understanding the roles of money, or when is the friedman rule optimal, and why?*

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Abstract

In this paper, we study the optimal steady state monetary policy in overlapping generations (OG) models. In contrast to economies populated by infinitely-lived representative agents (ILRA), the Friedman Rule is frequently not the policy that maximizes the welfare of two-period lived consumers. Our principal goal is to understand why the Friedman Rule is suboptimal in OG economies. To this end, we construct a mechanism–specifically, a monetary policy regime–that renders money useless in the sense of executing intergenerational transfers. Under this governmental regime, we show that the optimal monetary policy is the Friedman Rule. Our finding is robust to alternative rationales for valued fiat money; specifically, whether money is held voluntarily or involuntarily.

1   Introduction

The infinitely-lived representative agent (ILRA) and the overlapping-generations (OG) models are the principal models that researchers use to study economies in which fiat money is valued. One question draws considerable attention from these researchers; that is, What is the optimal steady state monetary policy? In ILRA model economies, the Friedman Rule is the unique optimal monetary policy.[1] The Friedman Rule stipulates that the government shrinks the stock of money at a rate such that return equality is satisfied. The optimal steady state monetary policy is more complicated in the OG setup. For one thing, there is more than one reasonable definition of optimality because the initial old and two-period lived agents, for instance, are heterogeneous groups and each group is affected differently by monetary policy. The upshot is that the policy deemed optimal under a particular definition may not be unique. More importantly for our purposes, the optimal policy (or policies) may be quite different from the Friedman rule.[2]

Our analysis proceeds in two stages. Throughout, the Friedman Rule serves as a benchmark, or reference point, as we compare the optimal steady state policies across the ILRA and OG setups. In the first stage, we review the optimality properties of the Friedman Rule in a standard OG model, under two different and widely used assumptions about money demand. In addition, we consider two alternative objectives for the government. Under the objective that seeks to maximize the welfare of agents alive in the initial period, we show that the Friedman Rule is the optimal policy, but it may not be unique. Under the objective that seeks to maximize the welfare of two-period lived agents, we demonstrate that the Friedman Rule is not optimal. In this case, the optimal policy may involve holding the money stock constant, or even allowing it to rise over time.

In the second stage of our analysis, we identify the key role of money in OG models that can account for why the Friedman Rule is not the optimal monetary policy. In order to confirm this identification, we devise an unusual government policy regime that eliminates one of money's non-trivial transaction role in the OG model. If the government adopts such a regime, we show that the Friedman Rule policy is unambiguously optimal.

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[1] For a partial listing, see Friedman (1969), Grandmont and Younes (1973) and Townsend (1980) and Kimbrough (1986). Phelps (1973) and others made the case that the Friedman Rule is suboptimal when non-distortionary taxes are excluded from the set of financing alternatives. However, Christiano, Chari, and Kehoe (1986) and Correia and Teles (1996, 1999) derive conditions in which the Friedman Rule survives the introduction of distortionary taxes.

[2] We are confining our attention to OLG models in which there are assets that dominate fiat currency in real return rate in any steady state in which the stock of fiat currency is held constant.

Freeman (1993) also attempts to explain why there is a difference between optimal policy prescriptions in the ILRA and OG setups. He studies an OG specification with a bequest motive that may or may not be active. With an operative bequest motive, the OG setup exactly mimics the ILRA model and Freeman demonstrates that the Friedman rule is optimal. However, in the version in which the bequest motive is not operative, he finds that the optimal policy is to hold the money stock constant. In both cases, he assumes that the government's goal is to maximize the steady-state utility of the two-period lived households.

Our paper differs from Freeman's in two ways. First, Freeman specifies a model in which fiat money is valued because it enters the utility function. Hence, money is held for voluntarily. We consider both voluntary (stochastic relocation) and involuntary (legal restriction) motives for valuing fiat money. We feel that by examining money's role in both structures, we can further illustrate the basic differences that exist between the ILRA and OG models.

Second, and more important, is the identification approach we take. We use the fact that money serves as a means of executing intergenerational transfers in the OG setup. Money's transaction role can account for why the Friedman Rule is suboptimal in the OG setup. Freeman recognized the intergenerational link. Indeed, he demonstrates that the intergenerational linkage, in the form of preferences for bequests, can obtain a standard OG setup that is a special case of the ILRA model. We agree that intergenerational linkages play the important role. Because money inherently embodies this intergenerational link, we develop a monetary policy regime that mutes money's role as a means of conducting intergenerational transfers. With this regime, we show that the Friedman Rule is the unique optimal monetary policy in the standard OG setup. Our chief contribution consists of three steps. First, money serves two concurrent roles in the OG setup: as a store of value and as a means of executing intergenerational transfers.[3] Second, we construct a mechanism such that money's role as a means of executing intergenerational transfers is trivialized. Third, we demonstrate that different policy prescriptions disappear under the new mechanism. In terms of the optimal steady state monetary policies, we can show that there are no differences between the two principal model economies used to do monetary policy analysis. Thus, our findings stress the role of money rather than preferences can account for differences between the ILRA and OG model setups.

We proceed as follows. In section 2, we offer a detailed discussion, highlighting the key features of the two classes of models. Our primary aim is to identify the principal characteristics of the two models.

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[3] We are not claiming that we are the first to recognize that money serves two roles concurrently in the OG setup. Rather, it is the first step in our logical argument.

Our goal is not to ignore particular insights offered by researchers, but to establish a broad set of common features that are present in the two classes of models. In section 3, we lay out an overlapping generations model with a standard monetary policy regime. We study the welfare properties of monetary policy when banks hold currency to satisfy reserve requirements. We describe the simulator monetary regime, and we repeat our optimal-policy analysis for that regime in Section 4. We also conduct numerical experiments on the size of the welfare effects of monetary policy under the two regimes. In section 5, we repeat the analysis conducted in sections 3 and 4 (except for the numerical experiments) in an environment where banks hold currency in order to help households overcome liquidity problems. In section 6, we present a brief summary and some concluding remarks. The proofs of our major results appear in the appendix to the paper.

2 The Friedman rule in two types of models: an overview

2.1 Step 1: Comparison of ILRA and OG models

2.1.1 ILRA models

Since the properties of the Friedman rule in ILRA models have been analyzed exhaustively, we will not conduct another such analysis in this paper. Instead, we confine ourselves to providing a brief description to account for why the Friedman rule is optimal in the ILRA model. For the purpose of this description, we will assume that real money balances enter the utility function of the representative agent. However, the description is easy to reformulate for other common money demand assumptions.

In the typical ILRA monetary model, the representative agent is endowed with a stock of fiat currency at the first date. The agent must decide to hold this currency at the first date, and all future dates. (The agent must also decide to hold any additions to the stock offered for sale by the government.) At the first date, the agent's endowment of currency finances its currency holdings. At later dates, the agent's previously held money balances–that is, money previously accumulated–finance money holdings during the current period.[4] The implication is that there is no need for the agent to reduce its consumption, or its holdings of real assets, in order to hold money. Since there is no inherent trade-off between money holding and other activities (consumption, saving) that produce current or future utility, the optimal steady state is one in which the real value of the agent's money holdings is large enough to allow the agent to reach the point of satiation: the point at which the marginal utility

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[4]In practice, the agent simply continues to hold the currency. But we are supposed to imagine the agent spending his currency on goods at the beginning of the period and then using his income to buy currency later in the period.

of real money balances is zero. The agent will continue to hold real money balances until their marginal utility is zero if and only if the agent perceives the opportunity cost of holding real balances as zero. It follows that in the optimal steady state, the real rate of return on money must be equal to the real rate of return on non-monetary assets. The nominal interest rates is zero for all stores of value.

Rate of return equality requires that the government set the gross growth rate of the nominal money stock at a value equal to the reciprocal of the gross real rate of return on nonmonetary assets. It accomplishes this by purchasing and retiring money at each date. These purchases are financed by levying a lump-sum tax on the representative agent. Since the real value of the agent's endowment of fiat money is equal to the present value of the sequence of lump-sum taxes, the money endowment does not add to the agent's net income. And since the agent's opportunity cost of holding money is zero, it faces a consumption-saving decision that is identical to the decision it would face in an otherwise-identical economy without monetary features. Thus, the optimal real allocation in the monetary economy is identical to the real allocation in the non-monetary economy.

Suppose that the government does not follow the Friedman Rule. If the growth rate of the stock of money is too high and the real return on money is too low, then the agent will perceive itself as facing a positive opportunity cost of holding money. It follows that the agent will not choose the satiation level of real balances. Welfare is lower because fewer real balances are held.[5]

2.1.2 OG models

A good way to begin gaining an understanding of the role of money in a standard OG monetary model is to consider the decision problem of the first group of agents that make a nontrivial decision – the first generation of two-period lived households (the initial young). Unlike the ILRA representative agent, these agents, are not endowed with fiat currency. Instead, they must purchase it from the agents who are endowed with it – the initial old households. Since the initial young households must trade goods for currency, rather than being endowed with it, each unit of real currency balances they purchase reduces their consumption and/or their holdings of non-monetary assets by a unit, relative to an equilibrium in

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[5]Additional channels emerge in alternative ILRA models in which money is valued because of, say, a legal restriction. More explicilty, the taxes levied on the agent will be smaller, and the present value of the tax stream will be smaller than the real value of the money endowment. As a result, the agent will perceive the money endowment as experiencing an increase in after-tax income. The agent will choose devote some of this increased income to consumption, producing a steady state in which the saving rate is too low and the capital stock is also too low. The fact that the capital stock is lower than in the optimal steady state will cause the levels of income and consumption to be lower, even thought the agent consumes a larger share of its income.

an analogous non-monetary economy.

What happens to the goods the initial young households use to purchase money? They are consumed by the members of the preceding generation. Similarly, when the initial young households sell their money holdings at the end of their second (and last) period of life, the currency is purchased by the young households from the next generation, who pay for it with goods from their incomes or endowments. Thus, in an monetary OG model, under a standard monetary regime, when agents purchase fiat money they are trading current consumption for future consumption by participating in intergenerational transfers. Since every unit of goods they devote to these transfers is a unit they cannot use to purchase real assets, there is a genuine trade-off between currency and real assets – which, as we have seen, is not true in an ILRA model.[6]

The existence of this trade-off implies that holding money has adverse welfare consequences, even under the Friedman Rule.[7] The basic reason for this is that the underlying return on intergenerational transfers is the rate at which the agent's aggregate income or endowment grows over time: usually, the population growth rate. (Agents may perceive themselves as getting a different return on money, but this will be due to government intervention.) In the OG models we study, the return is lower, in any steady state, than the rate of return on physical assets. Consequently, intergenerational transfers are an inefficient way for agents to trade current consumption for future consumption.

How does the Friedman rule work in an OG model? In an ILRA model, the government uses revenue from lump-sum taxes to retire money at a rate that drives the real return on money up to the rate of return on physical assets. In the OG case, however, equalizing these return rates is not unambiguously optimal. As we have seen, in OG models, under standard monetary policy regimes, the opportunity cost of holding money is not actually zero; agents holding more money get a lower average return rate on their asset portfolios because the return on intergenerational transfers is lower than the return on capital. Consequently, equalizing returns, which causes agents to perceive the opportunity cost of currency-holding as zero, will cause them to hold too much currency. Indeed, if the two-period-lived agents hold currency voluntarily, as they do under one of the two money demand

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[6]When a the representative agent in an ILRA model makes the decision to hold a unit of real balances during a period, it perceives itself as using part of its current income to purchase currency, instead of real assets, and thus as facing a tradeoff between the two. But this is an illusion: in equilibrium, the income the household uses to purchase currency is always provided by the currency it held during the preceding period.

[7]By "has adverse welfare consequences," we mean that the utility a two-period-lived household derives from consumption is always lower, in a steady state in a monetary OG model, than it would be in a steady state of an otherwise-identical model without monetary features.

assumptions we study in our paper, then the gross real return on money that is optimal for them is the gross population growth rate (unity, in our model), which is the return rate on intergenerational transfers. If, on the other hand, these agents hold money to satisfy a legal restriction, then the real return rate on currency that is optimal for them may be lower than the population growth rate, so that inflation may be optimal. This is possible because, under our assumptions, inflation reduces the size of the inefficient intergenerational transfer from the young agents to the old agents. When there is inflation, the young agents get a lump-sum transfer payment from the government that is funded by revenue from seigniorage. This revenue comes from the old agents, who get an artificially low return on their money. Thus, inflation finances transfers from the old agents to the young agents, and these old-to-young transfers partly offset the young-to-old transfers that occur when young agents buy currency from the old agents.[8]

The analysis we have just outlined indicates that the differences between the optimality properties of the Friedman rule in OG vs. ILRA models grow out of a fundamental difference in the role of money in the two models. The difference is that in OG models, money is a vehicle for intergenerational transfers, while in ILRA models, it is not. It may seem to follow that the differences between these models concerning the Friedman rule are inescapable consequences of differences in their structures – just as Freeman's results suggest. After all, in an ILRA model there is only one generation, so money cannot possibly be a vehicle for conducting intergenerational transfers.

The converse, however, may not be true: it may be possible to introduce fiat money into an OG model in a way that does not give money a non-trivial intergenerational-transfer role. In this paper, we will devise a method for introducing fiat money into our model that has this property, and we will demonstrate that, when this method is used, the Friedman Rule is unambiguously optimal.

2.2   Step 2: A preview of our formal analysis

In our formal analysis, we study a simple OG model in which goods can be stored using a technology that delivers a constant real return. The gross return on storage is higher than the gross population growth rate, which is unity. We assume, for simplicity, that all saving is intermediated, whether it is carried out by storing goods or holding fiat money.

This model can accommodate a variety of different money demand specifications. We will study specifications of two fundamentally different types. Under our first specification, fiat money derives its

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[8]The key assumption here is that the lump-sum taxes or transfers are paid or received by the young households, as opposed to old households.

value from a legal restriction: the financial intermediaries (banks) hold it so satisfy a reserve requirement. Under our second specification, fiat money is valued because banks use it to insure agents against liquidity shocks. More specifically, young agents face a risk of being relocated to a place where their banks cannot send them goods. The banks hold money, voluntarily, so they can send it to relocated agents. In both specifications, storage dominates money in real return, except under the Friedman Rule. In both specifications, the allocations supported by the Friedman Rule are Pareto optimal. In the reserve requirements specification, the Friedman Rule is the only optimal policy. In the random-relocation specification, any gross growth rate of the money stock between zero and the Friedman-Rule rate, inclusive, supports a Pareto optimum.

