

Center for the Study of Mathematics Curriculum

# The Intended Mathematics Curriculum as Represented in State-Level Curriculum Standards: Consensus or Confusion?

# EXECUTIVE SUMMARY WORKING DRAFT (April 14, 2006)

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# **Center for the Study of Mathematics Curriculum**

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#### **PREFACE**

Since 2001, many states have developed new, more specific mathematics curriculum frameworks outlining the intended curriculum, K-8. While some of these documents are intended to be "models" for districts to utilize in shaping local curriculum specifications, others are mandatory, specifying the mathematics all students within the state are expected to learn at particular grades. All appear to serve as guidelines for shaping annual state-wide grade level assessments. As a collection, the new state mathematics curriculum standards represent the mathematics students in the U.S. are expected to learn.

In developing the newest version of curriculum standards, many states provide increased levels of specificity over previous standards, in part due to NCLB requirements related to specification of performance standards and accompanying annual assessments in grades 3-8. While local control of educational decisions, including curriculum standards, is a hallmark of American education, increased accountability has focused more attention on state-level curriculum decisions. A recent survey indicates that the state-level curriculum documents are receiving as much, if not more, attention by school administrators and teachers as the textbooks purchased to support curriculum implementation (Reys, Dingman, Sutter, & Teuscher, 2005).

Given the higher profile of state-level curriculum standards documents, the Center for the Study of Mathematics Curriculum (CSMC), an NSF-funded Center for Learning and Teaching, set out to describe the level of consistency in learning goals across these documents. That is, to what extent are particular learning goals emphasized within state documents and what is the range of grade levels where these learning goals are emphasized?

This report represents the first detailed analysis of the grade placement of particular learning goals across all state-level curriculum documents published and current as of May 2005. One of the difficulties of this task was determining the intent of the learning expectations across states. Due to the vagueness of some learning expectations as well as different terminology used across state documents, interpretations were made that may not reflect the intent of the document. For any misinterpretation the authors of this report assume full responsibility.

The report documents the current situation regarding grade-level mathematics curriculum specification in the U.S. and highlights a general lack of consensus across states. As states continue to work to improve learning opportunities for all students, we hope this report will serve as a useful summary to inform future curriculum decisions. We also hope the report will stimulate discussion at the national level regarding roles and responsibilities of national agencies and professional organizations with regard to curriculum leadership. We believe that serious and collaborative work that results from such a discussion can contribute to a solution to the "mile wide and inch deep" U.S. curriculum, including national consensus regarding important learning goals.

#### **EXECUTIVE SUMMARY**

Since the passage of the federal *No Child Left Behind Act* (NCLB, 2001) state departments of education and local school districts have been scrambling to address the law's requirements. One major area of focus has been identification of student learning expectations in mathematics. These learning expectations, sometimes called curriculum standards, are referred to in recent state documents as grade-level learning expectations (GLEs). They convey the specific mathematics content that students at particular grades are expected to learn (and teachers are expected to teach).

NCLB requires that states adopt "challenging academic content standards" in mathematics, reading/language arts and science that (a) specify what children are expected to know and be able to do; (b) contain coherent and rigorous content; and (c) encourage the teaching of advanced skills (NCLB, 2001). Furthermore, states are required, beginning no later than school year 2005-2006, to measure the achievement of students against the state standards in grades 3 through 8. In fact, 39 states (the District of Columbia and the Department of Defense Education Agency are counted as states) have published new mathematics curriculum standards documents since 2002 (see Table 1). These new documents include learning expectations organized by grade for most, if not all, of the grades, K-8. The current set of state-level mathematics standards documents, including those that articulate grade-level learning expectations (GLE) or secondary course-level learning expectations (CLE) can be found at: http://mathcuriculumcenter.org/statestandards.

Table 1. Publication dates of most recent state-level mathematics curriculum documents (as of 2/1/06).

