Public Abstract

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Title: Potential theory and harmonic analysis methods for quasilinear and Hessian equations

The existence problem is solved, and global pointwise estimates of solutions are obtained for quasilinear and Hessian equations of Lane–Emden type, including the following two model problems:

$$-\Delta_p u = u^q + \mu, \qquad F_k[-u] = u^q + \mu, \qquad u \geq 0,$$

on $\mathbb{R}^n$, or on a bounded domain $\Omega \subset \mathbb{R}^n$. Here Δ_p is the p-Laplacian defined by $\Delta_p u = \operatorname{div}(\nabla u |\nabla u|^{p-2})$, and $F_k[u]$ is the k-Hessian defined as the sum of $k \times k$ principal minors of the Hessian matrix $D^2 u$ ($k = 1, 2, \ldots, n$); μ is a nonnegative measurable function (or measure) on Ω.

The solvability of these classes of equations in the renormalized (entropy) or viscosity sense has been an open problem even for good data $\mu \in L^s(\Omega)$, $s > 1$. Such results are deduced from our existence criteria with the sharp exponents $s = \frac{n(q-p+1)}{pq}$ for the first equation, and $s = \frac{n(q-k)}{2kq}$ for the second one. Furthermore, a complete characterization of removable singularities for each corresponding homogeneous equation is given as a consequence of our solvability results.

Our methods are based on systematic use of Wolff's potentials, dyadic models, and nonlinear trace inequalities. We make use of recent advances in potential theory and PDE due to Kilpeläinen and Malý, Trudinger and Wang, and Labutin. This enables us to treat singular solutions, nonlocal operators, and distributed singularities, and develop the theory simultaneously for quasilinear equations and equations of Monge-Ampère type.