For each specification, however, the policy that maximizes the steady state utility of the two-period-lived households – Freeman's (1993) "golden rule" policy – is different from the Friedman Rule. For the reserve-requirements specification, the golden rule policy features a (gross) money growth rate that is greater than unity, and, thus, produces inflation.[9] For the random-relocation specification, the golden rule policy features a constant money stock, which produces a constant price level.

In both these specifications, the government runs a monetary regime that introduces money into the model in a way that is standard in OG models. The initial old are endowed with fiat currency; when the first generation of two-period-lived agents is young, it uses goods from its endowment to purchase this money (through the banks); when this generation of agents is old, it sells this money to the next group of young agents (again, through the banks); and so on. However, we continue our analysis by studying the welfare properties of the Friedman Rule under an alternative monetary regime that we call the "ILRA simulator" (or simply "simulator") regime. Under this regime, the government runs a kind of reverse social security policy. At a given date, the government collects a lump-sum tax from the old agents and using the proceeds to pay a lump-sum transfer to the young agents. The value of this transfer is chosen so that, in equilibrium, the total transfer received by the young agents (which may be partly offset by a currency-retirement tax, or supplemented by a seigniorage-financed transfer) equals their real money demand; the total tax paid by the old agents, moreover, is equal to the real value of the fiat money they bring into the period.[10] Thus, the tax-transfer policy produces an old-to-young intergenerational transfer that exactly offsets the young-to-old transfer that occurs when the

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[9]This result is very closely related to a result reported by Bhattacharya and Haslag (2001).

[10]In other words, households make their money demand decisions taking these tax-transfer amounts as given. The government chooses these tax-transfer values to ensure that, in equilibrium, the young (old) households take the right decision and demand (supply) as much money as the transfer (tax) they get.

young agents purchase money from the old agents. In equilibrium, the young agents do not transfer goods to the old agents, on net, when they purchase money, even though they purchase money from the old agents: the old agents pay a transfer to the young agents which finances the money purchase. So agents acquire, hold and sell money without participating in intergenerational transfers – as in an ILRA model.

2.3   Step 3: preview of welfare analysis

We find that under the simulator regime, the Friedman Rule for monetary policy is uniquely and unambiguously optimal, under either of our money demand specifications. Thus, when we introduce money into an OG model in a way that is analogous to the way it is introduced in an ILRA model, we get the same results, regarding the optimal monetary policy, that we would get in an ILRA model. The Friedman Rule is suboptimal in the OG setup because of money's role as a means of executing intergenerational transfers. As Freeman finds, the intergenerational linkage is crucial. Instead of resolving the differences between the ILRA and OG setups by altering preferences, we modify the way in which money is introduced into the OG model. In doing so, we show that when money no longer serves as a means of executing intergenerational transfers–that is, we mute the intergenerational linkage in the OG setup embodied in money–the Friedman Rule is optimal.

We conclude the analytical portion of our paper by investigating the dependence of the size of the welfare costs or benefits of different monetary policy rules on the nature of the monetary regime (standard or simulator). We do this by conducting numerical experiments with a plausibly calibrated version of the model, using steady-state utility as our welfare criterion. We find that under the simulator regime, where Friedman-Rule deflation is optimal, when we use reserve-requirements money demand the welfare cost of a policy of zero inflation (a constant money stock) is very small: roughly 0.07 percent of steady state consumption. This result is quite consistent with results reported in the extensive literature on the welfare cost of inflation in ILRA models. When we use random-relocation money demand, the cost of zero inflation is much larger, but still relatively small: a bit more than 2.5 percent of consumption. Under the standard regime, where positive inflation is optimal, the welfare cost of the Friedman rule policy, relative to a zero-inflation policy, is quite large: on the order of 10 percent of steady state consumption, for both sets of money demand assumptions. Finally, we use a multi-period generalization of the model, with reserve-requirements money demand, to verify that these results are robust to increasing the number of periods in agents' lives.[11]

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[11]Our standard-regime results are broadly consistent with recent work by Bullard and Russell (2000), who report very

3   General Environment

We start by describing some of the important features of our model that are common to the two money demand assumption formulations, namely reserve requirements and stochastic relocation. The details of the individual setups will be provided below.

We consider an economy populated by an infinite sequence of overlapping generations. Time is discrete, and denoted by $t = 1, 2, \ldots$. At each date a unit-mass continuum of two-period-lived consumers is born ("generation t"). Consumers are endowed with y goods when young and nothing when old.[12] Generation-$t \geq 1$ consumers derive satisfaction by eating units of the single consumption good during each of the two periods of life. We assume these preferences are representable by an intertemporal utility function U with the form

$$U(c_{1t}, c_{2,t+1}) = \alpha\, u(c_{1t}) + (1 - \alpha)\, E_t\, u(c_{2,t+1}), \tag{1}$$

where $\alpha \in (0, 1)$ and u is an atemporal utility function with standard features. Here c_{1t} represents the consumption of a representative member of generation t when young and $c_{2,t+1}$ represents the consumption of the same household when it is old; note that $c_{2,t+1}$ may be state-contingent. In particular instances, we will employ the simple and widely used assumption that $u(c) = \ln c$.

At date $t = 1$, the economy is also populated by a unit-mass continuum of consumers that live only one period (the "initial old".) On behalf of the initial old, banks are endowed with the initial stock of fiat money, denoted $h_0 > 0$. At each date $t \geq 1$ there is a positive nominal stock of fiat money h_t in circulation at the end of the period. The price level at date t – units of fiat money exchanged per unit of the consumption good– is denoted p_t. We confine ourselves to the study of "monetary" equilibria in which p_t is finite for all $t \geq 1$. The gross real rate of return on money acquired at date t is $R_t^m \equiv p_t/p_{t+1}$. The gross inflation rate is $\Pi_t = 1/R_t^m = p_{t+1}/p_t$ and the net inflation rate is $\pi_t = p_{t+1}/p_t - 1$.

Starting at date $t = 1$, the government conducts monetary policy by changing the nominal stock of fiat money at gross rate $Z > 0$ per period, so that $h_t = Z\, h_{t-1}$ for all $t \geq 1$. Let $z \equiv Z - 1$. If $z > 0$ then the government uses the additional money it issues to purchase goods, which it gives to current

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large welfare costs of inflation in a calibrated multi-period OLG model. However, their approach to calibration is quite different from ours.

[12] It is equivalent to think of the endowment as goods or as productive time. In the latter interpretation, the y goods is interpreted as the factor payment received for inelastically supplying this productive time to the labor-only production technology.

young consumers in the form of lump-sum transfers. If $z < 0$ then the government collects lump-sum taxes from the current young consumers, which it uses to retire some of the money. We use τ_t to denote government's activities with $\tau > 0$ (< 0) referred to as a transfer (tax). Hereafter, we refer to this as the standard money-growth regime. The government's budget constraint is

$$\tau_t = \frac{h_t - h_{t-1}}{p_t} = \left(1 - \frac{1}{Z}\right) \frac{h_t}{p_t} \quad \forall t \geq 1. \tag{2}$$

The asset holdings of young households are intermediated, costlessly, by perfectly competitive banks. These banks hold portfolios of fiat money and physical assets, which consist of stored goods. The gross real return on the physical assets acquired at date t is denoted R_t. We confine ourselves to the study of equilibria in which $R_t \geq R_t^m$ for all $t \geq 1$. Households' deposits in the banks are denoted d_t. The banks divide their deposits between stored goods k_t and real balances of fiat currency m_t, so that

$$d_t = m_t + k_t. \tag{3}$$

Since banks are competitive profit-maximizers, the real return rate paid on the deposits equals the real return rate received on the bank's assets. The rate of return on bank deposits, which may be state contingent, is denoted R_t^d. In equilibrium, the zero-profit condition is

$$R_t^d = \frac{R_t^m m_t + R_t k_t}{d_t}. \tag{4}$$

In the market for fiat money, banks' demand real balances and the government supplies nominal balances. After deflating the nominal balances by the price level, we write the market-clearing condition as

$$m_t = \frac{h_t}{p_t} \quad \forall t \geq 1. \tag{5}$$

Each consumer faces the following pair of budget constraints:

$$c_{1t} + d_t \leq y - \tau_t \tag{6}$$

$$c_{2,t+1} \leq R_t^d d_t, \tag{7}$$

where the second constraint may be state contingent (see the model with stochastic relocation described below). Given our assumptions about the consumer's preferences, these constraints will be met with equality in equilibrium.

The initial old are endowed with a maturing deposit account at the bank. The real value of this deposit is denoted d_0. This value consists of the real value of the initial stock of fiat currency h_0 and the real return on the initial stock of physical assets $k_0 \geq 0$. That is,

$$R_0^d d_0 = \frac{h_0}{p_1} + R_0 k_0, \tag{8}$$

where R_0 is the gross real return rate on the initial physical asset holdings of the banks. The budget constraint of an initial-old consumer is simply

$$c_{21} \leq R_0^d d_0, \tag{9}$$

where c_{21} represents its consumption. These consumers are assumed to prefer more consumption to less, so this constraint will also be met with equality in equilibrium.

4 Linear storage with reserve requirements

In this particular specification, the two-period-lived consumers born at dates $t \geq 1$ face nonstochastic decision problems. Their endowment, y, consists of ω units of the consumption good. The physical asset in the economy is stored goods. Storage of a quantity of goods κ at any date $t \geq 1$ produces a real return of $X \kappa$ at date $t + 1$. We define $x \equiv X - 1$ and assume $X > 1$ for all $t \geq 1$.

Banks accept deposits at dates $t \geq 1$ hold fiat currency because they are legally required to do so. Their nominal fiat money holdings must comprise a fraction no lower than θ of the nominal value of their deposits. It follows that

$$m_t \geq \theta \, d_t. \tag{10}$$

Since the banks maximize their profits, in equilibrium we have $m_t = \theta \, d_t$ whenever $X > R_t^m$. And since we have already assumed $R_t \geq R_t^m$, equation (4) gives us

$$R_t^d = (1 - \theta) X + \theta R_t^m \tag{11}$$

in equilibrium.

We assume $k_0 = 0$, so c_{21}, the consumption of the initial old, is financed entirely by their real balances of fiat money. We have

$$c_{21} = m_0 \equiv \frac{h_0}{p_1}. \tag{12}$$

We confine ourselves to the study of steady state competitive equilibria (or simply "steady state equilibria") and their relationship to steady state allocations that may not be competitive equilibria.

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A steady state allocation consists of a consumption bundle (c_1^*, c_2^*) for all two-period-lived consumers, with $c_1^* \geq 0$ and $c_2^* \geq 0$, a non-negative consumption quantity c_{21}^* for the initial old, and a storage value $k^* \in [0, \omega]$, for each date $t \geq 1$, that are jointly feasible, in the sense of satisfying the resource constraints[13]

$$c_1^* = \omega - k^* - \psi, \qquad c_2^* = X\,k^* + \psi, \qquad c_{21}^* = \psi$$

where $\psi > 0 \; (< 0)$ is a lump-sum intergenerational transfer from the young to the old (from old to young).

4.1   Optimal monetary policy: reserve-requirement economy

Given a monetary policy rule Z^* and the associated nominal money supply sequence $\{h_t^*\}_{t=1}^{\infty}$, a steady state competitive equilibrium consists of a sequence of positive, finite price levels $\{p_t^*\}_{t=1}^{\infty}$, date-invariant positive market return rates R^{m*} and R^{d*}, a date-invariant tax or transfer τ^*, date-invariant nonnegative money and storage demand values m^* and k^*, and date-invariant nonnegative consumption and deposit values (c_1^*, c_2^*), c_{21}^* and d^*.[14]   Given the return rates and the transfer, the consumption and deposit values must maximize consumers' utility subject to met-with-equality versions of their budget constraints (6)-(7) for the two-period-lived consumers and (8)-(9) for the initial old. The values τ^* and m^* must satisfy equation (2) with m^* replacing h_t/p_t.   It follows from equation (5) and the money growth rule that $R^{m*} = 1/Z$.   The values m^*, k^* and d^* must satisfy equation (3) and condition (10).   If $X > R^m$ – the case where the reserve requirement is binding – then condition (10) must be satisfied with equality. The values R^{m*} and R^{d*} must satisfy equation (4).

Proposition 1 The unique Pareto optimal monetary policy is $Z = 1/X < 1$, so that $R^{m*} = X$ and $\tau^* < 0$.

Since the reserve requirement is not binding under this policy, it supports a continuum of steady state equilibria involving different levels of taxes and real money balances.[15]   Our first result is that

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[13]By requiring $k^* \in [0, \omega]$ we restrict our definition of stationary allocations to allocations that could be reached, starting from date 1, without storage of goods by the government or by old households.

[14]Throughout this analysis, we consider finite values of Z.

[15]This result is derived by Sargent and Wallace (1985) for a different but closely related model.   They go on to discuss the contrast between the indeterminacy the Friedman rule produces in many ILRA models and the indeterminacy it produces in their model.   In ILRA models, the levels of real balances and taxes are indeterminate, but the consumption allocation is uniquely determined.   In their model (and ours), the consumption allocation is also indeterminate.

the consumption allocations supported by all these steady states are Pareto optimal, and they are the only Pareto optimal allocations supportable as steady state competitive equilibria. This result is not new: versions of it have been stated and proved by Wallace (1980) and Smith (1991).

The logic behind this result is simple but revealing. In this economy, a hypothetical social planner has two options for reallocating goods from young consumers (who are endowed with goods) to old consumers: physical storage and intergenerational transfers. Storage is more productive than transfers: each unit of goods stored from the endowment of a young consumer makes $X > 1$ units of goods available to an old consumer. In contrast, a single unit of goods transferred from a young consumer yields only one unit to an old consumer. Storage, however, does not provide any benefits to the initial old consumers, who receive the very first transfer in the stationary sequence of intergenerational transfers. One might imagine the planner proceeding in two stages: first, deciding the sign and magnitude of the intergenerational transfer, and thus the size of the subsidy to (or tax on) the initial old; next, choosing a volume of storage that maximizes the utility of the two-period-lived consumers, given the size of the transfer. The planner chooses an allocation like the one displayed in Figure 1, at which the consumers' marginal rate of substitution between current and future consumption is equal to the rate of return on storage.