Year	Number	States
2006	1	MS
2005	10	AK, CA, CT, DC, HI, ID*, NV*, NY, ND, TX
2004	15	AR, DoDEA, GA, KY*, LA, ME, MD, MA, MI, MO, NH*, RI*, SD, VT, WA
2003	8	AL, AZ, KS, MN, NC, UT, WV, WY
2002	5	NJ, NM, OK, OR, VA
2001	3	OH, SC, TN
2000	2	IN, NE
Pre-2000	7	CO, DE, FL, IL, MT, PA, WI
None	1	IA
TOTAL	52	

<sup>\*</sup> Draft document

The state grade-level learning expectation (GLE) documents represent the intended curriculum within the respective state. However, the extent to which these documents present similar messages about content emphasis and grade placement is unclear. The purpose of this study was to describe the emphasis and grade-level placement of particular learning expectations as presented in state GLE documents and to document variations across states. It does not provide a comprehensive summary of the documents. Rather, attention to particular mathematical topics or themes in three strands (Number & Operation, Algebra, and Reasoning) was the focus of the study.

This report describes the amount of variation regarding specified grades at which states call for particular learning goals/expectations. That is, we examined the extent to which there is consensus across state documents on when students should study particular topics. We examined only state mathematics standards documents that included grade-by-grade learning expectations - 43 in all at the time of the study (see Table 2).

The extent to which the content emphasized at various grade-levels is the same or different has implications for the development of publisher-generated textbooks, teacher preparation and comparisons of student performance. Described here (and reported in more depth in the full report) are major findings of the analysis of three strands (Number and Operation, Algebra, Reasoning) across K-8 state GLE documents.

Table 2. Organization of mathematics Grade-level Learning Expectation (GLE) and Course-level Learning Expectation (CLE) standards documents by state and grade-level (as of 2/1/06).

Elementary/Middle School Learning Expectations				,	High School Learning Expectations	
GLE documents (Grades K-8)		Grade-Band documents	No GLE or Grade-band documents	CLE documents	Grade or Grade-band Learning Expectation documents	No GLE, Grade-Band or CLE documents
AL, AZ, AR, CT, DoDEA, DC, FL, GA, HI, ID, IN, KS, LA, MD, MI, MN, MS, MO, NV, NH, NM, NY, NC, ND, OH, OK, OR, RI, SC, SD, TN, TX, VT, VA, WA, WV, WY	AK (3-10) CA (K-7) ME (3-8) NJ (3-8) UT (K-7) KY (4-8)	CO <sup>1</sup> (K-4, 5-8, 9-12) DE (K-3, 4-5, 6-8, 9-10) IL <sup>1</sup> (Early elem., Late elem., middle/junior high, early HS, late HS) MA (1-2,3,3-4,5, 5-6,7,7-8) MT (K-4, 5-8, 9-12) NE (K-1, 2-4, 5-8, 9-12) PA <sup>1</sup> (K-3, 4-5, 6-8, 9-10) WI <sup>1</sup> (K-4, 5-8, 9-12)	IA	AL, AR, CA, DC, GA, HI, IN, KY, MA <sup>2</sup> , MD, MS, NY, NC, OK, TN, TX, UT, VA, WV	MO, OH (9, 10, 11, 12) AK, ID (9, 10) LA (9,10,11-12) DE, KS, WA, NH, RI (9-10) ND, MA <sup>2</sup> (9-10, 11-12) MN (9-11, 11-12) PA (11) AZ, CO, CT, DoDEA, FL, MT, NE, NV, NJ, NM, OR, SC, SD, VT, WI, WY (9-12) IL (Early HS, Late HS)	IA, ME, MI
37	6	8	1	19	31	3
	51		1	50 3		3

<sup>&</sup>lt;sup>1</sup>CO, IL, PA, WI have Assessment Frameworks dated 2003, 2004 or 2005 (CO (2003): 3-10, IL (2004): Grades 3-8,

PA (2004): Grades 3-8 & 11, WI (2005): Grades 3-8 & 10)

<sup>&</sup>lt;sup>2</sup> MA has both Course and Grade Band expectations for high school.

# Findings Regarding Number and Operation Strand

Learning expectations related to the Number and Operation strand account for about a third of the total number of GLEs across all the K-8 state documents and emphasis on this strand is most prominent in grades K-5. Within the strand, topics identified for analysis include: fluency with basic number combinations (basic facts), multi-digit whole number and fraction computation, estimation, and messages related to the role of calculators as computational tools. A summary of major findings follows.<sup>1</sup>

#### Basic Number Combinations

The term basic number combinations refers to the set of single-digit combinations (1+1, 1+2, ... 9+9; 1x1, 1x2, ... 9x9) whose sum (or product) students are expected to recall efficiently and accurately. Table 3 summarizes the grade at which 39 state documents (those that include at least grades K-6) indicate that basic number combination fluency is expected. The most common grade placement for fluency with both addition and subtraction combinations is grade 2. The most common grade placement for multiplication and division combinations is grade 4. Note that the range in grade-levels where fluency is expected is 2-3 years for each operation.

