Price Experimentation with Strategic Buyers*

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Abstract

A two-period model in which a monopolist endeavors to learn about the permanent demand parameter of a specific repeat buyer is presented. The buyer may strategically reject the seller's first-period offer for one of two reasons. First, in order to conceal information (i.e., to pool), a high-valuation buyer may reject high prices that would never be accepted by a low-valuation buyer. Second, in order to reveal information (i.e., to signal), a low-valuation buyer may reject low prices that would always be accepted by a high-valuation buyer. Given this, the seller often finds it optimal to post prices that reveal no useful information. Indeed, in the equilibrium where there is no signaling, the seller never charges an informative first-period price. Learning may occur in the equilibrium where there is maximal signaling, but the scope for learning appears to be quite limited even in this case. Indeed, in order to preempt information transmission through signaling, the seller may set a first-period price strictly below the buyer's lowest possible valuation.

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1 Introduction

In a classic paper, Rothschild (1974) showed that the pricing problem facing a monopolist with unknown demand is often analogous to a two-armed bandit problem.[1] The optimal policy for such a firm is, therefore, to experiment with prices in order to learn about its unknown demand parameters. It is, however, well-known that the optimal policy may not result in complete learning because of the opportunity cost of experimentation. In addition, the learning process may be severely hampered unless the firm possesses significant prior knowledge about the type of uncertainty confronting it. For instance, even when demand is deterministic, Aghion, Bolton, Harris, and Jullien (1991) show that strong conditions such as continuity and quasi-concavity of the profit function are required to guarantee that a monopolist will eventually learn all the relevant information about its demand. In this paper, a different caveat is added to the list of reasons that a monopolist may have difficulty in learning demand. It may serve customers who do not want their demands to be known!

In the prior literature on price experimentation, the possibility of strategic buyers has been largely ignored. Specifically, it has typically been assumed either that the monopolist faces a sequence of identical customers who exist in the market for only one period or that market demand is composed of a *large* number of *small* customers.[2] There are, however, many real-world situations in which buyers have a significant stake in what a firm learns about their demands. Any time that price discrimination is possible on an individual basis and repeat purchases are likely, buyers possess incentives for strategic manipulation of demand information. In any long-term supply relationship, the buyer wants the supplier to think that he has very elastic demand for the product, and the buyer may strategically reject some offers in order to influence the supplier's beliefs to this end.

In this paper, a two-period experimental pricing and learning environment is analyzed. There is assumed to be a single buyer whose underlying (permanent) demand parameter, λ, and current (transitory) valuation, v_t, are private information. The seller makes a take-it-or-leave-it offer in the first period and updates her belief about the value of λ based on the buyer's acceptance decision. In particular, acceptance (rejection) of a high price implies that the buyer's first-period valuation for the product, v_1, was high (low). This leads the seller to update her beliefs about λ and, therefore, to infer that the buyer's second-period valuation, v_2, is also likely to be high (low). It is shown that if the buyer is not strategic, then the informational value of a high first-period price can lead the seller to charge one when it would otherwise not be optimal to do so (i.e., to experiment).

Things are shown to be starkly different, however, when the buyer is strategic. In this case, if the buyer's first-period valuation is high, then he will often attempt to conceal information by

[1] Many authors have subsequently refined and extended this observation. See, for example, Aghion, Bolton, Harris, and Jullien (1991); Mirman, Samuelson, and Urbano (1993); Rustichini and Wolinsky (1995); Keller and Rady (1999).

[2] In a related paper, Segal (2003) considers a setting in which there is a finite number of buyers in the market from the outset. He shows that if the common distribution of buyers' valuations is unknown, then learning through price experimentation is dominated by a multi-unit auction which sets a price to each buyer on the basis of the demand distribution inferred statistically from other buyers' bids. Segal's analysis does not apply to the setting considered here where consumer types are assumed to be drawn from different distributions.

strategically rejecting high prices. A buyer with a high value of λ stands to gain the most from concealing a high first-period valuation for the good. This gives rise to "reverse screening" in the sense that only buyers with low values of λ and high valuations for the good will accept high first-period prices. When the first-period price is low, two limiting types of continuation equilibria exist, a Good equilibrium (for the seller) in which all buyer types purchase the product, and a Bad equilibrium in which a buyer with a relatively high value of λ but low valuation for the product strategically rejects the offer in order to signal his low first-period valuation. This signaling at low prices is the mirror image of the screening that occurs at high prices in the sense that strategic rejections at low prices reveal information while strategic rejections at high prices conceal it. When facing a strategic buyer, the seller typically finds it optimal to set a price that reveals no information about his demand parameter. In fact, in the Good PBE she never charges an informative first-period price, and in the Bad PBE she may even set an equilibrium first-period price strictly below the buyer's lowest possible valuation in order to forestall signaling, the antithesis of price experimentation!

The pricing and acceptance behavior exhibited in the model presented here can be viewed as a manifestation of the ratchet effect familiar from the regulation and agency literature.[3] Specifically, the fact that the seller cannot commit not to use the information it learns in the first period harms it. This lack of commitment generates the strategic rejections by the buyer that give rise to a weakly lower effective first-period demand for the good. This reduction in the probability of a first-period sale leaves the seller worse off as compared with a setting in which she could commit to price non-contingently. This paper also contributes to the burgeoning literature on behavior-based price discrimination.[4] In Internet retailing and many other market settings, firms now have the ability to track the purchasing behavior of individual customers and to tailor price offers to them.[5] To the extent that consumers are aware of this, the findings presented here indicate that they possess significant incentives to manipulate the information collected.

The paper closest to this study is Kennan (2001), where it is shown that persistent private information may lead to stochastic cycles in repeated labor negotiations. While several of the results presented here are reminiscent of Kennan's, the two models differ in important ways. First, in the setting studied by Kennan, a monopolist labor union attempts to learn the current valuation (low or high) of a monopsonist firm in full knowledge of the underlying stochastic process (an infinite-horizon Markov chain). In the present model, by contrast, the monopolist does not know the buyer's current valuation (low or high) and which of a continuum of two-period processes ($\lambda \in [0, 1]$) may have generated it.

In Kennan's model, the monopolist may use a screening offer in order to learn the buyer's current valuation and hence, the state of the process. In the present model, the seller may make an offer that

[3]See, for example, Laffont and Tirole (1988), and especially Hart and Tirole (1988).

[4]See, for example, Acquisti and Varian (2002), Taylor (2003), Shaffer and Zhang (2000), Fudenberg and Tirole (2000), and Villas Boas (1999).

[5]See Krugman (2000) and Streitfield (2000).

reveals the buyer's first-period valuation and allows her to update her beliefs about the underlying process but not to learn it with certainty. In Kennan's model, the seller eventually opts to separate types with a screening offer, while the monopolist in the present model seldom makes separating offers. Interestingly, in both Kennan's model and the present one, the monopolist sometimes makes equilibrium price offers strictly below the minimum valuation of the buyer. In both models, this occurs in order to preempt signaling by a low-value buyer who would reject an offer equal to his valuation. Such signaling would provide valuable information to the monopolist, but the value of the information obtained would not offset the reduced probability of making a current sale.

The infinite-horizon setting of persistent private information studied by Kennan allows him to uncover some interesting and important dynamic effects outside the scope of this investigation. The two-dimensional private information with a permanent component analyzed here, however, adheres more closely to the traditional experimental pricing paradigm where the focus is on learning the structural parameters underlying a noisy demand process.[6]

The model is presented in the next section. In Section 3, the bench-mark setting of a non-strategic buyer is characterized. The analysis at the core of the paper is presented in Section 4, where the first-period expected demand of a strategic buyer is derived. Sections 5 and 6 deal respectively with the best and worst equilibria for the seller and contain most of the economic results. Some brief concluding remarks are presented in Section 7. Proofs not appearing in the text have been relegated to Appendix A (lemmas and theorems) and Appendix B (examples).[7]

2 The Model

There are two risk-neutral players, a seller (S, she) and a buyer (B, he). In each period $t = 1, 2$, B demands one unit of a good which S may produce and sell to him. S's production cost is normalized to zero. B's valuation for the good in period t, v_t, equals 1 (a convenient normalization) with probability λ and equals ν with probability $1 - \lambda$, $\nu \in (0, 1)$. The demand parameter λ can be thought of as representing B's income or an underlying preference parameter. It is itself the realization of a random variable which is continuously distributed on $[0, 1]$ with probability density function $f(\lambda)$. Let $E[\lambda]$ denote the expected value of λ under this prior.

At the beginning of the game, B privately observes λ and v_1, and he privately observes v_2 at the beginning of the second period. Hence, in any given period, B's "type" has two components, a permanent structural component $\lambda \in [0, 1]$ and a noisy transitory component $v_t \in \{\nu, 1\}$.[8]

[6] In a recent working paper, Battaglini (2004) also studies a related setting in which a firm has a long-term relationship with a customer whose demand parameter is determined over time by a Markov process. Battaglini's findings, however, differ markedly from those presented here, primarily because he considers a setting with full-commitment in which the buyer completely reveals his private information at the outset of the relationship.

[7] Appendix B was included for the convenience of referees, but not intended for a publication.

[8] While B's type is multi-dimensional, the focus here is on pricing without commitment rather than on an optimal monopolistic screening mechanism (e.g., Armstrong 1996, Rochet and Chone 1998).

In each period $t = 1, 2$, B and S play an extensive-form game with the following stages.

1. B observes his valuation $v_t \in \{\nu, 1\}$.

2. S announces price $p_t \in \Re_+$ at which she is willing to sell the good to B.

3. B either accepts ($q_t = 1$) or rejects ($q_t = 0$) S's offer.

4. B's (contemporaneous) payoff is $q_t(v_t - p_t)$, and S's payoff is $q_t p_t$.

For ease of exposition, there is assumed to be no discounting.[9]

Note that while this game has a recursive structure, it is not a repeated game due to the presence of asymmetric information. Specifically, S updates her prior beliefs about λ from the first period to the second.

Let $h_S \equiv (p_1, q_1)$ be the history of first-period events observed by S, and let $h_B \equiv (\lambda, v_1, p_1, q_1)$ be the history of first-period events observed by B at the beginning of period 2. A behavior strategy for S is a pair of probability distributions, $(\Phi_1(p_1), \Phi_2(p_2; h_S))$, over all possible price offers. Similarly, a behavior strategy for B is a pair of functions, $(\gamma_1(\lambda, v_1, p_1), \gamma_2(v_2, p_2; h_B))$, where γ_t is the probability that B accepts S's offer in period t.

Let $f(\lambda|h_S)$ denote S's posterior beliefs about λ at the beginning of period 2. Likewise, let $E[\lambda|h_S]$ denote her updated expectation. The solution concept employed is efficient perfect Bayesian equilibrium (PBE); i.e., a PBE in which indifference about pricing or purchasing is resolved in favor of efficiency. (Since inefficient PBEs occur only for a non-generic set of parameters, the efficiency criterion is fairly innocuous and is suppressed in the discussion below.)

Observe that in the second period, B optimally accepts any price that does not exceed his valuation regardless of the history. Given this, S will either price at 1 and sell with probability $E[\lambda|h_S]$, or she will price at ν and sell with probability one. This is stated formally in the following lemma.

Lemma 1 (Second-Period Equilibrium Behavior) *In any PBE, B's strategy in period 2 is*

$$\gamma_2(v_2, p_2; h_B) = \begin{cases} 1, & \text{if } v_2 \geq p_2 \\ 0, & \text{if } v_2 < p_2. \end{cases}$$

Also, in any PBE, $\Phi_2(p_2; h_S)$ is a two-point distribution. Specifically, S offers $p_2 = \nu$ with probability $\theta(h_S)$ and $p_2 = 1$ with probability $1 - \theta(h_S)$ according to

$$\theta(h_S) = \begin{cases} 0, & \text{if } \nu < E[\lambda|h_S] \\ \text{some } \theta \in [0, 1], & \text{if } \nu = E[\lambda|h_S] \\ 1, & \text{if } \nu > [\lambda|h_S]. \end{cases}$$

[9] Discounting does not change the results qualitatively, but it complicates the analysis by adding a "kink" to expected first-period demand.