In a competitive equilibrium in this economy, an amount $m^* + \tau^*$ is extracted from each two-period-lived consumer to fund an intergenerational transfer: m^* through the bank's holdings of fiat money, which it purchases from the old consumers, and τ^* in the form of a lump-sum tax (or transfer, if negative) of goods. The consumers' actual rate of return on this intergenerational transaction is unity, since the tax or transfer is just large enough to make up the difference between unity and R^{m*}, the market gross return on money. However, the consumers view themselves as earning a return of R^{d*} on all their saving, where R^{d*} is the reserve-ratio-weighted average of the return rate on storage and the market return rate on money. They also view themselves as having an net endowment of $\omega - \tau^*$, instead of $\omega - (m^* + \tau^*)$. In equilibrium, the point they choose along the resulting budget set also lies along the social planner budget line associated with intergenerational transfer $\psi = m^* + \tau^*$. But if $R^m < X$, so that $R^d < X$, then they choose a bundle southeast of the optimal bundle – a bundle that is supported by too little saving, and too little storage, relative to the bundle that is optimal given a. They do this because a legal restriction (the reserve requirement) forces them to devote a fraction of any additional saving to money. The social planner does not face this restriction.

In a steady state equilibrium under the Friedman rule, $R^m = X = R^d$. So the household budget line coincides with the social planner budget line associated with the intergenerational transfer $\psi = m^* + \tau^*$.

As a result, Friedman rule equilibria are Pareto optimal. Another unique feature of Friedman rule equilibria is that banks are indifferent between holding fiat money and storing goods. As a result, the reserve requirement is not binding, and there are a continuum of real money values m and associated tax values τ consistent with equilibrium. The smallest m-value exactly satisfies the reserve requirement. The largest is equal to young households' after-tax saving, so that there is no storage whatever. All these equilibria are Pareto optimal, but as m (and $m + \tau$) increases, the welfare of the initial old consumers increases at the expense of the welfare of the two-period-lived consumers.

4.2   Welfare properties

Although non-Friedman-Rule steady states are not Pareto optimal, it does not follow that they produce lower levels of welfare for two-period lived consumers. With a lower return on money, there is a lower lump-sum taxes on young consumers, and they start producing transfers to these consumers when the associated inflation rates turn positive. Since the reserve requirement is binding, banks increase their currency holdings by only a fraction of the decrease in the direct intergenerational transfer caused by the tax. So a lower real return on money means a smaller net intergenerational transfer. Thus, as the inflation rate decreases the steady state allocations lie on social planner budget lines that would produce higher levels of utility for the two-period-lived consumers, if the planner was choosing the allocations. Of course, the distortion associated with lower return on money causes the consumers to choose points on these budget lines that are less attractive than the points the planner would choose. It seems quite conceivable, however, that two-period-lived consumers will realize an increase in welfare resulting from a lower real return on money, at least over some range. And it also seems possible that the gross real return on money that is optimal for these consumers might be less than unity, so that the optimal steady state involves a positive net rate of inflation.

An alternative way of making the same point would be to contrast the zero-inflation steady state to a steady state with positive inflation. In the zero-inflation steady state there is no direct intergenerational transfer (that is, there is no lump-sum tax on the young), but there is an indirect young-to-old transfer whose size is exactly equal to banks' real reserve holdings. Positive inflation rates induce a transfer in the opposite direction, as the reduction in the rate of return on money funds a transfer to the young consumers. Thus, inflation produces an old-to-young intergenerational transfer that partly reverses the young-to-old transfer associated with the reserve requirement.[16] And since intergenerational transfers

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[16]If the inflation rate is infinite, so that the real rate of return on currency is zero, then the two transfers exactly offset each other, so that the net intergenerational transfer is zero.

are an inefficient way for young consumers to obtain future consumption, inflation has the potential to produce welfare improvements for consumers other than the initial old.[17]

We are able to show, under logarithmic utility, a steady state with zero inflation always produces higher levels of welfare for the two-period-lived consumers than a steady state under the Friedman Rule. We are also able to show that the steady state that maximizes the welfare of the two-period-lived consumers always involves positive inflation.[18]

Proposition 2 Suppose $u(c) = \ln c$. Then a steady state competitive equilibrium with zero inflation produces higher utility for the two-period-lived households than any steady state equilibrium under the Friedman rule. However, there are steady state equilibria with positive inflation that produce higher utility, for these households, than the steady state equilibrium with zero inflation. In particular, if

$$\frac{\theta}{1-\theta} > \frac{x}{\alpha},$$

then a steady state equilibrium in which the (net) inflation rate is

$$\widetilde{\pi} \equiv \frac{(1-\theta)\,x}{\alpha\,\theta - (1-\theta)\,x}$$

produces higher welfare, for these households, than any other steady state equilibrium. Otherwise, a steady state equilibrium with a higher inflation rate always produces higher welfare than a steady state equilibrium with a lower inflation rate.

Proposition 2 establishes that if the required reserve ratio is sufficiently high, relative to the rate of return on storage, then there is a unique positive, finite inflation rate that is optimal for the two-period lived consumers. Otherwise, the optimal inflation rate is infinite, so that the optimal gross real rate of return on money is zero. The intuition here is as follows: given that the rate of return on storage is high, the enforced intergenerational transfer associated with the reserve requirement is very harmful to the two-period-lived consumers. Consequently, a policy that offsets it tends to be very beneficial to them. Such is the case when α is low, with consumers caring deeply about the amount of second-period consumption they lose by being forced to devote part of their saving to a low-return asset. However,

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[17] In an typical ILRA economy with a reserve requirement, an increase in the inflation rate increases the return distortion without providing any offsetting benefits, since the currency reserves represent neither an intergenerational transfer nor any transaction of any kind.

[18] A corollary to this proposition, which is stated and proved in Appendix A, is that increases in the inflation rate always hurt the initial old consumers by reducing the value of the net transfer they receive.

as the reserve ratio increases, the adverse effects of the return distortion increase, and the opportunity to store goods becomes less and less important to consumer welfare.

Figure 2a displays the steady state competitive equilibria under the Friedman rule and under a policy of zero inflation. Figure 2b displays the zero-inflation equilibrium and the equilibrium under the positive-inflation policy that is optimal for the two-period lived consumers. Notice that the net intergenerational transfer $m + \tau$ shrinks as the inflation rate rises, shifting the steady-state budget line to the right; in other words, the budget line is shifted away from the one that the social planner faces given the transfer to the initial old. When the real rate of return on money is lower than the Friedman rule rate, the consumer chooses a bundle that lies southeast of the best bundle along this line, saving (and storing) too little, given the transfer. This happens because the consumer views the tax or transfer τ as subtracting from or adding to its income, and thus bases its consumption decision on its after-tax income $\omega - \tau$. In reality, the present value of the consumer's income is $\widetilde{\omega} \equiv \omega - (m + \tau)\left(1 - \frac{1}{X}\right)$, where the second term represents the loss of income, in present value terms, associated with the net intergenerational transfer $m + \tau$. If $\tau < x\,m$ (because $R^m < X$) then $\widetilde{\omega} < \omega - \tau$ and the consumer's allocation has too much c_1 too little c_2 for a given transfer.[19]

The theoretical results indicate that the Friedman Rule is not the policy that maximizes steady state utility. The numerical exercises indicate that deviations from the Friedman Rule are quantitatively important. [20]

Our findings are consistent with those in the optimal seigniorage literature. Freeman (1987) studies an economy in which households save by depositing funds at financial intermediaries whose assets consist of physical investments (stored goods) and required reserves of fiat currency. The government imposes the reserve requirement in order to finance a real purchase via currency seigniorage. Freeman shows that the jointly (steady state) optimal choices for the reserve ratio and money growth rates is the lowest ratio feasible and an infinite money growth rate. Here, money facilitates intergenerational transfers

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[19]Under our preference assumptions, the optimal level of consumer first-period consumption is invariant to the rate of return on saving. Thus, the saving distortion created by inflation stems from the fact that the consumer mispercieves its income, not from the fact that it faces the "wrong" marginal rate of return.

[20]Our Proposition 2 is even more closely related to a result obtained by Bhattacharya and Haslag (2001). They use the same model, under the fixed-reserve-ratio assumption, to study a situation in which the government must finance a real purchase via some combination of first-period lump-sum taxes and inflation tax. They find that the combination that is optimal for the two-period-lived households always includes some use of the inflation tax. They interpret this finding as an optimal-tax result. But our analysis indicates that the optimality of positive inflation in this model is not a consequence of the distortions produced by other direct taxes.

at a pre-tax return equal to the population growth rate, while physical investment, under Freeman's assumptions, offers a higher return. Thus, money is an inefficient system for reallocating resources from young agents to old agents, and the monetary regime that is best for everyone (except the initial old agents) is the smallest reserve ratio that allows the government to raise the required revenue by taxing its benefits. In this reserve requirements regime, the gross return on reserves is zero, so that the reserves are confiscated by the government and provide no return to the banks or their depositors. The government confiscates the reserves by engineering a hyperinflation. Each period, the intermediaries buy all their reserves from the government, paying exactly the quantity of goods the government needs to finance its purchase. Next period, the hyperinflation has rendered the existing money stock worthless, so the intermediaries who purchase reserves must again buy all of them from the government.

The results highlight the principal driving feature in overlapping generations economies. In this paper, as in Freeman's analysis, money's chief role is as a means of participating in intergenerational transfers. In the standard monetary regime, the intergenerational transfer role is endemic to valued fiat money. Our next goal is to uncouple these features. in an effort to show that the Friedman Rule is the optimal steady state policy once the intergenerational transfer role is rendered trivial.

4.3 The simulator regime

The chief goal of this paper is to shed light on the crucial differences between the prototype ILRA model with valued fiat money and the OG model, emphasizing the differences between the two as models of money. In particular, we are interested in understanding the welfare properties, highlighting the Friedman Rule. To that end, we consider a special (and fairly non-standard) type of monetary regime in an overlapping generations economy. The simulator regime allows an overlapping generations with valued fiat money to simulate the welfare properties of a wide class of economies populated by infinitely lived agents; specifically, (i) the Friedman Rule is always the only Pareto optimum, (ii) every equilibrium under the Friedman Rule supports the same consumption allocation, and (iii) the Friedman Rule is always the unique optimum for the two-period-lived consumers. In contrast to the results in Proposition 2, zero or positive inflation is never are never optimal for two-period-lived consumers. Such contrast will then help us to illuminate the conditions under which Friedman Rule is the welfare-maximizing standard. It bears emphasis here that these three results are not true in the OG model under a "standard" monetary regime. To foreshadow, the simulator regime uncouples money's role as a "contrivance" for arranging intergenerational transfers from its existence as a valued asset.

The simulator regime may be described as follows. Starting at $t = 1$, the government imposes a

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lump-sum tax on the old consumers alive each period. The tax is equal to the equilibrium level of their money holdings. After the government collects the tax, it issues some more (inflation), or retires some (deflation), giving a lump-sum transfer to young consumers. More formally,

$$c_1 + s = \omega - a_1 \tag{13}$$

$$c_2 = R^d s - a_2 \tag{14}$$

and for the initial old

$$c_{21} = m_0 - a_{21} \tag{15a}$$

where a denotes the lump-sum aggregate transfer to young (a_1) or old (a_2). At $t = 1$, the economy is endowed with a stock of money, h_0, that is in the possession of the initial old. The initial old's money holdings are paid to the government as a tax, which the government turns around and gives to the date $t = 1$ young. The government can issue new money and distribute in a lump-sum fashion to the young. Alternatively, money may be purchased from the old.

The bank's balance-sheet identity is represented as follows:

$$s = m + k \tag{16}$$

and the reserve requirement is $m = \theta s$. Because the bank operates in a perfectly competitive setting, the zero-profit condition is $R^d = (1 - \theta) X + \theta R^m$. Period-by-period, the government runs a balanced budget. Formally,

$$\tau_1 = -(1 - R^m) m \tag{17}$$

and

$$t_1 = -R^m m, \quad t_2 = R^m m, \quad \text{and} \quad t_{21} = m_0 \tag{18}$$

so that the aggregate lump-sum transfers are represented as

$$
\begin{aligned}
a_1 &= \tau_1 + t_1 = -m, \\
a_2 &= t_2 = R^m m \\
a_{21} &= t_{21} = m_0.
\end{aligned}
$$

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Perhaps it is easiest to describe the simulator regime in practice. The government levies a regime tax, represented by the (18). For old consumers, the tax is equal to the equilibrium quantity of money held by the old. Indeed, the old consumers use their money holdings to pay the tax. The government then gives this money as a lump-sum transfer to young consumers. In equilibrium the young consumers hold the money.[21] Thus, under the simulator regime, no consumer has to, on net, give up goods to acquire money. Because there is no intergenerational transfer, the simulator regime mimics the role of money in economies populated by infinitely-lived agents. If identical monetary policies result in the same real return to money, it would be seem that consumers in overlapping generations economies would acquire and hold money on exactly the same terms as in the ILRA consumers.

However, money and monetary policy create a return distortion whenever the real rate of return on money is lower than the real rate of return on storage. To see this, suppose there is no inflation, so that $(1 - R^m) \, m = 0$ and $\tau_1 = -\tau_2 = -m$. In this case, the consumer's after-tax endowment bundle lies somewhere on a line with a slope of -1 extending southeast of the pretax bundle $(\omega, 0)$. Thus, the after-tax e-bundle lies above the extension of the no-reserve-requirement budget line (whose slope is $-X$) southeast from $(\omega, 0)$. On the other hand, the reserve-requirements budget line is flatter that the no-reserve-requirements line[22] – its slope is $R^d = (1 - \theta) \, X$ – and it intersects that line at a point that represents the reserve-requirements consumption bundle. This bundle will clearly lie southeast of the optimal bundle from the no-reserve-requirements economy. So the reserve requirement distorts the consumer's intertemporal consumption choice, causing it to consume to much when it is young and too little when it is old. Note that this is the same kind of distortion that is present in the ILRA model with valued money.

4.4 Welfare properties revisited

Suppose, however, that we implement the Friedman Rule, choosing $R^m = X$. In this case, the government budget constraint gives us $\tau_2 = X \, m$, which, in combination with $\tau_1 = -m$, is $\tau_2 = $

[21] An equivalent scenario can be characterized as follows. Suppose that old consumers pay the government in the form of goods. The government transfers these goods to young consumers. Young consumers trade goods for money held by the old consumers. Young consumers and old consumers both have net payments equal to zero. The young consumers end up with money and the old consumers have no money holdings.

