula to calculate the stock-out probability is

$$\eta = E\left[\left(\frac{D_2 - q)}{D_2}\right)^+\right],$$

where

$$D_2 = m_e\left(\overline{F}(p) - \overline{F}(p + \frac{\hat{x} - p}{\eta})\right) + M_i\overline{F}(p)$$

and

$$q = m_e\left(\overline{F}(p) - \overline{F}(p + \frac{\hat{x} - p}{\eta})\right) + q^*.$$

So we have

$$\eta = E\left[\left(\frac{M_i\overline{F}(p) - \exp\{\nu_i + \tau_i z_\beta\}\overline{F}(p)}{m_e\left(\overline{F}(p) - \overline{F}(p + \frac{\hat{x}-p}{\eta})\right) + M_i\overline{F}(p)}\right)^+\right]. \tag{4.1}$$

For given $\hat{x}$ and μ, (4.1) defines the endogenous stock-out probability. Denote $\eta_L(\hat{x})$ and $\eta_H(\hat{x})$ be the stock-out probability for $\mu = \mu_L$ and $\mu = \mu_H$, respectively[2].

In the second essay,

$$\eta = E\left[\left(\frac{M_i\overline{F}(p) - \exp\{\nu_i + \tau_i z_\beta\}\overline{F}(p)}{M_i\overline{F}(p)}\right)^+\right].$$

It is obvious that the stock-out probability η decreases when advance selling price premium is considered. The intuition is that with a certain amount of experienced consumers go to the second period and purchase, it softens the demand uncertainty and therefore reduces the stock-out probability.

With $\eta_L(\hat{x})$ and $\eta_H(\hat{x})$ decrease, according to Lemma 4 in the second essay, we have threshold values x_L and x_H both decrease. It implies that inexperienced consumers' optimal purchasing decisions are not affected in the situation $c < x_L < x_H < p$.

4.4.4 Equilibrium Analysis

If the retailer charges $\hat{x}$ in the advance selling season, he receives D_1 in the first period and faces a random demand in the second period. The random demand D_2 consists of two parts, a certain number of orders from experienced consumers and a random number of orders from inexperienced consumers. All the orders in the second period might face stock out. The retailer learns μ in the first period. The following lemma

[2] $\eta_L(\hat{x})$ and $\eta_H(\hat{x})$ are calculated to take different values.

describes the total expected profit from a price premium, where $\Pi(p)$ is the total expected profit when the advance selling price is set to be p.

Lemma 7 (Expected profit from a price premium)**.** *Advance selling at a price premium $\hat{x} > p$ yeilds the expected profit*

$$\Pi(\hat{x}) = m_e \left(\gamma \overline{F}_H(p + \frac{\hat{x} - p}{\eta_H(\hat{x})}) + (1-\gamma)\overline{F}_L(p + \frac{\hat{x} - p}{\eta_L(\hat{x})}) \right) (\hat{x} - p) + \Pi(p) \tag{4.2}$$

to the retailer.

The intuition is straightforward. Under both situations, advance selling at a price premium $x = \hat{x}$ and advance selling at p, the retailer learns μ in the first period and faces the same demand uncertainty in the second period. Therefore, the retailer produces the same quantity

$$Q = m_e \overline{F}(p) + q^*.$$

Consumers who buy when $x = \hat{x}$ also make purchases when $x = p$, either in the first period or in the second period. To the retailer, the only difference between the two situations is that, under $\hat{x}$,

$$m_e \overline{F} \left(p + \frac{\hat{x} - p}{\eta(\hat{x})} \right)$$

of the total number of transactions occurs at a higher price $\hat{x}$ while all other occur at p. Hence,

$$\Pi(\hat{x}) - \Pi(p) = m_e \left(\gamma \overline{F}_H \left(p + \frac{\hat{x} - p}{\eta_H(\hat{x})} \right) + (1-\gamma)\overline{F}_L \left(p + \frac{\hat{x} - p}{\eta_L(\hat{x})} \right) \right) (\hat{x} - p). \tag{4.3}$$

When $\hat{x}$ increases, the profit margin increases, however, the number of experienced consumers who pre-order decreases. The retailers faces a trade-off between high profit margin-low volume of pre-orders and low profit margin-high volume of pre-orders.

Lemma 8 (Properties of $\Pi(\hat{x})$)**.** *The expected profit from a price premium* $\Pi(\hat{x})$ *possesses the following properties:*

- $\Pi(\hat{x}) > \Pi(p)$ *for any given* $\hat{x} > p$*;*
- $\lim_{\hat{x}\to p} \Pi(\hat{x}) = \Pi(p)$*;*
- $\lim_{\hat{x}\to +\infty} \Pi(\hat{x}) = \Pi(p)$*.*

It is obvious from (4.3) that $\Pi(\hat{x}) > \Pi(p)$ for any given $\hat{x} > p$. In other words, the retailer is always better off by setting the advance selling price at a price premium than at $x = p$. Also, it can be shown that $\Pi(\hat{x})$ converges to $\Pi(p)$ as $\hat{x}$ approaches p from above, and $\Pi(\hat{x})$ converges to $\Pi(p)$ as $\hat{x}$ approaches $+\infty$. Let $\hat{x^*}$ be the optimal advance selling price premium, which maximizes (4.2). Thus, $p < \hat{x}^* < +\infty$, i.e., the optimal advance selling price premium, $\hat{x^*}$, exists.

Lemma 9 (Optimal advance selling price premium)**.** *The optimal advance selling price premium,* $\hat{x^*}$*, satisfies*

$$\frac{\partial m_e \left(\gamma \overline{F}_H \left(p + \frac{\hat{x^*} - p}{\eta_H(\hat{x^*})}\right) + (1-\gamma)\overline{F}_L \left(p + \frac{\hat{x^*} - p}{\eta_L(\hat{x^*})}\right)\right)(\hat{x^*} - p)}{\partial \hat{x}^*} = 0. \qquad (4.4)$$

Based on the properties of $\Pi(\hat{x})$, We can easily include $\Pi(\hat{x})$ in Figure 3.2 in the second essay by plotting a curve above $\Pi(p)$, which converges to $\Pi(p)$ as $\hat{x} \to +\infty$ and as $\hat{x} \to p$. Similarly, we have four patterns.

- Pattern 1: $\Pi(x)$ jumps down at both x_L and x_H.
- Pattern 2: $\Pi(x)$ jumps up at both x_L and x_H.

- Pattern 3: $\Pi(x)$ jumps up at x_L, jumps down at x_H.
- Pattern 4: $\Pi(x)$ jumps down at x_L, jumps up at x_H.

Under pattern 1 the optimal advance selling price x^* is either x_L, x_H, or $\hat{x}^*$; under pattern 2 it is $\hat{x}^*$; under pattern 3, it is either x_H or $\hat{x}^*$; under pattern 4, it is either x_L or $\hat{x}^*$. It follows that the retailer's optimal advance selling price can be any one of the three values: x_L, x_H, or $\hat{x}^*$.

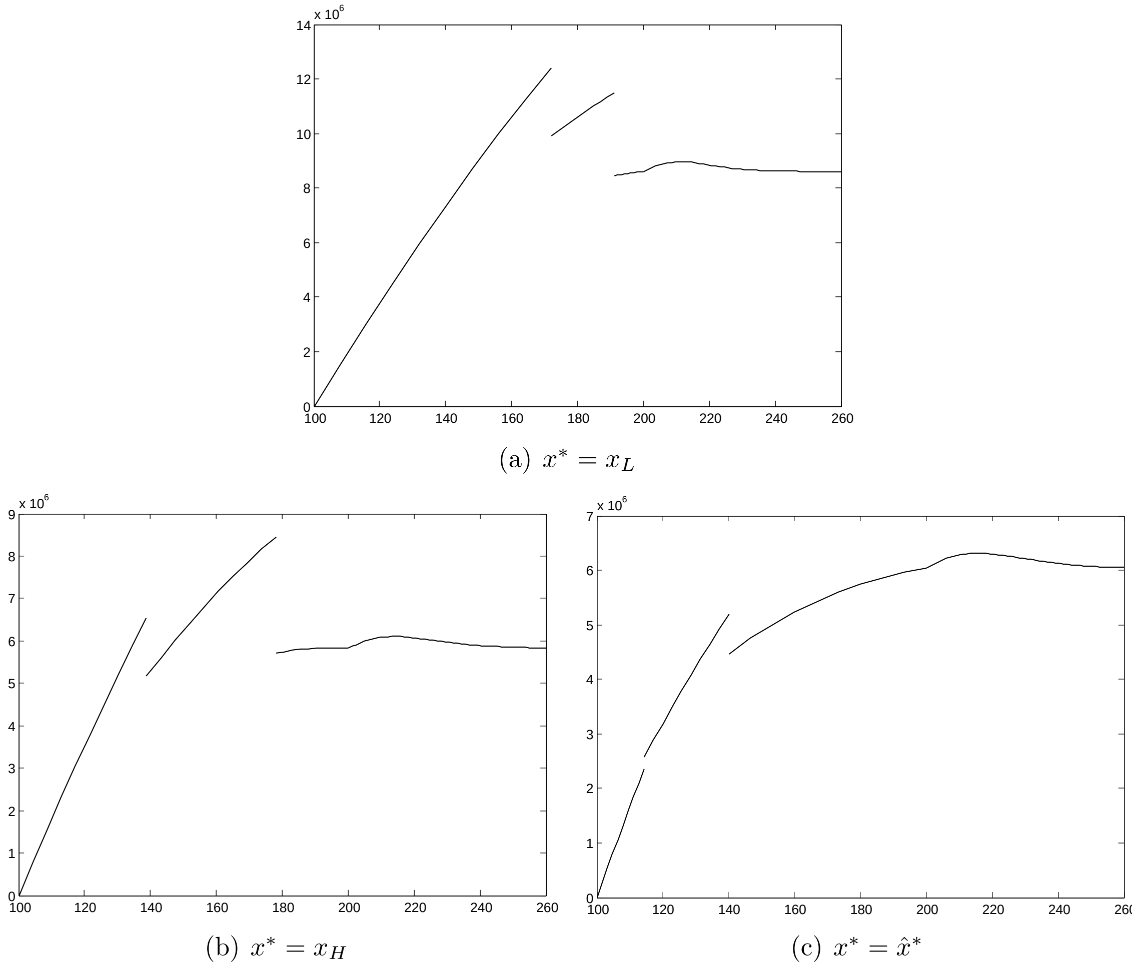
Proposition 6 (Optimal advance selling price when price premium is considered)**.** *When price premium for advance selling is considered, the optimal advance selling price is either x_L, x_H or $\hat{x}^*$.*

Figure 4.1 illustrates the three possibilities for the optimal advance selling price. In all three figures, $p = 200$, $c = 100$, $\tau_i = 1$, $m = 200000$, $\alpha = 0.5$ and $\gamma = 0.5$.