With this lemma in hand, it is possible to derive first-period equilibrium pricing and purchasing behavior, which is the focus of the ensuing sections.

3 The Classic Non-Strategic Buyer Setting

In this section, the seller's problem when facing a non-strategic buyer originally analyzed by Rothschild (1974) is recast in the simple two-period two-valuation framework of the current paper. In the next section, the straightforward solution to this problem is shown to contrast sharply with the case of a strategic buyer.

Formally, suppose – as in Rothschild (1974) – that S faces two stochastically equivalent (i.e., λ is the same), but distinct buyers, B_1 and B_2, who arrive sequentially. In this case, S learns about the demand parameter of B_2 by observing the purchasing behavior of B_1.

Denote by $\underline{E}$ the expected value of λ given $v_1 = \nu$, and $\overline{E}$ the expected value of λ given $v_1 = 1$. Straightforward calculations yield

$$\underline{E} \equiv E[\lambda|v_1 = \nu] = \frac{E[\lambda] - E\left[\lambda^2\right]}{1 - E[\lambda]}$$

and

$$\overline{E} \equiv E[\lambda|v_1 = 1] = \frac{E\left[\lambda^2\right]}{E[\lambda]}.$$

Observe that $\underline{E} < E[\lambda] < \overline{E}$.

Proposition 1 (Equilibrium with a Non-Strategic Buyer) *In any PBE, B_t accepts if and only if $v_t \geq p_t$. Equilibrium prices are given by:*

(i) $p_1 = p_2 = 1$, *if* $\nu < \underline{E}$;

(ii) $p_1 = 1$ *and* $p_2 = q_1 + (1 - q_1)\nu$, *if* $\nu \in [\underline{E}, \nu')$;

(iii) $p_1 = p_2 = \nu$, *if* $\nu \geq \nu'$,

where

$$\nu' \equiv \frac{E[\lambda] + E\left[\lambda^2\right]}{1 + E[\lambda]}.$$

Obviously, B_t will purchase the good if and only if the price does not exceed his valuation. Given this, S would never charge $p_t \neq \nu$ or 1. Note, however, that pricing at ν in the first period provides no information about the demand parameter λ because B_1 always accepts this offer. Pricing at 1 does reveal information because B_1 accepts if and only if $v_1 = 1$.

S finds it optimal to charge 1 in both periods if ν is low enough, and she charges ν in both periods if it is sufficiently high. In the intermediate range, $\nu \in [\underline{E}, \nu')$, S charges 1 in the first period, $p_2 = 1$ following acceptance and $p_2 = \nu$ following rejection. When $\nu \in (E[\lambda], \nu')$, S *experiments* by setting

$p_1 = 1$. That is, her first-period payoff would be maximized at $p_1 = \nu$, but she opts to set $p_1 = 1$ in order to obtain information about B_2's demand parameter, λ. It is straightforward to verify that this price experimentation lowers expected social surplus; i.e., the value of the information obtained by S is less than the concomitant loss in expected consumer surplus. In order to focus on settings where information is potentially valuable to S, the following necessary condition is assumed to hold throughout the remainder of the paper:

$$\underline{E} < \nu < \overline{E}.$$

4 The Strategic Buyer Setting

In this section, the case of a single buyer, B, who is interested in buying one unit of the good in each period is investigated. In particular, B's first-period equilibrium behavior on the continuation game following any price offer p_1 is derived.

From Lemma 1, the expected payoff to B from accepting an offer of p_1 is

$$v_1 - p_1 + \lambda\theta(p_1, 1)(1 - \nu),$$

and his expected payoff from rejecting is

$$\lambda\theta(p_1, 0)(1 - \nu).$$

This simple observation serves as proof of the following claim.

Lemma 2 (Dynamic Incentives) *In any PBE, B accepts p_1 if and only if*

$$v_1 - p_1 \geq \lambda(\theta(p_1, 0) - \theta(p_1, 1))(1 - \nu).$$

Next, note that in any PBE, it must be the case that $E[\lambda|p_1, q_1]$ is derived from B's first-period behavior given $\theta(p_1, q_1)$, and $\theta(p_1, q_1)$ is optimal for S given $E[\lambda|p_1, q_1]$. This interdependence between optimal actions and beliefs gives rise to the following lemma.

Lemma 3 (Beliefs and Actions) *Let $D_1(p_1)$ denote the probability S assigns to acceptance of her first-period offer. In any PBE, if $D_1(p_1) \in (0, 1)$, then*

$$E[\lambda|p_1, 1] \geq E[\lambda|p_1, 0],$$

$$\theta(p_1, 1) \leq \theta(p_1, 0).$$

Intuitively, since B is more likely to accept when $v_1 = 1$, S's beliefs about λ should be higher when she observes $q_1 = 1$ than when she observes $q_1 = 0$.

The following result is proven by applying Lemma 2 and Lemma 3.

Corollary 1 (Honest Rejections) *In any PBE, B always rejects $p_1 > v_1$.*

The next step is to determine when B with $v_1 = 1$ accepts an offer of $p_1 \in (\nu, 1]$. Lemmas 2 and 3 indicate that B with high λ has the most to gain from strategically rejecting such an offer (i.e., from pooling with B's possessing $v_1 = \nu$). Hence, suppose that B with $v_1 = 1$ accepts $p_1 \in (\nu, 1]$ if and only if $\lambda \leq \mu$ for some $\mu \in [0, 1]$. Then the expected value of λ conditional on acceptance is

$$\alpha(\mu) \equiv E[\lambda | v_1 = 1 \cap \lambda \leq \mu] = \frac{\int_0^\mu \lambda^2 f(\lambda)\,\mathrm{d}\lambda}{\int_0^\mu \lambda f(\lambda)\,\mathrm{d}\lambda},$$

and the expected value of λ conditional on rejection is

$$\rho(\mu) \equiv E[\lambda | \{v_1 = \nu\} \cup \{v_1 = 1 \cap \lambda > \mu\}] = \frac{E[\lambda] - \int_0^\mu \lambda^2 f(\lambda)\,\mathrm{d}\lambda}{1 - \int_0^\mu \lambda f(\lambda)\,\mathrm{d}\lambda}.$$

These functions have some important properties which are summarized in the following technical lemma.

Lemma 4 (Geometric Properties of α and ρ) *Functions α and ρ possess the following properties:*

(i) *α starts at $\alpha(0) = 0$ and increases monotonically until it ends at $\alpha(1) = \overline{E}$.*

(ii) *ρ starts at $\rho(0) = E[\lambda]$, increases until it crosses the 45-degree line, and then decreases until it ends at $\rho(1) = \underline{E}$.*

(iii) *α and ρ cross once, and at their intersection, $m_{\min}$, $\alpha(m_{\min}) = \rho(m_{\min}) = E[\lambda]$.*

Lemma 4 implies that there exists a unique number $m \in [m_{\min}, 1)$ defined as follows:

$$m \equiv \begin{cases} \rho^{-1}(\nu), & \text{if } \nu \in (\underline{E}, E[\lambda]) \\ \alpha^{-1}(\nu), & \text{if } \nu \in \big[E[\lambda], \overline{E}\big). \end{cases}$$

In Figure 1, m is shown for the case $\nu \in \big(E[\lambda], \overline{E}\big)$.

Given m, define the constant

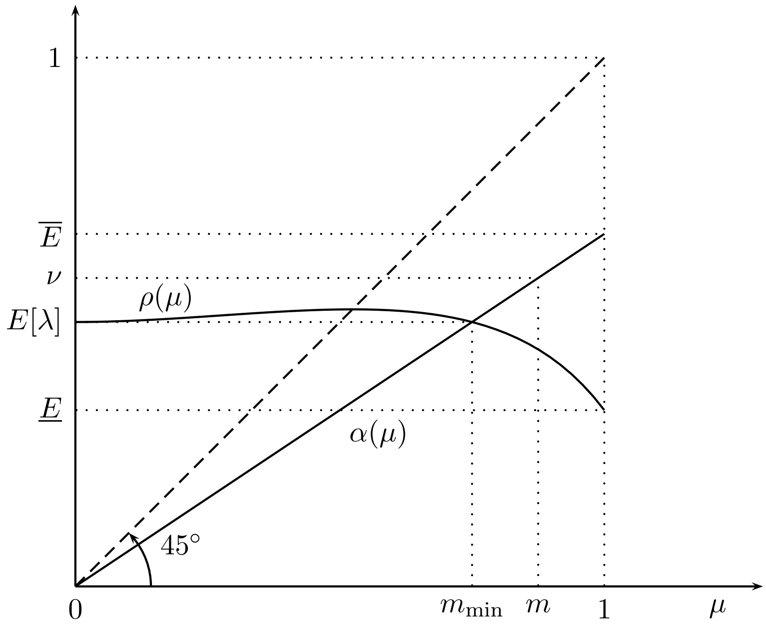
$$\overline{p} \equiv 1 - m(1 - \nu).$$

For any $p_1 \in (\nu, \overline{p}]$, the next result establishes that the marginal type for B that accepts when $v_1 = 1$ is given by the function

$$\mu(p_1) \equiv \frac{1 - p_1}{1 - \nu}.$$

Observe that $\mu(p_1)$ is monotone decreasing with $\mu(\nu) = 1$ and $\mu(\overline{p}) = m$.

Figure 1: Geometric Properties of α and ρ



Lemma 5 (Strategic Rejections) *Suppose $v_1 = 1$. In any PBE, the following must hold:*

(i) *If $p_1 \in (\nu, \overline{p}]$, then B accepts the price if and only if $\lambda \leq \mu(p_1)$. S sets $p_2 = 1$ following acceptance ($\theta(p_1, 1) = 0$) and $p_2 = \nu$ following rejection ($\theta(p_1, 0) = 1$).*

(ii) *If $p_1 \in (\overline{p}, 1]$, then B accepts the price if and only if $\lambda \leq m$. When $\nu < E[\lambda]$, S sets $p_2 = 1$ following acceptance ($\theta(p_1, 1) = 0$) and randomizes between ν and 1 following rejection according to*

$$\theta(p_1, 0) = \frac{1 - p_1}{m(1 - \nu)}.$$

When $\nu > E[\lambda]$, S sets $p_2 = \nu$ following rejection ($\theta(p_1, 0) = 1$) and randomizes between ν and 1 following acceptance according to

$$\theta(p_1, 1) = 1 - \frac{1 - p_1}{m(1 - \nu)}.$$

To understand this result, first consider $p_1 \in (\nu, \overline{p}]$. Given that B accepts p_1 if and only if $v_1 = 1$ and $\lambda \leq \mu(p_1)$, S updates her beliefs: $E[\lambda | p_1, 1] = \alpha(\mu(p_1)) \geq \nu$ and $E[\lambda | p_1, 0] = \rho(\mu(p_1)) \leq \nu$. Hence, S optimally sets $p_2 = 1$ following acceptance and $p_2 = \nu$ following rejection.

Notice that "reverse screening" occurs over this range in the sense that high prices induce rejection by high long-run types. Indeed, as p_1 increases, the "marginal" type, $\mu(p_1)$, falls and the set of types willing to accept shrinks. Acceptance and rejection, therefore, become less informative (i.e., $\alpha - \rho$ gets smaller). Once $p_1 = \overline{p}$, further increases in p_1 cannot induce more strategic rejection because acceptance would otherwise indicate a sufficiently low value of λ that S would prefer to set $p_2 = \nu$. In particular, for $p_1 \in (\overline{p}, 1]$, the marginal type must remain at m. This requires S to randomize between $p_2 = \nu$ and $p_2 = 1$ in order to keep B with $v_1 = 1$ and $\lambda = m$ indifferent between accepting and rejecting p_1.