Note that the description in the text, the equilibrium price level results in the sum of taxes and transfers netting to zero.

Clearly, the taxes and transfers have a fiscal policy component as well a monetary policy component.

[22] The reserve-requirement budget line is flatter than the no-reserve requirement budget line, that is, $-[(1 - \theta) \, X + \theta] > -X$.

$-X\tau_1$. So the after-tax endowment bundle lies southeast of $(\omega, 0)$ along the extension of the no-reserve-requirements budget line. And since $R^m = X$ implies $R^d = X$, the reserve-requirements budget line has the same slope as that line.[23] So the consumer makes the same consumption choice as in the no-reserve requirements economy. And while the level of money demand is not unique – because the reserve requirement is not binding – the budget line is unique, and so is the equilibrium consumption allocation.

Thus, under this government tax/transfer policy, different policies for the money growth rate have exactly the same welfare implications they have in the simple ILRA model. We have the following proposition.

Proposition 3 In the reserve-requirement economy, the Friedman Rule is the unique policy that supports the first-best optimum if combined with the ILRA simulator regime.

Proof. Under the ILRA simulator policy rule, the Friedman Rule is represented by $R^m = R^d = X$

so that

$$c_2 = Xd - \tau_2$$

and the lifetime budget constraint is

$$c_1 + \frac{c_2}{X} = \omega - \tau_1 - \frac{\tau_2}{X}. \tag{19}$$

With $\tau_1 = -m$ and $\tau_2 = R^m m = Xm$, (19) can be rewritten as

$$c_1 + \frac{c_2}{X} = \omega.$$

which is the same choice problem that the household faces in a non-monetary setting. ∎

The crux of Proposition 3 is that the Friedman Rule is again the unique optimum. The term simulator regime, therefore, is applicable because it "reclaims" a property that holds in a broad class of infinite-horizon representative agent economies. In the steady state ILRA model, consumers choose a quantity of real money balances at date t followed, at date $t+1$ by a nominal money supply contraction. With perfectly flexible prices, the quantity of real money balances stays across time and the Friedman Rule pins down the rate of the contraction that satisfies rate-of-return equality. Under the simulator regime, the government collects money balances lump-sum from old consumers and then returns a smaller quantity of money balances to young consumer. As with the ILRA model, the contraction is crucial for obtaining the rate-of-return equality.

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[23]See Figure 3 for a graphical representation of the budget line in the simulator model.

4.5 Some computational experiments

Here, our goal is to bring some quantitative evidence to bear on money's role as a means for conducting intergenerational transfers. The following computational experiments are structured as follows: we compute the steady-state welfare level under the two alternative policies. One of the policies produces zero inflation: we use this policy as our baseline. The other policy produces either a Friedman-Rule deflation – which turns out to be an inflation rate of -6.5 percent: see below – or an inflation rate of 10 percent. We conduct these experiments for both the standard and simulator regimes. Or theoretical results tell us that under the standard regime, steady state welfare is lower when implementing the Friedman Rule than it is when implementing zero inflation. Conversely, under the simulator regime, steady state welfare is lower when monetary policy produces zero inflation than it is when the policy follows the Friedman Rule.

The results of these welfare comparisons are presented in Table 1. The procedure we use to calculate them is as follows: First we compute the value of lifetime utility, using a standard CES utility function, for a two-period lived consumer under the zero-inflation policy. Denote this value $\bar{U}$. Next, we derive the consumption allocation $\{c_1^*, c_2^*\}$ for a steady state under one of our alternative policies. We proceed by solving the equation

$$\bar{U} = \sum_{j=1}^{2} \beta \frac{\left(\phi\, c_j^*\right)^{1-\gamma}}{1-\gamma}$$

for ϕ. Here, we interpret $(\phi - 1) \times 100$ as the percentage change in the consumption sequence that would make lifetime welfare under the alternative policy the same as under the zero-inflation policy.

We are also interested in quantifying the role that the number of periods in household's lifetime plays in determining the welfare implications of different policies. This analysis does not "approach" the infinite horizon. Rather, our aim is to extend the standard two-period lived consumer so that we can determine whether a consumers that live more periods can materially affect the welfare costs associated with a pair of different policies.[24]

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[24] See Bullard and Russell (1999) for a complete description of the model economy with n-period lived consumers. Details can also be obtained from the authors upon request.

Two things worth noting. First, in the n-period lived consumer, we solve for the steady state consumption sequence, denoted $\{c_j^*\}_{j=1}^{n}$ and we compare lifetime utility by solving the equation $\sum_{j=1}^{n} \beta^{j-1} \frac{(\phi c_j^*)^{1-\gamma}}{1-\gamma}$.

Second, note that storage and time-preference parameters are adjusted so that their annualized values are invariant to the number of periods in which a consumer lives.

Table 1

Welfare comparisons

between alternative monetary policies

	n = 2	
Monetary regime	Friedman rule vs. zero inflation	10% inflation vs. zero inflation
standard	−12.67	1.84
simulator	0.071	−0.023
	n = 60	
Monetary regime	Friedman rule vs. zero inflation	10% inflation vs. zero inflation
standard	−12.18	10.23
simulator	0.48	−1.99

For the results reported in Table 1, we use the following parameter values: $X = 1.07$, $\beta = 0.96$, $\gamma = 1$ (log utility), and $\theta = 0.1$.[25] Two principal results emerge from Table 1. First, under the standard regime the welfare costs of following the Friedman Rule, versus a zero-inflation policy, are quantitatively large. A two-period lived consumer would require a 12.67 percent increase in consumption, relative to Friedman-Rule level, to be indifferent between the Friedman Rule policy and a zero-inflation policy. Although the optimal policy turns out to be a hyperinflation, the welfare benefits of increasing the inflation rate beyond 10 percent are very small. However, the benefit from a policy that produces a 10 percent inflation rate, relative to a zero-inflation policy, is just over 10 percent of the higher-inflation consumption level.

When $n = 60$, the welfare cost of the Friedman Rule is almost unchanged from the two-period case: just over 12 percent of consumption. The benefit from a policy that produces 10 percent inflation is much larger than in the two-period case: a bit more than 10 percent of consumption. In this case, there is an interior optimal inflation rate: its value is close to 34 percent. The welfare benefit from the

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[25]More precisely, $X = 1.07$ and $\beta = 0.96$ are interpreted as annualized values and thus are used for $n = 60$. In all other cases ($n = 60$ is the only one reported), $X = 1.07^{60/n}$ and $\beta = 0.96^{60/n}$. We also annualize the inflation rate, so that a 10 percent (net) inflation rate translates to $R^m = 1.1^{-60/n}$, and so on. We confine ourselves to considering values of n that are multiples of 2. We assume the endowments are $\frac{y}{n}$ per priod for the first $\frac{n}{2}$ periods of a consumer's life and zero per period over the remaining $\frac{n}{2}$ periods.

optimal policy, relative to a zero-inflation policy, is more than 25 percent of consumption. (This value is not reported in Table 1.)

As the net real rate of return on money decreases, the difference between consumers' perceived income and their actual income gets larger and larger. It follows that the difference between their actual level of saving and the optimal level, given the net intergenerational transfer, also gets larger and larger. But the impact of this adverse "substitution effect" on the welfare of the two-period-lived consumers is more than offset by the beneficial "income effect" of the decrease in the size of the net intergenerational transfer. However, at some positive inflation rate – that is, at some net real money return rate less than zero – this balance of costs and benefits may reverse itself, and the level of welfare may start falling as the inflation rate continues to rise. In this case, there is a finite positive inflation rate that is optimal for the two-period-lived consumers.

The second lines of Table 1 report the welfare costs of inflation for the same monetary policy combinations, but under the simulator regime. As we have seen, under this regime the Friedman rule is optimal. As before, we begin by computing the percentage change in steady-state consumption that a two-period lived household would need to receive to be indifferent between zero inflation and the Friedman Rule. As the Table indicates, when $n = 2$ this value – the welfare cost of zero inflation – is only 0.071 percent. The cost of 10 percent inflation, relative to zero inflation is even smaller: only 0.023 percent. Thus, under the simulator regime the welfare cost of deviating from the optimal policy are small. When $n = 60$ the costs are much larger, but they are still small compared to their standard-regime counterparts.[26]

The results from the computational experiments offers insights into two related points. First, these results offer an account for why estimates of the welfare costs of inflation differ so for ILRA models of money and the OG models of money. More specifically, there is a large disparity between the numerical findings offered by Lucas (1988) and Cooley and Hansen (1989) using models populated by infinitely lived consumers and those presented in Bullard and Russel (1999) using models populated by finitely lived consumers. The comparison between the standard and simulator regimes suggest that once one eliminates money's role as a means for conducting intergenerational transfers, the welfare costs of inflation are substantially reduced.

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[26]Our results are in line with previous calculations. For instance, we fined a large welfare benefit from zero, or positive, inflation under the standard regime. Bullard and Russell report that changes in the inflation rate result in large changes in welfare in calibrated, multi-period overlapping generation economies. Under the simulator regime, we find that changes in the inflation rate results in small welfare effects, which is consistent with findings by researchers working with ILRA models with valued money.

Second, to our knowledge, these are the first experiments that can be used to infer the impact that money's role in conducting intergenerational transfers has on the welfare cost of inflation. The difference between the standard and simulator regimes is that the intergenerational transfer role is extricated from the simulator regime. Hence, we are isolating the impact that this role has for the welfare costs of inflation. The numerical evidence suggests that this role is quantitatively important.

4.6 Remarks

It remains to account for why the simulator regime mimics the optimality of the Friedman Rule. Under the simulator regime, consumers can acquire and hold money without participating in intergenerational transfers. Under the standard regime, money is an intergenerational transfer device so that society faces a trade-off between relatively productive capital and relatively unproductive intergenerational transfers.

Consider an off-the-shelf ILRA model in which fiat money is valued because real money balances enter directly into the utility function MIUF).[27] Suppose that the economy is in steady state following a Friedman Rule policy. The representative consumer carries over real balances.[28] Over time, the nominal quantity of money balances contracts as the government collects lump-sum taxes each period that are used to retire money. Such a nominal contraction results in the return to money being equal to the return on other assets.[29] Once we take the government budget constraint into account, note that the tax levied against the infinitely-lived consumer is used finance the retirement of money balances. Indeed, the consumer's initial stock of money balances is offset by future taxes of equal present value. Thus, the infinitely-lived consumer does not view the initial stock of money balances as an expansion of available resources for consumption; that is, the net income of the initial stock of money is zero. Because money has the same return as capital means that the consumer is not punished for holding the initial stock of money balances. Hence, the consumer holds the same quantity of real money balances, making the same consumption/saving decision that it would make in a non-monetary economy. The initial stock-taxation interpretation suggests a version of Ricardian Equivalence engineered exclusively

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[27]This is a slight departure from the overlapping generations model in which money is valued becuase of a reserve requirement. We consider a MIUF specification because it encompasses all other specifications in which fiat money is valued, including a reserve-requirement economy.

[28]In an MIUF setup, this level of money balances is the one that satiates the consumer. In the reserve-requirement, this would be the level that satisfies the reserve requirement. As the reader will see, the level does not meaningfully enter into our analysis of the policy effects. As such, we can consider the most general specification that rationalizes valued money without sacrificing the key insight of what happens in the ILRA monetary economy.

[29]Indeed, the consumer is willing to hold the satiation level of real money balances precisely becuase there is no opportunity cost associated with holding fiat money.

through manipulations of the money stock. The bottom line is that the Friedman Rule uniquely and simultaneously addresses both the rate-of-return issue and the perceived net income issue.

Consider the simulator regime in the overlapping generations setup in which monetary policy follows the Friedman Rule. Because a lifetime consists of two periods, the simulator regime ensures that Ricardian Equivalence holds over a consumer's lifetime; that is, the tax that retires money balances has to be levied when the consumer is old. There is no option of spreading the retirement over an infinite number of periods, as in the ILRA economy. Note that the young consumer receives an monetary endowment that is then collected when the same consumer is old. The return on this asset is exactly equal to the return paid on the other risk-free asset. As with the ILRA model, the simulator regime addresses the rate-of-return issue and the perceived net income issue. Under the simulator regime, the government effectively endows each generation of consumers with money so that there is no private or social cost in allocating goods between storage or money. Thanks to the implementation of the Friedman Rule, consumers do not face any private opportunity costs because the rate of return on money is the same as the rate of return on physical assets. Nor do consumers face any social trade-off between holding money and holding physical assets; consumers do not reduce their purchases of physical assets relative to a non-monetary economy because young consumers do not give goods, on net, to old consumers selling money. In short, there is no intergenerational transfer associated with the exchange of money for goods when the Friedman Rule policy is implemented under the simulator regime. In both the simulator regime and the ILRA policy, monetary policy is conducted so that the government's activities over a consumer's lifetime are matched.

Suppose that something other than the Friedman Rule is implemented under the simulator regime. If the present value of taxes is, for instance, less than monetary endowment. In other words, the money supply does not contract fast enough. We are back in a setting in which a physical asset offers a return that dominates money's rate-of-return. Consumers perceive an increase in their lifetime income because the net present value of their money endowment is positive. With log utility, the increase in perceived income results in an increase in consumption when young. Compared with Friedman Rule allocation, resources available for old-age consumption are smaller because of the lower return to money and the consumer's lifetime welfare is lower.

It remains to compare the standard and the simulator regimes with the overlapping generations model more carefully. Under the standard regime, a young consumer purchases money that is held until old age, using its goods endowment. The consumer gets some goods back when old when it sells money for the next generation's goods. However, the return to the intergenerational exchange–the social

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return–is inherently less than the return to physical assets. By forcing consumers to hold money, they are devoting part of their goods endowment to purchasing an inefficient asset. Under the simulator regime, money is an asset held by young consumers with a matching liability. Young consumers do not give any of their goods endowment to acquire money. Nor do old consumers give any goods when they sell money. Hence, the social return to holding money does not affect the consumer's allocation. When the Friedman Rule is the monetary policy, there is no private return distortion either. Thus, we can state the chief difference between the two regimes as follows: for a given monetary policy, the standard regime implicitly requires the consumer to participate in an intergenerational transfer scheme while the simulator regime relaxes the participation condition.

While our results are very similar to those obtained in Freeman, we can clearly state the different insights offered by our approach. With an operational bequest motive, date-t young consumers receive goods from the old generation that can used to acquire money balances. Moreover, as time progresses, the date-$t+1$ old consumers bequests goods to the young generation that can be used to acquire money balances. The implication is that consumers can acquire and hold money without participating in an intergenerational transfer. Freeman offers a preference-based approach to uncouple valued fiat money from the participation condition. Under the simulator regime, we uncouple the two by letting the government engineer the process of transferring money to the young and taxing away the money from the old.

