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Decaying dark energy in higher-dimensional gravity

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ABSTRACT

Aims. We use data from observational cosmology to put constraints on higher-dimensional extensions of general relativity in which the effective four-dimensional dark-energy density (or cosmological "constant") decays with time.

Methods. In particular we study the implications of this decaying dark energy for the age of the universe, large-scale structure formation, big-bang nucleosynthesis and the magnitude-redshift relation for type Ia supernovae.

Results. Two of these tests (age and the magnitude-redshift relation) place modest lower limits on the free parameter of the theory, a cosmological length scale L akin to the de Sitter radius. These limits will improve if experimental uncertainties on supernova magnitudes can be reduced around $z \sim 1$.

Key words. cosmology: theory – stars: supernovae: general

1. Introduction

In standard general relativity, dark energy is interchangeable with Einstein's cosmological constant Λ and the dark-energy density $\rho_\Lambda = \Lambda/(8\pi G)$ is constant. Observation tells us that, at the present time, this latter quantity is nonzero but many orders of magnitude smaller than expected based on calculations in quantum field theory. This mismatch has led some theorists to look at alternatives to standard general relativity in which Λ is a dynamical parameter whose value is not constant but might have decayed from large values in the early universe to those we see today (Overduin & Cooperstock 1998).

Here we look at one such alternative, the minimal extension of Einstein's theory to manifolds with one additional noncompact spacelike dimension (Overduin & Wesson 1997). Higher-dimensional gravity has been shown to be compatible with solar-system tests including the classical tests of general relativity (Kalligas et al. 1995; Liu & Overduin 2000) and experimental limits on violations of the equivalence principle (Overduin 2000). We wish to check here whether it is also consistent with basic cosmological tests that are sensitive to the Λ term. An exhaustive treatment of all cosmological data is beyond the scope of this introductory study.

Mashhoon & Wesson (2004) have shown that a gauge transformation in higher-dimensional gravity converts Λ from a constant of nature to a gauge-dependent measure of four-dimensional vacuum-energy density. Under reasonable assumptions (conformal flatness and geodesic motion in five dimensions), Λ decays *exponentially* with cosmic time. We investigate the cosmological implications of this kind of vacuum decay by solving numerically for the age $t(z)$ of the universe as a function of redshift. The bulk of the vacuum decays takes place near $z \sim 1$, too late to affect big-bang nucleosynthesis (BBN) or large-scale structure formation (LSS). Lower limits on the age of the universe, however, put weak constraints on the primary free parameter of the theory (a de Sitter-like length parameter L), and these bounds are improved somewhat by data on the magnitude-redshift relation for type Ia supernovae. Taken together with the solar-system tests, we tentatively conclude that available astrophysical data are consistent with a universe with one (or more) extra dimensions.

2. Higher-dimensional cosmology

A starting point for cosmological investigations in 5D theory is the metric in canonical form (Mashhoon et al. 1994):

$$\mathrm{d}S^2 = \frac{\ell^2}{L^2}\left[g_{\mu\nu}(x^\lambda,\ell)\mathrm{d}x^\mu \mathrm{d}x^\nu\right] - \mathrm{d}\ell^2, \quad (1)$$

where L is a constant with dimensions of length (akin to the de Sitter radius $L_d = \sqrt{3/\Lambda}$ in standard cosmology). The 5D line element contains the 4D one:

$$\mathrm{d}s^2 = g_{\mu\nu}(x^\lambda,\ell)\mathrm{d}x^\mu \mathrm{d}x^\nu. \quad (2)$$

There is no loss of generality to this point; five available degrees of coordinate freedom have been used to set the electromagnetic potentials ($g_{4\mu}$) to zero and to set the scalar potential (g_{44}) to a constant in Eq. (1). It is, however, necessary to retain ℓ-dependence in the 4D metric tensor in order to preserve this generality (Overduin & Wesson 1997).

