Observable frequency shifts via spin-rotation coupling

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Abstract

The phase perturbation arising from spin-rotation coupling is developed as a natural extension of the celebrated Sagnac effect. Experimental evidence in support of this phase shift, however, has yet to be realized due to the exceptional sensitivity required. We draw attention to the relevance of a series of experiments establishing that circularly polarized light, upon passing through a rotating half-wave plate, is changed in frequency by twice the rotation rate. These experiments may be interpreted as demonstrating the role of spin-rotation coupling in inducing this frequency shift, thus providing direct empirical verification of the coupling of the photon helicity to rotation. A neutron interferometry experiment is proposed which would be sensitive to an analogous frequency shift for fermions. In this arrangement, polarized neutrons enter an interferometer containing two spin flippers, one of which is rotating while the other is held stationary. An observable beating in the transmitted neutron beam intensity is predicted.

Theoretical interest in the influence of rotation on the phase of light passing through an optical interferometer already dates over a century [1]. Sagnac's observation of a phase shift proportional to the scalar product of the rotation frequency and the area of his interferometer [2] provided an empirical basis for a rich field of both fundamental and applied research into the influence of rotation on the phase of a quantum mechanical wave function [3].

The Sagnac effect may be regarded as a manifestation of the coupling of orbital angular momentum of a particle, $\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p}$, to rotation. Suppose any radiation propagates in vacuum around a rotating interferometer and has frequency ω_0 and wave vector $\boldsymbol{k}_0$ when measured in the corotating frame. An inertial observer O will observe that the wave vector of the radiation along the ith arm of the interferometer is (at first order in Ω) $\boldsymbol{k}_i = \boldsymbol{k}_0 + \omega_0\, \boldsymbol{\Omega} \times \boldsymbol{r}_i / c^2$ such that a phase shift arises:

$$\begin{aligned} \Delta\Phi &= \sum_i \Delta\boldsymbol{k}_i \cdot \Delta\boldsymbol{r}_i = \frac{\omega_0}{c^2} \sum_i \boldsymbol{\Omega} \cdot \boldsymbol{r}_i \times \Delta\boldsymbol{r}_i \\ &= \frac{2\omega_0}{c^2}\, \boldsymbol{\Omega} \cdot \boldsymbol{A} = \frac{1}{\hbar} \sum_i \boldsymbol{\Omega} \cdot \boldsymbol{L}_i\, \Delta t_i \quad , \end{aligned} \tag{1}$$

where we have used $\Delta\boldsymbol{r}_i = \Delta t_i\, \boldsymbol{v}_i$, for any particle in vacuum $\boldsymbol{v}_i = c^2 \boldsymbol{k}_i / \omega_i = c^2 \boldsymbol{p}_i / \omega_i \hbar$, and $\boldsymbol{A} \equiv 1/2 \sum_i \boldsymbol{r}_i \times \Delta\boldsymbol{r}_i$ is the area of the interferometer [4].

The Sagnac phase shift (1) is a scalar quantity that is independent of the motion of the observer. The same result therefore applies for an observer O' at rest in the corotating frame. An interpretation of this expression for O' is that the coupling of orbital angular momentum to rotation induces a frequency perturbation (relative to that measured by O) proportional to $\boldsymbol{\Omega} \cdot \boldsymbol{L}$. Summing this frequency perturbation over the time of flight of a particle around the interferometer in effect recovers the Sagnac phase shift. From the standpoint of our rotating observer, Eq. (1) may naturally be extended to include the intrinsic spin of a quantum mechanical particle through replacing the orbital angular momentum $\boldsymbol{L}$ with the total angular momentum $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$. This formalism consequently predicts that in the rotating frame, in addition to the Sagnac phase shift, a displacement of the interference fringes due to spin-rotation coupling will arise proportional to

$$\Delta\Phi_{\mathrm{SR}} = \frac{1}{\hbar} \sum_i \boldsymbol{\Omega} \cdot \boldsymbol{S}_i\, \Delta t_i \quad . \tag{2}$$

It has been shown how, with the addition of elements which reverse the spin of a neutron [5], or a photon [6], along specific sections of an interferometer, a phase shift arises due to the coupling of spin to rotation which agrees with Eq. (2). For a realistic experimental apparatus, however, such phase shifts are extremely small and this has precluded their direct observation to date. In this letter, we draw attention to a series of closely related experiments which have provided empirical confirmation of helicity-rotation coupling for photons. Their experimental design allows a natural extension to neutron interferometry which we describe, enabling a direct interferometric test of spin-rotation coupling for fermions.