Table 3. Grade placement of learning expectations related to fluency with basic number combinations for each operation.

Operation	Grade	Number of States (N=39)	Operation	Grade	Number of States (N=39)
	1	8		1	7
Addition	2	28	Subtraction	2	27
Addition	3 2		Subtraction	3	3
	Not specified	1		Not specified	2
	3	13		3	6
	4	22		4	20
Multiplication	5	1	Division	5	3
	6	1		6	1
	Not specified	2		Not specified	9

#### Multi-Digit Whole Number Computation

The grade at which students are introduced to multi-digit whole number computation and the grade at which fluency (proficiency with efficient and accurate methods) is expected varies considerably across the state GLE documents. For example, in some states students begin adding multi-digit numbers as early as Kindergarten while in other states this work begins in grade 3. Table 4 summarizes the grade at which students are expected to be fluent with multi-digit whole number computation for each operation. Forty-two state documents were reviewed for this analysis (those that include at least grades 3-7). As noted, the culminating GLE (where fluency is expected) for addition of multi-digit whole numbers ranges from grade 1 to grade 6 across the state documents. Multi-digit

<sup>&</sup>lt;sup>1</sup> A full report of the study will be available this summer.

multiplication is typically a focus at grades 3 or 4 with fluency expected one year later (in grades 4 or 5). Multi-digit whole number division begins as early as grade 3 in some states with an expectation of fluency most typically at grade 5.

When particular learning expectations are examined, further variation is evident. For example, some state documents specify that students should be fluent in adding 2- or 3-digit numbers and others specify very large numbers (one state specifies computational fluency with 9-digit numbers).

Table 4. Grade placement for culminating learning expectations related to fluency with whole number computation for each operation

Operation	Grade	Number of States (N=42)	Operation	Grade	Number of States (N=42)
	1	1		1	1
	2	3		2	2
	3	14		3	15
Addition	Addition 4	15	Subtraction	4	15
	5	5		5	5
	6	3		6	3
	N/S*	1		N/S	1
	3	2		3	0
	4	21		4	12
Multiplication	5	15	Division	5	23
	6	3		6	6
	N/S	1		N/S	1

<sup>\*</sup>Not specified within state document.

## Fraction Computation

Attention to fractions within the school mathematics curriculum spans the full K-8 continuum and includes the introduction and development of the concept of a fraction, multiple representations of fractions, equivalence of fractions, conversions between fraction, decimal and percent forms, and computation with fractions. As with whole number computation, state documents differ in their trajectory regarding the development of computational fluency with fractions. Table 5 provides a summary of the grade-level at which states introduce computation with fractions. Table 6 summarizes the grade-level where students are expected to be fluent computing with fractions. Once again, expectations span several years and highlight lack of consensus among states.

Table 5. Number of states and grade-level when state GLE documents <u>introduce</u> computation with fractions.

Grade	Addition & Subtraction of Fractions	Multiplication of Fractions	Division of Fractions
1 <sup>st</sup> grade	2 states		
2 <sup>nd</sup> grade			
3 <sup>rd</sup> grade	7 states		
4 <sup>th</sup> grade	22 states	1 state	1 state
5 <sup>th</sup> grade	9 states	10 states	6 states
6 <sup>th</sup> grade	1 state	25 states	27 states
7 <sup>th</sup> grade	1 state	5 states	6 states
8 <sup>th</sup> grade		1 state	1 state
Not specified			1 state

Table 6. Number of states and grade-level when state standards indicate <u>expectation of</u> fluency with addition, subtraction, multiplication and division of fractions.

	Addition and Subtraction of Fractions	Multiplication of Fractions	Division of Fractions
4 <sup>th</sup> grade	1 state		
5 <sup>th</sup> grade	15 states	2 states	1 state
6 <sup>th</sup> grade	20 states	25 states	24 states
7 <sup>th</sup> grade	6 states	13 states	14 states
8 <sup>th</sup> grade		1 state	1 state

## Role of Calculators

A recent report published by the Thomas B. Fordham Foundation (2005) criticized state standards documents for their "over-reliance" on calculators. Our review of the state documents does not support this finding. We examined the set of K-8 learning expectations in each state document, compiling each GLE that included one or both terms, "calculator" and/or "technology." Eleven of the 42 state documents make no mention of either term within the set of learning expectations. Another 18 of 42 state documents include ten or fewer references to calculators/technology. The mean number of GLEs referencing calculators in the 31 state documents that do reference either term is 12.8 per state (1.4 per grade).