Under the restriction to 5D conformal flatness, and the natural assumption that all test particles (massive as well as massless) move along null geodesics in 5D (i.e., $\mathrm{d}S^2 = 0$), Mashhoon & Wesson (2004) have shown that Λ in 4D drops exponentially with proper time s:

$$\Lambda = \frac{3}{L^2}\frac{1}{(1-\mathrm{e}^{\mp s/L})^2}. \quad (3)$$

Physically, this variation arises because we require the 5D field equations to satisfy general covariance in five, not four dimensions. The canonical metric (1) is invariant with respect to translations along the ℓ-axis, so ℓ- or gauge-dependence then necessarily appears in the 4D field equations. We have used the 4D metric to re-express this dependence in terms of proper time s rather than ℓ. There are two cases: in the first ("−" sign in the exponent), Λ decays asymptotically to the small finite value $3/L^2$ as $s \to \infty$, while in the second ("+" sign) it vanishes in this limit. Measurements tell us that Λ is small at present, but are not precise enough to discriminate between a constant value and one that is still decaying on cosmological timescales. Therefore we retain both possibilities in what follows.

One way to constrain proposals of this kind is to ask what Λ decays *into*. If matter or radiation, then strong constraints can be placed on the theory using experimental limits on cosmic background radiation (Overduin et al. 1993; Overduin & Wesson 2003, 2004). In this paper, we will investigate the more conservative scenario in which Λ does *not* decay into matter or radiation, so that the evolution of the matter density ($\rho_{\rm M}$) and radiation energy density ($\rho_{\rm R}$) proceed just as in standard 4D cosmology. It should be noted that such an assumption requires, in principle, the existence of some other field to which Λ is coupled and into which its energy can be transferred, in accordance with the 4D conservation law (Overduin & Cooperstock 1998):

$$\nabla^{\nu}(8\pi G\mathcal{T}_{\mu\nu} - \Lambda g_{\mu\nu}) = 0. \tag{4}$$

In order to compare Eq. (3) to observation, we need to convert from proper time s to ordinary cosmic time t. A physically-motivated argument due to Mashhoon is the following: translations along the ℓ-axis do not only introduce gauge-dependence into Λ; they also give rise to an apparent "fifth force" in the equations of motion (Mashhoon & Wesson 2004; Wesson 2005). This force can however be made to vanish by an appropriate choice of affine parameter, so making the motion geodesic in the usual 4D sense (Seahra & Wesson 2001). This choice can be shown to lead to the following relation between s and t, assuming that the motion in 5D is null, that the galaxies are comoving in 4D spacetime, and that $\mathrm{d}t/\mathrm{d}s > 0$ over $0 < s < \infty$:

$$e^{\pm s/L} = 1 \pm (1/\alpha)\mathrm{e}^{t/L}. \tag{5}$$

Here α (expressed this way for later convenience) is an unknown positive constant whose value is to be fixed by cosmological boundary conditions, and the signs are to be read in the same way as in Eq. (3); i.e., the upper ("+") sign here corresponds to the upper ("−") sign there. Putting Eq. (5) into the decay law (3), we find that:

$$\Lambda(t) = \frac{3}{L^2}\left(1 \pm \alpha\mathrm{e}^{-t/L}\right)^2, \tag{6}$$

where the sign order again corresponds to that in Eqs. (3) and (5).

Equation (6) provides the starting-point for our investigation. Motivated by the 5D results, we study dark-energy decay of the functional form given by Eq. (6) in the context of 4D cosmology from $t = 0$ at the big bang to $t \to \infty$. This kind of exponential dark-energy decay appears to be unique in the literature (Overduin & Cooperstock 1998). In the limit $t \to 0$, we observe that Λ originates with a finite value of $3(1 \pm \alpha)^2/L^2$ and decays asymptotically toward $3/L^2$ as $t \to \infty$. Motivated by the cosmological-constant problem, we might expect very large values of α with steep drop-offs at early times. We shall find, however, that α is typically within a few orders of magnitude of unity, rendering departures from standard 4D cosmology rather mild.

3. Decaying dark energy

Evaluating Eq. (6) at the present time t_0, restoring physical units and expressing the results in terms of the critical density, we obtain:

$$\Omega_{\Lambda,0} = \frac{\Lambda(t_0)c^2}{3H_0^2} = \left(\frac{c}{H_0 L}\right)^2\left(1 \pm \alpha\mathrm{e}^{-c\,t_0/L}\right)^2, \tag{7}$$

where H_0 is the present value of Hubble's parameter. It will be convenient to rescale all dynamical parameters in dimensionless terms, so we define:

$$\tau \equiv H_0\,t, \quad \mathcal{L} \equiv H_0 L/c, \quad \mathcal{H} \equiv H/H_0. \tag{8}$$