To complete the characterization of equilibrium play for B, it remains to consider $p_1 \leq \nu$. The analysis over this range, however, is more complicated because B's equilibrium behavior is not unique. In particular, there exists a lower bound $\underline{p}$ such that B accepts in any PBE if $p_1 \leq \underline{p}$. For any $p_1 \in (\underline{p}, \nu]$, however, either pooling or signaling may occur in equilibrium. To illustrate this, the two extreme equilibria that involve minimal signaling (i.e., all types accept $p_1 \leq \nu$) and maximal signaling (i.e., strategic rejections by some types for all $p_1 \in (\underline{p}, \nu]$) are derived. For ease of exposition, these equilibria are called (using S's perspective) respectively *the Good PBE* and *the Bad PBE*, with the understanding that they actually bracket a continuum of intermediate cases. In particular, an *Intermediate PBE* obtains if the Good PBE holds for some proper subset of prices $p_1 \in (\underline{p}, \nu]$ and the bad PBE holds for the complementary subset of prices in this range.

Interestingly, B's behavior in the Bad PBE for prices $p_1 \in (\underline{p}, \nu]$ is the mirror image of his behavior for prices in $(\nu, \overline{p}]$. Specifically, for $p_1 \in (\nu, \overline{p}]$, strategic rejections *conceal* information through pooling. On the other hand, for $p_1 \in (\underline{p}, \nu]$, it is shown below that strategic rejections *reveal* information through signaling.

The following result is proven by applying Lemma 2 and Lemma 3.

Corollary 2 (Honest Acceptances) *In any PBE, B with $v_1 = 1$ always accepts $p_1 \leq \nu$.*

The next step is to determine when B with $v_1 = \nu$ accepts $p_1 \leq \nu$. Lemmas 2 and 3 indicate that B with high λ has the most to gain from strategically rejecting such an offer (i.e., from separating from B's possessing $v_1 = 1$). Hence, suppose that B with $v_1 = \nu$ accepts $p_1 \leq \nu$ if and only if $\lambda \leq \widehat{\mu}$ for some $\widehat{\mu} \in [0, 1]$. Then the expected value of λ conditional on acceptance is

$$\widehat{\alpha}\left(\widehat{\mu}\right) \equiv E[\lambda | \{v_1 = 1\} \cup \{v_1 = \nu \cap \lambda \leq \widehat{\mu}\}] = \frac{E[\lambda] - \int_{\widehat{\mu}}^1 \lambda(1-\lambda) f(\lambda)\,\mathrm{d}\lambda}{1 - \int_{\widehat{\mu}}^1 (1-\lambda) f(\lambda)\,\mathrm{d}\lambda},$$

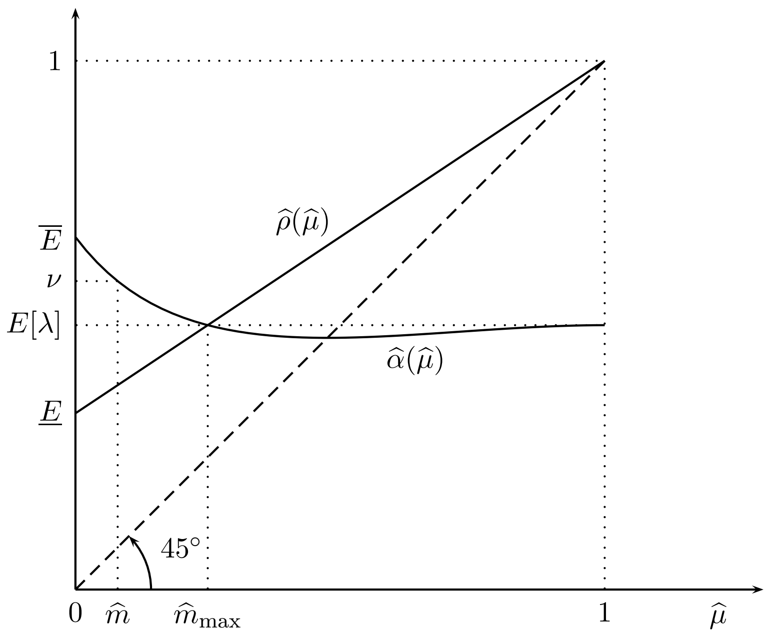
and the expected value of λ conditional on rejection is

$$\widehat{\rho}(\widehat{\mu}) \equiv E[\lambda | v_1 = \nu \cap \lambda > \widehat{\mu}] = \frac{\int_{\widehat{\mu}}^1 \lambda(1-\lambda) f(\lambda)\,\mathrm{d}\lambda}{\int_{\widehat{\mu}}^1 (1-\lambda) f(\lambda)\,\mathrm{d}\lambda}.$$

These functions have some important properties which are summarized in the following technical lemma.

Lemma 6 (Geometric Properties of $\widehat{\alpha}$ and $\widehat{\rho}$) *Functions $\widehat{\alpha}$ and $\widehat{\rho}$ possess the following properties:*

Figure 2: Geometric Properties of $\widehat{\alpha}$ and $\widehat{\rho}$



(i) *$\widehat{\alpha}$ starts at $\widehat{\alpha}(0) = \overline{E}$, decreases until it crosses the 45-degree line, and then increases until it ends at $\widehat{\alpha}(1) = E[\lambda]$.*

(ii) *$\widehat{\rho}$ starts at $\widehat{\rho}(0) = \underline{E}$ and increases monotonically until it ends at $\rho(1) = 1$.*

(iii) *$\widehat{\alpha}$ and $\widehat{\rho}$ cross once, and at their intersection, $\widehat{m}_{\max}$, $\widehat{\alpha}(\widehat{m}_{\max}) = \widehat{\rho}(\widehat{m}_{\max}) = E[\lambda]$.*

Lemma 6 implies that there exists a unique number $\widehat{m} \in (0, \widehat{m}_{\max}]$ defined as follows:

$$\widehat{m} \equiv \begin{cases} \widehat{\rho}^{-1}(\nu), & \text{if } \nu \in (\underline{E}, E[\lambda]) \\ \widehat{\alpha}^{-1}(\nu), & \text{if } \nu \in \big[E[\lambda], \overline{E}\big). \end{cases}$$

In Figure 2, $\widehat{m}$ is shown for the case $\nu \in (E[\lambda], \underline{E})$.

Given $\widehat{m}$, define the constant

$$\underline{p} \equiv \nu - \widehat{m}(1 - \nu).$$

For any $p_1 \in (\underline{p}, \nu]$ the next result establishes that the marginal type for B that accepts in the Bad PBE when $v_1 = \nu$ is given by the function

$$\widehat{\mu}(p_1) \equiv \frac{\nu - p_1}{1 - \nu}.$$

Observe that $\widehat{\mu}(p_1)$ is monotone decreasing with $\widehat{\mu}(\underline{p}) = \widehat{m}$ and $\widehat{\mu}(\nu) = 0$.

Lemma 7 (The Good PBE and the Bad PBE) *Suppose $v_1 = \nu$. There is a Good PBE in which B always accepts $p_1 \leq \nu$, S sets $p_2 = 1$ ($\theta(p_1, 1) = 0$) if $\nu < E[\lambda]$ and $p_2 = \nu$ ($\theta(p_1, 1) = 0$) if $\nu > E[\lambda]$. There is also a Bad PBE in which the following holds:*

(i) *If $p_1 \in (\underline{p}, \nu]$, then B accepts if and only if $\lambda \leq \widehat{\mu}(p_1)$. S sets $p_2 = 1$ following acceptance ($\theta(p_1, 1) = 0$) and $p_2 = \nu$ following rejection ($\theta(p_1, 0) = 1$).*

(ii) *If $p_1 \leq \underline{p}$, then B always accepts the price. S sets $p_2 = 1$ ($\theta(p_1, 1) = 0$) if $\nu < E[\lambda]$ and $p_2 = \nu$ ($\theta(p_1, 1) = 1$) if $\nu > E[\lambda]$.*

To understand the Bad PBE, first consider $p_1 \in (\underline{p}, \nu]$. Given that B rejects if and only if $v_1 = \nu$ and $\lambda > \widehat{\mu}(p_1)$, S updates her beliefs: $E[\lambda|p_1, 0] = \widehat{\rho}(\widehat{\mu}(p_1)) \leq \nu$ and $E[\lambda|p_1, 1] = \widehat{\alpha}(\widehat{\mu}(p_1)) \geq \nu$. Hence, S optimally sets $p_2 = \nu$ following rejection and $p_2 = 1$ following acceptance.

Notice that as p_1 falls, $\widehat{\mu}(p_1)$ increases and the set of long-run types willing to strategically reject shrinks. Acceptance and rejection, therefore, become less informative (i.e., $\widehat{\alpha} - \widehat{\rho}$ gets smaller). For $p_1 \leq \underline{p}$, signaling is not possible because rejection would indicate a sufficiently high value of λ that S would prefer to set $p_2 = 1$. Lemma 7 shows that any $p_1 \leq \underline{p}$ must induce acceptance by all types of B in any PBE.

With Lemmas 2 through 7 in hand, it is possible to formulate the expected first-period demand by B and the expected values of λ conditional on acceptance and rejection of the first-period price.

Proposition 2 (Expected First-Period Demand and Posterior Beliefs) *The probability that B accepts p_1 and the expected values of λ conditional on acceptance/rejection of p_1 are as follows:*

(i) *If $p_1 > 1$, then $D_1(p_1) = 0$ and $E[\lambda|p_1, 0] = E[\lambda]$.*

(ii) *If $p_1 \in (\overline{p}, 1]$, then $D_1(p_1) = \int_0^m \lambda f(\lambda)\,\mathrm{d}\lambda$ and*

$$E[\lambda|p_1, q_1] = \begin{cases} \alpha(m), & \text{if } q_1 = 1 \\ \rho(m), & \text{if } q_1 = 0. \end{cases}$$

(iii) *If $p_1 \in (\nu, \overline{p}]$, then $D_1(p_1) = \int_0^{\mu(p_1)} \lambda f(\lambda)\,\mathrm{d}\lambda$ and*

$$E[\lambda|p_1, q_1] = \begin{cases} \alpha(\mu(p_1)), & \text{if } q_1 = 1 \\ \rho(\mu(p_1)), & \text{if } q_1 = 0. \end{cases}$$

(iv) *If $p_1 \leq \nu$ and the Good PBE obtains, then $D_1(p_1) = 1$ and $E[\lambda|p_1, 1] = E[\lambda]$. If the Bad PBE obtains, then*

$$D_1(p_1) = \begin{cases} 1 - \int_{\widehat{\mu}(p_1)}^1 (1-\lambda) f(\lambda)\,\mathrm{d}\lambda, & \text{if } p_1 \in (\underline{p}, \nu] \\ 1, & \text{if } p_1 \leq \underline{p} \end{cases}$$

and

$$E[\lambda|p_1, q_1] = \begin{cases} \widehat{\alpha}(\widehat{\mu}(p_1)), & \text{if } q_1 = 1 \text{ and } p_1 \in (\underline{p}, \nu] \\ \widehat{\rho}(\widehat{\mu}(p_1)), & \text{if } q_1 = 0 \text{ and } p_1 \in (\underline{p}, \nu] \\ E[\lambda], & \text{if } q_1 = 1 \text{ and } p_1 \leq \underline{p}. \end{cases}$$

Note that the strategic rejections in the Bad PBE for $p_1 \in (\underline{p}, \nu]$ and in all PBE for $p_1 \in (\nu, 1]$ imply that $D_1(p_1)$ is strictly lower than the expected demand of a non-strategic buyer at these prices. In particular, Proposition 1 indicates that a non-strategic buyer accepts $p_1 \leq \nu$ with probability one and accepts $p_1 \in (\nu, 1]$ with probability $E[\lambda]$. Since strategic rejections also impede learning, it follows immediately that S is weakly worse off when facing a strategic buyer.

