Nonlinear Spin-Polarized Transport through a Ferromagnetic Domain Wall

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(Received 25 January 2002; published 13 August 2002)

A domain wall separating two oppositely magnetized regions in a ferromagnetic semiconductor exhibits, under appropriate conditions, strongly nonlinear *I-V* characteristics similar to those of a *p-n* diode. We study these characteristics as functions of wall width and temperature. As the width increases or the temperature decreases, direct tunneling between the majority spin bands reduces the effectiveness of the diode. This has important implications for the zero-field quenched resistance of magnetic semiconductors and for the design of a recently proposed spin transistor.

DOI: 10.1103/PhysRevLett.89.098302 PACS numbers: 85.75.–d

It has recently been reported that some doped semiconductors, such as $Ga_{1-x}Mn_xAs$ [1] and $Ti_{1-x}Co_xO_2$ [2], undergo ferromagnetic transitions at temperatures as high as 110 and 300 K, respectively, while others (*n*-doped $Zn_{1-x}Mn_xSe$ [3]) are almost completely spin polarized by the application of a relatively modest magnetic field. These findings have raised hopes for the realization of semiconductor-based magnetoelectronic devices [4].

In a ferromagnetic semiconductor, the up- and down-spin components of just *one* carrier type are quite analogous to majority and minority carriers in ordinary doped semiconductors. Accordingly, a domain wall separating two ferromagnetic regions with opposite magnetizations is the analog of a *p-n* junction, while two consecutive domain walls correspond to a *p-n-p* transistor. In a recent paper [5] we have exploited this analogy to show that nonlinear amplification of a spin-polarized charge current is, indeed, possible in the "*p-n-p*" configuration, and can be controlled by a magnetic field or by a voltage applied to the "base" region between the two domain walls. However, the analysis of Ref. [5] was based on the assumption that the probability of a carrier flipping its spin while crossing the domain wall is negligible. This corresponds to assuming the resistivity of the domain wall is large compared to that of the bulk material.

The resistance of a domain wall between ferromagnetic materials has been examined several times from different perspectives since the pioneering work of Cabrera and Falicov [6]. These authors found that the resistance was very small, and later calculations [7,8] have supported that result for metallic magnets. A far different regime is possible, however, when the spin polarization is or approaches 100%. For example, experimental and theoretical results [9] indicate that domain walls in $La_{0.7}Ca_{0.3}MnO_3$ may dominate the resistance in thin films. Magnetic semiconductor systems, due to their very small bandwidths, are also likely to be 100% spin polarized, and thus their domain walls should be highly resistive in the absence of spin-flip transport processes across them.

In this Letter we present an analytical theory of the nonlinear *I-V* characteristics of a magnetic domain wall *taking into account spin-flip processes.* The main issue is the competition between minority spin injection, which is responsible for the nonlinear spin-diode behavior, and majority spin transmission, which tends to suppress it. We shall show that the latter dominates when either the temperature is low or the domain wall is thick. Because the thickness of a domain wall can now be directly measured [10] and, in principle, geometrically controlled [11], our theory should therefore be useful both in designing devices such as the one proposed in [5] and in understanding the zero-field quenched resistance and the low-field magnetoresistance of magnetic semiconductors.

Our model is schematically depicted in Fig. 1(a). The two ferromagnetic regions $F1$ and $F2$ are separated by a domain wall region of width d, $-d/2 < x < d/2$. The exchange field $\vec{B}(x)$ has the form

$$\vec{B}(x) = B_0[\cos\theta(x)\hat{x} + \sin\theta(x)\hat{y}], \tag{1}$$

where $\hat{x}$ and $\hat{y}$ are unit vectors in the direction of x and y, and the angle $\theta(x)$ varies linearly from $\theta = \pi/2$ in $F2$ to $\theta = -\pi/2$ in $F1$ [12]. We assume that d, while possibly large in comparison to a typical carrier wavelength, is smaller than the mean free path: therefore, the motion of the carriers through the domain wall is *ballistic.*

The logic of our calculation is as follows. In regions $F1$ and $F2$ we assume electrical charge neutrality and use the standard drift-diffusion theory to establish the form of the quasichemical potentials $\mu_\uparrow$ and $\mu_\downarrow$ and the currents $J_\uparrow$ and $J_\downarrow$ in the presence of a steady charge current $J_q = J_\uparrow + J_\downarrow$ [13]. These forms depend on three unknown constants: the quasichemical potentials of minority spin carriers on each side of the domain wall, i.e., $\mu_\uparrow(-d/2)$ and $\mu_\downarrow(d/2)$, and the value of the voltage V that develops across the domain wall due to the current flow. To fix the values of the three constants we impose *matching conditions,* which follow from the assumption that the electron transport through the