In terms of these quantities Eq. (7) determines the age of the universe $\tau_0 \equiv H_0\,t_0$ in terms of the two free parameters α and $\mathcal{L}$ as follows:

$$\tau_0 = \mathcal{L}\ln\left(\frac{\pm\alpha}{\mathcal{L}\sqrt{\Omega_{\Lambda,0}} - 1}\right). \tag{9}$$

We note from Eq. (9) that the two cases corresponding to the "+" and "−" signs are separated by a "critical" case with $\mathcal{L}_{\rm crit} \equiv 1/\sqrt{\Omega_{\Lambda,0}}$. For values of $\mathcal{L} > \mathcal{L}_{\rm crit}$ we must use the "+" solution, while the "−" solution is operative if $\mathcal{L} < \mathcal{L}_{\rm crit}$. (An alternative solution, corresponding to the negative root of Eq. (7), is also available in principle, since Eq. (6) has three possible square roots. However, the root that results in the alternative solution should be excluded as it does not have the same limit as Eq. (6) for $t \to \infty$. This explains why the alternative solution is not taken into consideration here.) In the limit $\mathcal{L} \to \mathcal{L}_{\rm crit}$ precisely, it is apparent from Eqs. (7) with the definitions (8) that $\alpha = 0$ and the theory goes over to standard cosmology with Λ =const. The theory also goes over to standard cosmology in the limit $L \to \infty$, which from Eq. (7) means that α must go as L^2 for large L, as we will confirm numerically below. The physical length corresponding to $\mathcal{L}_{\rm crit}$ is just the de Sitter radius of standard cosmology, $L_{\rm crit} = c/(H_0\sqrt{\Omega_{\Lambda,0}})$, which takes the value $L_{\rm crit} = 4.9$ Gpc for WMAP values of H_0 and $\Omega_{\Lambda,0}$ (Spergel et al. 2003).

We now fix the value of α by requiring consistency between Eq. (9) and the age of the universe τ_0 as obtained by numerical integration of the Friedmann-Lemaître equation:

$$\tau(z) = \tau_0 - \int_0^z \frac{\mathrm{d}z'}{(1+z')\,\mathcal{H}[z', \tau(z')]}. \tag{10}$$

Here z is redshift and the standard expression for Hubble's parameter is modified following Eqs. (6) and (7) so that:

$$\mathcal{H}[z, \tau(z)] = \Big[\Omega_{\rm M,0}(1+z)^3 + (1 - \Omega_{\rm M,0} - \Omega_{\Lambda,0})(1+z)^2 + \mathcal{L}^{-2}\left(1 \pm \alpha\mathrm{e}^{-\tau(z)/\mathcal{L}}\right)^2\Big]^{1/2}. \tag{11}$$

Consistency is to be enforced by putting the boundary condition $\tau(\infty) = 0$ into Eq. (10); i.e., for each value of L we solve numerically for a value of α satisfying:

$$\int_0^{\infty} \frac{\mathrm{d}z'}{(1+z')\,\mathcal{H}[z', \tau(z')]} = \mathcal{L}\ln\left(\frac{\pm\alpha}{\mathcal{L}\sqrt{\Omega_{\Lambda,0}} - 1}\right). \tag{12}$$

We adopt WMAP values for the present values of the density parameters, assuming spatial flatness: $\Omega_{\rm M,0} = 0.135/h_0^2 = 0.27$