S's continuation payoff from offering p_1 to a strategic buyer is

$$\Pi_S(p_1) = D_1(p_1)(p_1 + \max\{E[\lambda|p_1, 1], \nu\}) + (1 - D_1(p_1)) \max\{E[\lambda|p_1, 0], \nu\}.$$

The final step in deriving a PBE of the entire game is to determine the value of p_1 that maximizes $\Pi_S(p_1)$. Of course, the solution to this problem depends on what PBE obtains for continuation games with $p_1 \leq \nu$. The Good and the Bad PBE are investigated respectively in the next two sections.

5 The Good Equilibrium

In this section, the Good PBE in which B always accepts first-period offers $p_1 \leq \nu$ is fully characterized. The main finding is that in this equilibrium, S never offers a first-period price that yields her valuable information about B's structural demand parameter, λ.

To ease notation, for any $\mu \in [0, 1]$, define

$$I(\mu) \equiv \int_0^{\mu} \lambda f(\lambda)\, \mathrm{d}\lambda.$$

First, suppose S is considering offering $p_1 \in (\overline{p}, 1]$. Observe that the marginal type m that accepts such an offer is calibrated so that the value of the information conveyed to S is zero. In particular, if $\nu > E[\lambda]$, then

$$E[\lambda|p_1, 1] = \alpha(m) = \nu > \rho(m) = E[\lambda|p_1, 0].$$

Thus, charging $p_2 = \nu$ (which is optimal under the prior) maximizes S's expected second-period return following a first-period acceptance *and* rejection. On the other hand, if $\nu < E[\lambda]$, then

$$E[\lambda|p_1, 1] = \alpha(m) > \nu = \rho(m) = E[\lambda|p_1, 0].$$

Thus, charging $p_2 = 1$ (which is optimal under the prior) maximizes S's expected second-period return following a first-period acceptance *and* rejection. Hence, S's continuation payoff from offering $p_1 \in (\overline{p}, 1]$ can be written as

$$\Pi_S(p_1) = I(m)p_1 + \max\{E[\lambda], \nu\},$$

which is maximized at $p_1 = 1$.

Second, suppose S is considering offering $p_1 \in (\nu, \overline{p}]$. The information conveyed by B's purchasing decision is valuable to S for prices in this range because

$$E[\lambda|p_1, 1] = \alpha(\mu(p_1)) \geq \nu \geq \rho(\mu(p_1)) = E[\lambda|p_1, 0].$$

In the second period, S optimally sets $p_2 = 1$ following acceptance and $p_2 = \nu$ following rejection. Hence, her continuation payoff is

$$\Pi_S(p_1) = I(\mu(p_1))(p_1 + E[\lambda|p_1, 1]) + (1 - I(\mu(p_1)))\nu.$$

Finally, suppose S is considering offering $p_1 \leq \nu$. Since B always accepts such an offer, S's continuation payoff is

$$\Pi_S(p_1) = p_1 + \max\{E[\lambda], \nu\},$$

which is maximized at $p_1 = \nu$.

Summarizing the above, in the Good PBE, S charges some $p_1 \in \{1, \nu\} \cup (\nu, \overline{p})$, depending on the primitives of the model, ν and $f(\lambda)$. In fact, Propositions 3 and 4 below show that S never sets a first-period price $p_1 \in (\nu, \overline{p})$ that yields valuable information.

Proposition 3 (Optimal Prices when Beliefs are Pessimistic) *Suppose $\nu \geq E[\lambda]$. Then, S offers $p_1 = p_2 = \nu$ in the Good PBE.*

Recall from Proposition 1 that when B is not strategic, it is optimal for S to experiment by charging $p_1 = 1$ if $\nu \in [E[\lambda], \nu')$. When B is strategic, however, S runs a substantially increased risk that he will reject any offer $p_1 > \nu$. Specifically, for any informative offer $p_1 \in (\nu, \overline{p})$, Proposition 3 shows that the information rent S must yield to B dominates the value of the information obtained, and S, therefore, opts not to experiment. Since S learns nothing by setting $p_1 = \nu$ in the Good PBE, she also sets $p_2 = \nu$ when her initial beliefs are pessimistic. This, of course, maximizes welfare because B buys the good in both periods with probability one.

Proposition 4 (Optimal Prices when Beliefs are Optimistic) *Suppose $\nu < E[\lambda]$. Then in the Good PBE:*

(i) *If $\nu < \nu''$, S offers $p_1 = p_2 = 1$.*

(ii) *If* $\nu \in [\nu'', E[\lambda])$*, S offers* $p_1 = \nu$ *and* $p_2 = 1$*,*

where ν'' *is implicitly defined by*

$$I\left(\rho^{-1}\left(\nu''\right)\right) = \nu''.$$

The story behind this result is similar to the previous one. The information rent outweighs the value of the information obtained, and S, therefore, prefers prices that yield no valuable information. In the Good PBE, there are two potentially optimal prices for which this is true, $p_1 = \nu$ and $p_1 = 1$. Propositions 3 and 4 indicate that S always sets one of these prices, and hence, she never obtains valuable information about B's demand parameter λ from his first-period purchasing decision.

Proposition 4 indicates that if $\nu < E[\lambda]$, then S always sets $p_2 = 1$ on the equilibrium path. The fact that she is not committed to do this, however, causes serious erosion in her first-period expected profit. If S could commit to offer $p_2 = 1$, then B would not strategically reject offers of $p_1 = 1$. This would raise S's profit whenever $\nu < E[\lambda]$ and would raise total welfare when $\nu < \nu''$.

Finally, while it seems plausible that S might be able to commit not to raise prices, a commitment not to lower them is not renegotiation proof. Note, however, that it is the commitment not to lower prices that has strategic value to S when $\nu < E[\lambda]$. In particular, if S commits to $p_2 = 1$ and B rejects $p_1 = 1$, then S regrets her commitment to a high second-period price (and so does B).

6 The Bad Equilibrium

In this section, the Bad PBE described in Lemma 7 is investigated. While it is not possible to provide a full characterization for this equilibrium, some general results are obtained. Also, two examples are presented that illustrate a variety of first-period equilibrium pricing behavior. Specifically, Example 1 shows that – unlike in the Good PBE – S may offer a first-period price that reveals valuable information about B's demand parameter. Example 2, however, shows that this is not necessarily the case and that S may even elect to set the uninformative first-period price $p_1 = \underline{p}$ which is less than B's lowest possible valuation for the good, ν.

To ease notation, for any $\widehat{\mu} \in [0, 1]$, define

$$\widehat{I}(\widehat{\mu}) \equiv 1 - \int_{\widehat{\mu}}^{1} (1 - \lambda) f(\lambda) \,\mathrm{d}\lambda.$$

For first-period prices $p_1 > \nu$, B's purchasing behavior and S's expected profit are the same in the Good and the Bad PBE. Suppose S is considering offering $p_1 \in (\underline{p}, \nu]$. The information revealed by B's purchasing decision over this range of prices is valuable to S in the Bad PBE. Specifically, in the second period, S optimally sets $p_2 = 1$ following acceptance and $p_2 = \nu$ following rejection because

$$E[\lambda | p_1, 1] = \widehat{\alpha}(\widehat{\mu}(p_1)) > \nu > \widehat{\rho}(\widehat{\mu}(p_1)) = E[\lambda | p_1, 0].$$

Given this, S's continuation payoff from offering $p_1 \in (\underline{p}, \nu]$ is

$$\Pi_S(p_1) = \widehat{I}(\widehat{\mu}(p_1))(p_1 + \widehat{\alpha}(\widehat{\mu}(p_1))) + \left(1 - \widehat{I}(\widehat{\mu}(p_1))\right)\nu.$$

Finally, suppose S is considering offering $p_1 \leq \underline{p}$. Since B always accepts such an offer, S's continuation payoff is

$$\Pi_S(p_1) = p_1 + \max\{E[\lambda], \nu\},$$

which is obviously maximized at $p_1 = \underline{p}$.

Summarizing the above, in the Bad PBE, S charges some $p_1 \in \{\underline{p}, 1\} \cup (\underline{p}, \overline{p})$, depending on the primitives of the model, ν and $f(\lambda)$. Before exploring S's first-period pricing decision any further, it is worth establishing that the Bad PBE is indeed bad from S's perspective.

Proposition 5 (The Bad PBE v.s. the Good PBE) *The expected payoff to S is (weakly) lower in the Bad PBE than in the Good PBE.*

This result is easily understood. For $p_1 > \nu$, S's continuation payoff $\Pi_S(p_1)$ is the same in the Bad PBE and in the Good one. Also, S does strictly worse by offering $p_1 = \underline{p}$ in the Bad PBE than by offering $p_1 = \nu$ in the Good one because $\underline{p} < \nu$ and B purchases with probability one in both cases. The only remaining question is whether S can do better by charging an informative price $p_1 \in (\underline{p}, \nu]$ in the Bad PBE than by charging the uninformative price $p_1 = \nu$ in the Good one. It turns out that the value of the information obtained by charging $p_1 \in (\underline{p}, \nu]$ in the Bad PBE never makes up for the lower probability of sale, and hence, S does strictly worse at these prices.

Propositions 3 and 4 of the previous section established that S never sets an informative first-period price in the Good PBE. The following example shows that this is not true for the Bad PBE.

Example 1 (Maximal Learning) Suppose $F(\lambda) = 1 - \sqrt{1-\lambda}$. In this case, $E[\lambda] = 2/3$, $\underline{E} = 2/5$ and $\overline{E} = 4/5$. Consider "neutral" beliefs, $\nu = E[\lambda] = 2/3$. Then, $\widehat{m} = \widehat{m}_{\max} = 4/9$ and $m = m_{\min} < 0.91$.

As discussed above, one of three situations is optimal for S in the Bad PBE: either she sets an informative first-period price $p_1 \in (\underline{p}, \overline{p})$, or she sets the uninformative low price $p_1 = \underline{p}$, or she sets the uninformative high price $p_1 = 1$.

Straightforward algebra shows that S's continuation payoff over $p_1 \in (\underline{p}, \overline{p})$ is maximized at $p_1 = \nu$. B accepts $p_1 = \nu$ if and only if $v_1 = 1$. This corresponds to full separation, $E[\lambda|\nu, 0] = \underline{E}$ and $E[\lambda|\nu, 1] = \overline{E}$. In the second period, S, therefore, offers $p_2 = q_1 + (1-q_1)\nu$. Hence, her expected payoff from $p_1 = \nu$ is

$$\Pi_S(\nu) = E[\lambda]\left(\nu + \overline{E}\right) + (1 - E[\lambda])\nu = \frac{6}{5}.$$

Next, note that

$$\underline{p} = \nu - \widehat{m}(1-\nu) = 14/27.$$

B always accepts $p_1 = \underline{p}$. Since S learns nothing, she is content to set $p_2 = \nu$. Hence, her expected payoff from $p_1 = \underline{p}$ is

$$\Pi_S(\underline{p}) = \underline{p} + \nu = \frac{32}{27} < \frac{6}{5}.$$

Finally, if S charges $p_1 = 1$, then B accepts when $v_1 = 1$ and $\lambda \leq m$. The information conveyed by B's purchasing decision has no value to S, so she is content to offer $p_2 = \nu$ in the second period. Hence, her expected payoff from $p_1 = 1$ is

$$\Pi_S(1) = I(m) + \nu < I(0.91) + \frac{2}{3} = \frac{3127}{3000} < \frac{6}{5}.$$

It is, therefore, optimal for S to offer $p_1 = \nu$, and maximal learning occurs on the equilibrium path.

The next proposition demonstrates that the Bad PBE may be either worse or better for B than the Good PBE depending on parameter values. In particular, the set of parameters for which S sets $p_1 = 1$ is larger in the Bad PBE. On the other hand, there exist parameter values for which S finds it optimal to induce complete pooling by setting a price lower than ν, something she would never do in the Good PBE.