The phenomenon of spin-rotation coupling is of basic interest since it reveals the inertial properties of intrinsic spin. In the formal realization of the invariance of quantum systems under inhomogeneous Lorentz transformations, the irreducible unitary representations of the inhomogeneous Lorentz group are indispensable for the description of physical states. These representations are characterized by means of mass and spin. The inertial properties of mass in moving frames of reference are already well known: for instance via Coriolis, centrifugal and other mechanical effects [7]. The coupling of intrinsic spin with rotation reveals the rotational inertia of intrinsic spin.

The underlying physics of spin-rotation coupling may intuitively be illustrated through a thought experiment. Imagine our observer, O', rotates with angular velocity $\mathbf{\Omega}$ parallel to the direction of propagation of a plane linearly polarized monochromatic electromagnetic wave whose electric field can be written in the coordinates of a reference inertial frame I as:

$$\boldsymbol{E} = E_0\,\hat{\mathbf{x}}\,e^{-i\omega t+ikz}, \tag{3}$$

where E_0 is a constant amplitude, $\mathbf{k} = k\hat{\mathbf{z}}$ is the wave vector and $\omega = ck$. The coordinate system of O' is related to I by

$$\hat{\mathbf{x}} + i\hat{\mathbf{y}} = e^{\pm i\Omega t}\,(\hat{\mathbf{x}}' \pm i\hat{\mathbf{y}}')\,; \quad \hat{\mathbf{z}} = \hat{\mathbf{z}}'; \quad t = t', \tag{4}$$

such that, from the viewpoint of the rotating observer,

$$\boldsymbol{E} = E_0\,(\cos\Omega t\,\hat{\mathbf{x}}' - \sin\Omega t\,\hat{\mathbf{y}}')\,e^{-i\omega t+ikz}, \tag{5}$$

with the direction of linear polarization appearing to rotate in a clockwise sense about the $z-$axis. Our treatment ignores certain relativistic complications and focuses attention instead on the simple fact that from the viewpoint of the rotating observer, O', the direction of linear polarization that is fixed in the inertial frame I must drift in a clockwise sense with frequency Ω about the direction of propagation.

Linearly polarized light represents a coherent superposition of right circularly polarized (RCP) and left circularly polarized (LCP) waves,

$$\boldsymbol{E} = \frac{1}{2} E_0 \left(\hat{\mathbf{x}} + i \hat{\mathbf{y}} \right) e^{-i\omega t + ikz} + \frac{1}{2} E_0 \left(\hat{\mathbf{x}} - i \hat{\mathbf{y}} \right) e^{-i\omega t + ikz}. \tag{6}$$

From the viewpoint of the rotating observer the radiation field may also be written as a sum of RCP and LCP components

$$\boldsymbol{E} = \frac{1}{2} E_0 \left(\hat{\mathbf{x}}' + i \hat{\mathbf{y}}' \right) e^{-i(\omega - \Omega)t + ikz} + \frac{1}{2} E_0 \left(\hat{\mathbf{x}}' - i \hat{\mathbf{y}}' \right) e^{-i(\omega + \Omega)t + ikz}, \tag{7}$$

as these eigenstates of the radiation field remain invariant under rotation but their frequencies undergo the characteristic ‘Zeeman’ splitting that has a simple physical interpretation. In a RCP (LCP) wave, the electric and magnetic fields rotate in the positive (negative) sense about the direction of propagation with frequency ω. Since the observer rotates in the positive sense with frequency Ω, it perceives the effective frequency of the RCP (LCP) wave to be $\omega - \Omega$ ($\omega + \Omega$) with respect to time t. The proper time along the worldline of O' is $\tau = t/\gamma$, where γ is the Lorentz factor such that the frequencies of the RCP and LCP light as measured by O' are