- In Figure 4.1(a), I used $s = 55$, $\sigma = 80$, $\mu_L = 200$ and $\mu_H = 240$. In this case the optimal advance selling price is x_L.
- In Figure 4.1(b), I used $s = 50$, $\sigma = 80$, $\mu_L = 150$ and $\mu_H = 210$. In this case the optimal advance selling price is x_H.
- In Figure 4.1(c), I used $s = 80$, $\sigma = 130$, $\mu_L = 140$ and $\mu_H = 180$. In this case the optimal advance selling price is $\hat{x}^*$.

4.5 Analysis of advance selling price premium

After we relax the assumption $x \leq p$, setting advance selling price at p is dominated by any price premium. The retailer will choose from either x_L, x_H or $\hat{x}^*$. It is very



(a) $x^* = x_L$

(b) $x^* = x_H$

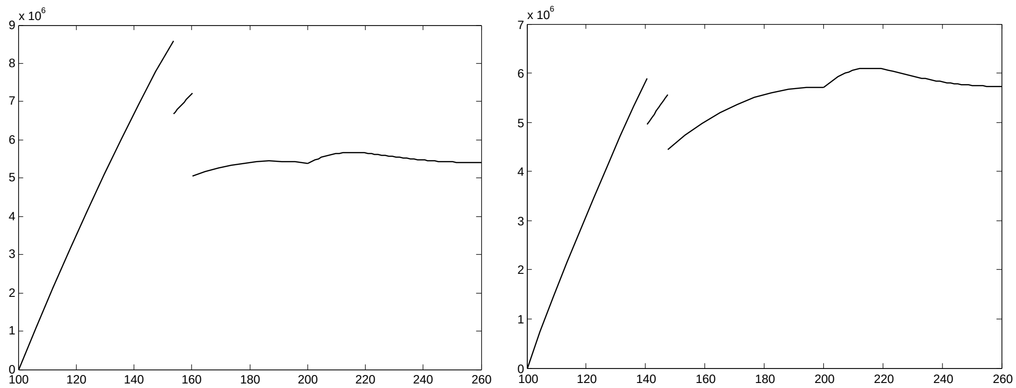
(c) $x^* = \hat{x}^*$

Figure 4.1: Optimal advance selling price when price premium is considered

interesting to see how the optimal advance selling price changes after an advance selling price premium is considered.

First, if the optimal advance selling price is p in the situation $x \leq p$, after considering advance selling price premium, the optimal advance selling price will be $\hat{x}^*$. See Figure 4.1(c). It comes directly from the result that $\Pi(\hat{x}) > \Pi(p)$.

Second, if the optimal advance selling price is x_L under $x \leq p$, considering advance selling price premium, x^* might change to $\hat{x}^*$. In Figure 4.2(a), the optimal advance



(a) x^* remains at x_L (b) x^* changes to $\hat{x}^*$

Figure 4.2: The optimal advance selling price is x_L under $x \leq p$

selling price remains at x_L after advance selling price premium is considered. In contrast, in Figure 4.2(b), the optimal advance selling price changes to $\hat{x}^*$. To explore what cause the retailer to charge a price premium instead of a deep price discount, we need to study the consumers's purchasing behaviors together with the demand uncertainty to the retailer at both prices, x_L and $\hat{x}^*$.

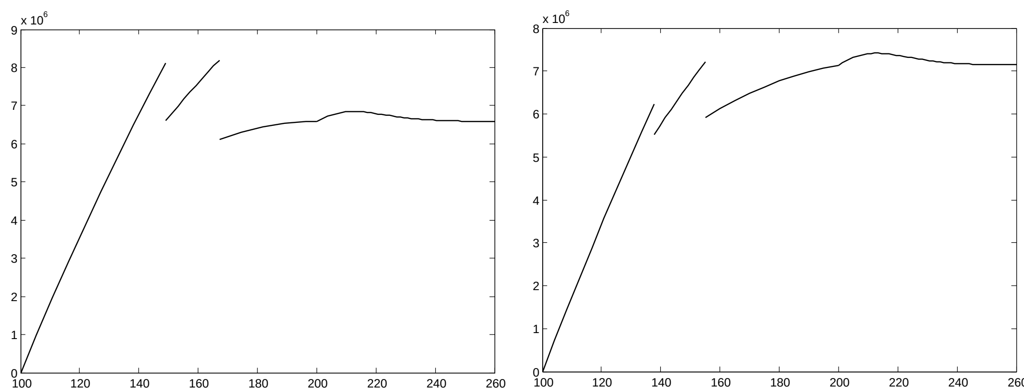
- When $x = x_L$, experienced consumers with $v > x_L$ and all inexperienced consumers pre-order. The retailer faces no demand uncertainty in the second period. He produces exactly the same number of products as the pre-orders.

- When $x = \hat{x}^*$, experienced consumers with $v > p$ buy this product either at p or $\hat{x}^*$. Inexperienced consumers with $v > p$ buy in the regular selling season. The retailer faces a demand uncertainty which comes from the number of inexperienced consumers. For each unit of unsold products, the salvage value is s.

Therefore, if we have more experienced consumers in the market, if there are more consumers with higher valuations, if the salvage value for unsold products increases, or if the uncertainty of the number of inexperienced consumers decreases, it is more likely for the retailer to charge $x^* = \hat{x}^*$ instead of $x^* = x_L$. Thus, we have the following lemma.

Lemma 10 (Conditions for x^* changes from x_L to $\hat{x}^*$)**.** *After considering advance selling price premium, the retailer may switch from charging a deep price discount x_L to a price premium $\hat{x}^*$ if one or more of the following conditions satisfied: 1) α is high, 2) σ is high, 3) s is high or 4) τ_i is small.*

Last, if the optimal advance selling price is x_H under $x \leq p$, considering advance selling price premium, x^* might change to $\hat{x}^*$. In Figure 4.3(a), the optimal advance



(a) x^* remains at x_H (b) x^* changes to $\hat{x}^*$

Figure 4.3: The optimal advance selling price is x_H under $x \leq p$

selling price remains at x_H after advance selling price premium is considered. In contrast, in Figure 4.3(b), the optimal advance selling price changes to $\hat{x}^*$. As we did before, we need to study the consumers's purchasing behaviors together with the demand uncertainty to the retailer at both prices, x_H and $\hat{x}^*$.

- When $x = x_H$, experienced consumers with $v > x_H$ pre-order, and all inexperienced consumers pre-order at x_H if and only if $\mu = \mu_H$. With probability γ, $\mu = \mu_H$, the retailer faces no demand uncertainty in the second period. He produces exactly the same number of products as the pre-orders. With probability $1 - \gamma$, $\mu = \mu_L$, the retailer faces a demand uncertainty which comes from the number of inexperienced consumers. For each unit of unsold products, the salvage value is s.

- When $x = \hat{x}^*$, experienced consumers with $v > p$ buy this product either at p or $\hat{x}^*$. Inexperienced consumers with $v > p$ buy in the regular selling season. The retailer faces a demand uncertainty which comes from the number of inexperienced consumers. For each unit of unsold products, the salvage value is s.

Therefore, if we have more experienced consumers in the market, if there are more consumers with higher valuations, if the salvage value for unsold products increases, if the uncertainty of the number of inexperienced consumers decreases, or if the probability that $\mu = \mu_H$ is lower, it is more likely for the retailer to charge $x^* = \hat{x}^*$ instead of $x^* = x_H$. Thus, we have the following lemma.

Lemma 11 (Conditions for x^* changes from x_H to $\hat{x}^*$)**.** *After considering advance selling price premium, the retailer may switch from charging a moderate price discount x_H to a price premium $\hat{x}^*$ if one or more of the following conditions are satisfied: 1) α is high, 2) σ is high, 3) s is high, 4) τ_i is small or 5) γ is small.*

There are an extensive numerical simulations conducted to support the findings in Lemma 10 and Lemma 11.

4.6 Sensitivity Analysis under Price Premium

In this section I consider how the retailer's optimal advance selling price x^* and the expected profit $\Pi(x^*)$ are affected by some important parameters of the model when $x^* = \hat{x}^*$, such as salvage value s, the number uncertainty of inexperienced consumers τ_i, the relative number of experienced consumers α and the valuation uncertainty of inexperienced consumers σ. Let $\Pi^* \equiv \Pi(x^*)$. Intuitively, we should expect that a decrease in the salvage value s and an increase of the valuation uncertainty σ would result in a lower Π^*. Surprisingly, we find that Π^* might actually increase.