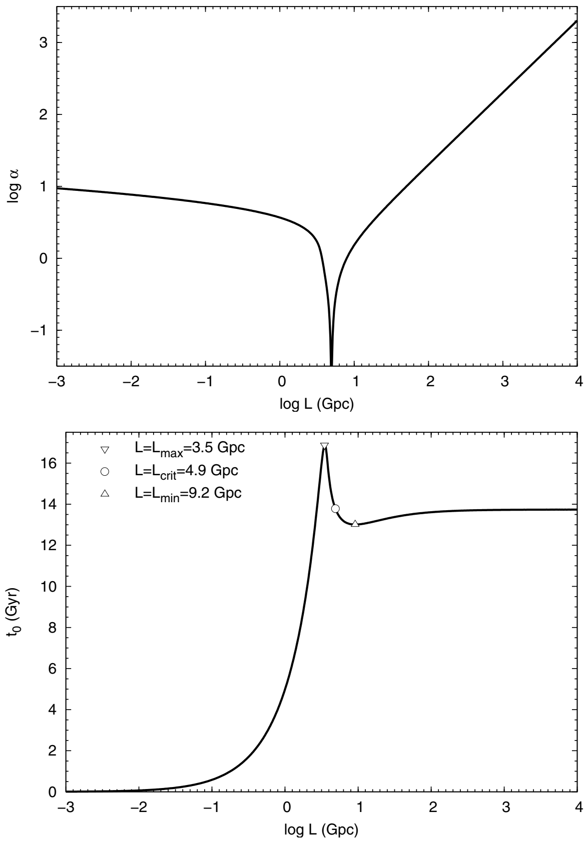

Fig. 1. (*Top*) Values of the constant α as a function of L, assuming WMAP values of $\Omega_{M,0}$ and $\Omega_{\Lambda,0}$. (*Bottom*) Age of the universe as a function of L for the same values of $\Omega_{M,0}$ and $\Omega_{\Lambda,0}$. Labelled points correspond to the critical case separating the two classes of solutions of Eq. (9), and to universes of maximum and minimum age in the two regimes (see text for discussion)

(with $h_0 \equiv H_0/100$ km s^{-1} Mpc^{-1} = 0.71) and $\Omega_{\Lambda,0} = 1 - \Omega_{M,0} = 0.73$ (Spergel et al. 2003). That leaves us with only one adjustable parameter in the theory: the de Sitter-like length L.

Figure 1 (top) shows how this cosmological consistency requirement produces values of α between approximately 0.1 and 1000 for a range of L-values with $L > 5$ Gpc. As expected, $\alpha \rightarrow 0$ in the limit $L \rightarrow L_{crit}$ and $\alpha \propto L^2$ for large L.

Figure 1 (bottom) shows the corresponding age of the universe $t_0 = \tau_0/H_0$ (as obtained from either Eq. (9) or Eq. (10) in the limit $z \rightarrow \infty$) for the same range of values of L as in Fig. 1 (top). The shape of this plot can be understood physically as follows: The age of the universe goes over to that of a standard flat ΛCDM model with $\Omega_{M,0} = 0.27$ (i.e., 13.7 Gyr) in both the "critical" case $L = L_{crit}$ and the limit $L \rightarrow \infty$, as expected. To the right and left of the critical case are the "+" and "−" regimes in Eq. (6) respectively. The presence of a local minimum and a maximum here is due to the L-dependence of the dark-energy density: the behavior near these extrema is dominated by the exponential term $\pm\alpha \exp(-ct/L)$ in Eq. (6), whose magnitude in this region is comparable to unity.

To understand why the age of the universe in this theory is always lower than that of standard cosmology for $L > L_{crit}$, whereas it can climb somewhat above this value for $L < L_{crit}$, we recall that the dark-energy density is always pinned at its observed value at present. In the "+" regime ($L > L_{crit}$), it can only grow in the past direction relative to its value in standard cosmology, increasing the total density of matter plus dark energy. Since the square of the expansion rate is proportional to total density (from the Friedmann equation) the expansion rate goes up and the expansion timescale goes down. Hence, for given values of $\Omega_{M,0}$ and $\Omega_{\Lambda,0}$ the age of the universe in this theory is always smaller than that in standard cosmology for $L > L_{crit}$. In the "−" regime, by contrast, there is a range of L-values for which the dark-energy density in the past direction is slightly *lower* than that in the equivalent standard cosmology (i.e., with the same boundary conditions); hence one obtains a slightly older universe in this region.

These departures from standard cosmology can be significant: for $L > L_{crit}$ the age of the universe drops from 13.7 to a local minimum of 13.0 Gyr at $L_{min} = 9.2$ Gpc, while for $L < L_{crit}$ it climbs to a possible maximum of 16.9 Gyr at $L_{max} \equiv 3.5$ Gpc, before dropping rapidly toward zero as $L \rightarrow 0$ (assuming WMAP values of $\Omega_{M,0}$ and $\Omega_{\Lambda,0}$ as usual). This curve is sufficient to set a robust lower bound $L > 2.2$ Gpc from the fact that the age of the oldest observed stars in globular clusters are at least 11 Gyr (Wanajo et al. 2002; Schatz et al. 2002). The range of acceptable L values could be further constrained by other limits on the age of the universe from a variety of observations, including upper bounds (though the latter are necessarily less robust since we may not be able to see the oldest members of any target population). In any case we will obtain stronger constraints below.