Proposition 6 (Prices in the Bad PBE) *Suppose the Bad PBE obtains, then:*

(i) *There exists $\epsilon > 0$ such that if $\nu < \nu'' + \epsilon$, then S offers $p_1 = p_2 = 1$.*

(ii) *There exists $\xi > 0$ such that if $\nu > \overline{E} - \xi$, then S offers $p_1 = \underline{p}$ and $p_2 = \nu$.*

Recall from Proposition 4 that when $\nu < \nu''$, S charges $p_1 = 1$ in the Good PBE. This price also maximizes S's continuation payoff in this region of the parameter space when the Bad PBE obtains. Indeed, since charging $p_1 = \nu$ is strictly worse for S in the Bad PBE, the region over which it is optimal for her to set $p_1 = 1$ is somewhat larger; i.e., $\nu < \nu'' + \epsilon$.

The most striking aspect of Proposition 6 is that if beliefs are sufficiently pessimistic, then it is optimal for S to set a first period price of $\underline{p}$, which is always strictly less than B's valuation for the good. Specifically, as ν approaches $\overline{E}$: the value of any information S can obtain goes to zero; $\underline{p}$ goes to ν; and the probability that B accepts $p_1 \in (\underline{p}, \nu]$ goes to $E[\lambda] < 1$. When ν is sufficiently close to $\overline{E}$, S, therefore, prefers to sell at $\underline{p}$ with certainty rather than at a marginally higher price with significantly lower probability.

Proposition 6 shows that for arbitrary beliefs, $F(\lambda)$, there exist values of ν for which S sets an uninformative first-period price (either $p_1 = 1$ or $p_1 = \underline{p}$) in the Bad PBE. The following example demonstrates that there exist prior beliefs, $F(\lambda)$, such that S sets an uninformative first-period price for any value of $\nu \in (\underline{E}, \overline{E})$. The example also illustrates that $p_1 = \underline{p}$ does not require ν to be close to $\overline{E}$ or even pessimistic beliefs.

Example 2 (No Learning) Suppose $F(\lambda) = \lambda$ (the uniform distribution). In this case, $E[\lambda] = 1/2$, $\underline{E} = 1/3$ and $\overline{E} = 2/3$. In Appendix B it is shown that when beliefs are pessimistic, $\nu \in [1/2, 2/3)$, then S's continuation payoff is maximized at $p_1 = \underline{p}$. When beliefs are optimistic, $\nu \in (1/3, 1/2)$, then there exists ν''' such that S offers $p_1 = 1$ if $\nu \in (1/3, \nu''')$, and she offers $p_1 = \underline{p}$ if $\nu \in [\nu''', 1/2)$. In other words, learning does not occur on the equilibrium path for any value of ν.

7 Conclusion

This paper investigated a new wrinkle in an old question. In particular, it built on the monopoly price-experimentation literature pioneered by Rothschild (1974) by considering the impact of strategic buyers on the ability of firms to learn about demand. When market demand is composed of many anonymous individuals or when each individual purchases the good only once, strategic considerations are not relevant. Increasingly, however, firms are able to use innovations in information technology to track customers and make them personalized offers that depend on their history of prior purchases. In such settings, consumers have incentives to manipulate the beliefs of sellers in order to induce them to offer low prices.

In the context of the model presented above, it was shown that a buyer may strategically reject a seller's first-period offer for one of two reasons. First, in order to conceal information (i.e., to pool), a high-valuation buyer may reject high prices that would never be accepted by a low-valuation buyer. Second, in order to reveal information (i.e., to signal), a low-valuation buyer may reject low prices that would always be accepted by a high-value buyer. Given this behavior, the seller often finds it optimal to post prices that generate no useful information. Indeed, in the Good equilibrium (where there is no signaling), it was shown that the seller never charges a first-period price that yields valuable information. Learning may occur, however, in the Bad equilibrium (where there is maximal signaling). Nevertheless, the scope for learning by the seller appears to be quite limited even in this case. Indeed, it was shown that in order to preempt signaling, in the Bad equilibrium the seller may actually set a first-period price strictly below the buyer's lowest possible valuation.

The model presented here is probably the simplest one in which learning by a monopolist in the context of a strategic buyer can be meaningfully addressed. It would, therefore, be interesting both theoretically and practically to explore the robustness of the findings presented here to other specifications such as longer time horizons or more sophisticated parameterizations of demand. While such generalizations will undoubtedly involve serious technical hurdles, the value of such research appears large and growing.

Appendix A

PROOF OF PROPOSITION 1: B_t accepts if and only if $p_t \leq v_t$. Given this, S would never charge $p_t \neq \nu$ or 1.

Case 1: $\nu < \underline{E}$**.** If S sets $p_1 = 1$, then the expected values of λ conditional on acceptance and rejection are $\overline{E}$ and $\underline{E}$, respectively. In either case, S sets $p_2 = 1$ by Lemma 1. On the other hand, if S sets $p_1 = \nu$, then the expected value of λ conditional on acceptance is $E[\lambda]$, and the expected value conditional on rejection is immaterial since rejection does not occur in equilibrium. Hence, S sets $p_2 = 1$. It is optimal for S to set $p_1 = p_2 = 1$ rather than $p_1 = \nu$ and $p_2 = 1$ since

$$2E[\lambda] > \nu + E[\lambda]$$

holds for $\nu < \underline{E}$.

Case 2: $\nu \in [\underline{E}, E[\lambda])$**.** If S sets $p_1 = 1$, then she optimally sets $p_2 = \nu$ following rejection and $p_2 = 1$ following acceptance. On the other hand, if S sets $p_1 = \nu$, then she optimally sets $p_2 = 1$. It is optimal for S to charge $p_1 = 1$ and $p_2 = q_1 + (1 - q_1)\nu$ rather than $p_1 = \nu$ and $p_2 = 1$ since

$$E[\lambda]\left(1 + \overline{E}\right) + (1 - E[\lambda])\nu > \nu + E[\lambda]$$

holds for $\nu \in [\underline{E}, E[\lambda])$.

Case 3: $\nu \in \left[E[\lambda], \overline{E}\right)$**.** If S sets $p_1 = 1$, then she optimally sets $p_2 = \nu$ following rejection and $p_2 = 1$ following acceptance. On the other hand, if S sets $p_1 = \nu$, then she optimally sets $p_2 = \nu$. It is optimal for S to charge $p_1 = 1$ and $p_2 = q_1 + (1 - q_1)\nu$ rather than $p_1 = p_2 = \nu$ if and only if

$$E[\lambda]\left(1 + \overline{E}\right) + (1 - E[\lambda])\nu > 2\nu,$$

or

$$\nu < \nu'.$$

Case 4: $\nu \geq \overline{E}$**.** If S sets $p_1 = 1$, then she optimally sets $p_2 = \nu$ following rejection as well as acceptance. On the other hand, if S sets $p_1 = \nu$, then she optimally sets $p_2 = \nu$. It is optimal for S to charge $p_1 = p_2 = \nu$ rather than $p_1 = 1$ and $p_2 = \nu$ since

$$2\nu > E[\lambda] + \nu$$

holds for $\nu \geq \overline{E}$. □

PROOF OF LEMMA 3: The proof consists of 3 steps.

Step 1. By way of contradiction, suppose $\theta(p_1, 1) = \theta(p_1, 0)$ and $E[\lambda|p_1, 1] < E[\lambda|p_1, 0]$. Lemma 2 implies the following:

1. If $p_1 > 1$, then B always rejects the first-period offer. $D_1(p_1) = 0$ in this case.
2. If $p_1 \in (\nu, 1]$, then B accepts if and only if $v_1 = 1$. Hence, $E[\lambda|p_1, 1] = \overline{E} > E[\lambda|p_1, 0] = \underline{E}$, which contradicts the supposition.
3. If $p_1 \leq \nu$, then B always accepts the first-period offer. $D_1(p_1) = 1$ in this case.

Step 2. By way of contradiction, suppose $\theta(p_1, 1) > \theta(p_1, 0)$. Lemma 2 implies the following:

1. If $p_1 > 1$, then B accepts if and only if $v_1 = 1$ and $\lambda \geq \lambda'$, where

$$\lambda' = \frac{p_1 - 1}{(\theta(p_1, 1) - \theta(p_1, 0))(1 - \nu)}.$$

   (If calculated $\lambda' \geq 1$, then B always rejects the first-period offer, $D_1(p_1) = 0$.) Hence, $E[\lambda|p_1, 1] \geq \overline{E} > E[\lambda] \geq E[\lambda|p_1, 0]$. This along with Lemma 1 implies that $\theta(p_1, 1) = 0 \leq \theta(p_1, 0)$ must hold, which contradicts the supposition.

2. If $p_1 \in (\nu, 1]$, then B rejects if and only if $v_1 = \nu$ and $\lambda < \lambda'$, where

$$\lambda' = \frac{p_1 - \nu}{(\phi(p_1, 1) - \phi(p_1, 0))(1 - \nu)}.$$

   (If calculated $\lambda' \geq 1$, then B accepts p_1 if and only if $v_1 = 1$.) Hence, $E[\lambda|p_1, 1] \geq E[\lambda] > \underline{E} \geq E[\lambda|p_1, 0]$. This along with Lemma 1 implies that $\theta(p_1, 1) \leq \theta(p_1, 0) = 1$ must hold, which contradicts the supposition.

3. If $p_1 \leq \nu$, then B always accepts the first-period offer. $D_1(p_1) = 1$ in this case.

Step 3. By way of contradiction, suppose $\theta(p_1, 1) \neq \theta(p_1, 0)$ and $E[\lambda|p_1, 1] < E[\lambda|p_1, 0]$. This implies either $E[\lambda|p_1, 1] < \nu \leq E[\lambda|p_1, 0]$ or $E[\lambda|p_1, 1] \leq \nu < E[\lambda|p_1, 0]$. Thus, it must be $\theta(p_1, 1) > \theta(p_1, 0)$, which cannot happen in equilibrium by Step 2. □

Proof of Lemma 4: Each part is proven in turn.

(i) Differentiating α gives

$$\alpha'(\mu) = \frac{\mu f(\mu) \int_0^\mu (\mu - \lambda)\lambda f(\lambda)\, \mathrm{d}\lambda}{(\int_0^\mu \lambda f(\lambda)\, \mathrm{d}\lambda)^2}.$$

This is strictly positive for $\mu > 0$.

(ii) Differentiating ρ gives

$$\rho'(\mu) = \frac{\mu f(\mu)[(E[\lambda] - \int_0^\mu \lambda^2 f(\lambda)\, \mathrm{d}\lambda) - \mu(1 - \int_0^\mu \lambda f(\lambda)\, \mathrm{d}\lambda)]}{(1 - \int_0^\mu \lambda f(\lambda)\, \mathrm{d}\lambda)^2}.$$

This is positive for sufficiently small $\mu > 0$. Hence, ρ is initially increasing. Moreover, setting the above expression equal to zero establishes that ρ has a unique critical point where it crosses the 45-degree line. Hence, ρ is increasing up to this point and decreasing thereafter.

(iii) Equating $\alpha(m_{\min})$ and $\rho(m_{\min})$ and performing simple algebra reveals $\alpha(m_{\min}) = E[\lambda]$. □

Proof of Lemma 5: Each part is proven in turn.

(i) Consider $p_1 \in (\nu, \overline{p}]$. Given B's strategy to accept p_1 if and only if $v_1 = 1$ and $\lambda \leq \mu(p_1)$, S's posterior beliefs are $E[\lambda|p_1, 0] = \rho(\mu(p_1)) \leq \nu$ and $E[\lambda|p_1, 1] = \alpha(\mu(p_1)) \geq \nu$. It then follows from Lemma 1 that setting $\theta(p_1, 0) = 1$ and $\theta(p_1, 1) = 0$ is optimal. By Lemma 2, B with $v_1 = 1$ optimally accepts p_1 if and only if

$$1 - p_1 \geq \lambda(\theta(p_1, 0) - \theta(p_1, 1))(1 - \nu) = \lambda(1 - \nu),$$

or $\lambda \leq \mu(p_1)$.