Table 4.2 shows the results of the sensitivity analysis, which is conducted to focus on small changes in the parameter values so that the retailer's optimal choice stays the same. All of the directional changes in the parameters (the first row) are chosen to "hurt" the retailer on an intuitive basis. Therefore, all cases in which Π^* increases ($\Pi^* \uparrow$) represent counterintuitive results.

Table 4.2: Sensitivity analysis results when $x^* = \hat{x}^*$

optimal choice x^* \ parameter change	$s \downarrow$	$\tau_i \uparrow$	$\alpha \downarrow$	$\sigma \uparrow$
$\hat{x}^*$	$x^* \uparrow$ $\Pi^* \uparrow\downarrow$	$x^* \downarrow$ $\Pi^* \downarrow$	$x^* \uparrow$ $\Pi^* \downarrow$	$x^* \uparrow$ $\Pi^* \uparrow\downarrow$

4.6.1 When s decreases

When s becomes lower, the retailer reduces his output to avoid too many unsold products at the end of the regular selling season, which raises the stock-out probability for consumers who wait until the regular selling season. The number of inexperienced consumers who buy in the regular selling season does not change. However, some experienced consumers will switch to pre-order due to higher stock-out probability while the total number of experienced consumers who buy, either in the first period or in the second period, does not change.

First, look at how x^* changes. We know that x^* maximizes (4.3); that is, x^* maximizes

$$\Pi(\hat{x}) - \Pi(p) = m_e \left(\gamma \overline{F}_H \left(p + \frac{\hat{x} - p}{\eta_H(\hat{x})} \right) + (1-\gamma) \overline{F}_L \left(p + \frac{\hat{x} - p}{\eta_L(\hat{x})} \right) \right) (\hat{x} - p),$$

where

$$\left(\gamma \overline{F}_H \left(p + \frac{\hat{x} - p}{\eta_H(\hat{x})} \right) + (1-\gamma) \overline{F}_L \left(p + \frac{\hat{x} - p}{\eta_L(\hat{x})} \right) \right) \tag{4.5}$$

describes the proportion of experienced consumers who pre-order, and

$$(\hat{x} - p) \tag{4.6}$$

describes the additional profit earned from each pre-order. Note that (4.5) is a decreasing function of $\hat{x}$ while (4.6) is an increasing function of $\hat{x}$. With the decrease of s, some experienced consumers who buy in the regular selling season switch to pre-order, which therefore softens the decreasing power in (4.5). Thus, x^* increases.

Next, to the retailer, the total expected profit might increase because that some experienced consumers switch to pre-order at $x^* > p$. In addition, the optimal advance selling price x^* increases. However, the decreased salvage value does bite the retailer because of the lower volume of production and higher stock-out probability, which might decreases the number of sales. The following examples shows the two directions for $\Pi(x^*)$, where x^* always increases. All examples in Subsection 4.6 are constructed with $p = 200$, $c = 100$, $m = 200000$ and $\gamma = 0.5$.

Example 7. *I use $\mu_L = 160$, $\mu_H = 210$, $\sigma = 150$, $\alpha = 0.5$ and $\tau_i = 1$. s is decreased from 55 to 50. When $s = 55$, the optimal advance selling price $x^* = 234.29$ and $\Pi(x^*) = 7.62 \times 10^6$. When $s = 50$, the optimal advance selling price $x^* = 235.92$ and $\Pi(x^*) = 7.58 \times 10^6$. Thus, x^* increases and $\Pi(x^*)$ decreases.*

Example 8. *I use $\mu_L = 170$, $\mu_H = 200$, $\sigma = 120$, $\alpha = 0.9$ and $\tau_i = 1$. s is decreased from 55 to 50. When $s = 55$, the optimal advance selling price $x^* = 208.16$ and $\Pi(x^*) = 8.95 \times 10^6$. When $s = 50$, the optimal advance selling price $x^* = 209.80$ and $\Pi(x^*) = 8.96 \times 10^6$. In this example, both x^* and $\Pi(x^*)$ increase.*

4.6.2 When τ_i increases

When τ_i increases, the uncertainty of the random demand from inexperienced consumers increases. So the retailer will produce more. The consumer market structure stays the same, i.e., the numbers of experienced consumers and inexperienced consumer do not change. Also, the purchasing behaviors of consumers do not change. Thus, the stock-out probability seems to be smaller. As a result, some experienced consumers will switch to order in the regular selling season instead of pre-ordering due to lower stock-out probability.

As we known in Subsection 4.6.1, x^* maximizes $\Pi(\hat{x}) - \Pi(p)$. In addition, there are two opposite powers inside $\Pi(\hat{x}) - \Pi(p)$. With the increase of τ_i, some experienced consumers who pre-order switch to buy in the regular selling season, which therefore boost the decreasing power in $\Pi(\hat{x}) - \Pi(p)$. Thus, x^* decreases.

Next, look at the total expected profit. In (4.2), $\Pi(p)$ describes the profit by setting the advance selling price at p. As shown in the second essay, $\Pi(p)$ decreases with τ_i. $\Pi(\hat{x}) - \Pi(p)$ describes the additional profit from experienced consumers who pre-order in the first period. Because both the number of experienced consumers who pre-order and the optimal advance selling price x^* decrease, $\Pi(\hat{x}) - \Pi(p)$ decreases. Thus, $\Pi(x^*)$ decreases. The following examples show that both $\Pi(x^*)$ and x^* decreases with τ_i.

Example 9. *I use* $\mu_L = 170$, $\mu_H = 200$, $\sigma = 120$, $\alpha = 0.8$ *and* $s = 60$. τ_i *is increased from 0.5 to 0.55. When* $\tau_i = 0.5$, *the optimal advance selling price* $x^* = 214.69$ *and* $\Pi(x^*) = 9.12 \times 10^6$. *When* $\tau_i = 0.55$, *the optimal advance selling price* $x^* = 213.06$ *and* $\Pi(x^*) = 9.05 \times 10^6$.

Example 10. *I use* $\mu_L = 140$, $\mu_H = 160$, $\sigma = 100$, $\alpha = 0.5$ *and* $s = 60$. τ_i *is increased from 0.5 to 0.55. When* $\tau_i = 0.5$, *the optimal advance selling price* $x^* = 226.12$ *and* $\Pi(x^*) = 5.75 \times 10^6$. *When* $\tau_i = 0.55$, *the optimal advance selling price* $x^* = 226.11$ *and* $\Pi(x^*) = 5.65 \times 10^6$.

4.6.3 When α decreases

When α decreases, the number of experienced consumers decreases, while the expected number of inexperienced consumers increases accordingly. With a decreased number of experienced consumers, the number of pre-orders decreases. Also, the number of experienced consumers who purchase in the regular selling season decreases. From the analysis in the second essay, if there are no experienced consumers in the second period, the stock-out probability is independent of α. With a smaller number of experienced consumers who purchase in the second period, it is straightforward to get a higher stock-out probability based on (4.1). Thus, the stock-out probability increases when α decreases.

The increased stock-out probability makes some experienced consumers switch to pre-order. It therefore softens the decreasing power of (4.5). As a result, x^* increases.

The total expected profit is composed of two parts, $\Pi(p)$ and $\Pi(\hat{x}) - \Pi(p)$. As shown in the second essay, $\Pi(p)$ decreases when α decreases. With regard to $\Pi(\hat{x}) - \Pi(p)$, it decreases because the total number pre-orders decreases. The decreasing of the total expected profit is very intuitive. With less experienced consumer, the demand uncertainty in the second period increases, which brings the retailer a higher risk of over-stock or under-stock and thus hurts him. The results of numerical simulations positively support the findings in this subsubsection. Examples are provided below.

Example 11. *I use* $\mu_L = 140$, $\mu_H = 170$, $\sigma = 110$, $\tau_i = 1$ *and* $s = 60$. α *is decreased from 0.45 to 0.40. When* $\alpha = 0.45$, *the optimal advance selling price* $x^* = 221.22$ *and* $\Pi(x^*) = 4.71 \times 10^6$. *When* $\alpha = 0.40$, *the optimal advance selling price* $x^* = 222.86$ *and* $\Pi(x^*) = 4.49 \times 10^6$.

Example 12. *I use* $\mu_L = 170$, $\mu_H = 210$, $\sigma = 120$, $\tau_i = 0.8$ *and* $s = 60$. α *is decreased from 0.75 to 0.70. When* $\alpha = 0.75$, *the optimal advance selling price* $x^* = 214.69$ *and* $\Pi(x^*) = 8.89 \times 10^6$. *When* $\alpha = 0.70$, *the optimal advance selling price* $x^* = 216.33$ *and* $\Pi(x^*) = 8.69 \times 10^6$.

4.6.4 When σ increases

In this subsection we consider how an increase in σ affects the retailer. Parameter σ captures the valuation in consumers valuations. An increase in σ also means that there are more consumers (experienced and inexperienced) with high valuations.

First, look at the optimal advance selling price premium. The number of experienced consumers does not change. With the increase of σ, there are more experienced with high valuations. Intuitively, it is more profitable for the retailer to charge a higher advance selling price. An extensive numerical simulations also indicate that x^* increases with σ.