As a first step we use Eq. (10) to compute the age $t(z) = \tau(z)/H_0$ as a function of redshift. The presence of explicit time-dependence under the integral sign makes this a difficult equation analytically, but it can be solved numerically. Results are shown in Fig. 2 (top). The $L = L_{crit}$, $L = L_{min}$ and $L = L_{max}$ cases are again indicated together with a representative sampling of other models.

The quantity $t(z)$ plotted in Fig. 2 (top) is helpful in checking whether the theory is consistent with the formation of LSS by gravitational instability in the early matter-dominated era and BBN at the end of the radiation-dominated era. For these processes to work successfully, the density of dark energy must not rise so quickly in the past direction that it becomes comparable to or greater than the matter density during LSS formation, or comparable to or greater than the radiation energy density during BBN. These latter two quantities are given by:

$$\begin{aligned} \rho_R(z) &= \rho_{crit,0}\,\Omega_{R,0}(1+z)^4 \\ \rho_M(z) &= \rho_{crit,0}\,\Omega_{M,0}(1+z)^3, \end{aligned} \tag{13}$$

where $\rho_{crit,0} = 3H_0^2/(8\pi G) = 1.88 \times 10^{-29} h_0^2$ g cm^{-3} is the critical density and $\Omega_{R,0} = 4.17 \times 10^{-5}/h_0^2$ from COBE data plus standard neutrino physics (Overduin & Wesson 2003). By comparison, the density of dark energy is given by Eq. (6) as:

$$\rho_\Lambda(z) = \rho_{crit,0}\,\mathcal{L}^{-2}\left(1 \pm \alpha e^{-\tau(z)/\mathcal{L}}\right)^2, \tag{14}$$

with $\tau(z)$ as shown in Fig. 2 (top).

Inserting $\tau(z)$ into the dark-energy density (14), and comparing with the matter and radiation energy densities, we obtain the plots shown in Fig. 2 (bottom). A representative sampling of models with $L > 3.5$ Gpc is included, with the $L = L_{crit}$, L_{min} and L_{max} cases labelled as before. It is clear from these plots that departures from standard cosmology are too small to have a significant effect on either LSS or BBN. For $L < L_{crit}$ the density of dark energy *drops* relative to that in standard cosmology. For $L > L_{crit}$, even the largest possible increase in dark-energy density (corresponding to the shortest-lived universe with $L = L_{min}$) is far too modest relative to the matter density at $z \gtrsim 1$ to interfere with structure formation, and completely negligible relative to radiation-energy density at $z \gtrsim 1000$. Thus these tests do not place meaningful constraints on the theory.

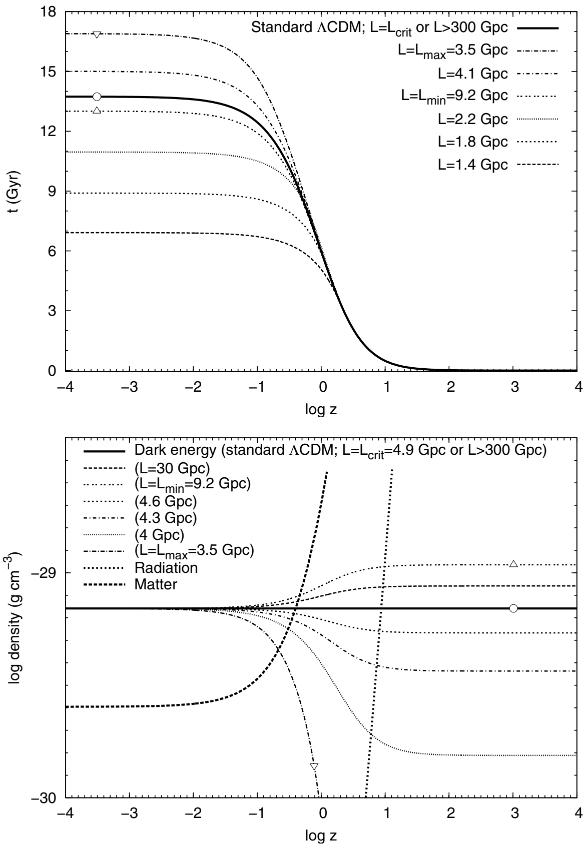

Fig. 2. (*Top*) Age of the universe as a function of redshift for various values of L, assuming WMAP values of $\Omega_{M,0}$ and $\Omega_{\Lambda,0}$. (*Bottom*) Densities of radiation, matter and dark energy as a function of redshift for various values of L, assuming the COBE value of $\Omega_{R,0}$ and WMAP values of $\Omega_{M,0}$ and $\Omega_{\Lambda,0}$. The three labelled points correspond to universes with critical, maximum and minimum values of L, as in Fig. 1 (*bottom*).