5 Linear storage with random relocation

In this section, we study an overlapping generations model in which fiat money is valued because of limited communication. Our primary goal is verify that our results are not sensitive to the assumption that money is valued because of the legal restriction. Instead, fiat money is valued because of limited communication. By looking at a model in which money is voluntarily held, we can illustrate and better understand the overlapping generations and ILRA models of money.

Following Bencivenga and Smith (1991), we model limited communication by dividing the economy into two regions. For consumers born at dates $t \geq 1$, half the generation begin their lives in one of these regions; the others begin their lives in the other region. (For the sake of simplicity, we re-normalize the populations so that a unit mass is present at both locations.) At the end of the first period of their lives, after they have made their consumption and saving decisions, a fraction ϕ of the consumers are relocated to the other region. When the young consumers make their decisions they do not know

whether they will be relocated or not, but they do know the value of ϕ. The initial old consumers are divided equally across the two regions.

A consumer that is relocated cannot collect the return on any physical assets, or physical assets that are being held on the consumer's behalf, since goods cannot be transported across regional boundaries. However, a consumer can carry fiat money from one location to the other. In addition, banks can transport fiat money across regional boundaries. So if a relocated consumer owns a claim on fiat money from a bank in its home region – a claim that may be contingent on the consumer's having been relocated – then the bank can pay the claim.

Under the circumstances, there are two strategies a consumer can use to save. First, it can save on its own, storing some quantity of goods and acquiring some quantity of money. The drawbacks of this strategy are that if the consumer is relocated then it must abandon its physical assets, and if the consumer is not relocated then it regrets having acquired the money, whose real return rate, in most steady states, is lower than the real rate of return on physical assets.

An alternative strategy is for the consumer to deposit funds in a perfectly competitive bank. The bank pools the goods deposited by all the consumers and uses them to acquire a portfolio of stored goods and money. It issues claims to the consumers whose nature, timing and size are contingent on their relocation status. If a consumer does not get relocated, then it gets a return rate on its deposit that is funded by the goods the bank has stored. If the consumer gets relocated, then it gets a return on its deposit that takes the form of money; a payment funded by the bank's holdings of money.

It turns out that the latter strategy always dominates the former one – see Bencivenga and Smith (1991) – and we analyze the economy applying the same approach.

If we let R_n represent the gross real deposit return rate received by a consumer from generation t that is not relocated, and R_r the rate for a consumer from generation t that is relocated, then the budget constraints of the consumer are equation (6) plus the following version of equation (7):

$$c_{2n,t+1} = R_{nt}\, d_t \tag{20}$$
$$c_{2r,t+1} = R_{rt}\, d_t$$

The consumer takes R_{nt}, R_{rt} and τ_t as given and chooses d_t in order to maximize

$$U(c_{1t}, c_{2,t+1}) = \alpha\, u(c_{1t}) + (1 - \alpha)\, [(1 - \phi)\, u(c_{2n,t+1}) + \phi\, u(c_{2r,t+1})] \, . \tag{21}$$

Given the deposit d_t entrusted to it by each young consumer, the competitive, zero-profits bank

chooses values m_t, k_t, and γ_t that maximize consumers' second-period expected utility

$$V(c_{2,t+1}) \equiv (1-\phi)\, u(c_{2n,t+1}) + \phi\, u(c_{2r,t+1}) \tag{22}$$

subject to budget constraint (20) and the zero-profits return conditions

$$
\begin{aligned}
R_{nt} &= \frac{X\, k_t + (1-\gamma_t)\, R_t^m\, m_t}{(1-\phi)\, d_t} \\
R_{rt} &= \frac{\gamma_t\, R_t^m\, m_t}{\phi\, d_t}
\end{aligned}
\tag{23}
$$

Here $\gamma_t \geq 0$ represents the fraction of the bank's real money balances that it pays out to consumers who are relocated. In order to attract deposits, the bank must choose γ_t, m_t and k_t to given d_t. We assume, as in the preceding section, that $R_t^m \leq X$. If $R_t^m = X$ then it is readily seen that γ_t, m_t and k_t are indeterminate. In this case, the bank chooses these values so that $R_{nt} = R_{rt}$; it follows that $0 \leq k_t \leq (1-\phi)\, d_t$. If $R_t^m < X$ then it is readily seen that the bank always chooses $\gamma_t = 1$.

In equilibrium, the value of d_t that the bank takes as given is the optimal value for the households, given the values of R_{nt} and R_{rt} that are produced when the bank chooses m_t and k_t optimally, based on the value of d_t chosen by the consumers.

We confine ourselves to the study of steady state competitive equilibria and how they compare to steady state allocations that are chosen by the benevolent social planner. A steady state allocation consists of consumption values c_1^*, c_{2n}^* and c_{2r}^* for the two-period-lived consumers, and c_{21}^* for the initial old, and a storage value $k^* \in [0, \omega]$ that are feasible, in the sense of satisfying the resource constraints

$$
\begin{aligned}
c_1^* &= \omega - k^* - \psi \\
c_{2n}^* &= \frac{X\, k^*}{1-\phi} \\
c_{2r}^* &= \frac{\psi}{\phi} \\
c_{21}^* &= \psi.
\end{aligned}
\tag{24}
$$

Recall that ψ is the lump-sum intergenerational transfer. This definition implicitly assumes that the social planner faces essentially the same constraints as the banks: it cannot give the returns on physical assets stored at date t to the consumers who are relocated at date $t+1$, but it can take goods from young consumers in a location and transfer them to old consumers who have been moved to that location. The result is a Pareto optimal allocation in which consumption is smoothed across relocation states.

5.1 Optimal monetary policy: random-relocation economy

Consider the equilibrium outcome in a decentralized economy. Given a monetary policy rule z^* and the associated nominal money supply sequence $\{h_t^*\}_{t=1}^{\infty}$, a steady state competitive equilibrium consists of a sequence of positive, finite price levels $\{p_t^*\}_{t=1}^{\infty}$, date-invariant positive market return rates R_n^* and R_r^*, a date-invariant tax or transfer τ^*, date-invariant nonnegative real money and real storage demand values m^* and k^*, a date-invariant non-negative value γ^* and date-invariant nonnegative consumption and deposit values $c_1^*, c_{2n}^*, c_{2r}^*, c_{21}^*$ and d^*. Given the return rates and the transfer, the consumption and deposit values must maximize consumers' intertemporal expected utility (21) subject to the consumer's budget constraints; specifically (6) and (20). Given the deposit value, the bank's choices for k^*, m^* and γ^* must maximize consumers' second-period expected utility (22) subject to the budget constraints (20), the bank's budget constraint (3) and the deposit return conditions (17). The value m^* and τ^* must satisfy equation (2), with m^* replacing h_t/p_t, as in the preceding economy. Again, equation (5) implies that we must have $R^{m*} = 1/Z$.

The key difference between this economy and the legal-restrictions version is that money is part of efficient allocation even though it is rate-of-return dominated. Money is held voluntarily to insure against the risk of being relocated. Stated differently, banks have access to two technologies for transferring goods into the future on consumers' behalf: storing goods, which is effective only for consumers who are not relocated, and acquiring money balances, which is effective whether or not consumers are relocated. If $R^m < X$ then it is inefficient for banks to hold money for the purpose of financing payments to consumers who are not relocated – as in the economy of the preceding section. Money is the only way they can finance payments to consumers who are relocated.

Since money is a device for conducting intergenerational transfers, the "natural" rate of return on money is the natural return rate on intergenerational transfers, which is unity. Consequently, it seems reasonable to expect that the Pareto optimal allocations are those supported by steady state competitive equilibria in which money has a zero inflation rate. It turns out, however, that such equilibria are not the only Pareto optimal steady state equilibria. Indeed, any equilibrium in which in which the inflation rate is lower than zero – a category that includes the Friedman Rule equilibria – is Pareto optimal. We characterize the set Pareto optimal allocations in the following proposition.

Proposition 4 A consumption allocation supported by a steady state competitive equilibrium in which $R^m \in [1, X]$ cannot be Pareto dominated by any other steady state allocation. A consumption allocation supported by a steady state competitive equilibrium in which $R^m < 1$ can be Pareto dominated by another

steady state allocation.

Proposition 4 indicates that in terms of Pareto-optimality properties, the random-relocation-money-demand economies studied in this section are both similar to, but also different from the reserve-requirement economies studied in the preceding section. In both economies, Friedman rule equilibria are Pareto optimal, and equilibria with positive inflation rates are not Pareto optimal. But in the reserve-requirements economies, Friedman rule equilibria are the unique Pareto optima, while in the random-relocation economies there are many other Pareto optimal equilibria, including equilibria with zero inflation.

It is useful to explain why the set of Pareto optimal equilibria in the random-relocation economies than in the reserve-requirement economies. In both model economies, consumers and banks are making portfolio, distributing saving between money and the physical asset. Clearly, at date $t = 1$, the money-holding decision impacts the welfare of the initial old consumers. Consider the reserve-requirement economy. For a given level of real money balances (and the associated intergenerational transfer), consumers save too little in the sense that the return to deposits is less than the return to storage. In the random-relocation economy, the equilibrium allocation chosen by banks and consumers is the same as the solution to the social planner's problem when the inflation rate is zero. For non-zero inflation rates, the equilibrium allocations will, in general, differ from the social planner's choices. For instance, with negative (positive) inflation, too much (little) real money is held relative to what the social planner would choose. Despite the welfare losses to two-period lived consumers that come with holding too much money, the initial old consumers gain. Each equilibria with $\pi \in [-X, 0]$ satisfies the definition of Pareto optimality. It is only when inflation is positive that an increase in real money balances would result in two-period lived consumers and the initial old would both attain higher welfare levels. If the return rate on money is too low, real money balances are less than the social planner's choice. Because of the intergenerational friction, two-period consumers conduct too few intergenerational transfers, even though their welfare would be increased if these transfers could be conducted directly and if the level of transfers could be set by general agreement among all the generations of consumers. Thus, in the random-relocation economies, equilibria with positive inflation are Pareto suboptimal.

5.2 Welfare properties – the random-relocation economy

Here, we find the monetary policy that maximizes steady state welfare in the random-relocation economy with following proposition.

Proposition 5 Suppose $u(c) = \ln c$. Then the steady state competitive equilibrium that is optimal for the two-period-lived households is the one with zero (net) inflation.

Thus, the optimal steady state monetary policy results in a lower steady state inflation rate in the random-relocation economies than in the reserve-requirement economies. The intuition is straightforward. Note that in the reserve-requirement economies, the involuntary nature of the demand for real money balances combined with the inefficient intergenerational transfer from the young consumers to the old consumers that accompanies money holdings resulted in making the inflation tax desirable.[30] Indeed, the inflation tax financed a transfer from the old to the young that partly offset the inefficient so that steady state welfare is positively related to inflation evaluated at $\pi = 0$. In the random-relocation economies, however, the intergenerational transfer from young consumers to old consumers serves an indispensable purpose. Hence, steady state welfare falls in response to any policy that works to offset the intergenerational-transfer effect.

To further illustrate our point, consider the following graphical exposition. Figure 3 displays two steady-state consumption bundles: one with zero inflation and one with positive inflation. For simplicity, we assume, consumers value only old age consumption; that is, $\alpha = 0$. The vertical axis represents consumption in the relocated state (state r) and the horizontal axis represents consumption in the non-relocated state (state n). Since the social planner's stationary budget set coincides with the consumer's budget set when there is zero inflation, it follows that zero inflation maximizes steady-state utility. We denote the consumption bundle for the zero-inflation policy as $\overline{c}_2$. For non-zero inflation rates, one can determine the utility maximizing consumption bundle from the intersection of the social planner's budget line and the consumer's budget line. For instance, if the inflation rate is positive, then the consumer receives a transfer and perceives its income to be greater than ω, but it faces a budget line with a slope of $R^m/X < 1/X$. The consumer's budget line intersects the social planner's line to the right of $\overline{c}_2$, so that non-movers consume more, relative to the zero-inflation allocation, and movers consume less.

Now, consider the graphical result in an economy with log utility. Two-period lived consumers divide their saving, putting $(1 - \phi)\%$ of their income into the physical asset and using the remaining income to acquire money. An inflation tax reduces the return on their money relative to a zero-inflation policy. In the face of this lower real return, two-period lived consumers hold a portfolio that is heavier in physical assets and lighter in real money balances. Since the level of the physical asset and the

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[30]Under the Friedman Rule, money holdings are not interpreted as involuntary. Here, are comments are restricted to those equilibria in which the monetary policy is not the Friedman Rule.

net intergenerational transfer amount could have been chosen under a zero-inflation policy and were not chosen, we know that the consumption bundle under a positive-inflation policy results in lower utility. Moreover, because the decrease in utility follows from the fact that consumers' portfolio has a higher level of the physical asset and the intergenerational transfer is smaller, we know that the initial old would be better off under a lower inflation rate. Thus, the consumption bundle chosen under the positive inflation rate is Pareto dominated by the consumption bundle chosen under the zero-inflation policy.

Lastly, we consider a case in which monetary policy results in a negative inflation rate. Two-period-lived consumers pay a tax to finance money retirement. Consumers treat the tax as given. Consumers perceive that their disposable income is below ω, putting to few goods into physical assets and increasing the level of net intergenerational transfer. The increase in the net transfer benefits the initial old at the expense of the two-period-lived consumers. The equilibrium allocation remains Pareto optimal but does not maximize steady state welfare.

5.3   Simulator regime in a the random-relocation economy

We now examine the optimal steady state policy under the simulator regime. In the simulator regime, all young consumers receive a monetary endowment and all old consumers must pay a tax that is proportional to the monetary endowment. Not surprisingly, we find that the optimality of the Friedman Rule is re-established in the simulator regime. Our result may then be stated as follows.

Proposition 6 In the random relocation economy, the Friedman Rule supports the first-best optimum in the random relocation model if combined with the ILRA simulator regime.

Under the simulator regime, relocated consumers can use money to finance old-age consumption without participating in intergenerational transfers. Here, money has two roles: (i) it is a device for intergenerational transfers; and (ii) it provides insurance against being relocated. At first glance, these two roles are intertwined in such a way that eliminating the intergenerational transfer role–as the simulator does–could undermine its insurance feature.