$$\omega' = \gamma(\omega \mp \Omega). \tag{8}$$

In this expression the Lorentz factor accounts for time dilation, which is consistent with the transverse Doppler effect. In addition ‘angular Doppler terms’ ($\mp\gamma\Omega$) arise due to the observer’s rotation. Writing Eq. (8) in terms of energy as $\mathcal{E}' = \gamma(\mathcal{E} \mp \hbar\Omega)$ illustrates that the deviation from the simple transverse Doppler effect stems from the coupling of the spin of a circularly polarized photon to the rotation of the observer, since a RCP (LCP) photon carries an intrinsic spin of $\hbar(-\hbar)$ along its direction of propagation [8].

Now replace the concept of a rotating observer that measures the frequency components of circularly polarized light with the atoms constituting a slowly rotating half-wave plate (HWP). Suppose RCP light falls perpendicular to the surface of this optical element, illustrated in Fig. 1. Eq. (8) now describes the frequency of the radiation that drives the motion of electrons within this material. As such, RCP light will cause electrons to oscillate with frequency $\omega'_{RCP} \approx \omega - \Omega$ in the rotating frame. Furthermore, the action of the HWP is to transform RCP to LCP light such that light transmitted through the HWP will become LCP and will have the same frequency, $\omega'_{LCP} \approx \omega - \Omega$, *in the rotating frame of reference.* Through the inverse transformation of Eq. (8), *i.e.* $\omega_{LCP} \approx \omega'_{LCP} - \Omega$, the transmitted light in I will be both LCP *and shifted in frequency by*

$$\Delta\omega_{RCP \to LCP} = -2\,\Omega, \tag{9}$$

hence the medium absorbs energy, linear momentum and angular momentum from the radiation field. Conversely, for LCP radiation passing through the same system the relevant spin-rotation frequency shift would involve $\omega'_{LCP} \approx \omega + \Omega$ such that $\Delta\omega_{LCP \to RCP} = 2\Omega$. It follows that for linearly polarized light no net transfer of energy, momentum or angular momentum occurs!

These results can be extended to more general spin states via the superposition principle. For instance, if in Fig. 1 the rotating HWP is replaced by a rotating quarter-wave plate, the outgoing radiation will be a superposition of a RCP component with frequency ω and a LCP component with frequency $\omega - 2\Omega$.

Identical conclusions to these heuristic arguments have been drawn from Maxwell's equations when considered in the rotating frame of reference [6, 9]. Furthermore, and of central importance to this letter, a series of experiments have been performed which confirm the frequency shift predicted above. A helicity-dependent rotational frequency shift was first observed using microwave radiation [10]. This effect has subsequently been investigated in the optical regime by several authors [11]. These studies provide direct experimental verification of the phenomenon of helicity-rotation coupling for electromagnetic radiation.

With these experimental results in mind, the connection between the *frequency shift*, Eq. (9), and the *constant optical phase shift* predicted by Eq. (2), can be clarified in a simple configuration as follows. Let an optical interferometer be set in rotation as illustrated in Fig. 2. When viewed from I, RCP light having passed through the first HWP becomes LCP and is shifted in frequency by -2Ω, Eq. (9), as the HWP rotates with the interferometer at angular velocity $\mathbf{\Omega}$. Multiplying this frequency shift by the time of flight of a photon between the two HWP's, $\Delta t = l/c$, gives in effect what amounts to a helicity-rotation phase shift $\Delta\Phi = \oint \boldsymbol{k} \cdot d\boldsymbol{r} = (\omega_+ - \omega_-) l/c = 2\Omega\, l/c$, where $\omega_+ = \omega$ and $\omega_- = \omega - 2\Omega$. This same phase shift is expected at the detector in the rotating frame and is given by Eq. (2), the factor of two arising as $\boldsymbol{\Omega} \cdot \boldsymbol{S} = \pm\hbar$ for RCP or LCP light, respectively, in the rotating frame.