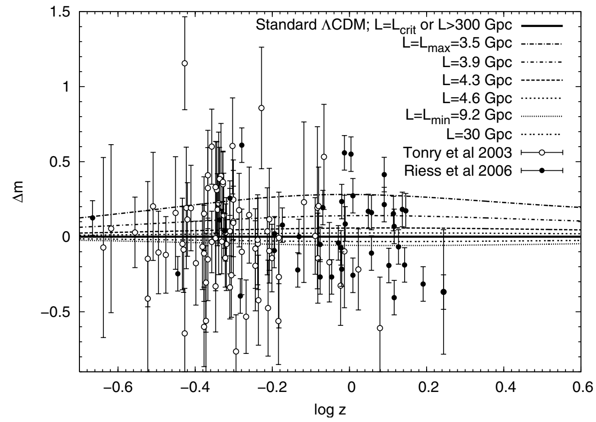

Fig. 3. Magnitude-redshift relation for various values of L, as compared to observational data on type Ia supernovae

Figure 2 (bottom) shows that the largest departures from standard theory are found near $z \sim 1$, raising the possibility that stronger constraints might be obtained by use of the SNIa magnitude-redshift relation. Supernovae are now being routinely monitored at $z \sim 1$ (Riess et al. 2006), providing a sensitive testbed for alternative theories of gravity with time-varying dark-energy density (see for example Fukui 2006).

The magnitude-redshift formulae are derived in the Appendix. We focus on the magnitude residual $\Delta m(z)$, or difference in apparent magnitude relative to a fiducial model, which we take here as the standard flat ΛCDM model with WMAP values of $\Omega_{M,0}$ and $\Omega_{\Lambda,0}$. Predictions are plotted as curves (for various values of L) in Fig. 3, where they are compared with measurements for 92 medium-redshift SNIa at $z > 0.1$ by Tonry et al. (2003) and 23 high-redshift SNIa at $z \sim 1$ as compiled by Riess et al. (2006).

The heavy solid line in Fig. 3 indicates the fiducial or standard ΛCDM model (straight line $\Delta m = 0$), which overlaps with the present theory in the case $L = L_{\rm crit}$ and all cases with $L > 300$ Gpc (to within the precision of the plot). For smaller values of L, the theory begins to depart from standard cosmology with maximum deviations near $z \sim 1$, confirming the usefulness of SNIa as probes of the theory. For L near (or greater than) the critical value $L_{\rm crit} = 4.9$ Gpc, the experimental uncertainties are too large to discriminate usefully between theoretical values of L. The data are, however, good enough to disfavor smaller values of L, improving significantly on the age constraint and tightening observational bounds on the theory to $L > 4.3$ Gpc.

4. Discussion and conclusions

We have taken the basic extension of general relativity from 4 to 5 dimensions and asked what observational consequences follow from its decaying cosmological "constant", or density of dark energy. The four main consequences involve the age of the universe, structure formation, nucleosynthesis and the magnitude-redshift relation for type Ia supernovae. There are, of course, large literatures on all four of these subjects. We have therefore presented our results as possible departures from the current standard model. The theory is consistent with all four classes of data at present. The best way to separate 4D and 5D cosmology by observational means in the future would appear to be by use of better supernovae data at $z \gtrsim 1$.

Another possible test of the theory might come from analysis of the angular power spectrum of fluctuations in the cosmic microwave background (CMB). Qualitative considerations, however, suggest that the sensitivity of such a test would not be competitive with those discussed above for the kind of theory considered here. The main feature of the CMB power spectrum is the angular position of the first acoustic peak, $\ell_{\rm peak}$. This quantity depends only weakly on dark-energy density (see Fig. 1 of White 1998, or the analytic approximation in Cornish 2000). What $\ell_{\rm peak}$ really measures is the *sum* of matter and dark-energy densities, i.e. spatial curvature. We have assumed throughout that $\Omega_{M,0} + \Omega_{\Lambda,0} = 1$ (i.e. $k = 0$), as in the standard ΛCDM model, so we would not expect a significant shift in $\ell_{\rm peak}$. Figure 2 (bottom) shows that dark-energy density changes by at most $\sim\pm 40\%$ between $z \approx 1$ and $z \approx 10$ relative to standard ΛCDM cosmology. Following Cornish (2000) the change in Λ might shift $\ell_{\rm peak}$ by at most $\sim 40\%/35 \sim 1\%$, comparable to the current level of experimental uncertainty in this parameter (Page et al. 2003). Physically, the reason why the CMB constrains this theory less strongly than SNIa is because it probes higher redshifts where the density of matter is so much higher than that of dark energy that a modest change in the latter has little effect. More detailed study is warranted, however, particularly in light of the increase in experimental precision expected from the Planck mission. We hope to return to this issue in future work.