In the 31 documents that reference one or both terms, we identified a total of 430 learning expectations (less than 3 percent of the total number of learning expectations) utilizing either term. After eliminating GLEs that referred specifically to computer software (34 in all), a total of 396 GLEs were coded for analysis. As might be expected, the number of GLEs referring to "technology" or "calculators" increases as the grades increase from K-8 (see Table 7). As noted, the largest concentration of references to calculators/technology is in the middle grades. In fact, 211 of the 396 (53 percent) calculator-related GLEs identified are found at grades 6, 7, or 8.

Table 7. References to "calculators" or "technology" within learning expectations by grade level across 31 state GLE documents (those which include at least one reference to these terms).

Grade	Total number of references	Mean number of references per
	across all documents	document
K	8	0.26
1	20	0.65
2	27	0.87
3	36	1.16
4	44	1.42
5	50	1.61
6	59	1.90
7	66	2.13
8	86	2.77
Gr. K-8	396	1.42

In addition to counting the number of references, we coded the implied or stated role or purpose of calculator/technology within the GLEs. At the K-2 level, emphasis is on using tools (calculator or technology) to develop or demonstrate conceptual understanding. For grades 3-5 the most common role is for developing concepts and/or solving problems. For grades 6-8 the most common role of the calculator/technology specified in the state documents is to solve problems and/or display data.

In summary, attention to calculators and technology in all but a very few state documents is limited and focused on use as tools for conceptual development and problem solving rather than as an alternative to computational fluency. In fact, all of the documents referring to calculators/technology are explicit in emphasizing that these tools do not replace the need for computational fluency.

#### Findings Regarding Algebra Strand

Within the K-8 algebra strand, five general categories of GLEs that accounted for approximately 90 percent of the learning expectations were identified: Patterns; Functions; Equations, Expressions and Inequalities (EEI); Properties; and Relationships Between Operations.

Figure 1 shows the total number of algebra expectations in three categories (Patterns, Functions, and EEI) that account for the greatest proportion of GLEs in the Algebra strand (about 80 percent). The graph shows that emphasis begins in Kindergarten and steadily increases over grades K to 5 followed by a more dramatic increase in grades 6 to 8. Figure 2 shows the number of expectations for each sub-strand separately. When the three areas of Patterns, Functions, and EEI are graphed on the same axes, the dominance of Pattern GLEs in grades K to 3 with a steady decline over grades 4 to 8 is apparent. The topics of Function and EEI steadily increase in emphasis (as judged by the number of GLEs) from grades K-4 with dramatic increases in EEI from grades 5 to 8 and Functions from grades 6 to 8. Overall, the emphasis on EEI is predominant over both Functions and Patterns in grade 4 to 8.

Figure 1. Total number of GLEs in Patterns, Functions, and EEI across grade levels.

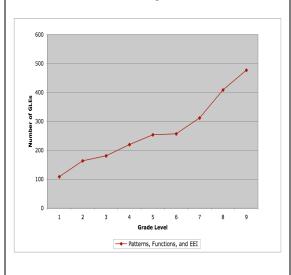
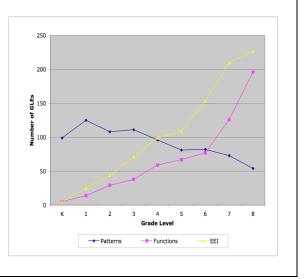


Figure 2. Number of GLEs in Patterns, Functions, and EEI across grade levels.



The number of learning expectations focused on Functions and EEI are an indication of the nature of algebra in the articulated school mathematics curriculum. The EEI strand represents what might be called "symbolic algebra" which suggests that algebra, particularly in the later grades, is focused on the development of symbolic algebra, or an equation-solving-driven algebra, more than on a function-based algebra.