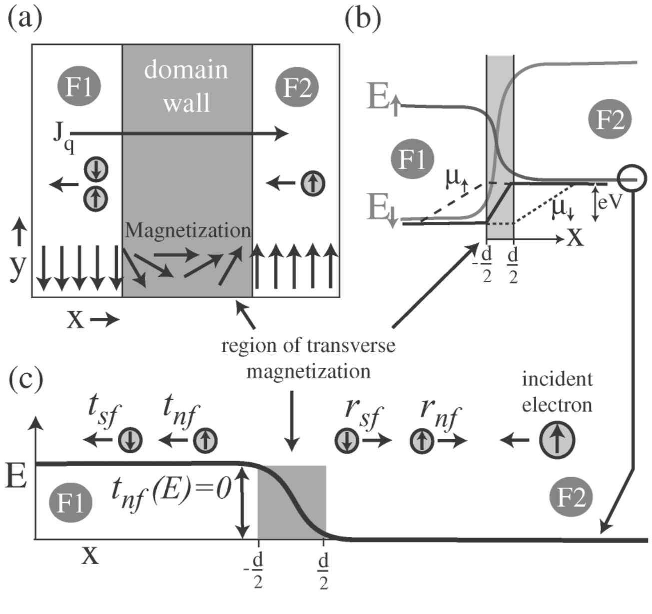

FIG. 1. (a) Schematic representation of a domain wall. (b) Qualitative behavior of the quasichemical potentials and the electrostatic potential (solid line). Note that the nonequilibrium voltage drop occurs within the interfacial region, while the nonequilibrium populations extend up to a distance of order of the spin diffusion length from it. (c) Reflection and transmission processes for an electron incident on the domain wall. Transmission without spin flip is not possible in the shaded energy range.

domain wall is ballistic. In solving the Schrödinger equation in the domain wall region, we assume that the voltage drop is smaller than the exchange spin splitting $\Delta = g\mu_B B$, say, $eV \lesssim 0.5\Delta$, so that the electric field in the region can be neglected without serious consequences. Finally, throughout the analysis, we assume that the carrier density is low enough, or the temperature sufficiently high, to justify the use of nondegenerate statistics.

Let $\Delta n_\sigma \equiv n_\sigma - n_\sigma^{(0)}$ be the deviations, due to the current flow, of the up- and down-spin densities in $F1$ and $F2$ from their equilibrium values $n_\sigma^{(0)}$. Since, by charge neutrality, $\Delta n_\uparrow \simeq -\Delta n_\downarrow$, we see that the *relative* change in the majority spin density is much smaller than the corresponding relative change in the minority spin density. This implies that the quasichemical potential of majority spin carriers (electrons for definiteness) is essentially unaffected by the current and is given by $\mu_\downarrow \simeq 0$ in $F1$ and $\mu_\uparrow \simeq eV$ in $F2$ [see Fig. 1(b)]. In contrast to this, the quasichemical potential of minority spin electrons presents a significant variation in a region of the order of the spin diffusion length $\sim L_s$ on either side of the domain wall. Because of the low density of minority spin carriers, the minority current $J_<$ ($J_< = J_\uparrow$ in $F1$ and $J_< = J_\downarrow$ in $F2$) is almost entirely a diffusion current given by the classical relation $J_<(x) = eDdn_<(x)/dx$, where D is the diffusion constant. According to the standard drift-diffusion equations, the density deviations relax exponentially to zero as one moves away from the domain wall: $\Delta n_\sigma(x) = \Delta n_\sigma(\pm d/2)e^{-|x\mp d/2|/L_s}$ where the lower sign holds in $F1$ and the upper sign in $F2$. Thus we have

$$J_<(\pm d/2) = \mp \frac{eD\Delta n_<(\pm d/2)}{L_s}, \tag{2}$$

where the density of minority carriers is related to their quasichemical potentials by the Boltzmann relation

$$\Delta n_<(\pm d/2) = n_<^{(0)}\{e^{\frac{\mu_<(\pm d/2)-\mu_<(\pm\infty)}{k_BT}} - 1\}. \tag{3}$$