(ii) Consider $p_1 \in (\overline{p}, 1]$ and suppose $\nu > E[\lambda]$. Given B's strategy to accept p_1 if and only if $v_1 = 1$ and $\lambda \leq m$, S's posterior beliefs are $E[\lambda|p_1, 0] = \rho(m) < \nu$ and $E[\lambda|p_1, 1] = \alpha(m) = \nu$. It then follows from Lemma 1 that setting $\theta(p_1, 0) = 1$ and any $\theta(p_1, 1) \in [0, 1]$ is optimal. Mixing probability $\theta(p_1, 1)$ is calibrated to make B with $v_1 = 1$ and $\lambda = m$ indifferent between accepting and rejecting,

$$1 - p_1 = m(1 - \theta(p_1, 1))(1 - \nu).$$

Now suppose $\nu < E[\lambda]$. Given B's strategy to accept p_1 if and only if $v_1 = 1$ and $\lambda \leq m$, S's posterior beliefs are $E[\lambda|p_1, 0] = \rho(m) = \nu$ and $E[\lambda|p_1, 1] = \alpha(m) > \nu$. It then follows from Lemma 1 that setting $\theta(p_1, 1) = 0$ and any $\theta(p_1, 0) \in [0, 1]$ is optimal. Mixing probability $\theta(p_1, 0)$ is calibrated to make B with $v_1 = 1$ and $\lambda = m$ indifferent between accepting and rejecting,

$$1 - p_1 = m\theta(p_1, 0)(1 - \nu).$$

□

Proof of Lemma 6: Each part is proven in turn.

(i) Differentiating $\widehat{\alpha}$ gives

$$\widehat{\alpha}'(\widehat{\mu}) = \frac{-(1 - \widehat{\mu})f(\widehat{\mu})\left[\left(E[\lambda] - \int_{\widehat{\mu}}^{1} \lambda(1 - \lambda)f(\lambda)\,\mathrm{d}\lambda\right) - \widehat{\mu}\left(1 - \int_{\widehat{\mu}}^{1}(1 - \lambda)f(\lambda)\,\mathrm{d}\lambda\right)\right]}{\left(1 - \int_{\widehat{\mu}}^{1}(1 - \lambda)f(\lambda)\,\mathrm{d}\lambda\right)^2}.$$

This is negative for small $\widehat{\mu} > 0$. Hence, $\widehat{\alpha}$ is initially decreasing. Moreover, setting the above expression equal to zero establishes that $\widehat{\alpha}$ has a unique critical point where it crosses the 45-degree line. Hence, $\widehat{\alpha}$ is decreasing up to this point and increasing thereafter.

(ii) Differentiating $\widehat{\rho}$ gives

$$\widehat{\rho}'(\widehat{\mu}) = \frac{(1-\widehat{\mu})f(\widehat{\mu})\int_{\widehat{\mu}}^{1}(\lambda-\widehat{\mu})(1-\lambda)f(\lambda)\,\mathrm{d}\lambda}{\left(\int_{\widehat{\mu}}^{1}(1-\lambda)f(\lambda)\,\mathrm{d}\lambda\right)^2}.$$

This is strictly positive for $\widehat{\mu} < 1$.

(iii) Equating $\widehat{\alpha}$ and $\widehat{\rho}$ and performing simple algebra reveals $\widehat{\alpha}(\widehat{m}_{\max}) = E[\lambda]$. □

Proof of Lemma 7: First, consider the Good PBE. Given B's strategy to accept $p_1 \leq \nu$, S's beliefs remain unchanged, $E[\lambda|p_1, 1] = E[\lambda]$ and $E[\lambda|p_1, 0] = E[\lambda]$ (immaterial). It then follows from Lemma 1 that setting $\theta(p_1, 1) = \theta(p_1, 0) = 0$ if $E[\lambda] > \nu$ and $\theta(p_1, 1) = \theta(p_1, 0) = 1$ if $E[\lambda] < \nu$ is optimal. By Lemma 2, B with $v_1 = \nu$ has no incentives to reject p_1 as

$$\nu - p_1 \geq \lambda(\theta(p_1, 0) - \theta(p_1, 1))(1-\nu) = 0$$

holds for all $\lambda \in [0, 1]$.

Second, consider the Bad PBE.

(i) Suppose $p_1 \in (\underline{p}, \nu]$. Given B's strategy to reject p_1 if and only if $v_1 = \nu$ and $\lambda > \widehat{\mu}(p_1)$, S's beliefs are $E[\lambda|p_1, 1] = \widehat{\alpha}(\widehat{\mu}(p_1)) > \nu$ and $E[\lambda|p_1, 0] = \widehat{\rho}(\widehat{\mu}(p_1)) < \nu$. It then follows from Lemma 1 that setting $\theta(p_1, 1) = 0$ and $\theta(p_1, 0) = 1$ is optimal. By Lemma 2, B with $v_1 = \nu$ optimally accepts p_1 if and only if

$$\nu - p_1 \geq \lambda(\theta(p_1, 0) - \theta(p_1, 1))(1-\nu) = \lambda(1-\nu),$$

or $\lambda \leq \widehat{\mu}(p_1)$.

(ii) Suppose $p_1 \leq \underline{p}$. Signaling is not possible in this case, all types of B must accept p_1. S's beliefs remain unchanged, she optimally sets $\theta(p_1, 1) = \theta(p_1, 0) = 0$ if $E[\lambda] > \nu$ and $\theta(p_1, 1) = \theta(p_1, 0) = 1$ if $E[\lambda] < \nu$. Obviously, B with $v_1 = \nu$ has no incentives to reject p_1. □

Proof of Proposition 3: Offering $p_1 = \nu$ dominates $p_1 = 1$ if and only if

$$2\nu > I(m) + \nu.$$

But, this follows from $\nu \geq E[\lambda] > I(m)$.

Thus, it is left to show that $p_1 = \nu$ dominates all $p_1 \in (\nu, \overline{p}]$, or

$$2\nu > I(\mu)(1 - \mu(1-\nu) + \alpha(\mu)) + (1 - I(\mu))\nu$$

for all $\mu \in [m, 1)$. The condition may be recast as

$$(\nu - I(\mu)(1 - \nu(1-\mu))) + (I(\mu)\mu - I(\mu)\alpha(\mu)) > 0.$$

The first term of this expression is positive. The second term is also positive since

$$I(\mu)\mu - I(\mu)\alpha(\mu) = \int_0^{\mu} (\mu - \lambda)\lambda f(\lambda)\, \mathrm{d}\lambda > 0$$

for all $\mu \in [m, 1)$. □

Proof of Proposition 4: Offering $p_1 = \nu$ dominates all $p_1 \in (\nu, \overline{p}]$ if and only if

$$\nu + E[\lambda] > I(\mu)(1 - \mu(1-\nu) + \alpha(\mu)) + (1 - I(\mu))\nu$$

for all $\mu \in [m, 1)$. The condition may be recast as

$$(E[\lambda] - I(\mu)) + I(\mu)\nu(1-\mu) + (I(\mu)\mu - I(\mu)\alpha(\mu)) > 0.$$

The first two terms are non-negative, the third term is positive.

Thus, it is left to compare S's continuation payoffs from charging $p_1 = 1$ and ν. Offering $p_1 = 1$ dominates $p_1 = \nu$ if

$$I(m) + E[\lambda] > \nu + E[\lambda],$$

or

$$I\left(\rho^{-1}(\nu)\right) > \nu.$$

Taking the limit as ν goes to $\underline{E}$ yields $E[\lambda] > \underline{E}$. Thus, when ν is close to $\underline{E}$, S optimally offers $p_1 = 1$.

On the other hand, offering $p_1 = \nu$ is optimal if

$$\nu > I\left(\rho^{-1}(\nu)\right).$$

Taking the limit as ν goes to $E[\lambda]$ yields $E[\lambda] > I(m_{\min})$. Thus, when ν is close to $E[\lambda]$, S optimally offers $p_1 = \nu$.

Finally, observe that

$$\Delta(\nu) \equiv I\left(\rho^{-1}(\nu)\right) - \nu$$

is decreasing in ν. Thus, there is a cut-off value ν'' such that S's continuation payoff is higher under $p_1 = 1$ if $\nu < \nu''$, and it is higher under $p_1 = \nu$ if $\nu \in (\nu'', E[\lambda])$. □

Proof of Proposition 5: Suppose that in the Bad PBE, S offers $p_1 > \nu$, then she can get the same payoff in the Good PBE by adopting the same strategy. Hence, suppose $p_1 \in \{\underline{p}\} \cup (\underline{p}, \nu]$. There are two cases to consider.

Case 1: $\nu \geq E[\lambda]$. S's expected payoff in the Good PBE is 2ν. First, suppose S offers $p_1 = \underline{p}$ in the Bad PBE, then her expected payoff is $\underline{p} + \nu < 2\nu$.

Second, suppose S offers $p_1 \in (\underline{p}, \nu]$. It can be shown that

$$\widehat{I}(\widehat{\mu})(\nu - \widehat{\mu}(1-\nu) + \widehat{\alpha}(\widehat{\mu})) + \left(1 - \widehat{I}(\widehat{\mu})\right)\nu < 2\nu,$$

for all $\widehat{\mu} \in [0, \widehat{m})$. The condition may be recast as

$$\widehat{I}(\widehat{\mu})\widehat{\mu}(1-\nu) + (\nu - E[\lambda]) + \int_{\widehat{\mu}}^{1} \lambda(1-\lambda) f(\lambda)\, \mathrm{d}\lambda > 0.$$

The first two terms are non-negative, the third term is positive.

Case 2: $\nu < E[\lambda]$. S's expected payoff in the Good PBE is at least $\nu + E[\lambda]$. First, suppose S offers $p_1 = \underline{p}$ in the Bad PBE, then her expected payoff is $\underline{p} + E[\lambda] < \nu + E[\lambda]$.

Second, suppose S offers $p_1 \in (\underline{p}, \nu]$. It can be shown that

$$\widehat{I}(\widehat{\mu})(\nu - \widehat{\mu}(1-\nu) + \widehat{\alpha}(\widehat{\mu})) + \left(1 - \widehat{I}(\widehat{\mu})\right)\nu < \nu + E[\lambda],$$

for all $\widehat{\mu} \in [0, \widehat{m})$. The condition may be recast as

$$\widehat{I}(\widehat{\mu})\widehat{\mu}(1-\nu) + \int_{\widehat{\mu}}^{1} \lambda(1-\lambda) f(\lambda)\, \mathrm{d}\lambda > 0.$$

The first terms is positive, the second term is non-negative. □

Proof of Proposition 6: Each part is proven in turn.

(i) Combined with Proposition 4, Proposition 5 implies that if $\nu \leq \nu''$, then offering $p_1 = 1$ strictly dominates $p_1 = \underline{p}$ and all $p_1 \in (\underline{p}, \overline{p})$. Thus, there exists $\epsilon > 0$ such that when $\nu < \nu'' + \epsilon$, offering $p_1 = 1$ is optimal in the Bad PBE.

(ii) Suppose $\nu > E[\lambda]$. Offering $p_1 = \underline{p}$ dominates all $p_1 \in (\underline{p}, \nu]$ if

$$2\nu > \widehat{I}(\widehat{\mu})(\nu - \widehat{\mu}(1-\nu) + \widehat{\alpha}(\widehat{\mu})) + \left(1 - \widehat{I}(\widehat{\mu})\right)\nu$$

for all $\widehat{\mu} \in [0, \widehat{m})$. Taking the limit as ν goes to $\overline{E}$ yields $\overline{E} > E[\lambda]\overline{E}$.

Second, offering $p_1 = \underline{p}$ dominates all $p_1 \in (\nu, \overline{p}]$ if

$$2\nu > I(\mu)(1 - \mu(1 - \nu) + \alpha(\mu)) + (1 - I(\mu))\nu$$

for all $\mu \in [m, 1)$. Taking the limit as ν goes to $\overline{E}$ yields $\overline{E} > E[\lambda]\overline{E}$.