Next, consider how $\Pi(x^*)$ is affected. As σ increases the valuation distribution functions become more dispersed in that more consumers have valuations farther away from the mean value than before. As a result, if $p > \mu$ then more consumers have valuations above p and if $p < \mu$ then less consumers have valuations above p. On one hand, if $p < \mu_L$, with σ increases, less consumers will make purchases, therefore the total expected profit may decrease. On the other hand, if $p > \mu_H$, more consumers will make purchases, therefore the total expected profit may increase. Obviously, either an increase or a decrease in the retailer's expected profit is possible if $\mu_L < p < \mu_H$. Thus, the directional change in the retailer's expected profit depends on the value of p in relation to μ_L and μ_H, all of which are exogenously given in our model. The following examples shows the two directions for $\Pi(x^*)$, while x^* always increases.

Example 13. *I use* $\mu_L = 210$*,* $\mu_H = 240$*,* $\alpha = 0.5$*,* $\tau_i = 1$ *and* $s = 80$*.* σ *is increased from 130 to 140. When* $\sigma = 130$*, the optimal advance selling price* $x^* = 211.43$ *and* $\Pi(x^*) = 9.45 \times 10^6$*. When* $\sigma = 140$*, the optimal advance selling price* $x^* = 213.06$ *and* $\Pi(x^*) = 9.39 \times 10^6$*. In this example,* x^* *increases and* $\Pi(x^*)$ *decreases.*

Example 14. *I use* $\mu_L = 140$*,* $\mu_H = 160$*,* $\alpha = 0.5$*,* $\tau_i = 1$ *and* $s = 80$*.* σ *is increased from 110 to 120. When* $\sigma = 110$*, the optimal advance selling price* $x^* = 214.69$ *and* $\Pi(x^*) = 5.38 \times 10^6$*. When* $\sigma = 120$*, the optimal advance selling price* $x^* = 216.33$ *and* $\Pi(x^*) = 5.62 \times 10^6$*. In this example, both* x^* *and* $\Pi(x^*)$ *increase.*

4.7 Conclusion

In this paper, I investigate advance selling price premium for the retailer before he releases a new product. Consumers are divided into two groups. While inexperienced consumers do not know their valuations in the advance selling season, experienced consumers know their valuations from the outset. All consumers are assumed to be strategic. When pre-orders are available, they make the decisions to pre-order or not by comparing the expected payoffs from pre-orders and waiting.

I consider there is an advance selling price premium in the case $c < x_L < x_H < p$. Since $x_H < p < x$, all inexperienced consumers decide to wait till the regular selling season and make purchases if $v > p$. Experienced consumers with $v > p$ make purchases, either in the first period or in the second period. With regard to the retailer, he is uncertain about the consumer valuation distribution at first. Since implementing advance selling strategy brings more profits to him, he will offer pre-orders to the consumers before the regular selling season. With the pre-orders he receives, he can infer the consumer valuation distribution. However, he is still uncertain about the number of inexperienced consumers in the market. In this case, this retailer faces a Newsvendor Problem. He needs to decide how much to produce before the regular selling season.

The main results of this paper are summarized below.

- The retailer learns from pre-orders which softens the Newsvendor Problem. However, the uncertainty about the number of inexperienced consumers still remains.

- When $x^* = \hat{x}^*$, experienced consumers with $v > p$ purchase the product, either in the first period or in the second period. All inexperienced consumers wait till the second period.

- Advance selling at a price premium $\hat{x}$ strictly dominates advance selling at p.

- There are three types of advance selling strategies for a retailer: advance selling at a price premium, advance selling with a deep discount and advance selling with a moderate discount.

- After considering advance selling price premium, the retailer's optimal advance selling strategy might change from a price discount to a price premium.

- The sensitivity analysis in regard to changes in some parameters of the model yields several interesting results, some intuitive and some counterintuitive. For example, as the salvage value decreases, the firm's expected profit may decrease (intuitive), but may also increase (counterintuitive).

For future research, several issues are worthy of investigation. First, it would be interesting to introduce competition into the model and study the retailers' optimal advance selling strategy. Under competition, retailers are more likely to adopt advance selling at a discount to win consumers.

Second, studying how some other factors affect retailers' optimal advance selling strategy could be another direction. For example, advertising and information noises, return policies, and brand loyalty.

Last, it would be interesting to study a dynamic model in which consumers arrive in the advance selling season at different times and they can update their valuations based on prior pre-orders.

Chapter 5

Summary and concluding remarks

As an effective selling strategy for retailers to better match the demand with supply, advance selling has been widely implemented in many product categories. This dissertation focuses on the study of advance selling and provides theoretical and numerical analysis under different business frameworks.

The first essay studies advance selling of a completely new products when assuming the retailer is also uncertain about the consumers' valuations distribution in addition to the market size. Pre-orders in the first period allows the retailer to learn the distribution of consumers' valuations, with which he is able to update the forecast of future demand. In this essay, the conditions under which the retailer should sell in advance are provided. In the second essay, advance selling is studied in the presence of experienced consumers, with applications to the products with early generations. With experienced consumers in the model, the retailer can always learn the distribution of consumers' valuations. We show that the firm will always adopt advance selling and that the optimal pre-order price may or may not be at a discount to the regular selling price. The third essay extend the analysis in the second essay to study advance selling with price premium. I show that advance selling at no discount is dominated by advance selling at a premium. Also, numerical examples are conducted to show that when a retailer is more likely to implement advance selling at a premium rather than a price discount.

In the future, there are a number of directions are worthy of further investigation. One direction is to study advance selling in a duopoly model with strategic consumers. It is expected that retailers are more likely to adopt advance selling at a discount to win consumers under competition. Another direction is to consider a dynamic model in which consumers' arrival times in the advance selling season are different. Prior pre-orders can work as a signal for consumers to update their valuations. Besides theoretical studies on advance selling, it offers a promising direction to study advance selling empirically.

Appendix A

Proofs in Chapter 2

A.1 Explicit expressions for Π^0, equation (2.4):

From equation (2.3) we know that

$$\Pi^0 = \gamma \Pi_H(Q_0) + (1-\gamma)\Pi_L(Q_0).$$

First, I will show that $\Pi_H(Q^0) = (p-c)Q^0 - (p-s)A_H$.

$$\begin{aligned}
\Pi_H(Q_0) &= E_{D_2}\left[p \min\left\{Q^0, D_2\right\} + s(Q^0 - D_2)^+ - cQ^0\right] \\
&= (p-c)E(D_2) - (c-s)E_{D_2}[(Q^0 - D_2)^+] - (p-c)E_{D_2}[(D_2 - Q^0)^+] \\
&= (p-c)\overline{F}_H(p)m_i - (c-s)E_{D_2}[(Q^0 - D_2)^+] - (p-c)E_{D_2}[(D_2 - Q^0)^+],
\end{aligned}$$

where $D_2 = M_i \overline{F}_H(p) \sim \mathrm{LN}\left(\nu_i + \ln \overline{F}_H(p), \tau_i^2\right)$. In the second term,

$$
\begin{aligned}
E_{D_2}[(Q^0 - D_2)^+] &= \int_{-\infty}^{Q^0} (Q^0 - D_2) g(D_2)\, \mathrm{d}D_2 \\
&= Q^0 \int_{-\infty}^{Q^0} g(D_2)\, \mathrm{d}D_2 - \int_{-\infty}^{Q^0} D_2 g(D_2)\, \mathrm{d}D_2 \\
&= Q^0 G(Q^0) - \int_{-\infty}^{Q^0} D_2 \frac{1}{D_2\sqrt{2\pi\tau_i^2}} \exp\left\{ -\frac{(\ln D_2 - (\nu_i + \ln \overline{F}_H(p)))^2}{2\tau^2} \right\} \mathrm{d}D_2 \\
&= Q^0 \Phi(T_H) - \int_{-\infty}^{Q^0} \frac{1}{\sqrt{2\pi\tau_i^2}} \exp\left\{ -\frac{(\ln D_2 - (\nu_i + \ln \overline{F}_H(p)))^2}{2\tau^2} \right\} \mathrm{d}D_2.
\end{aligned}
$$

Applying change of variable $u = \ln D_2$ yields

$$
\begin{aligned}
&Q^0 \Phi(T_H) - \overline{F}_H(p) m_i \int_{-\infty}^{\ln Q^0} \frac{1}{\sqrt{2\pi\tau_i^2}} \exp\left\{ -\frac{(u - (\nu_i + \ln \overline{F}_H(p) + \tau_i^2))^2}{2\tau_i^2} \right\} \mathrm{d}u \\
&= Q^0 \Phi(T_H) - \overline{F}_H(p) m_i \Phi\left(\frac{\ln Q^0 - (\nu_i + \ln \overline{F}_H(p)) + \tau_i^2)}{\tau_i} \right) \\
&= Q^0 \Phi(T_H) - \overline{F}_H(p) m_i \Phi(T_H - \tau_i).
\end{aligned}
$$

Similarly, in the third term,

$$
\begin{aligned}
E_{D_2}[(D_2 - Q^0)^+] &= \int_{Q^0}^{+\infty} (D_2 - Q^0) g(D_2)\, \mathrm{d}D_2 \\
&= \int_{Q^0}^{+\infty} D_2 g(D_2)\, \mathrm{d}D_2 - Q^0 \int_{Q^0}^{+\infty} g(D_2)\, \mathrm{d}D_2 \\
&= \overline{F}_H(p) m_i \left(1 - \Phi(T_H - \tau_i)\right) - Q^0 \left(1 - \Phi(T_H)\right).
\end{aligned}
$$