We have considered thus far the simplest configuration for the measurement of frequency shifts due to helicity-rotation coupling, since the direction of propagation has been along the axis of rotation. The general expression for spin-rotation coupling relating the energy measured by a rotating observer to measurements performed in I can be written as

$$\mathcal{E}' = \gamma(\mathcal{E} - \hbar M \Omega), \tag{10}$$

where M is the total (orbital plus spin) 'magnetic' quantum number along the axis of rotation; that is, $M = 0, \pm 1, \pm 2, \ldots$ for a scalar or a vector field while $M \mp \frac{1}{2} = 0, \pm 1, \pm 2, \ldots$ for a Dirac field. In the JWKB approximation, Eq. (10) can be written as $\mathcal{E}' = \gamma(\mathcal{E} - \boldsymbol{\Omega} \cdot \boldsymbol{J}) = \gamma(\mathcal{E} - \boldsymbol{v} \cdot \boldsymbol{p}) - \gamma \boldsymbol{S} \cdot \boldsymbol{\Omega}$, so that in the absence of intrinsic spin we recover the classical expression for the energy of a particle as measured in the rotating frame with $\boldsymbol{v} = \boldsymbol{\Omega} \times \boldsymbol{r}$. Spin-rotation coupling, however, violates the underlying assumption of locality in special relativity: that the results of any measurement performed by an accelerating observer (in this case the measurement of frequency) are locally equivalent to those of a momentarily comoving inertial observer, but agrees with an extended form of the locality hypothesis. This is a nontrivial axiom since there exist definite acceleration scales of time and length that are associated with an accelerated observer. Discussion of this extension to the standard Doppler formula, and its wider implications on the theory of relativity, have been presented elsewhere [5, 6, 12, 13].

Observational support for this energy shift for fermions has been provided via a small frequency offset in high-precision experiments due to the nuclear spin of Mercury coupling to the rotation of the Earth [13, 14]. More general experimental arrangements which test Eq. (10) can also be envisioned. In fact, an experimental configuration [15] recently demonstrated [16] that linearly polarized light, when prepared as an eigenstate of the orbital angular momentum operator L_z, also suffers a frequency shift upon passing through a rotating Dove prism. These observations can be explained on the basis of Eq. (10); moreover, it would be interesting to repeat such experiments using circularly polarized radiation in order to see the combined coupling of the orbital plus spin angular momentum of the field to rotation.

In analogy with the observed frequency shift when circularly polarized light passes through a rotating HWP, Eq. (10) indicates that similar experiments should be possible using polarized neutrons. To this end let neutrons propagating through a uniformly rotating spin flipper be polarized with their spin $|\uparrow\rangle \parallel \mathbf{\Omega}$, as illustrated in Fig. 3. Repetition of the arguments leading to Eq. (9) gives, in the JWKB approximation, a frequency shift (measured in I) for the transmitted neutrons equal to

$$\Delta\omega_{|\uparrow\rangle\rightarrow|\downarrow\rangle} = -2M\Omega = -\Omega, \tag{11}$$

as the incident neutron in state $|\uparrow\rangle$ has $M = 1/2$. It is assumed here that the average energy of the neutron in the spin flipper remains constant, i. e. there is no intrinsic frequency shift associated with the spin flipper; otherwise, the additional shift should also be taken into account. Moreover, our result for the neutron frequency shift can be extended to more general spin states. The simplest spin flipper consistent with our assumption would be a coil producing a uniform static magnetic field B normal to the polarization axis of the neutrons [17]. If t is the interval of time that it takes neutrons of speed v_n to traverse the length of the coil, the probability of spin flip upon passage is $\sin^2\zeta$, where $\zeta = -\mu_n Bt/\hbar$ and μ_n is the neutron magnetic moment. The neutron spin would therefore flip for $\zeta = (2N+1)\pi/2$, $N = 0, 1, 2, \ldots$. The length L of an appropriate coil can thus be obtained from $L = (2N+1)L_0$, where $L_0 = v_n t$ for $\zeta = \pi/2$; hence, $L_0 = \pi\hbar v_n/(2|\mu_n|B)$. For $B = 500$ G and $v_n/c \simeq 10^{-5}$, we find $L_0 \simeq 1$ mm; in this case, thermal neutrons of wavelength $\lambda \simeq 1$ Å would take less than a μsec to traverse the coil. Should the coil be rotated slowly, the various approximations involved in our treatment could be justified.