Third, offering $p_1 = \underline{p}$ dominates $p_1 = 1$ if

$$2\nu > I(\mu) + \nu.$$

Taking the limit as ν goes to $\overline{E}$ yields $\overline{E} > E[\lambda]$.

Thus, there exists $\xi > 0$ such that if $\nu > \overline{E} - \xi$, then offering $p_1 = \underline{p}$ is optimal in the Bad PBE. □

Appendix B

Proof of Example 1: For any $x \in [0,1]$ it is notationally convenient to define

$$\begin{aligned}
J_1(x) &\equiv \int_0^x \lambda f(\lambda)\,\mathrm{d}\lambda, \\
J_2(x) &\equiv \int_0^x \lambda^2 f(\lambda)\,\mathrm{d}\lambda, \\
\widehat{J}_1(x) &\equiv \int_x^1 (1-\lambda) f(\lambda)\,\mathrm{d}\lambda, \\
\widehat{J}_2(x) &\equiv \int_x^1 \lambda(1-\lambda) f(\lambda)\,\mathrm{d}\lambda.
\end{aligned}$$

Note that $I(x) = J_1(x)$ and $\widehat{I}(x) = 1 - \widehat{J}_1(x)$. Employing the change of variables $w = \sqrt{1-\lambda}$ yields

$$\begin{aligned}
J_1(x) &= \int_0^x \frac{\lambda}{2\sqrt{1-\lambda}}\,\mathrm{d}\lambda = \int_{\sqrt{1-x}}^1 \frac{\left(1-w^2\right)2w}{2w}\,\mathrm{d}w \\
&= \int_{\sqrt{1-x}}^1 \left(1-w^2\right)\,\mathrm{d}w = \left(w - \frac{1}{3}w^3\right)\Big|_{\sqrt{1-x}}^1 = \frac{1}{3}\left(2-(2+x)\sqrt{1-x}\right),
\end{aligned}$$

$$\begin{aligned}
J_2(x) &= \int_0^x \frac{\lambda^2}{2\sqrt{1-\lambda}}\,\mathrm{d}\lambda = \int_{\sqrt{1-x}}^1 \frac{\left(1-w^2\right)^2 2w}{2w}\,\mathrm{d}w = \int_{\sqrt{1-x}}^1 \left(1-2w^2+w^4\right)\,\mathrm{d}w \\
&= \left(w - \frac{2}{3}w^3 + \frac{1}{5}w^5\right)\Big|_{\sqrt{1-x}}^1 = \frac{1}{15}\left(8-\left(8+4x+3x^2\right)\sqrt{1-x}\right),
\end{aligned}$$

$$\begin{aligned}
\widehat{J}_1(x) &= \int_x^1 \frac{(1-\lambda)}{2\sqrt{1-\lambda}}\,\mathrm{d}\lambda = \int_x^1 \frac{1}{2}\sqrt{1-\lambda}\,\mathrm{d}\lambda \\
&= -\frac{1}{3}\left(\sqrt{1-\lambda}\right)^{\frac{3}{2}}\Big|_x^1 = \frac{1}{3}(1-x)\sqrt{1-x},
\end{aligned}$$

$$\begin{aligned}
\widehat{J}_2(x) &= \int_x^1 \frac{\lambda(1-\lambda)}{2\sqrt{1-\lambda}}\,\mathrm{d}\lambda = \int_x^1 \frac{1}{2}\lambda\sqrt{1-\lambda}\,\mathrm{d}\lambda = \int_0^{\sqrt{1-x}} \frac{1}{2}\left(1-w^2\right)w2w\,\mathrm{d}w \\
&= \left(\frac{1}{3}w^3 - \frac{1}{5}w^5\right)\Big|_0^{\sqrt{1-x}} = \frac{1}{15}(2+3x)(1-x)\sqrt{1-x}.
\end{aligned}$$

Therefore,

$$
\begin{aligned}
E[\lambda] &= J_1(1) = \frac{2}{3},\\
\underline{E} &= \frac{E[\lambda] - J_2(1)}{1 - E[\lambda]} = \frac{2/3 - 8/15}{1 - 2/3} = \frac{2}{5},\\
\overline{E} &= \frac{J_2(1)}{E[\lambda]} = \frac{8/15}{2/3} = \frac{4}{5},\\
\alpha(\mu) &= \frac{J_2(\mu)}{J_1(\mu)} = \frac{8 - \left(8 + 4\mu + 3\mu^2\right)\sqrt{1-\mu}}{5\left(2 - (2+\mu)\sqrt{1-\mu}\right)},\\
\rho(\mu) &= \frac{E[\lambda] - J_2(\mu)}{1 - J_1(\mu)} = \frac{2 + \left(8 + 4\mu + 3\mu^2\right)\sqrt{1-\mu}}{5\left(1 + (2+\mu)\sqrt{1-\mu}\right)},\\
\widehat{\alpha}(\widehat{\mu}) &= \frac{E[\lambda] - \widehat{J}_2(\widehat{\mu})}{1 - \widehat{J}_1(\widehat{\mu})} = \frac{10 - (2 + 3\widehat{\mu})(1-\widehat{\mu})\sqrt{1-\widehat{\mu}}}{5\left(3 - (1-\widehat{\mu})\sqrt{1-\widehat{\mu}}\right)},\\
\widehat{\rho}(\widehat{\mu}) &= \frac{\widehat{J}_2(\widehat{\mu})}{\widehat{J}_1(\widehat{\mu})} = \frac{2 + 3\widehat{\mu}}{5},\\
\widehat{m} &= \widehat{\rho}^{-1}\left(\frac{2}{3}\right) = \frac{1}{3}\left(5 \times \frac{2}{3} - 2\right) = \frac{4}{9}.
\end{aligned}
$$

The value of m cannot be calculated explicitly. Observe that

$$
\alpha(m) = \frac{2}{3} < \alpha(0.91) = \frac{8 - \left(8 + 4 \times 0.91 + 3 \times (0.91)^2\right) \times 0.3}{5 \times (2 - (2 + 0.91) \times 0.3)} = \frac{7679}{11500}.
$$

Since α is an increasing function, the above inequality implies $m < 0.91$.

S's continuation payoff over $p_1 \in (\underline{p}, \overline{p}]$ is maximized at $p_1 = \nu$. To see this, first consider prices $p_1 \in [\nu, \overline{p})$. In terms of $\mu \in (m, 1]$, S's continuation payoff can be written as

$$
\begin{aligned}
\pi_S(\mu) &= I(\mu)(1 - \mu(1-\nu) + \alpha(\mu)) + (1 - I(\mu))\nu\\
&= I(\mu)(1-\mu)(1-\nu) + I(\mu)\alpha(\mu) + \nu.
\end{aligned}
$$

Differentiating with respect to μ yields

$$
\begin{aligned}
\pi_S'(\mu) &= (\mu f(\mu)(1-\mu) - I(\mu))(1-\nu) + \mu^2 f(\mu)\\
&= (\mu f(\mu) - I(\mu))(1-\nu) + \mu^2 f(\mu)\nu.
\end{aligned}
$$

The first term is positive since f is an increasing function and, therefore,

$$
\mu f(\mu) > \int_0^\mu f(\lambda)\,\mathrm{d}\lambda > \int_0^\mu \lambda f(\lambda)\,\mathrm{d}\lambda = I(\mu).
$$

The second term is also positive. Thus, S's continuation payoff is maximized at $\mu = 1$, which corresponds to $p_1 = \nu$.

Next, consider prices $p_1 \in (\underline{p}, \nu]$. In terms of $\widehat{\mu} \in [0, \widehat{m})$, S's continuation payoff can be written as

$$\begin{aligned}\widehat{\pi}_S(\widehat{\mu}) &= \widehat{I}(\widehat{\mu})(\nu - \widehat{\mu}(1-\nu) + \widehat{\alpha}(\widehat{\mu})) + \left(1 - \widehat{I}(\widehat{\mu})\right)\nu \\ &= -\widehat{I}(\widehat{\mu})\widehat{\mu}(1-\nu) + \widehat{I}(\widehat{\mu})\widehat{\alpha}(\widehat{\mu})) + \nu.\end{aligned}$$

Differentiating with respect to $\widehat{\mu}$ yields

$$\begin{aligned}\widehat{\pi}'_S(\widehat{\mu}) &= -\left((1-\widehat{\mu})f(\widehat{\mu})\widehat{\mu} + \widehat{I}(\widehat{\mu})\right)(1-\nu) + \widehat{\mu}(1-\widehat{\mu})f(\widehat{\mu}) \\ &= (1-\widehat{\mu})f(\widehat{\mu})\widehat{\mu}\nu - \widehat{I}(\widehat{\mu})(1-\nu) \\ &= \frac{(1-\widehat{\mu})\widehat{\mu}}{2\sqrt{1-\widehat{\mu}}} \times \frac{2}{3} - \left(1 - \frac{1}{3}(1-\widehat{\mu})\sqrt{1-\widehat{\mu}}\right) \times \frac{1}{3} \\ &= -\frac{1}{9}\left(3 - (2\widehat{\mu}+1)\sqrt{1-\widehat{\mu}}\right) < 0.\end{aligned}$$

Thus, S's continuation payoff is maximized at $\widehat{\mu} = 0$, which corresponds to $p_1 = \nu$.

As it was shown above, S's continuation payoff over $p_1 \in (\underline{p}, \overline{p}]$ is maximized at $p_1 = \nu$. B accepts $p_1 = \nu$ if and only if $v_1 = 1$; maximum information is conveyed, $E[\lambda|\nu, 0] = \underline{E}$ and $E[\lambda|\nu, 1] = \overline{E}$. S offers $p_2 = q_1 + (1-q_1)\nu$ in the second period; her expected payoff is

$$\Pi_S(\nu) = E[\lambda]\left(\nu + \overline{E}\right) + (1 - E[\lambda])\nu = \frac{2}{3}\left(\frac{2}{3} + \frac{4}{5}\right) + \frac{1}{3} \times \frac{2}{3} = \frac{6}{5}.$$

Another potentially optimal price is

$$p_1 = \underline{p} = \nu - \widehat{m}(1-\nu) = \frac{2}{3} - \frac{4}{9} \times \frac{1}{3} = \frac{14}{27}.$$

B accepts the price; S offers $p_2 = \nu$ in the second period. S's expected payoff is

$$\Pi_S(\underline{p}) = \underline{p} + \nu = \frac{14}{27} + \frac{2}{3} = \frac{32}{27} < \frac{6}{5}.$$

Finally, suppose S charges $p_1 = 1$. B accepts the price if and only if $v_1 = 1$ and $\lambda \leq m$; the information conveyed has no value to S. S offers $p_2 = \nu$ in the second period; her expected payoff is

$$\Pi_S(1) = I(m) + \nu < I(0.91) + \frac{2}{3} = \frac{1}{3}(2 - (2 + 0.91) \times 0.3) + \frac{2}{3} = \frac{3127}{3000} < \frac{6}{5}.$$

□

Proof of Example 2: For uniformly distributed prior beliefs:

$$\begin{aligned}
J_1(x) &\equiv \int_0^x \lambda \, \mathrm{d}\lambda = \frac{x^2}{2}, \\
J_2(x) &\equiv \int_0^x \lambda^2 \, \mathrm{d}\lambda = \frac{x^3}{3}, \\
\widehat{J}_1(x) &\equiv \int_x^1 (1-\lambda) \, \mathrm{d}\lambda = \frac{1}{2}\left(1 - 2x + x^2\right), \\
\widehat{J}_2(x) &\equiv \int_x^1 \lambda(1-\lambda) \, \mathrm{d}\lambda = \frac{1}{6}\left(1 - 3x^2 + 2x^3\right).
\end{aligned}$$