It will be argued below that the quasichemical potential of minority spin electrons on each side of the domain wall adjusts to the quasichemical potential of majority spin electrons on the opposite side, so that

$$\mu_\uparrow(-d/2) \simeq eV, \qquad \mu_\downarrow(d/2) \simeq 0 \tag{4}$$

[see Fig. 1(b)]. Since $\mu_\uparrow(-\infty) = 0$ and $\mu_\downarrow(\infty) = eV$, this leaves us with only one unknown, namely, the potential drop eV. It will also be shown that, for nondegenerate carriers, the spin current $J_s(x) \equiv J_\uparrow(x) - J_\downarrow(x)$ satisfies the condition

$$\frac{J_s(-d/2)}{J_s(d/2)} = \frac{\bar{t}_- + \bar{t}_+ e^{-eV/k_BT}}{\bar{t}_+ + \bar{t}_- e^{-eV/k_BT}}, \tag{5}$$

where $\bar{t}_\pm = \bar{t}_{nf} \pm \bar{t}_{sf}$, and $\bar{t}_{sf}$ and $\bar{t}_{nf}$ are population-averaged transmission coefficients, with and without spin flip [see Fig. 1(c)], which will be defined more precisely below. Thus, the spin current is conserved across a sharp domain wall ($\bar{t}_+ = \bar{t}_-$), and changes sign across a smooth one ($\bar{t}_+ = -\bar{t}_-$).

Combining Eqs. (2)–(5) with current conservation we arrive at our main results. First

$$\frac{J_q}{J_0} = \sinh\left(\frac{eV}{k_BT}\right)\left[1 + \frac{\bar{t}_{sf}}{\bar{t}_{nf}}\tanh^2\left(\frac{eV}{2k_BT}\right)\right], \tag{6}$$

where $J_0 \equiv 2eDn_<^{(0)}/L_s$. For $\bar{t}_{sf} = 0$ this reduces to the equation [14] derived in [5], while for $\bar{t}_{nf} = 0$ we get $V = 0$ as expected for a ballistic conductor. In the linear regime $eV/k_BT \ll 1$ this formula leads to the well-known interfacial resistance of Fert and Valet [15]. Second, in the immediate vicinity of the domain wall the spin current is given by

$$\frac{J_s}{J_0} = 2\sinh^2\left(\frac{eV}{2k_BT}\right)\left[1 \pm \frac{\bar{t}_{sf}}{\bar{t}_{nf}}\tanh\left(\frac{eV}{2k_BT}\right)\right], \tag{7}$$

where the upper sign holds in $F2$ and the lower sign in $F1$. We see that spin-flip processes cause the appearance of an odd-in-voltage component of the spin current, whereas, for $\bar{t}_{sf} = 0$, the spin current is an even function of V [5]. Notice that even for $V \to 0$ the spin current is a nonlinear function of V.

Shown in Fig. 2(a) is the spin current in $F1$, in Fig. 2(b) the charge current, and in Fig. 2(c) the ratio of the two. The curves correspond to several different values of $\bar{t}_{nf}/\bar{t}_{sf}$.

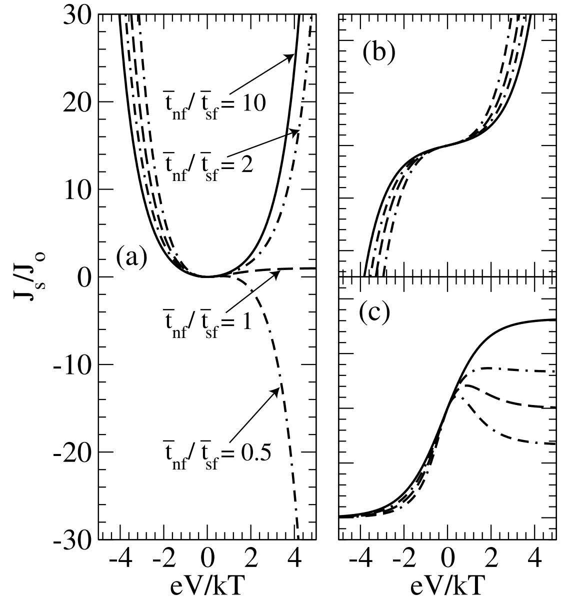

FIG. 2. (a) Spin current in $F1$, (b) charge current, (c) ratio of spin current to charge current vs voltage for $\bar{t}_{nf}/\bar{t}_{sf} = 10, 2, 1, 0.5$. For $\bar{t}_{nf}/\bar{t}_{sf} > 1$, transport is dominated by minority spin injection in the entire range $0 < eV < 4kT$.