Thus, we have

$$
\begin{aligned}
\Pi_H(Q^0) &= (p - c)\overline{F}_H(p) m_i - (c - s)[Q^0 \Phi(T_H) - \overline{F}_H(p) m_i \Phi(T_H - \tau_i)] \\
&\quad - (p - c)[\overline{F}_H(p) m_i \left(1 - \Phi(T_H - \tau_i)\right) - Q^0 \left(1 - \Phi(T_H)\right)] \\
&= (p - c) Q^0 + (p - s)\overline{F}_H(p) m_i \Phi(T_H - \tau_i) - (p - s) Q^0 \Phi(T_H) \\
&= (p - c) Q^0 - (p - s) A_H.
\end{aligned}
$$

Similarly, $\Pi_L(Q^0) = (p-c)Q^0 - (p-s)A_L$. Thus, put both $\Pi_H(Q^0)$ and $\Pi_L(Q^0)$ into equation (2.3), we have

$$\begin{aligned}\Pi^0 &= \gamma\big((p-c)Q^0 - (p-s)A_H\big) + (1-\gamma)\big((p-c)Q^0 - (p-s)A_L\big) \\ &= (p-c)Q^0 - (p-s)\left(\gamma A_H + (1-\gamma)A_L\right).\end{aligned}$$

A.2 Derivations of (2.6) and (2.7):

As shown in Silver, Pyke, and Peterson (1998), the solution to the Newsvendor Problem

$$\pi(q) = p\,\mathrm{E}\left[\min\{q, D_2\}\right] + s\,\mathrm{E}\left[(q-D_2)^+\right] - cq$$

is q^* that satisfies

$$\Pr(D_2 \le q^*) = \beta. \tag{A.1}$$

Moreover,

$$\pi(q^*) = (p-s)\int_0^{q^*} D_2 g(D_2)\mathrm{d}D_2.$$

With $D_2 \sim LN\left(\nu, \tau^2\right)$, equation (A.1) becomes

$$\Phi\left(\frac{\ln q^* - \nu}{\tau}\right) = \beta,$$

or

$$q^* = \exp\{\nu + \tau z_\beta\}.$$

Next,

$$\pi(q^*) = (p-s)\int_0^{\exp\{\nu+\tau z_\beta\}} \frac{1}{\sqrt{2\pi\tau^2}}\exp\left\{-\frac{(\ln D_2 - \nu)^2}{2\tau^2}\right\}\,\mathrm{d}D_2.$$

Applying the change of variable $u = \ln D_2$ yields

$$\begin{aligned}
\pi(q^*) &= (p-s)\int_{-\infty}^{\nu+\tau z_\beta} \frac{1}{\sqrt{2\pi\tau^2}} \exp\left\{-\frac{(u-\nu)^2}{2\tau^2}+u\right\}\,\mathrm{d}u \\
&= (p-s)\exp\left\{\frac{(\nu+\tau^2)^2-\nu^2}{2\tau^2}\right\}\int_{-\infty}^{\nu+\tau z_\beta}\frac{1}{\sqrt{2\pi\tau^2}}\exp\left\{-\frac{(u-(\nu+\tau^2))^2}{2\tau^2}\right\}\,\mathrm{d}u \\
&= (p-s)\exp\left\{\nu+\frac{\tau^2}{2}\right\}\Phi(z_\beta-\tau) \\
&= (p-s)\,(1-\Phi(\tau-z_\beta))\exp\left\{\nu+\frac{\tau^2}{2}\right\}.
\end{aligned}$$

A.3 Derivation of (2.8):

$$\begin{aligned}
\eta &= \int_{q^*}^{+\infty}\frac{D_2-q^*}{D_2}g(D_2)\,\mathrm{d}D_2 = 1-G(q^*)-\int_{q^*}^{+\infty}\frac{q^*}{D_2}g(D_2)\,\mathrm{d}D_2 \\
&= 1-G(\exp\{\nu+\tau z_\beta\})-\int_{\exp\{\nu+\tau z_\beta\}}^{+\infty}\frac{\exp\{\nu+\tau z_\beta\}}{D_2}\frac{1}{D_2\sqrt{2\pi\tau^2}}\exp\left\{-\frac{(\ln D_2-\nu)^2}{2\tau^2}\right\}\,\mathrm{d}D_2.
\end{aligned}$$

Applying the change of variable $u = \ln D_2$ yields

$$\begin{aligned}
\eta &= 1-\beta-\int_{\nu+\tau z_\beta}^{+\infty}\frac{\exp\{\nu+\tau z_\beta\}}{\exp\{u\}}\frac{1}{\exp\{u\}\sqrt{2\pi\tau^2}}\exp\left\{-\frac{(u-\nu)^2}{2\tau^2}\right\}\exp\{u\}\,\mathrm{d}u \\
&= 1-\beta-\exp\{\nu+\tau z_\beta\}\int_{\nu+\tau z_\beta}^{+\infty}\frac{1}{\sqrt{2\pi\tau^2}}\exp\left\{-\frac{(u-\nu)^2}{2\tau^2}-u\right\}\,\mathrm{d}u \\
&= 1-\beta-\exp\{\nu+\tau z_\beta\}\exp\left\{-\nu+\frac{\tau^2}{2}\right\}\int_{\nu+\tau z_\beta}^{+\infty}\frac{1}{\sqrt{2\pi\tau^2}}\exp\left\{-\frac{(u-(\nu-\tau^2))^2}{2\tau^2}\right\}\,\mathrm{d}u \\
&= 1-\beta-\exp\left\{\tau z_\beta+\frac{\tau^2}{2}\right\}\left(1-\Phi\left(\frac{\nu+\tau z_\beta-(\nu-\tau^2)}{\tau}\right)\right) \\
&= 1-\beta-\exp\left\{\tau z_\beta+\frac{\tau^2}{2}\right\}(1-\Phi(z_\beta+\tau)).
\end{aligned}$$

A.4 Explicit expressions for η_L and η_H:

Denote $\nu_L = \nu_i + \ln \overline{F}_L(p)$, under $D_2 = M_i \overline{F}_L(p) \sim \mathrm{LN}\,(\nu_L, \tau_i^2)\,,$

$$\begin{aligned}
\eta_L &= E\left[\left(\frac{D_2 - Q^0}{D_2}\right)^+\right] \\
&= \int_{Q^0}^{+\infty} \frac{D_2 - Q^0}{D_2} g(D_2)\,\mathrm{d}D_2 \\
&= \int_{Q^0}^{+\infty} g(D_2)\,\mathrm{d}D_2 - \int_{Q^0}^{+\infty} \frac{Q^0}{D_2} g(D_2)\,\mathrm{d}D_2.
\end{aligned}$$

The first term can be simplified to

$$1 - G(Q^0) = 1 - \Phi\left(\frac{\ln Q^0 - \nu_L}{\tau_i}\right).$$

Applying a variable change $u = \ln D_2$, the second term can be simplified to

$$\begin{aligned}
&\int_{Q^0}^{+\infty} \frac{Q^0}{D_2} \frac{1}{D_2\sqrt{2\pi\tau_i^2}} \exp\left\{-\frac{(\ln D_2 - \nu_L)^2}{2\tau_i^2}\right\}\,\mathrm{d}D_2 \\
&\quad = Q^0 \exp\left\{\frac{\tau_i^2}{2} - \nu_L\right\} \int_{\ln Q^0}^{+\infty} \frac{1}{\sqrt{2\pi\tau_i^2}} \exp\left\{-\frac{(u - \nu_L + \tau_i^2)^2}{2\tau_i^2}\right\}\,\mathrm{d}u \\
&\quad = Q^0 \exp\left\{\frac{\tau_i^2}{2} - \nu_L\right\} \left(1 - \Phi\left(\frac{\ln Q^0 - \nu_L + \tau_i^2}{\tau_i}\right)\right).
\end{aligned}$$

Thus,

$$\eta_L = 1 - \Phi\left(\frac{\ln Q^0 - \nu_L}{\tau_i}\right) + Q^0 \exp\left\{\frac{\tau_i^2}{2} - \nu_L\right\} \left(1 - \Phi\left(\frac{\ln Q^0 - \nu_L + \tau_i^2}{\tau_i}\right)\right).$$

Similarly,

$$\eta_H = 1 - \Phi\left(\frac{\ln Q^0 - \nu_H}{\tau_i}\right) + Q^0 \exp\left\{\frac{\tau_i^2}{2} - \nu_H\right\}\left(1 - \Phi\left(\frac{\ln Q^0 - \nu_H + \tau_i^2}{\tau_i}\right)\right),$$

where $\nu_H = \nu_i + \ln \overline{F}_H(p)$.

A.5 Proof of Lemma 1:

As shown in Silver, Pyke, and Peterson (1998), the solution to the Newsvendor Problem

$$\pi(q) = p\,\mathrm{E}\left[\min\{q, D_2\}\right] + s\,\mathrm{E}\left[(q - D_2)^+\right] - cq$$

is q^* that satisfies $\Pr(D_2 \le q^*) = \beta$.

With advance selling, the following results hold:

$$\begin{cases} \Pr(D_2 \le q_H^*) = \beta & \text{if } D_2 = M_i\overline{F}_H(p), \\ \Pr(D_2 \le q_L^*) = \beta & \text{if } D_2 = M_i\overline{F}_L(p), \end{cases}$$

and $q_L^* < q_H^*$.

Following the same logic, in the case of no advance selling, the optimal quantity Q^0 should satisfy the condition that

$$\Pr(D_2 \le Q^0) = \beta.$$

Since the random regular selling season demand D_2 will be $M_i\overline{F}_H(p)$ with probability

γ, and $M_i\overline{F}_L(p)$ with probability $1-\gamma$,

$$\begin{cases} \Pr(D_2 \leq q_H^*) > \beta, \\ \Pr(D_2 \leq q_L^*) < \beta. \end{cases}$$

As a result, $q_L^* < Q^0 < q_H^*$.