Thus, it is important to demonstrate that these roles are sufficiently separable so as to better understand how the simulator regime works. Note that money provides insurance because it is serves as the generally acceptable medium of exchange in the random-relocation economies. When a consumer is relocated, the consumer's claims against goods stored by banks are worthless. Banks in the home locations cannot pay claims across locational boundaries and banks in the new location do not hold the

claims of the relocated consumer. Meanwhile, money is an anonymous asset that is accepted in any location.

A key point is that money provides the insurance feature to relocated consumers as long as there is some demand for money. The source of the demand for money does not have to involve intergenerational transfers. Under the standard regime, only young consumers demand money. Consequently, money transactions necessarily involve intergenerational transfers. Under the simulator regime, old consumers demand money to meet their obligations to the government. The old consumer's money demand is independent of their relocation status. Intragenerational transfers can arise, for instance, as old, relocated consumers sell money to old consumers who were not relocated. The intragenerational exchange involves goods possessed by non movers and money held by movers. Intragenerational trade means that the insurance feature is provided without any consumer necessarily engaging in an inefficient intergenerational transfer. The simulator regime, therefore, solves both the insurance problem and the inefficiency problem that accompanies intergenerational transfers. It is possible to attain the first-best solution, provided the Friedman rule is implemented.

It is also important to note another important feature of the simulator regime when the Friedman rule is implemented. Namely, consumers do not perceive an increase in their lifetime income when the Friedman Rule is implemented. In present value, the taxes paid by old consumers are exactly offset by transfers paid to young consumers. Consumers do not react to any perceived-income effect. Accordingly,such, there are no mistakes that cause the consumer to alter their consumption-saving bundle because of belief that lifetime income is larger or smaller than it actually is.

Our results do not extend to models in which intergenerational transfers are essential to money's usefulness. Sargent (1987) looks at model economies in which intergenerational transfers provide some insurance against aggregate risks in returns paid by physical assets. In those models, fiat money is one way the government can provide this insurance without undertaking more fiscal intervention. If the simulator regime were imposed in this kind of setup, the insurance role of money would eliminated along with participation in intergenerational transfers. Steady-state welfare would be adversely affected by eliminating the insurance role of money.

6 Summary and Conclusions

In this paper, we seek to identify the basic difference between the infinitely lived representative agent (ILRA) model and the overlapping generations (OG) model as models of money. The point of departure

for our investigation is the fact that in most monetary ILRA models, there is a unique optimal monetary policy: the Friedman rule. Under this policy, the government contracts the stock of money at a rate that drives the real rate of return on fiat money up to the level of the real return rate on nonmonetary assets, so that the nominal interest rate on all assets is zero. Although there has been much less research on the optimal monetary policy in overlapping generations models, the literature that does exist suggests that the monetary policy that maximizes steady state welfare often involves a money growth rate that is much higher than the Friedman Rule rate.

In our formal analysis, we study a simple overlapping generations model under two different sets of assumptions about the source of money demand. Initially, the regime our government uses to introduce money into the economy is the same one that has been studied in most of the literature: we call it the "standard monetary regime." Under our first set of money demand assumptions, we find that the monetary policy that maximizes steady state welfare features a money stock that grows over time and produces a positive inflation rate. Under our second set of assumptions, we find that the monetary policy that is optimal, in the sense we have just described, features a constant stock of money and produces a net real return rate on money of zero (that is, zero inflation). Thus, under both sets of assumptions, the optimal monetary policy is quite different from the Friedman Rule.

Careful examination of these results leads us to the conclusion that the key reason they are different from the results of analogous policy experiments in ILRA models is the fact that in our OG model, fiat money is a vehicle through which agents conduct intergenerational transfers. This diagnosis leaves us with two remaining goals. The first goal is simply to confirm its accuracy. The second goal is to determine whether the intergenerational-transfer role of fiat money is an inherent feature of monetary OG models, or, alternatively, whether it is a consequence of decisions made, by the modeler, about the form of the government's monetary regime.

Our strategy for accomplishing both goals is to attempt to devise a monetary regime that allows money to be valued, but eliminates its intergenerational-transfer role. We call this alternative regime the "ILRA simulator regime" because it introduces money into our OG model in essentially the same way it is usually introduced into ILRA models – models in which intergenerational transfers are rendered trivial. We find that under the simulator regime, the Friedman rule is the optimal monetary policy under either set of money demand assumptions. This finding indicates that our diagnosis is correct. It also indicates that there is a sense in which the key difference between the ILRA and OG models, as models of money, is an artifact of the way monetary regimes are usually set up in the two models, rather than being caused by differences in their structure.

We extend our formal analysis by conducting of computational experiments designed to determine how the welfare costs or benefits of different monetary policies, in the reserve-requirements version of our model, depend on whether money is introduced in a way that gives it an intergenerational-transfer role. We do this by setting the parameters of the model at empirically plausible values and comparing steady state welfare, for different monetary policies, under both set of money demand assumptions and both types of monetary regimes. We find that under the standard regime, where the optimal policy involves positive inflation, the welfare benefit from a zero-inflation policy, relative to a Friedman-rule deflation, is roughly 10 percent of steady-state consumption. This is true under either of our money demand assumptions. Under the simulator regime, where the optimal policy is the Friedman rule, a zero-inflation policy results in a welfare loss of less than 0.1 percent of steady state consumption when money demand comes from the first source we study, but a loss of more than 2.5 percent of consumption when money demand comes from the second source.

We conclude this portion of our analysis by performing similar computational experiments using a multi-period generalization of our model. (We can do this only for the case of reserve-requirements money demand.) The results of these experiments indicate that our findings about the sizes of the welfare costs and benefits of alternative monetary policies, and about their dependence on the nature of the monetary regime, do not depend, in any important way, on the number of periods in agents' lives.

Our analysis of the welfare properties of monetary policy in an OG model has pointed up important qualitative and quantitative differences between the OG model and the ILRA model, as models of money. It has also demonstrated that the underlying source of these differences is the fact that in most monetary OG models, fiat money is a vehicle through which agents conduct intergenerational transfers. Finally, it has shown that this intergenerational-transfer feature of monetary OG models is not inherent in the structure of the model: instead, it is an artifact of the way monetary regimes have usually been constructed in the model. We have made this point by showing that it is possible to construct an alternative monetary regime under which fiat money is valued without playing any intergenerational-transfer role.

Our findings suggest several potential avenues for future research. For one thing, money's role as a means of intergenerational transfers is closley linked to money's role as a medium of exchange. Consider a general equilibrium model in which money's role as the medium of exchange could be extracted as we have done in the simulator regime. Our results hint that the Friedman Rule is intimately tied to money's role as a store of value. Suppose additional roles for money are considered. The key question is, is the Friedman Rule robust under these additional roles?