Therefore,

$$\begin{aligned}
E[\lambda] &= J_1(1) = \frac{1}{2}, \\
\underline{E} &= \frac{E[\lambda] - J_2(1)}{1 - E[\lambda]} = \frac{1/2 - 1/3}{1 - 1/2} = \frac{1}{3}, \\
\overline{E} &= \frac{J_2(1)}{E[\lambda]} = \frac{1/3}{1/2} = \frac{2}{3}, \\
\alpha(\mu) &= \frac{J_2(\mu)}{J_1(\mu)} = \frac{2}{3}\mu, \\
\rho(\mu) &= \frac{E[\lambda] - J_2(\mu)}{1 - J_1(\mu)} = \frac{3 - 2\mu^3}{3\left(2 - \mu^2\right)}, \\
\widehat{\alpha}(\widehat{\mu}) &= \frac{E[\lambda] - \widehat{J}_2(\widehat{\mu})}{1 - \widehat{J}_1(\widehat{\mu})} = \frac{2 + 3\widehat{\mu}^2 - 2\widehat{\mu}^3}{3\left(1 + 2\widehat{\mu} - \widehat{\mu}^2\right)}, \\
\widehat{\rho}(\widehat{\mu}) &= \frac{\widehat{J}_2(\widehat{\mu})}{\widehat{J}_1(\widehat{\mu})} = \frac{1 - 3\mu^2 + 2\mu^3}{3\left(1 - 2\mu + \mu^2\right)} = \frac{2\widehat{\mu} + 1}{3}, \\
m_{\min} &= \alpha^{-1}(E[\lambda]) = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}, \\
\widehat{m}_{\max} &= \widehat{\rho}^{-1}(E[\lambda]) = \frac{3}{2} \times \frac{1}{2} - \frac{1}{2} = \frac{1}{4}.
\end{aligned}$$

Also,

$$m = \begin{cases} \rho^{-1}(\nu), & \text{if } \nu \in \left(\frac{1}{3}, \frac{1}{2}\right) \\ \frac{3}{2}\nu, & \text{if } \nu \in \left[\frac{1}{2}, \frac{2}{3}\right), \end{cases}$$

$$\widehat{m} = \begin{cases} \frac{3\nu - 1}{2}, & \text{if } \nu \in \left(\frac{1}{3}, \frac{1}{2}\right) \\ \widehat{\alpha}^{-1}(\nu), & \text{if } \nu \in \left[\frac{1}{2}, \frac{2}{3}\right). \end{cases}$$

For any $\nu \in (1/3, 2/3)$, S's continuation payoff over $p_1 \in (\underline{p}, \overline{p}]$ is maximized at $p_1 = \nu$. To see this, first consider prices $p_1 \in [\nu, \overline{p})$. In terms of $\mu \in (m, 1]$, S's continuation payoff can be written as

$$\pi_S(\mu) = I(\mu)(1-\mu)(1-\nu) + I(\mu)\alpha(\mu) + \nu.$$

Differentiating with respect to μ yields

$$\begin{aligned}\pi_S'(\mu) &= (\mu f(\mu) - I(\mu))(1-\nu) + \mu^2 f(\mu)\nu \\ &= \left(\mu - \frac{\mu^2}{2}\right)(1-\nu) + \mu^2\nu > 0.\end{aligned}$$

Thus, S's continuation payoff is maximized at $\mu = 1$, which corresponds to $p_1 = \nu$.

Next, consider prices $p_1 \in (\underline{p}, \nu]$. In terms of $\widehat{\mu} \in [0, \widehat{m})$, S's continuation payoff can be written as

$$\widehat{\pi}_S(\widehat{\mu}) = -\widehat{I}(\widehat{\mu})\widehat{\mu}(1-\nu) + \widehat{I}(\widehat{\mu})\widehat{\alpha}(\widehat{\mu})) + \nu.$$

Differentiating with respect to $\widehat{\mu}$ yields

$$\begin{aligned}\widehat{\pi}_S'(\widehat{\mu}) &= (1-\widehat{\mu})f(\widehat{\mu})\widehat{\mu}\nu - \widehat{I}(\widehat{\mu})(1-\nu) \\ &= (1-\widehat{\mu})\widehat{\mu}\nu - \left(1 - \frac{1}{2}\left(1 - 2\widehat{\mu} + \widehat{\mu}^2\right)\right)(1-\nu) \\ &= \frac{1}{2}\left(1 + 4\widehat{\mu} - 3\widehat{\mu}^2\right)\nu - \frac{1}{2}\left(1 + 2\widehat{\mu} - \widehat{\mu}^2\right) \\ &< \frac{1}{2}\left(1 + 4\widehat{\mu} - 3\widehat{\mu}^2\right) \times \frac{2}{3} - \frac{1}{2}\left(1 + 2\widehat{\mu} - \widehat{\mu}^2\right) \\ &= -\frac{1}{6}\left(1 - 2\widehat{\mu} + 3\widehat{\mu}^2\right) = -\frac{1}{6}\left((1-\widehat{\mu})^2 + 2\widehat{\mu}^2\right) < 0.\end{aligned}$$

Thus, S's continuation payoff is maximized at $\widehat{\mu} = 0$, which corresponds to $p_1 = \nu$.

Hence, for any $\nu \in (1/3, 2/3)$ S's continuation payoff over $p_1 \in (\underline{p}, \overline{p}]$ is maximized at $p_1 = \nu$. B accepts $p_1 = \nu$ if and only if $v_1 = 1$; maximum information is conveyed, $E[\lambda|\nu, 0] = \underline{E}$ and $E[\lambda|\nu, 1] = \overline{E}$. S offers $p_2 = q_1 + (1-q_1)\nu$ in the second period; her expected payoff is

$$\Pi_S(\nu) = E[\lambda]\left(\nu + \overline{E}\right) + (1 - E[\lambda])\nu = \frac{1}{2}\left(\nu + \frac{2}{3}\right) + \frac{1}{2}\nu = \frac{1}{3} + \nu.$$

Another potentially optimal price is

$$p_1 = \underline{p} = \nu - \widehat{m}(1-\nu) = \begin{cases} \nu - \frac{3\nu - 1}{2}(1-\nu), & \text{if } \nu \in \left(\frac{1}{3}, \frac{1}{2}\right) \\ \nu - \widehat{\alpha}^{-1}(\nu)(1-\nu), & \text{if } \nu \in \left[\frac{1}{2}, \frac{2}{3}\right). \end{cases}$$

B accepts the price; S charges $p_2 = 1$ if $\nu < E[\lambda]$ and $p_2 = \nu$ if $\nu \geq E[\lambda]$. S's expected payoff is

$$\begin{aligned}\Pi_S(\underline{p}) = \underline{p} + \max\{E[\lambda], \nu\} &= \begin{cases} \nu - \frac{3\nu - 1}{2}(1-\nu) + \frac{1}{2}, & \text{if } \nu \in \left(\frac{1}{3}, \frac{1}{2}\right) \\ \nu - \widehat{\alpha}^{-1}(\nu)(1-\nu) + \nu, & \text{if } \nu \in \left[\frac{1}{2}, \frac{2}{3}\right) \end{cases} \\ &= \begin{cases} 1 - \nu + \frac{3\nu^2}{2}, & \text{if } \nu \in \left(\frac{1}{3}, \frac{1}{2}\right) \\ 2\nu - \widehat{\alpha}^{-1}(\nu)(1-\nu), & \text{if } \nu \in \left[\frac{1}{2}, \frac{2}{3}\right). \end{cases}\end{aligned}$$

Finally, suppose S charges $p_1 = 1$. B accepts the price if and only if $v_1 = 1$ and $\lambda \leq m$; the information conveyed has no value to S. S offers $p_2 = 1$ if $\nu < E[\lambda]$ and $p_2 = \nu$ if $\nu \geq E[\lambda]$; her expected payoff is

$$\Pi_S(1) = I(m) + \max\{E[\lambda], \nu\} = \begin{cases} \frac{1}{2} + \frac{\left(\rho^{-1}(\nu)\right)^2}{2}, & \text{if } \nu \in \left(\frac{1}{3}, \frac{1}{2}\right) \\ \nu + \frac{9}{8}\nu^2, & \text{if } \nu \in \left[\frac{1}{2}, \frac{2}{3}\right). \end{cases}$$

Consider pessimistic beliefs, $\nu \in [1/2, 2/3)$. Charging $p_1 = \underline{p}$ yields higher continuation payoff to S than $p_1 = \nu$ if and only if

$$2\nu - \widehat{\alpha}^{-1}(\nu)(1-\nu) > \frac{1}{3} + \nu,$$

or

$$\nu - \widehat{\alpha}^{-1}(\nu)(1-\nu) > \frac{1}{3}.$$

Observe that the left hand side is increasing in ν. Therefore,

$$\nu - \widehat{\alpha}^{-1}(\nu)(1-\nu) \geq \frac{1}{2} - \frac{1}{4} \times \frac{1}{2} = \frac{3}{8} > \frac{1}{3};$$

i.e., $p_1 = \underline{p}$ dominates $p_1 = \nu$.

Charging $p_1 = \underline{p}$ yields higher continuation payoff to S than $p_1 = 1$ if and only if

$$2\nu - \widehat{\alpha}^{-1}(\nu)(1-\nu) > \nu + \frac{9}{8}\nu^2,$$

or

$$\widehat{\alpha}^{-1}(\nu) < \frac{\nu - 9\nu^2/8}{1-\nu}.$$

Differentiating the right hand side with respect to ν yields

$$\frac{d}{d\nu}\left(\frac{\nu - 9\nu^2/8}{1-\nu}\right) = \frac{9}{8} - \frac{1}{8(1-\nu)^2} > \frac{9}{8} - \frac{1}{8(1-2/3)^2} = 0.$$

Therefore,

$$\widehat{\alpha}^{-1}(\nu) \leq \frac{1}{4} < \frac{7}{16} = \frac{1/2 - 9(1/2)^2/8}{1-1/2} \leq \frac{\nu - 9\nu^2/8}{1-\nu};$$

i.e., $p_1 = \underline{p}$ dominates $p_1 = 1$.

As it was shown above, S optimally offers $p_1 = \underline{p}$ when beliefs are pessimistic. Now consider optimistic beliefs, $\nu \in (1/3, 1/2)$. Charging $p_1 = \underline{p}$ yields higher continuation payoff to S than $p_1 = \nu$ if and only if

$$1 - \nu + \frac{3\nu^2}{2} > \frac{1}{3} + \nu,$$

or

$$(3\nu - 2)^2 > 0;$$

i.e., $p_1 = \underline{p}$ dominates $p_1 = \nu$.

Charging $p_1 = \underline{p}$ yields higher continuation payoff to S than $p_1 = 1$ if and only if

$$1 - \nu + \frac{3\nu^2}{2} > \frac{1}{2} + \frac{\left(\rho^{-1}(\nu)\right)^2}{2}.$$

Define the function

$$\Lambda(\nu) \equiv \left(1 - \nu + \frac{3\nu^2}{2}\right) - \left(\frac{1}{2} + \frac{\left(\rho^{-1}(\nu)\right)^2}{2}\right) = \frac{1}{2}\left(1 + 3\nu^2 - 2\nu - \left(\rho^{-1}(\nu)\right)^2\right).$$

Differentiating with respect to ν yields

$$\Lambda'(\nu) = 3\left(\nu - \frac{1}{3}\right) + \frac{\rho^{-1}(\nu)}{(-1) \times \rho'\left(\rho^{-1}(\nu)\right)} > 0.$$

Also, observe that

$$\Lambda\left(\frac{1}{3}\right) = \frac{1}{2}\left(1 + 3 \times \frac{1}{3^2} - 2 \times \frac{1}{3} - 1^2\right) = -\frac{1}{3},$$

$$\Lambda\left(\frac{1}{2}\right) = \frac{1}{2}\left(1 + 3 \times \frac{1}{2^2} - 2 \times \frac{1}{2} - \left(\frac{3}{4}\right)^2\right) = \frac{3}{32}.$$

Therefore, there exists ν''' such that S offers $p_1 = 1$ in equilibrium if $\nu \in (1/3, \nu''')$, and she offers $p_1 = \underline{p}$ if $\nu \in [\nu''', 1/2)$. $\square$

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