#### Algebra Curriculum for Grades K-8

In algebra few expectations reach mastery over grades K-8. However, there is ample evidence that states vary substantially in the grade-levels at which they concentrate on particular algebra topics. For example, the levels at which states expect the commutative property of multiplication to be taught vary from grade 2 to 8 with grades 3 and 4 having the greatest concentration of states. The levels at which states expect knowledge of variables ranges from kindergarten to grade 8 with the greatest emphasis in grades 4-7. In general, the EEI and Function GLEs are concentrated at grades 6 to 8, while Patterns are concentrated at the lower grades.

In order to have a metric that would represent a minimal level of agreement, we took 21 states (half the 42 state documents analyzed) as our benchmark. When we held this standard for the "common" K-8 algebra curriculum, very few topics made the cut (see Table 8). Table 8 does not tell the whole story, but it does give a picture of the core algebra concepts on which at least 21 states agree should be taught somewhere in grades K-8.

Table 8. Algebra topics/concepts in at least 21 of 42 state documents analyzed.

	Classification of Objects	Repeating Patterns
	Sorting of Objects	Numeric Patterns
Patterns	Rule/Generalization	Geometric Figure/Shape Patterns
	Growing and Shrinking Patterns	Sequences
	Patterns involving Skip Counting	
	Rule/Generalization	Linear Functions
Functions	Change	Slope
	Independent/Dependent Variables	Nonlinear Functions
Expressions,	Variables	1-Step Equations
Equations, and	Expressions	2-Step Equations
Inequalities	Formulae	Inequalities
mequanties	Number Sentences/Equations	
	Commutative Property of Addition	Distributive Property
Proportios	Commutative Property of Multiplication	Additive Identity
Properties	Associative Property of Addition	Multiplicative Identity
	Associative Property of Multiplication	Inverse (Additive plus multiplicative)
Palationshins	Addition and Subtraction as Inverse	Multiplication as Repeated Addition
Relationships Between	Operations	Division as Repeated
	Multiplication and Division as Inverse	Subtraction
Operations	Operations	Order of Operations

While state standards documents include learning expectations related to algebra concepts in lower grade levels, the migration is not as apparent as the rhetoric in the U.S. would imply. There is a gradual buildup to more symbolic algebra at grades 7 and 8, but the work at the lower grades seems to be more conceptual with gradual exposure to ideas. This analysis shows that there is a core of agreement on topics included in K-8 among at least half the states. However, there appears to be little overall agreement across documents in the algebra expectations for a particular grade level. In fact, there were no concepts or topics in algebra for which all 42 states at a given grade level include an expectation specific enough to code for the concept or topic. The greatest agreement reflected in our analysis is that 39 of the 42 states state an expectation that students should study algebraic expressions in grade 7. The next highest level of agreement is that 32 states expect students to study variables at grade 5 and expressions at grade 8. The major result from our analysis is the *lack of agreement* on what should be expected at each grade level in the sub-strands of algebra.

#### Findings Regarding Reasoning Strand

The importance of reasoning is clearly recognized as a K-8 learning goal based on a review of the state curriculum documents. In some state documents a "reasoning" strand provides the organizational structure for conveying intended emphasis on reasoning. Other state documents weave goals related to the development of reasoning throughout the content strands. However, there appears to be no consistency across state documents related to emphasis on reasoning at particular grade levels.

The major emphasis of our analysis focused on learning expectations pertaining to reasoning for verification. Learning expectations related to verification were identified then coded into categories as noted in Table 9. The majority of these learning

expectations were primarily found in three content strands: Data Analysis/Probability, Algebra and Geometry. Table 9 summarizes the number of state standards documents that address one or more of these topics by grade. For example, 17 state documents include at least one GLE related to prediction in grade 1. As noted, prediction is a common theme across grade levels and all categories of reasoning receive greater emphasis in grades 4-8 than in K-3.

Table 9. Number of state standards documents, by grade, that include GLEs in each category of the reasoning framework.

Reasoning Focus	K	1	2	3	4	5	6	7	8
Prediction	8	17	24	24	24	26	22	25	27
Generalization	2	1	8	5	9	10	10	12	12
Verification	2	1	2	5	7	7	6	6	13
Justification	1	1	8	12	14	23	20	19	24
Conclusion/Inference	1	6	9	12	13	16	15	16	17
Conjecture	0	0	1	2	5	7	6	13	10
Testing	1	1	4	6	12	10	6	9	7
Argument	0	0	1	0	2	6	3	7	11
Evaluation	0	0	3	2	2	7	9	9	14

In general, we find that reasoning is not well articulated or integrated across K-8 standards documents. When reasoning GLEs are organized within a separate strand they tend to be broad and general, and isolated from specific content. Idaho's grade 6 GLE provides an example: *Formulate conjectures and discuss why they must be or seem to be true*. Since this GLE is not content-specific, it may be hard to interpret and/or implement at the classroom level.