Finally, we have shown that the answers to important policy questions depend on whether fiat currency is a vehicle for conducting intergenerational transfers. This finding suggests that there is a need for research designed to determine whether, and to what extent, government currency actually plays this role. One approach to answering this question may involve determining whether, and to what extent, government currency is backed, in the sense of intermediating real assets. It can be shown that in an OG model, under certain conditions, a monetary regime featuring fully backed currency is equivalent to a simulator regime – a regime in which money has no intergenerational-transfer role. On the other hand, a regime featuring currency that it entirely unbacked is equivalent to a standard regime.
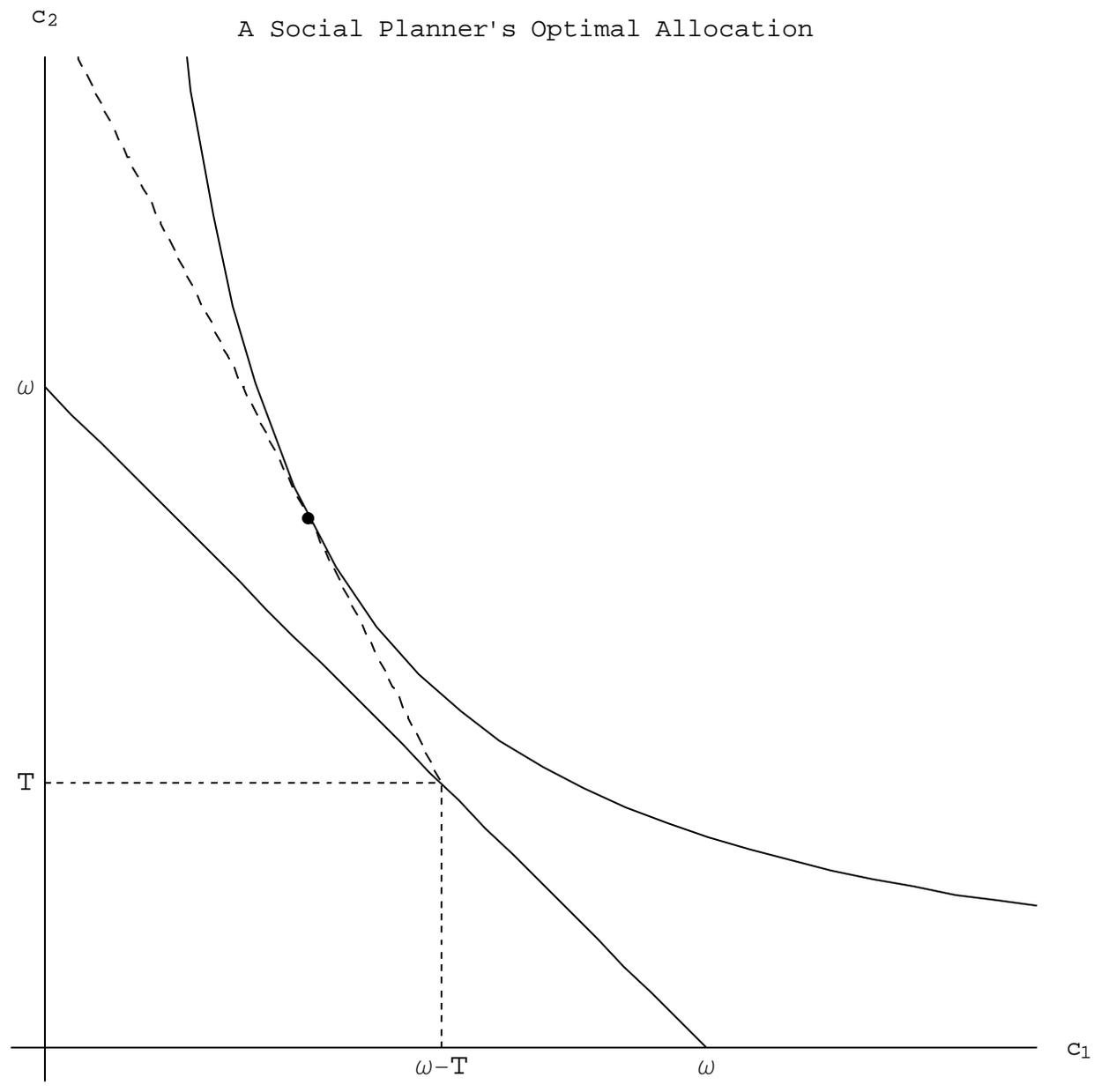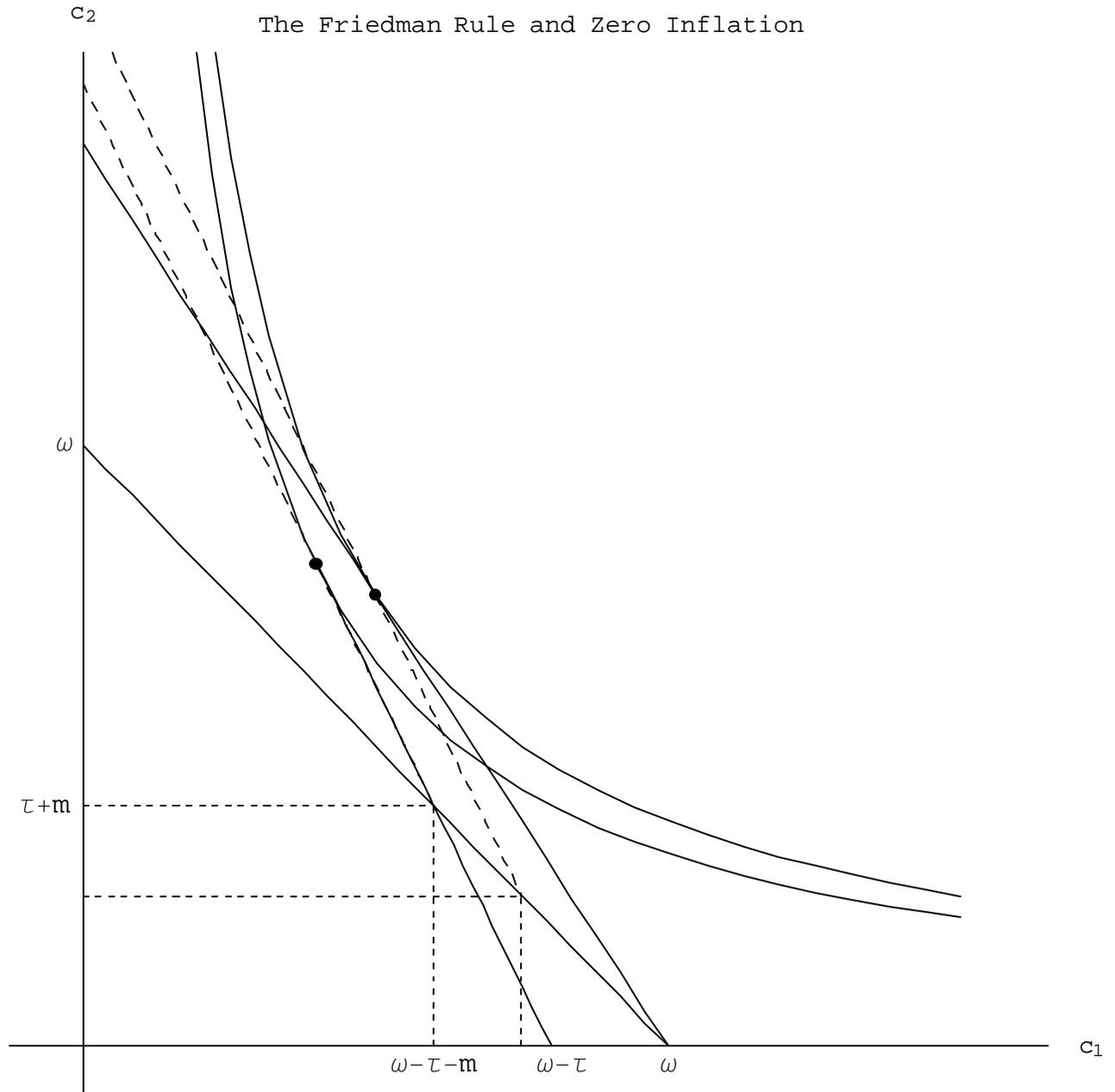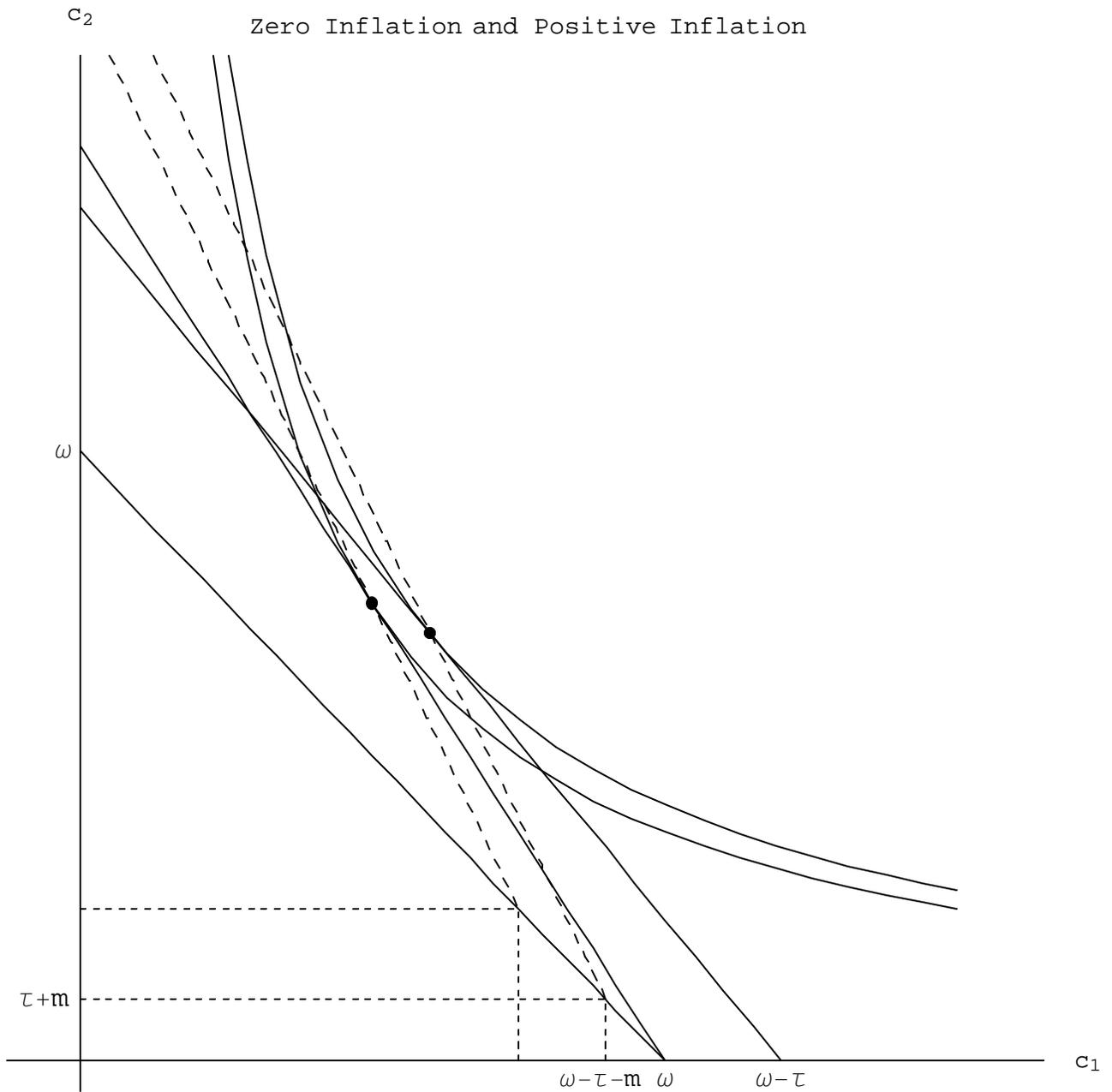
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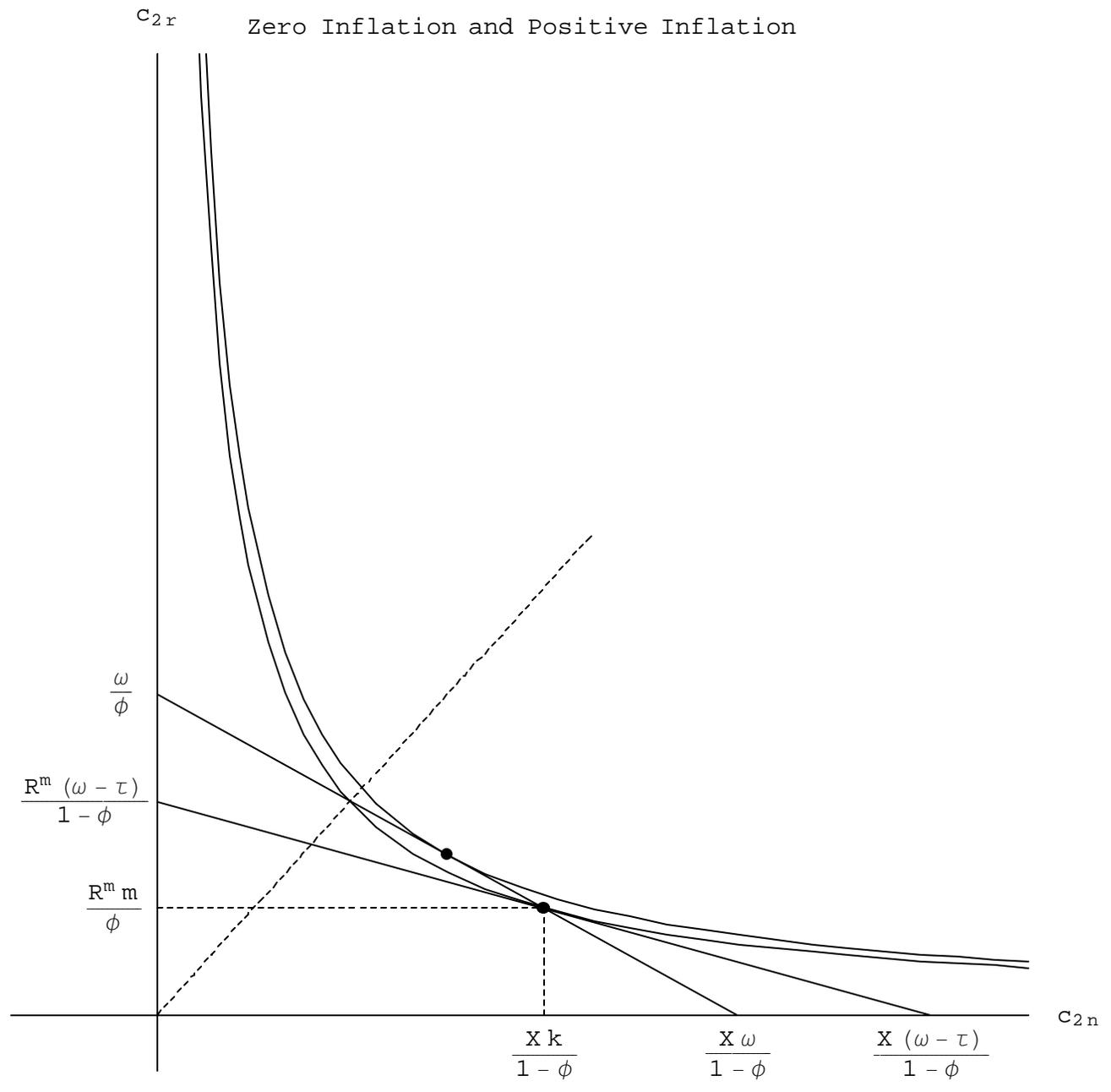
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Figure1.nb 1

A Social Planner's Optimal Allocation

The Friedman Rule and Zero Inflation

c_2

Zero Inflation and Positive Inflation

ω

$\tau + m$

$\omega - \tau - m$ ω $\omega - \tau$

c_1

Figure3.nb 1

Zero Inflation and Positive Inflation

Appendix

Proof of Proposition 1:

To the contrary, consider a stationary competitive equilibrium with $R^m < X$. Thus,

$$c_1^* = \omega - \tau^* - d^*$$
$$c_2^* = R^{d*} d^*$$

where $d^* = m^* + k^*$, with $m^* = \theta\, d^*$ and $k^* = (1 - \theta)\, d^*$, and $R^{d*} = (1 - \theta)\, X + \theta\, R^m < X$. We have

$$-\tau^* = \frac{h_1 - h_0}{p_1^*},$$

so that

$$m_0^* \equiv \frac{h_0}{p_1^*} = \frac{h_1}{p_1^*} + \tau^* = m^* + \tau^*.$$

Thus, the initial old receive a "transfer" of $m_0^* = m^* + \tau^*$.

The social planner faces a stationary resource constraint, represented by

$$c_1 = \omega - a - k$$
$$c_2 = X\, k + a$$

where a represents an intergenerational transfer from the young consumers to the old consumers. Suppose the social planner imposes a transfer of $\widehat{a} = m_0^*$. If the social planner also chooses $\widehat{k} = k^*$ then it duplicates the competitive equilibrium allocation:

$$\widehat{c}_1 = \omega - (m^* + \tau^*) - k^* = \omega - \tau^* - d^* = c_1^*$$
$$c_2 = X\, k^* + (m^* + \tau^*) = X\, k^* + m^* + (R^{m*} - 1)\, m^* = R^{d*} d^*.$$

However, that allocation occurs at a bundle on the social planner's budget line whose indifference curve has a slope of $-R^{d*}$ at that bundle. The social planner, by increasing k and decreasing c_1, holding a fixed, can produce allocations along a line northwest of that bundle with a slope of $-X$. If the increase in k is sufficiently small then these allocations must produce higher utility for the two-period-lived consumers. Thus, the competitive equilibrium allocation is not Pareto optimal.

In a stationary competitive equilibrium with $R^m = X$ – a Friedman rule allocation – we have

$$c_1^* = \omega - \tau^* - d^*$$
$$c_2^* = X\, d^*.$$

Again, the initial old receive a "transfer" of $m_0^* = m^* + \tau^*$. Suppose the social planner imposes an intergenerational transfer of $\widehat{a} = m_0^*$ from the young to the old. Given this transfer, the social planner's budget constraint is

$$c_1 = \omega - \widehat{a} - k$$
$$c_2 = X\, k + \widehat{a}$$

as above. Notice that both constraints have a slope of $-X$. Moreover, if the social planner chooses $\widehat{k} = k^*$ then the two allocations coincide. It follows that the constraints coincide. Thus, the consumers' choice of k is the choice the social planner would make, given $\widehat{a}$.

Any increase in a must make the consumer worse off, since the indifference curve through the optimal bundle has a slope of $-X$ at the bundle, while increasing a moves the allocation northwest along a line with a slope of -1. And any decrease in a makes the initial old worse off. Thus, the competitive equilibrium allocation is Pareto optimal. ¤

Proof of Proposition 2:

With log utility, consumption and deposit demand functions are

$$
\begin{aligned}
c_1 &= \alpha(\omega - \tau) \\
c_2 &= R^d (1 - \alpha)(\omega - \tau)
\end{aligned}
$$

and

$$
d = (1 - \alpha)(\omega - \tau),
$$

where

$$
R^d = (1 - \theta) X + \theta R^m .
$$

In an equilibrium with $R^m < X$ we have $m = \theta d$. Since we also have $\tau = (R^m - 1) m$ in equilibrium, we have

$$
\tau = (R^m - 1) \theta d = (R^m - 1) \theta (1 - \alpha)(\omega - \tau) .
$$

Define $r^m \equiv R^m - 1$. Then

$$
\begin{aligned}
\tau &= r^m \theta (1 - \alpha)(\omega - \tau) \Leftrightarrow \frac{\omega}{\tau} - 1 = \frac{1}{r^m \theta (1 - \alpha)} \\
&\Leftrightarrow \frac{\omega}{\tau} = \frac{1}{r^m \theta (1 - \alpha)} + 1 = \frac{1 + r^m \theta (1 - \alpha)}{r^m \theta (1 - \alpha)} .
\end{aligned}
$$

So

$$
\tau = \omega \frac{r^m \theta (1 - \alpha)}{1 + r^m \theta (1 - \alpha)}
$$

and

$$
\omega - \tau = \frac{\omega}{1 + r^m \theta (1 - \alpha)}
$$

Thus

$$
c_1 = \frac{\alpha \omega}{1 + r^m \theta (1 - \alpha)} .
$$

and

$$
\begin{aligned}
c_2 &= \frac{(1 - \alpha) \omega}{1 + r^m \theta (1 - \alpha)} \left[(1 - \theta) X + \theta R^m\right] \\
&= \frac{(1 - \alpha) \omega}{1 + r^m \theta (1 - \alpha)} \left[(1 - \theta) X + \theta (1 + r^m)\right] .
\end{aligned}
$$

So

$$
\begin{aligned}
U(r^m) &= \alpha \log c_1 + (1 - \alpha) \log c_2 \\
&= \alpha \log \frac{\alpha \omega}{1 + r^m \theta (1 - \alpha)} + (1 - \alpha) \log \frac{(1 - \alpha) \omega}{1 + r^m \theta (1 - \alpha)} \left[(1 - \theta) X + \theta (1 + r^m)\right] \\
&= \alpha \left(\log \alpha \omega - \log \left[1 + r^m \theta (1 - \alpha)\right]\right) \\
&\quad + (1 - \alpha) \left\{\log (1 - \alpha) \omega + \log \left[(1 - \theta) X + \theta (1 + r^m)\right] - \log \left[1 + r^m \theta (1 - \alpha)\right]\right\}
\end{aligned}
$$

and

$$
U'(r^m) = -\frac{\alpha \theta (1 - \alpha)}{1 + r^m \theta (1 - \alpha)} + (1 - \alpha) \left[\frac{\theta}{(1 - \theta) X + \theta (1 + r^m)} - \frac{\theta (1 - \alpha)}{1 + r^m \theta (1 - \alpha)}\right] .
$$

Thus, $U'(r^m)$ is proportional to

$$\frac{1}{(1-\theta)X + \theta(1+r^m)} - \frac{1-\alpha}{1+r^m\,\theta(1-\alpha)} - \frac{\alpha}{1+r^m\,\theta(1-\alpha)},$$

which is

$$\frac{1}{(1-\theta)X + \theta(1+r^m)} - \frac{1}{1+r^m\,\theta(1-\alpha)}.$$

It follows that $U'(r^m) < 0$ iff

$$\frac{1}{1+r^m\,\theta(1-\alpha)} > \frac{1}{(1-\theta)X + \theta(1+r^m)}$$

which is

$$(1-\theta)X + \theta(1+r^m) > 1 + r^m\,\theta(1-\alpha)$$

or

$$(1-\theta)x + r^m\,\theta\,\alpha > 0.$$

Notice that this is always true when $r^m \geq 0$. Thus, zero inflation ($r^m = 0$) is always better than the Friedman rule ($r^m = x$), but it is always worse than a little inflation.