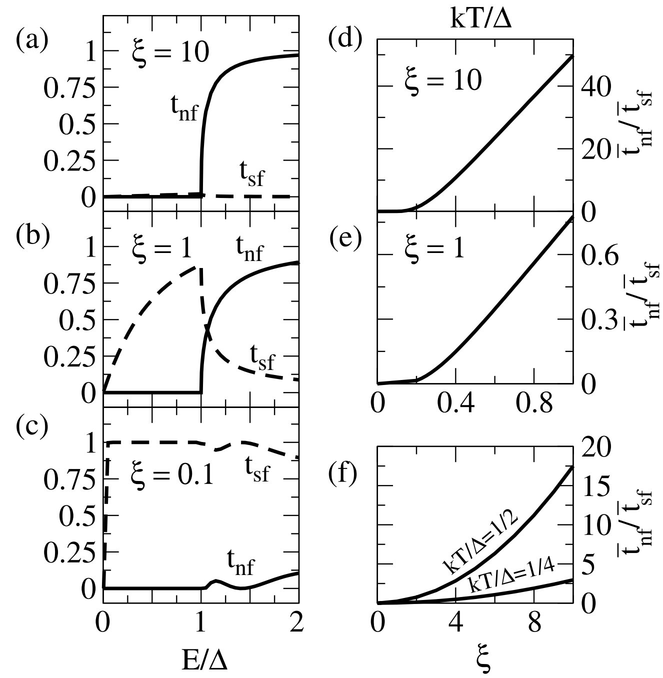

FIG. 3. (a)–(c) Energy dependence of transmission coefficients for $\xi = 10$, 1, and 0.1, respectively. The zero of the energy coincides with the lowest spin-split band. (d),(e) Ratio of the population-averaged non-spin-flip to spin-flip transmission coefficients ($\bar{t}_{nf}/\bar{t}_{sf}$) vs temperature for $\xi = 10$ and 1, respectively. (f) $\bar{t}_{nf}/\bar{t}_{sf}$ versus ξ for $k_BT/\Delta = 0.25$ and 0.5.

The trends for the spin and charge currents described above are evident in Fig. 2; specifically the charge current is always odd in V, whereas the spin current is even in the absence of spin flip. When spin flip dominates, the spin current becomes odd as well. The spin current in $F2$ is related to that in $F1$ according to the following relation: $J_s(F2; V) = -J_s(F1; -V)$. As $\bar{t}_{nf}/\bar{t}_{sf}$ becomes smaller, the "leakage current" between the two majority bands becomes significant, and the odd in V term in the spin current begins to dominate. Over the entire range shown of $\bar{t}_{nf}/\bar{t}_{sf}$ the relationship between J_q and V is highly nonlinear, indicating ballistic transport. Thus ballistic transport itself is not a sufficient condition for maintaining spin polarization in transport across a domain wall.

As discussed in the introduction, we calculate the transmission/reflection coefficients from the exact numerical solution of the Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} - \frac{\Delta}{2}\begin{pmatrix} \sin\theta(x) & \cos\theta(x) \\ \cos\theta(x) & \sin\theta(x) \end{pmatrix}\right]\begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} = E\begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix}, \tag{8}$$

where the electric potential drop is neglected in comparison to the spin splitting Δ. The technique of solution is the same as used in Ref. [7]. Sample results are shown in Figs. 3(a)–3(c) for three different values of the dimensionless parameter $\xi = \hbar\pi/2d\sqrt{2m\Delta} = 10$, 1, and 0.1, corresponding to sharp, intermediate, and smooth domain walls, respectively. Recent experiments [10] suggest the width of domain walls in artificial nanostructures can be as small as 1 nm, giving $\xi \sim 1$ for an effective mass m equal to the electron mass and a spin splitting $\Delta = 100$ meV. Domain walls thinner than 20 nm have already been inferred in thin GaMnAs layers [16].

Figures 3(d)–3(f) show the behavior of the key ratio $\bar{t}_{nf}/\bar{t}_{sf}$ as a function of temperature and thickness. As expected, $\bar{t}_{nf}$ vanishes at low temperature, because, in this limit, there are no incident states above the exchange barrier to provide minority spin injection. The spin diode is a thermally activated device (as a *p*-*n* diode is); thus higher temperature is favorable to its performance. Figures 3(d) and 3(e) support this view by showing that minority spin injection dominates only above a certain temperature, which depends on domain wall thickness, but is typically larger than $0.2\Delta/k_B$ (note that at this temperature the system is $> 99\%$ spin polarized).