A.6 Proof of Lemma 2:

First, I prove that $\eta_L < \eta^*$ for $D_2 = M_i\overline{F}_L(p) \sim \mathrm{LN}\left(\nu_i + \ln \overline{F}_L(p), \tau_i^2\right)$:

$$\begin{aligned}
\eta^* - \eta_L &= E\left[\left(\frac{D_2 - q_L^*}{D_2}\right)^+\right] - E\left[\left(\frac{D_2 - Q^0}{D_2}\right)^+\right] \\
&= \int_{q_L^*}^{+\infty} \frac{D_2 - q_L^*}{D_2} g(D_2)\,\mathrm{d}D_2 - \int_{Q^0}^{+\infty} \frac{D_2 - Q^0}{D_2} g(D_2)\,\mathrm{d}D_2 \\
&= \int_{q_L^*}^{Q^0} \frac{D_2 - q_L^*}{D_2} g(D_2)\,\mathrm{d}D_2 + \int_{Q^0}^{+\infty} \left(\frac{D_2 - q_L^*}{D_2} - \frac{D_2 - Q^0}{D_2}\right) g(D_2)\,\mathrm{d}D_2 \\
&= \int_{q_L^*}^{Q^0} \frac{D_2 - q_L^*}{D_2} g(D_2)\,\mathrm{d}D_2 + \int_{Q^0}^{+\infty} \frac{Q^0 - q_L^*}{D_2} g(D_2)\,\mathrm{d}D_2 > 0
\end{aligned}$$

Next, I prove that $\eta^* < \eta_H$ for $D_2 = M_i\overline{F}_H(p) \sim \mathrm{LN}\left(\nu_i + \ln \overline{F}_H(p), \tau_i^2\right)$:

$$\begin{aligned}
\eta_H - \eta^* &= E\left[\left(\frac{D_2 - Q^0}{D_2}\right)^+\right] - E\left[\left(\frac{D_2 - q_H^*}{D_2}\right)^+\right] \\
&= \int_{Q^0}^{+\infty} \frac{D_2 - Q^0}{D_2} g(D_2)\,\mathrm{d}D_2 - \int_{q_H^*}^{+\infty} \frac{D_2 - q_H^*}{D_2} g(D_2)\,\mathrm{d}D_2 \\
&= \int_{Q^0}^{q_H^*} \frac{D_2 - Q^0}{D_2} g(D_2)\,\mathrm{d}D_2 + \int_{q_H^*}^{+\infty} \left(\frac{D_2 - Q^0}{D_2} - \frac{D_2 - q_H^*}{D_2}\right) g(D_2)\,\mathrm{d}D_2 \\
&= \int_{Q^0}^{q_H^*} \frac{D_2 - Q^0}{D_2} g(D_2)\,\mathrm{d}D_2 + \int_{q_H^*}^{+\infty} \frac{q_H^* - Q^0}{D_2} g(D_2)\,\mathrm{d}D_2 > 0.
\end{aligned}$$

A.7 Proof of Lemma 3:

I need to prove $x_L < x_H$ for all possible values of η in these two equations.

Let $x_L|_{\eta=\eta^*}$ and $x_L|_{\eta=\eta_L}$ denote x_L when $\eta = \eta^*$ and $\eta = \eta_L$, respectively. Similarly, let $x_H|_{\eta=\eta^*}$ and $x_H|_{\eta=\eta_H}$ denote x_H when $\eta = \eta^*$ and $\eta = \eta_H$.

I need to show that

$$\max\{x_L|_{\eta=\eta^*}, x_L|_{\eta=\eta_L}\} < \min\{x_H|_{\eta=\eta^*}, x_H|_{\eta=\eta_H}\}.$$

From equations (2.11) and (2.12), it is easy to see that both x_L and x_H increase with η. Since $\eta_L < \eta^* < \eta_H$ (Lemma 2), it follows

$$x_L|_{\eta=\eta^*} = \max\{x_L|_{\eta=\eta^*}, x_L|_{\eta=\eta_L}\}$$

and

$$x_H|_{\eta=\eta^*} = \min\{x_H|_{\eta=\eta^*}, x_H|_{\eta=\eta_H}\}.$$

Below I show that $x_L|_{\eta=\eta^*} < x_H|_{\eta=\eta^*}$. We have that

$$x_H|_{\eta=\eta^*} - x_L|_{\eta=\eta^*} = \mu_H - \mu_L - (1-\eta^*)\left[\int_p^{+\infty}(v-p)f_H(v)\,\mathrm{d}v - \int_p^{+\infty}(v-p)f_L(v)\,\mathrm{d}v\right]$$

Since $f_L(v)$ is a parallel shift of $f_H(v)$, the term in the square brackets can be rewritten as

$$\int_p^{+\infty}(v-p)f_H(v)\,\mathrm{d}v - \int_{p+\mu_H-\mu_L}^{+\infty}(v-p-\mu_H+\mu_L)f_H(v)\,\mathrm{d}v.$$

And it follows that

$$
\begin{aligned}
\int_p^{+\infty} (v-p) f_H(v)\,\mathrm{d}v - \int_{p+\mu_H-\mu_L}^{+\infty} (v-p-\mu_H+\mu_L) f_H(v)\,\mathrm{d}v \\
&= \int_p^{p+\mu_H-\mu_L} (v-p) f_H(v)\,\mathrm{d}v + \int_{p+\mu_H-\mu_L}^{+\infty} (\mu_H-\mu_L) f_H(v)\,\mathrm{d}v \\
&< \int_p^{p+\mu_H-\mu_L} (\mu_H-\mu_L) f_H(v)\,\mathrm{d}v + \int_{p+\mu_H-\mu_L}^{+\infty} (\mu_H-\mu_L) f_H(v)\,\mathrm{d}v \\
&= (\mu_H-\mu_L) \int_p^{+\infty} f_H(v)\,\mathrm{d}v \\
&< \mu_H-\mu_L.
\end{aligned}
$$

Therefore,

$$
x_H|_{\eta=\eta^*} - x_L|_{\eta=\eta^*} > \mu_H-\mu_L-(1-\eta^*)(\mu_H-\mu_L) = \eta^*(\mu_H-\mu_L) \geq 0.
$$

so

$$
\max\{x_L|_{\eta=\eta^*}, x_L|_{\eta=\eta_L}\} < \min\{x_H|_{\eta=\eta^*}, x_H|_{\eta=\eta_H}\}.
$$

Therefore, x_L is always less than x_H.

A.8 Proof of Proposition 2:

The retailer should implement advance selling if and only if

$$
\Pi^* = \max\{m_i(x_L-c)-k; \gamma m_i(x_H-c)+(1-\gamma)\pi_L-k; \Pi^0-k\} \geq \Pi^0.
$$

That is, the retailer will implement advance selling if and only if he gets more profits through advance selling.

$$\begin{aligned}
&\Pi^* = \max\{m_i(x_L - c) - k; \gamma m_i(x_H - c) + (1-\gamma)\pi_L - k; \Pi^0 - k\} \geq \Pi^0 \\
&\Longleftrightarrow \max\{m_i(x_L - c) - k; \gamma m_i(x_H - c) + (1-\gamma)\pi_L - k\} \geq \Pi^0 \\
&\Longleftrightarrow m_i(x_L - c) - k \geq \Pi^0, \text{ or } \gamma m_i(x_H - c) + (1-\gamma)\pi_L - k \geq \Pi^0 \\
&\Longleftrightarrow c \leq x_L - \frac{\Pi^0 + k}{m_i}, \text{ or } c \leq x_H - \frac{\Pi^0 + k - (1-\gamma)\pi_L}{\gamma m_i},
\end{aligned}$$

i.e.,

$$c \leq \mu_L - (1-\eta^*) \int_p^{+\infty} (v-p) f_L(v)\, \mathrm{d}v - \frac{\Pi^0 + k}{m_i} = c_1,$$

or

$$c \leq \mu_H - (1-\eta_H) \int_p^{+\infty} (v-p) f_H(v)\, \mathrm{d}v - \frac{\Pi^0 + k - (1-\gamma)\pi_L}{\gamma m_i} = c_2.$$

That is, $c \leq \max\{c_1, c_2\}$.

Appendix B

Proofs in Chapter 3

B.1 Explicit expressions for x_L and x_H:

Below we show that

$$\int_p^{+\infty} (v-p)f(v)\,\mathrm{d}v = (\mu - p)\overline{F}(p) + \sigma^2 f(p).$$

The explicit expressions for x_L and x_H will follow immediately from this equality. Applying the change of variable $z = (v-\mu)/\sigma$ to the left-hand side yields

$$\begin{aligned}
\int_p^{+\infty} (v-p)f(v)\,\mathrm{d}v &= \int_{\frac{p-\mu}{\sigma}}^{+\infty} (\mu + \sigma z - p)\phi(z)\,\mathrm{d}z \\
&= (\mu - p)\left(1 - \Phi\left(\frac{p-\mu}{\sigma}\right)\right) + \int_{\frac{p-\mu}{\sigma}}^{+\infty} \frac{\sigma z}{\sqrt{2\pi}} \exp\left\{-\frac{z^2}{2}\right\}\,\mathrm{d}z \\
&= (\mu - p)\overline{F}(p) + \frac{\sigma}{\sqrt{2\pi}} \int_{\frac{p-\mu}{\sigma}}^{+\infty} \exp\left\{-\frac{z^2}{2}\right\}\,\mathrm{d}\frac{z^2}{2}.
\end{aligned}$$

Applying the change of variable $u = z^2/2$ yields

$$\begin{aligned}(\mu - p)\overline{F}(p) &+ \frac{\sigma}{\sqrt{2\pi}} \int_{\frac{(p-\mu)^2}{2\sigma^2}}^{+\infty} \exp\{-u\}\ \mathrm{d}u \\ &= (\mu - p)\overline{F}(p) + \frac{\sigma}{\sqrt{2\pi}} \exp\left\{-\frac{(p-\mu)^2}{2\sigma^2}\right\} \\ &= (\mu - p)\overline{F}(p) + \sigma^2 f(p).\end{aligned}$$

B.2 Proof of Lemma 4:

It follows immediately from (3.1) and (3.2) that x_L and x_H increase in η, so we have part (iii) of Lemma 4.