For $U' = 0$ we would need

$$-\alpha\,\theta\,r^m = (1-\theta)x \Leftrightarrow r^m_{\max} = -\frac{(1-\theta)x}{\alpha\,\theta} < 0.$$

For $r^m_{\max} > -1$ we need

$$\frac{\theta}{1-\theta} > \frac{x}{\alpha}.$$

This will always be true if x is close enough to zero. As x gets really large, however, the required value of θ approaches 1 from below. Note that if $\alpha = 0$ then the optimal policy is always a hyperinflation.
¤

Proof of Proposition 4:

For stationary allocations of the type we describe, the budget constraints of a social planner are

$$
\begin{aligned}
c_1 &= (\omega - a) - k \\
c_{21} &= \frac{(1+x)\,k}{1-\phi} \\
c_{22} &= \frac{a}{\phi},
\end{aligned}
$$

where a represents the transfer from the current young consumers to the current old consumers who are relocated. (Since $x > 0$, it is never optimal to give any part of the intergenerational transfer to old consumers who are not relocated.) Note that each initial old consumer receives a transfer of a.

Given a, which determines the welfare of the initial old consumers, the social planner chooses k to maximize

$$E\{U\} = \alpha\,u(c_1) + (1-\alpha)\,[(1-\phi)\,u(c_{21}) + \phi\,u(c_{22})].$$

The first-order condition is

$$-\alpha\,u'(c_1) + (1-\alpha)\,(1+x)\,u'(c_{21}) = 0. \tag{A1}$$

This equation governs the relationship between k and a.

Now

$$
\begin{aligned}
\frac{dU}{da} &= -\alpha\, u'(c_1)\left(1 + \frac{dk}{da}\right) + (1 - \alpha)\left[(1 + x)\, u'(c_{21})\frac{dk}{da} + u'(c_{22})\right] \\
&= -\alpha\, u'(c_1) + (1 - \alpha)\, u'(c_{22}) + \frac{dk}{da}\left[-\alpha\, u'(c_1) + (1 - \alpha)(1 + x)\, u'(c_{21})\right] \\
&= \quad [\text{using equation (A1)}] \quad -\alpha\, u'(c_1) + (1 - \alpha)\, u'(c_{22}).
\end{aligned}
$$

So $\frac{dU}{da} > 0$ requires

$$
(1 - \alpha)\, u'(c_{22}) > \alpha\, u'(c_1) \Leftrightarrow 1 > \frac{\alpha}{1 - \alpha}\frac{u'(c_1)}{u'(c_{22})} = \frac{\alpha}{1 - \alpha}\frac{u'((\omega - a) - k)}{u'(\frac{a}{\phi})}.
$$

It follows that the social planner will never choose a-values that satisfy this condition; stated differently, the associated allocations are not Pareto optimal. It also follows that a-values that do not satisfy this condition are associated with allocations that are Pareto optimal.

In a stationary competitive equilibrium, the consumer faces budget constraints repreented by

$$
\begin{aligned}
c_1 &= (\omega - \tau) - d \\
c_{21} &= R_1^d\, d \\
c_{22} &= R_2^d\, d.
\end{aligned}
$$

The consumer chooses d so that

$$
-\alpha\, u'(c_1) + (1 - \alpha)\left[(1 - \phi)\, R_1^d\, u'(c_{21}) + R_2^d\, \phi\, u(c_{22})\right] = 0. \tag{A2}
$$

The bank faces a fixed d and

$$
R_1^d = \frac{(1 + x)(d - m)}{(1 - \phi)\, d} \tag{A3}
$$

$$
R_2^d = \frac{R^m\, m}{\phi\, d}\,; \tag{25}
$$

it chooses m to maximize

$$
(1 - \phi)\, R_1^d\, u(c_{21}) + \phi R_2^d\, u(c_{22})
$$

given d. The first order condition is

$$
(1 - \alpha)\left[-(1 + x)\, u'(c_{21}) + R^m\, u'(c_{22})\right] = 0. \tag{A4}
$$

In equilibrium, conditions (25) and (A4) yield

$$
-\alpha\, u'(c_1) + (1 - \alpha)\left[\frac{(1 + x)(d - m)}{d}\, u'(c_{21}) + \frac{R^m\, m}{d}\, u(c_{22})\right] = 0. \tag{A5}
$$

Condition (A2) can be rewritten

$$
R^m\, u'(c_{22}) = (1 + x)\, u'(c_{21}) \Leftrightarrow u'(c_{21}) = \frac{R^m}{1 + x}\, u'(c_{22}),
$$

so condition (A5) becomes

$$
-\alpha\, u'(c_1) + (1 - \alpha)\left[\frac{R^m(d - m)}{d}\, u'(c_{22}) + \frac{R^m\, m}{d}\, u(c_{22})\right] = 0
$$

which is

$$-\alpha\, u'(c_1) + (1-\alpha)\, R^m\, u'(c_{22}) = 0$$

or

$$R^m = \frac{\alpha}{1-\alpha}\frac{u'(c_1)}{u'(c_{22})} = \frac{\alpha}{1-\alpha}\frac{u'((\omega-\tau)-d)}{u'(\frac{R^m\, m}{\phi})}. \tag{A6}$$

Finally, we have

$$\tau = (R^m - 1)\, m\,,$$

so that

$$R^m\, m = m + \tau\,. \tag{A7}$$

Equations (A6) and (A7) give us

$$R^m = \frac{\alpha}{1-\alpha}\frac{u'\left((\omega-(m+\tau))-k\right)}{u'(\frac{m+\tau}{\phi})},$$

where $m + \tau$ is the net intergenerational transfer.

Note that for any value $\widehat{a}$, if the government chooses $\widehat{R^m} = 1/\widehat{z}$ such that

$$\widehat{R^m} = \frac{\alpha}{1-\alpha}\frac{u'(\omega-\widehat{a}-k)}{u'(\frac{\widehat{a}}{\phi})},$$

then there is a competitive equilibrium with $R^{m*} = \widehat{R^m}$ and $m^* + \tau^* = \widehat{a}$ that supports the same allocation. Thus, if $R^{m*} < 1$ then we have

$$1 > \frac{\alpha}{1-\alpha}\frac{u'((\omega-(m^*+\tau^*))-k^*)}{u'(\frac{m^*+\tau^*}{\phi})},$$

and the competitive equilibrium is not Pareto optimal: increasing the intergenerational transfer would make everyone better off. Otherwise, the competitive equilibrium is Pareto optimal. ¤

Proof of Proposition 5:

With log utility, a consumers's demand function for deposits is

$$d = (1-\alpha)\,(\omega - \tau)$$

and the consumer's consumption demand functions

$$c_1 = \alpha\,(\omega - \tau)$$

$$\begin{aligned}
c_{21} &= (1+x)\,(1-\alpha)\,(\omega-\tau)\\
c_{22} &= (1+r^m)\,(1-\alpha)\,(\omega-\tau).
\end{aligned}$$

They also yield bank asset demand functions

$$\begin{aligned}
k &= (1-\phi)\,d\\
m &= \phi\,d.
\end{aligned}$$

It follows that in equilibrium we must have

$$\begin{aligned}
k &= (1-\phi)\,(1-\alpha)\,(\omega-\tau)\\
m &= \phi\,(1-\alpha)\,(\omega-\tau).
\end{aligned}$$

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In equilibrium, the government budget constraints gives us

$$\tau = r^m \, m = r^m \, \phi \, (1 - \alpha) \, (\omega - \tau).$$

Thus,

$$\tau = r^m \, \phi \, (1 - \alpha) \, (\omega - \tau) \Leftrightarrow \frac{\omega}{\tau} - 1 = \frac{1}{r^m \, \phi \, (1 - \alpha)}$$

$$\Leftrightarrow \frac{\omega}{\tau} = \frac{1}{r^m \, \phi \, (1 - \alpha)} + 1 = \frac{1 + r^m \, \phi \, (1 - \alpha)}{r^m \, \phi \, (1 - \alpha)}.$$

So

$$\tau = \omega \, \frac{r^m \, \phi \, (1 - \alpha)}{1 + r^m \, \phi \, (1 - \alpha)},$$

and

$$\omega - \tau = \frac{\omega}{1 + r^m \, \phi \, (1 - \alpha)}.$$

It follows that

$$c_1 = \frac{\alpha \, \omega}{1 + r^m \, \phi \, (1 - \alpha)}$$

$$c_{21} = \frac{(1 - \alpha) \, \omega}{1 + r^m \, \phi \, (1 - \alpha)} \, (1 + x)$$

$$c_{22} = \frac{(1 - \alpha) \, \omega}{1 + r^m \, \phi \, (1 - \alpha)} \, (1 + r^m).$$

So, in equilibrium,

$$
\begin{aligned}
V &\equiv E\{U\} = \alpha \ln c_1 + (1 - \alpha) \left[(1 - \phi) \ln c_{21} + p \ln c_{22} \right] \\
&= \alpha \ln \frac{\alpha \, \omega}{1 + r^m \, \phi \, (1 - \alpha)} + \\
&\quad (1 - \alpha) \left[(1 - \phi) \ln \frac{(1 - \alpha) \, \omega}{1 + r^m \, \phi \, (1 - \alpha)} \, (1 + x) + \phi \ln \frac{(1 - \alpha) \, \omega}{1 + r^m \, \phi \, (1 - \alpha)} \, (1 + r^m) \right] \\
&= \alpha \left\{ \ln \alpha \, \omega - \ln [1 + r^m \, \phi \, (1 - \alpha)] \right\} + \\
&\quad (1 - \alpha) \left\{ \begin{array}{l} (1 - \phi) \, (\ln (1 - \alpha) \, \omega \, (1 + x) - \ln [1 + r^m \, \phi \, (1 - \alpha)]) + \\ \phi \, (\ln (1 - \alpha) \, \omega + \ln (1 + r^m) - \ln [1 + r^m \, \phi \, (1 - \alpha)]) \end{array} \right\}
\end{aligned}
$$

and

$$V'(r^m) = -\frac{\alpha \, \phi \, (1 - \alpha)}{1 + r^m \, \phi \, (1 - \alpha)} + (1 - \alpha) \left[-(1 - \phi) \frac{\phi \, (1 - \alpha)}{1 + r^m \, \phi \, (1 - \alpha)} - \phi \left(\frac{\phi \, (1 - \alpha)}{1 + r^m \, \phi \, (1 - \alpha)} - \frac{1}{1 + r^m} \right) \right].$$

Thus, $V'(r^m)$ is (positively) proportional to

$$-\frac{\alpha}{1 + r^m \, \phi \, (1 - \alpha)} - (1 - \phi) \frac{1 - \alpha}{1 + r^m \, \phi \, (1 - \alpha)} - \phi \, \frac{1 - \alpha}{1 + r^m \, \phi (1 - \alpha)} + \frac{1}{1 + r^m},$$

which reduces to

$$\frac{1}{1 + r^m} - \frac{1}{1 + r^m \, \phi \, (1 - \alpha)}.$$

It follows that $V'(r^m) \lessgtr 0$ as $r^m \gtrless 0$, which means that zero inflation produces higher welfare, for the two-period-lived consumers, than any other inflation rate. ¤

Proof of Proposition 6:

With the simulator policy, the budget constraints for young and old are represented by

$$c_1 + d = \omega - \tau_1 \tag{26}$$

$$c_{2n} = R^d d - \tau_2 \tag{27}$$

$$c_{2r} = R^d d - \tau_2 \tag{28}$$

$$c_{21} = m_0 - \tau_{21} \tag{29}$$

In addition, the bank has a balance-sheet identity and two feasibility constraints represented by

$$d = m + k \tag{30}$$

$$R^d d = \frac{X}{1 - \phi} k \tag{31}$$

and

$$R^d d = \frac{R^m}{\phi} m \tag{32}$$

The Friedman Rule fixes the gross real return to money such that $R^m = X$. Thus, $R^d d = \frac{X}{1-\phi} k$ and $R^d d = \frac{X}{\phi} m$ for non-movers and movers, respectively. Solve equation (30) for the capital stock and substitute into the agent's ex ante old-age budget constraint, yielding

$$E\, c_2 = (1 - \phi) \left[\frac{X}{1 - \phi}(d - m) - \tau_2 \right] + \phi \left[\frac{X}{\phi} m - \tau_2 \right] = Xd - \tau_2 \tag{33}$$

Note that expected old-age consumption is invariant to the bank's choice of money and capital. The bank can hold old-age consumption constant over time–$c_{21} = c_{2t} \; \forall t \geq 2$–if $\frac{X}{1-\phi}(d - m) = \frac{X}{\phi} m \Leftrightarrow m = \phi d, \; k = 1 - \phi d$. Given the bank's portfolio allocation, we obtain

$$R_n^d d = \frac{X}{1 - \phi} k = Xd$$

$$R_r^d d = \frac{X}{\phi} m = Xd$$

The agent's budget constraints can be simplified to

$$c_1 + d = \omega - \tau_1$$

$$c_2 = Xd - \tau_2$$

With $\tau_1 = -m$ and $\tau_2 = R^m m = Xm$, the agent's lifetime budget constraint is

$$\begin{aligned} c_1 + \frac{c_2}{X} &= \omega - \tau_1 - \frac{\tau_2}{X} \\ &= \omega \end{aligned}$$

which is the same choice problem faced by the social planner, producing the riskless social optimum.