We now come to the justification of the matching condition (5) and the calculation of the quasichemical potential offset. We begin with the former. In the spirit of the Landauer-Büttiker formalism we treat the ferromagnetic regions $F1$ and $F2$ as two reservoirs of spin-polarized electrons at chemical potentials $\mu_1 = 0$ and $\mu_2 = eV$ which inject down- and up-spin electrons, respectively, into the domain wall region. The small density of minority spin carriers is neglected in the following argument. The components of the current due to electrons with energies in the range $(E, E + dE)$ on the two sides of the domain wall are given (in units of e/h) by

$$\begin{aligned} j_{1\downarrow}(E) &= -[1 - r_{nf}(E)]f_{1\downarrow}(E) + t_{sf}(E)f_{2\uparrow}(E), \\ j_{1\uparrow}(E) &= r_{sf}(E)f_{1\downarrow}(E) + t_{nf}(E)f_{2\uparrow}(E), \\ j_{2\uparrow}(E) &= [1 - r_{nf}(E)]f_{2\uparrow}(E) - t_{sf}(E)f_{1\downarrow}(E), \\ j_{2\downarrow}(E) &= -r_{sf}(E)f_{2\uparrow}(E) - t_{nf}(E)f_{1\downarrow}(E), \end{aligned} \quad (9)$$

where r_{nf} and r_{sf} are the non-spin-flip and spin-flip reflection probabilities, related to t_{nf} and t_{sf} by the unitarity condition $r_{nf} + r_{sf} + t_{nf} + t_{sf} = 1$, and $f_{1\sigma}$ and $f_{2\sigma}$ are shorthand notations for the equilibrium distributions of σ-spin carriers in $F1$ (label 1) and $F2$ (label 2), respectively. Notice that, assuming Boltzmann statistics, $f_{1\downarrow} = f_{2\uparrow}e^{-eV/k_BT}$ because $\mu_{2\uparrow} - \mu_{1\downarrow} = eV$ (see Fig. 1). We also find that the spin-flip reflection coefficient r_{sf} is extremely small at all energies and thicknesses, and can therefore be safely neglected. With this approximation, combined with the unitarity condition, it is easy to show that the energy-resolved currents are given by $j_{s1(2)}(E) = [t_{-(+)}(E) + t_{+(-)}(E)e^{-eV/k_BT}]f_{2>}(E)$. Noting that $f_{2>}(E) \propto e^{-E/k_BT}$ and integrating over energy we see that the total current $J_{s1} = \int_0^\infty j_{s1}(E)e^{-E/k_BT}dE$ is equal to $A(\bar{t}_- + \bar{t}_+e^{-eV/k_BT})$ where the average transmission coefficients are defined as

$$\bar{t}_{nf(sf)} = \frac{\int_0^\infty t_{nf(sf)}(E)e^{-E/k_BT}dE}{\int_0^\infty e^{-E/k_BT}dE}, \quad (10)$$

and A is a constant. Similarly $J_{s2} = A(\bar{t}_+ + \bar{t}_-e^{-eV/k_BT})$. The ratio J_{s1}/J_{s2} is thus given by Eq. (5).

To justify the quasichemical potential offset condition, Eq. (4), we notice that, neglecting the small spin-flip reflection terms in Eq. (9), the minority spin currents $j_{1\uparrow}$ in $F1$ and $j_{2\downarrow}$ in $F2$ arise entirely from the injection of majority spin electrons from the opposite side. In view of our ballistic assumption, it is therefore natural to ascribe to the minority carriers the same chemical potential that the majority carriers have on the opposite side of the domain wall.

In summary, we have shown that both the thickness and the temperature have a profound influence on the nonlinear transport properties of a ferromagnetic domain wall. We have derived analytical formulas, Eqs. (6) and (7), for the charge and spin currents of this "magnetic junction" under physical assumptions similar to the ones from which the Shockley equations of a classical *p-n* junction are derived. These formulas indicate a new transport regime, where charge transport is ballistic, but spin polarization is lost. Equations (6) and (7), together with microscopic calculation of the population-averaged transmission coefficients, can be used to assess the effectiveness of unipolar spin-diode devices in realistic circumstances.

We gratefully acknowledge support from NSF Grant No. DMR-0074959 and from DARPA/ARO DAAD19-01-1-0490.

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[13] The electron spin is, strictly speaking, neither "up" nor "down" with respect to the local magnetization, but in a mixture of the two orientations. However, the relative phase of the up and down components of the wave function varies rapidly in space due to the large difference between the up- and down-spin wave vectors $|k_\uparrow - k_\downarrow| \simeq \frac{2m\Delta}{\hbar^2(k_\uparrow + k_\downarrow)}$. The rapid dephasing causes the transverse component of the spin density to average to zero on a length scale much shorter than the longitudinal spin diffusion length. Therefore the carriers' spin can be safely assumed to be either parallel or antiparallel to the local magnetization.
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