One can imagine a variety of interferometric configurations using rotating spin flippers. If the arrangement is such as to produce a constant phase shift then, in effect, such experiments would be similar to the configuration suggested a decade ago [5]. Because this phase shift is very small a large-scale neutron interferometer is required for its possible realization. It is therefore interesting to conceive an interference experiment that would observe a beating between two different de Broglie frequencies. A beat frequency of $\simeq 2 \times 10^{-2}$ Hz has previously been measured in a neutron interferometry experiment [18] involving the passage of neutrons through stationary rf coils driven at slightly different frequencies.

As illustrated in Fig. 4, we propose to place identical spin flippers along each of the two separated neutron beams such that an intensity maximum is recorded when both coils are aligned parallel. Keeping the interferometer stationary in the inertial frame of the laboratory, I, we then rotate one of the coils with angular velocity $\mathbf{\Omega}$ parallel to the neutron wave vector. From Eq. (11), a shift in frequency of this beam by $\Delta\omega = -\Omega$ will be induced such that a time-dependent interference intensity envelope of the form $I \propto [1 + \cos(\Omega t + \phi_0)]$ arises, where ϕ_0 is the constant phase shift between the two interferometer arms [19]. The frequency components of this intensity modulation may easily be recovered by recording the intensity as a function of time and taking the Fourier transform of the output [20]. A sinusoidal modulation of the intensity arising from spin-rotation coupling will cause sideband structure to appear in the resulting spectra, with the peaks separated by Ω, the rate of rotation of the spin-flipper. As the proposed experimental apparatus closely resembles that used by Allman *et al.* [17] to measure simultaneously geometric and dynamical phase shifts, we believe that the potential observation of a spin-rotation coupling induced frequency shift for fermions falls entirely within the sphere of current technology.

All of the experimental work to date has involved rotation frequencies $\Omega \ll \omega$ and the interpretation of the experiments has been based on certain intuitive considerations [9, 10, 11, 15]. The present identification of the origin of these results in terms of spin-rotation coupling makes it possible to discuss the general situation for arbitrary Ω and spin, as well as whether RCP radiation can stand completely still for $\omega = \Omega$ in Eq. (8). This situation is reminiscent of the pre-relativistic Doppler formula for linear motion, which predicted that electromagnetic radiation would stand completely still relative to an observer moving with speed c along the direction of propagation of the wave. This circumstance proved an important motivating factor in Einstein's development of the theory of relativity [21]. These issues have been the subject of a number of theoretical investigations and it is hoped that further experimental studies can shed light on future developments toward a nonlocal theory of accelerated systems [22].

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Figure Captions

Fig. 1. Frequency redshift via a rotating half-wave plate (HWP).

Fig. 2. Schematic plot of a rotating interferometer. A half-wave plate flips the initial helicity along one path while a second half-wave plate flips it back before recombination in order that interference can take place. The distance between the HWPs is l.

Fig. 3. Schematic depiction of the passage of longitudinally polarized neutrons through a uniformly rotating spin flipper. We assume that the average energy of the neutron does not change while in the spin flipper. The rotational energy shift, $\mathcal{E}_f - \mathcal{E}_i = -\hbar\Omega$, provides a new way to moderate neutrons. Note that if the sense of rotation is reversed, then there would be a gain in energy by $\hbar\Omega$.

Fig. 4. A neutron interferometer in an inertial frame of reference. Longitudinally polarized neutrons pass through a slowly rotating spin flipper along one arm and a static spin flipper along the other arm resulting in a beat phenomenon at the detector.