(i) Below we show that

$$x_H - x_L = \mu_H - \mu_L - (1-\eta)\left[\int_p^{+\infty} (v-p) f_H(v)\ \mathrm{d}v - \int_p^{+\infty} (v-p) f_L(v)\ \mathrm{d}v\right] > 0$$

for all η and σ. Since $f_L(v)$ is a parallel shift of $f_H(v)$, the term in the square brackets can be rewritten as

$$\begin{aligned}\int_p^{+\infty} (v-p) f_H(v)\ \mathrm{d}v &- \int_{p+\mu_H-\mu_L}^{+\infty} (v - p - \mu_H + \mu_L) f_H(v)\ \mathrm{d}v \\ &= \int_p^{p+\mu_H-\mu_L} (v-p) f_H(v)\ \mathrm{d}v + \int_{p+\mu_H-\mu_L}^{+\infty} (\mu_H - \mu_L) f_H(v)\ \mathrm{d}v \\ &< \int_p^{p+\mu_H-\mu_L} (\mu_H - \mu_L) f_H(v)\ \mathrm{d}v + \int_{p+\mu_H-\mu_L}^{+\infty} (\mu_H - \mu_L) f_H(v)\ \mathrm{d}v \\ &= (\mu_H - \mu_L) \int_p^{+\infty} f_H(v)\ \mathrm{d}v < \mu_H - \mu_L.\end{aligned}$$

Therefore,

$$x_H - x_L > \mu_H - \mu_L - (1-\eta)(\mu_H - \mu_L) = \eta(\mu_H - \mu_L) \geq 0.$$

(ii) In order to prove that x_L decreases in σ, we need to show that $\int_p^{+\infty}(v-p)f_L(v)\ \mathrm{d}v$ increases in σ.

$$\begin{aligned}
\int_p^{+\infty}(v-p)f_L(v)\ \mathrm{d}v &= \int_p^{+\infty} vf_L(v)\ \mathrm{d}v - p\int_p^{+\infty} f_L(v)\ \mathrm{d}v \\
&= \int_{-\infty}^{+\infty} vf_L(v)\ \mathrm{d}v - \int_{-\infty}^{p} v\ \mathrm{d}F_L(v) - p(1-F_L(p)) \\
&= \mu_L - \left(vF_L(v)\ |_{-\infty}^{p} - \int_{-\infty}^{p} F_L(v)\ \mathrm{d}v\right) - p + pF_L(p) \\
&= \mu_L + \int_{-\infty}^{p} F_L(v)\ \mathrm{d}v - p.
\end{aligned}$$

Hence,

$$\frac{\partial}{\partial\sigma}\left(\int_p^{+\infty}(v-p)f_L(v)\ \mathrm{d}v\right) = \frac{\partial}{\partial\sigma}\left(\int_{-\infty}^{p} F_L(v)\ \mathrm{d}v\right).$$

Suppose $p < \mu_L$, then

$$\begin{aligned}
\frac{\partial}{\partial\sigma}\left(\int_{-\infty}^{p} F_L(v)\ \mathrm{d}v\right) &= \frac{\partial}{\partial\sigma}\left(\sigma\int_{-\infty}^{\frac{p-\mu_L}{\sigma}} \Phi(z)\ \mathrm{d}z\right) \\
&= \int_{-\infty}^{\frac{p-\mu_L}{\sigma}} \Phi(z)\ \mathrm{d}z - \frac{\sigma(p-\mu_L)}{\sigma^2}\Phi\left(\frac{p-\mu_L}{\sigma}\right) > 0. \quad \text{(B.1)}
\end{aligned}$$

Next, suppose $p > \mu_L$.

$$\int_{-\infty}^{p} F_L(v)\ \mathrm{d}v = \int_{-\infty}^{2\mu_L - p} F_L(v)\ \mathrm{d}v + \int_{2\mu_L-p}^{\mu_L} F_L(v)\ \mathrm{d}v + \int_{\mu_L}^{p} F_L(v)\ \mathrm{d}v. \quad \text{(B.2)}$$

Applying the change of variable $u = 2\mu_L - v$ to the second integral yields

$$\int_{\mu_L}^{p} F_L(2\mu_L - u)\,\mathrm{d}u.$$

By symmetry of the normal distribution $F_L(v) + F_L(2\mu_L - v) = 1$, thus the sum of the last two integrals in (B.2) equals $p - \mu_L$. Hence,

$$\frac{\partial}{\partial\sigma}\left(\int_{-\infty}^{p} F_L(v)\,\mathrm{d}v\right) = \frac{\partial}{\partial\sigma}\left(\int_{-\infty}^{2\mu_L - p} F_L(v)\,\mathrm{d}v\right).$$

The upper limit of the integration on the right-hand side is less than μ_L, hence, by (B.1) the above derivative is positive.

Thus, we have showed that x_L decreases in σ. The proof that x_H decreases in σ is similar.

B.3 Proof of Lemma 6:

The following properties of the standard normal distribution will be used in the proof:

$$\phi(u)\left(\frac{1}{u} - \frac{1}{u^3}\right) < 1 - \Phi(u) < \frac{\phi(u)}{u}, \quad u > 0 \tag{B.3}$$

and

$$\phi(u)\left(\frac{1}{|u|} - \frac{1}{|u|^3}\right) < \Phi(u) < \frac{\phi(u)}{|u|}, \quad u < 0. \tag{B.4}$$

(i) First,

$$\eta(\beta, 0) = 1 - \beta - (1 - \Phi(z_\beta)) = 1 - \beta - (1 - \beta) = 0.$$

Next,

$$\lim_{\tau_i \to +\infty} \eta(\beta, \tau_i) = 1 - \beta - \lim_{\tau_i \to +\infty} \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}(1 - \Phi(z_\beta + \tau_i)).$$

It follows from (B.3) that $1 - \Phi(u) = \frac{\phi(u)}{u}(1 + o(u^{-1}))$. Hence, the above expression can be rewritten as

$$\begin{aligned}
\lim_{\tau_i \to +\infty} \eta(\beta, \tau_i) &= 1 - \beta - \lim_{\tau_i \to +\infty} \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\} \frac{\phi(z_\beta + \tau_i)}{z_\beta + \tau_i} \\
&= 1 - \beta - \lim_{\tau_i \to +\infty} \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\} \frac{\exp\left\{-\frac{(z_\beta + \tau_i)^2}{2}\right\}}{\sqrt{2\pi}(z_\beta + \tau_i)} \\
&= 1 - \beta - \lim_{\tau_i \to +\infty} \frac{\exp\left\{-\frac{z_\beta^2}{2}\right\}}{\sqrt{2\pi}(z_\beta + \tau_i)} = 1 - \beta.
\end{aligned}$$

Finally,

$$\begin{aligned}
\frac{\partial \eta}{\partial \tau_i} &= \frac{\partial}{\partial \tau_i}\left(-\exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}(1 - \Phi(z_\beta + \tau_i))\right) \\
&= -(z_\beta + \tau_i)\exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}(1 - \Phi(z_\beta + \tau_i)) + \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}\phi(z_\beta + \tau_i) \\
&= (z_\beta + \tau_i)\exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}\left[\frac{\phi(z_\beta + \tau_i)}{z_\beta + \tau_i} - (1 - \Phi(z_\beta + \tau_i))\right].
\end{aligned}$$

By (B.3) the term in the square brackets is positive, so $\partial\eta/\partial\tau_i > 0$.