These cosmological results are complementary to earlier ones based on the classical solar-system tests and ones involving the equivalence principle. It has been known for some time that the basic 5D extension of 4D Einstein theory is consistent with these tests (Kalligas et al. 1995; Liu & Overduin 2000; Overduin 2000; Wesson 2006 for review). It is important to recall here that standard cosmological models which are *curved* in 4D may be smoothly embedded in models which are *flat* in 5D. (For a review see the books by Wesson 1999, 2006, for an account of the embeddings from an astrophysical viewpoint see Lachieze-Rey 2000.) The data may therefore be suggesting not only that the universe has a fifth dimension, but that its structure may be much simpler than previously thought.

Appendix

The apparent magnitude m of a source at redshift z is defined by:

$$m(z) = M + K(z) + 5\log\left[d_{\rm L}(z)/10\ {\rm pc}\right], \tag{15}$$

where M is absolute magnitude, $K(z)$ is the K-correction due to frequency shift and the luminosity distance $d_{\rm L}(z)$ is given by:

$$d_{\rm L}(z) = \frac{c(1+z)}{H_0\,\sigma_k}\,\mathcal{S}_k\left[\sigma_k \int_0^z \frac{{\rm d}z'}{\mathcal{H}(z')}\right]. \tag{16}$$

Here the constant σ_k and function $\mathcal{S}_k$ are defined so that $\sigma_k \equiv \{\sqrt{\Omega_{\rm M,0} - \Omega_{\Lambda,0} - 1}, 1, \sqrt{1 - \Omega_{\rm M,0} - \Omega_{\Lambda,0}}\}$ and $\mathcal{S}_k[X] \equiv \{\sin X, X, \sinh X\}$ respectively for $k = \{+1, 0, -1\}$. Equation (15) contains terms such as M, $K(z)$ and H_0 that are independent of the background cosmology and hence not of interest to us here. We remove those terms by focusing, not on the apparent magnitude itself, but on the difference or "residual" magnitude relative to a fiducial model, which we take here to be the standard flat ΛCDM model with WMAP values of $\Omega_{\rm M,0}$ and $\Omega_{\Lambda,0}$. That is, we focus on the observational quantity

$$\Delta m(z) = m(z) - m_{\rm fid}(z) = 5\log\left[\frac{\mathcal{D}(z)}{\mathcal{D}_{\rm fid}(z)}\right], \tag{17}$$

where

$$\begin{aligned}\mathcal{D}(z) &\equiv \int_0^z \frac{{\rm d}z'}{\mathcal{H}[z', \tau(z')]}\\ \mathcal{D}_{\rm fid}(z) &\equiv \int_0^z \frac{{\rm d}z'}{\sqrt{\Omega_{\rm M,0}(1+z')^3 + \Omega_{\Lambda,0}}}\,,\end{aligned} \tag{18}$$

and where the modified Hubble parameter $\mathcal{H}$ is specified as before by Eq. (11). To compare our predictions with the magnitude residuals measured by Tonry et al. (2003) for 92 SNIa at $z > 0.1$, we write $\Delta m(z) = 5(y \pm \delta y) - 5\log\left[c(1+z)\mathcal{D}_{\rm fid}(z)\right]$, where z, y and δy are read from Cols. 7–9 of Table 15 in that paper [$y = \log(d_{\rm L}H_0)$]. To incorporate the new and invaluable survey of 23 SNIa at $z \sim 1$ compiled by Riess et al. (2006), we note that $y = \log(d_{\rm L}H_0) = \frac{1}{5}(\mu_0 - C)$ where μ_0 is read from Col. 3 of that paper and C is a constant whose value is fixed by requiring that both samples give consistent values of $d_{\rm L}$ for SN1997ff at $z = 1.755$, implying that $C = 15.825$.

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