(ii) First,

$$\eta(0, \tau_i) = 1 - \lim_{z_\beta \to -\infty} \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}(1 - \Phi(z_\beta + \tau_i)).$$

It follows from (B.4) that $\Phi(u) = \frac{\phi(u)}{|u|}(1 + o(u^{-1}))$. Hence, the above expression can be rewritten as

$$\begin{aligned}
\eta(0,\tau_i) &= 1 - \lim_{z_\beta \to -\infty} \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}\left(1 - \frac{\phi(z_\beta+\tau_i)}{|z_\beta+\tau_i|}\right) \\
&= 1 - \lim_{z_\beta \to -\infty} \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}\left(1 - \frac{\exp\left\{-\frac{(z_\beta+\tau_i)^2}{2}\right\}}{\sqrt{2\pi}|z_\beta+\tau_i|}\right) \\
&= 1 - \lim_{z_\beta \to -\infty} \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\} + \lim_{z_\beta \to -\infty} \frac{\exp\{-\frac{z_\beta^2}{2}\}}{\sqrt{2\pi}|z_\beta+\tau_i|} = 1.
\end{aligned}$$

Next,

$$\eta(1,\tau_i) = -\lim_{z_\beta \to +\infty} \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}(1 - \Phi(z_\beta+\tau_i)).$$

Since $1 - \Phi(u) = \frac{\phi(u)}{u}(1 + o(u^{-1}))$,

$$\begin{aligned}
\eta(1,\tau_i) &= -\lim_{z_\beta \to +\infty} \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}\frac{\phi(z_\beta+\tau_i)}{z_\beta+\tau_i} \\
&= -\lim_{z_\beta \to +\infty} \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}\frac{\exp\left\{-\frac{(z_\beta+\tau_i)^2}{2}\right\}}{\sqrt{2\pi}(z_\beta+\tau_i)} \\
&= -\lim_{z_\beta \to +\infty} \frac{\exp\left\{-\frac{z_\beta^2}{2}\right\}}{\sqrt{2\pi}(z_\beta+\tau_i)} = 0.
\end{aligned}$$

Finally, we can write η as a function of z_β and τ_i,

$$\eta(z_\beta,\tau_i) = 1 - \Phi(z_\beta) - \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}(1 - \Phi(z_\beta+\tau_i)).$$

Differentiating the above expression with respect to z_β yields

$$\begin{aligned}\frac{\partial \eta}{\partial z_\beta} &= -\phi(z_\beta) - \tau_i \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}(1-\Phi(z_\beta+\tau_i)) + \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}\phi(z_\beta+\tau_i)\\ &= -\tau_i \exp\left\{\tau_i z_\beta + \frac{\tau_i^2}{2}\right\}(1-\Phi(z_\beta+\tau_i)) < 0.\end{aligned}$$

Since z_β is increasing in β, it follows immediately that $\partial\eta/\partial\beta$ is also negative.

B.4 Derivatives of $\Pi^A(x_L)$, $\Pi^B(x_H)$, and $\Pi^C(p)$ with respect to α:

Rewriting the expressions (3.8) through (3.10) as functions of α yields

$$\Pi^A(x;\alpha) = \alpha m\left(\gamma\overline{F}_H(x) + (1-\gamma)\overline{F}_L(x)\right)(x-c) + (1-\alpha)m(x-c),$$

$$\Pi^B(x;\alpha) = \alpha m\left(\gamma\overline{F}_H(x) + (1-\gamma)\overline{F}_L(x)\right)(x-c) + \gamma(1-\alpha)m(x-c) + (1-\gamma)\pi_L,$$

and

$$\Pi^C(x;\alpha) = \alpha m\left(\gamma\overline{F}_H(x) + (1-\gamma)\overline{F}_L(x)\right)(x-c) + \gamma\pi_H + (1-\gamma)\pi_L,$$

where

$$\pi_L = (p-s)\left(1-\Phi(\tau_i - z_\beta)\right)(1-\alpha)m\overline{F}_L(p)$$

and

$$\pi_H = (p-s)\left(1-\Phi(\tau_i - z_\beta)\right)(1-\alpha)m\overline{F}_H(p).$$

Next, we differentiate $\Pi^A(x_L;\alpha)$ with respect to α,

$$\frac{\partial \Pi^A(x_L;\alpha)}{\partial \alpha} = m\left(\gamma \overline{F}_H(x_L) + (1-\gamma)\overline{F}_L(x_L)\right)(x_L - c) - m(x_L - c) < 0,$$

as

$$\gamma \overline{F}_H(x_L) + (1-\gamma)\overline{F}_L(x_L) < 1.$$

Differentiating $\Pi^B(x_H;\alpha)$ with respect to α yields

$$\frac{\partial \Pi^B(x_H;\alpha)}{\partial \alpha} = m\left(\gamma \overline{F}_H(x_H) + (1-\gamma)\overline{F}_L(x_H)\right)(x_H - c) - \gamma m(x_H - c).$$

The derivative can be positive or negative (we constructed examples). Finally,

$$\begin{aligned}
\frac{\partial \Pi^C(p;\alpha)}{\partial \alpha} &= m\left(\gamma \overline{F}_H(p) + (1-\gamma)\overline{F}_L(p)\right)\left((p-c) - (p-s)\left(1 - \Phi(\tau_i - z_\beta)\right)\right) \\
&= m\left(\gamma \overline{F}_H(p) + (1-\gamma)\overline{F}_L(p)\right)\left((p-s)\Phi(\tau_i - z_\beta) - (c-s)\right) \\
&> m\left(\gamma \overline{F}_H(p) + (1-\gamma)\overline{F}_L(p)\right)\left((p-s)\Phi(-z_\beta) - (c-s)\right)
\end{aligned}$$

Substituting $\Phi(-z_\beta) = 1 - \Phi(z_\beta) = 1 - \beta = (c-s)/(p-s)$ into the above inequality yields

$$\partial \Pi^C(p)/\partial \alpha > m\left(\gamma \overline{F}_H(p) + (1-\gamma)\overline{F}_L(p)\right)\left((c-s) - (c-s)\right) = 0.$$

B.5 Derivative of $\Pi^A(x_L)$ with respect to σ:

The expected profit function in region A, given by (3.8), depends on σ:

$$\begin{aligned}\Pi^A(x;\sigma) &= m_e\left(\gamma\overline{F}_H(x) + (1-\gamma)\overline{F}_L(x)\right)(x-c) + m_i(x-c)\\ &= m_e\left(\gamma\left(1-\Phi\left(\frac{x-\mu_H}{\sigma}\right)\right) + (1-\gamma)\left(1-\Phi\left(\frac{x-\mu_L}{\sigma}\right)\right)\right)(x-c) + m_i(x-c).\end{aligned}$$

For any x in region A, $x < x_L < \mu_L$,

$$\frac{\partial}{\partial\sigma}\left(1-\Phi\left(\frac{x-\mu_H}{\sigma}\right)\right) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left\{-\frac{(x-\mu_H)^2}{2\sigma^2}\right\}\frac{x-\mu_H}{\sigma^2} < 0$$

and

$$\frac{\partial}{\partial\sigma}\left(1-\Phi\left(\frac{x-\mu_L}{\sigma}\right)\right) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left\{-\frac{(x-\mu_L)^2}{2\sigma^2}\right\}\frac{x-\mu_L}{\sigma^2} < 0.$$

It follows that

$$\frac{\mathrm{d}\Pi^A(x_L;\sigma)}{\mathrm{d}\sigma} = \frac{\partial\Pi^A(x_L;\sigma)}{\partial\sigma} + \frac{\partial\Pi^A(x_L;\sigma)}{\partial x_L}\frac{\partial x_L}{\partial\sigma} < 0,$$

because the first term is negative as was shown above, and the second term is negative because $\partial\Pi^A(x_L;\sigma)/\partial x > 0$ by Assumption 1 and $\partial x_L/\partial\sigma < 0$ by Lemma 4(ii).

Appendix C

Proofs in Chapter 4

C.1 Proof of Lemma 7:

If $\mu = \mu_H$, advance selling at price $\hat{x}$ yields

$$m_e \left(\overline{F}_H(p + \frac{\hat{x} - p}{\eta_H(\hat{x})}) \right) (\hat{x} - c) + m_e \left(\overline{F}_H(p) - \overline{F}_H(p + \frac{\hat{x} - p}{\eta_H(\hat{x})}) \right) (p - c) + \pi_H^* \quad \text{(C.1)}$$

to the retailer; If $\mu = \mu_L$, advance selling at price $\hat{x}$ yields

$$m_e \left(\overline{F}_L(p + \frac{\hat{x} - p}{\eta_L(\hat{x})}) \right) (\hat{x} - c) + m_e \left(\overline{F}_L(p) - \overline{F}_L(p + \frac{\hat{x} - p}{\eta_L(\hat{x})}) \right) (p - c) + \pi_L^* \quad \text{(C.2)}$$

to the retailer. We know that $\mu = \mu_H$ with probability γ and $\mu = \mu_L$ with probability $1 - \gamma$. We can first multiply (C.1) by γ and (C.2) by $1 - \gamma$, then add them together, which yields

$$\begin{aligned}\Pi(\hat{x}) &= m_e\left(\gamma\overline{F}_H(p+\frac{\hat{x}-p}{\eta_H(\hat{x})})+(1-\gamma)\overline{F}_L(p+\frac{\hat{x}-p}{\eta_L(\hat{x})})\right)(\hat{x}-c)\\ &+ m_e\left(\gamma\overline{F}_H(p)+(1-\gamma)\overline{F}_L(p)\right)(p-c)+\gamma\pi_H^*+(1-\gamma)\pi_L^*\\ &= m_e\left(\gamma\overline{F}_H(p+\frac{\hat{x}-p}{\eta_H(\hat{x})})+(1-\gamma)\overline{F}_L(p+\frac{\hat{x}-p}{\eta_L(\hat{x})})\right)(\hat{x}-p)+\Pi(p).\end{aligned}$$

C.2 Proof of Lemma 8:

(i) From (4.3), it is obvious that $\Pi(\hat{x}) > \Pi(p)$ for any given $\hat{x} > p$.

(ii) $\lim_{\hat{x}\to p} m_e\left(\gamma\overline{F}_H(p+\frac{\hat{x}-p}{\eta_H(\hat{x})})+(1-\gamma)\overline{F}_L(p+\frac{\hat{x}-p}{\eta_L(\hat{x})})\right)(\hat{x}-p)=0.$

(iii) $\lim_{\hat{x}\to+\infty} m_e\left(\gamma\overline{F}_H(p+\frac{\hat{x}-p}{\eta_H(\hat{x})})+(1-\gamma)\overline{F}_L(p+\frac{\hat{x}-p}{\eta_L(\hat{x})})\right)(\hat{x}-p)=0.$

C.3 Proof of Lemma 9:

The retailer maximizes his total expected profit from a price premium, that is, he maximizes (4.3). Lemma 9 follows directly from the maximization of (4.3).

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