In summary, most state standards documents give attention to reasoning, incorporating learning expectations related to reasoning either within a separate, designated strand or by weaving messages about reasoning throughout the content strands. However, most state standards fail to address reasoning aspects in a thorough and comprehensive manner across grade levels and content strands. In addition, clarity and specificity of reasoning learning expectations vary across and within state documents.

# Recommendations Regarding Specification of Learning Expectations

Findings from this study confirm that state mathematics curriculum documents vary along several dimensions including grain size (level of specificity of learning outcomes), language used to convey learning outcomes (understand, explore, memorize, etc.), and the grade placement of particular learning expectations. We offer here some suggestions for groups that engage in future efforts to specify grade-level mathematics learning expectations.

• *Identify major goals or focal points at each grade level, K-8.* At each grade, we recommend a general statement of major goals for the grade be stated. These general goals may specify emphasis on a few strands of mathematics or a few topics within strands. These general goals should be coordinated across all

- grades, K-8, to ensure curricular coherence and comprehensiveness. Offering these major goals will provide guidance to teachers in appropriation of instructional time. It may also help reduce superficial treatment of many mathematical topics, a common criticism of the U.S. mathematics curriculum.
- Limit the number of learning goals per grade to focus instruction and deepen learning. The set of learning expectations per grade level should be manageable given the school year. Along with the statement of general goals and priorities for a particular grade, we suggest that the set of learning expectations per grade be limited to 20-25. This number is similar to curriculum standards documents in other countries and may help authors of standards develop an appropriate grain size for communicating learning goals.
- Organize learning expectations by grade and by strand. We recommend that the standards be organized by grade and by content strand. Further, we recommend that standards give attention to both content strands (e.g., Number & Operation, Geometry, Measurement, Algebra, Data Analysis & Probability) and important mathematical processes (e.g. Problem Solving, Reasoning, Representations, and Connections among mathematical concepts and procedures).
- Develop clear statements of learning expectations focusing on mathematics content to be learned. We recommend that learning expectations be expressed succinctly, coherently, and with optimum brevity, limiting the use of educational terms (jargon) that may not communicate clearly to the intended audience of teachers, school leaders, and parents. GLE statements should focus on the mathematics to be learned rather than pedagogy to be employed in presenting the mathematics. The set of learning expectations for a grade should include mathematics to be learned at that grade level (not just what will be assessed). If particular GLEs will be the focus of annual assessments, these should be clearly identified.
- Limit the use of examples within learning expectations. Some state GLE documents include examples (occasionally or frequently, depending on the document) to clarify the learning expectation and others do not. In some documents the examples also include messages regarding suggested pedagogy. We recommend that the use of examples be limited in standards documents. Instead we urge authors to provide clarity within the statement of the GLE. If additional information and/or guidance is needed for particular audiences (e.g. teachers or parents), we suggest that a supplement (or companion document) to the curriculum standards document be developed for this particular purpose.
- Involve people with a broad spectrum of expertise. Many different constituent groups have valuable knowledge and expertise to contribute to the development of curriculum standards. These groups include: classroom teachers, mathematics educators, mathematicians, curriculum supervisors, and researchers in related fields such as educational and developmental psychology and cognitive science.
- Collaborate to promote consensus. Fifty states with 50 state standards documents increases the likelihood of large textbooks that treat many topics superficially. In order to increase the likelihood of focused curriculum materials, states will need to work together to create some level of consensus about important curriculum

goals at each grade. This can be accomplished through state consortiums such as the New England Consortium mentioned earlier, through collaborative efforts sponsored by groups such as the National Council of Supervisors of Mathematics, the Association of State Supervisors of Mathematics or the Council of Chief State School Officers. It can also be accomplished if states build their curriculum standards from a "core curriculum" offered by national groups such as the National Council of Teachers of Mathematics, the College Board and/or Achieve, Inc. In fact, we recommend that a consortium of national groups collaborate to propose a national core curriculum that focuses on priority goals for each grade, K-8. In this way, states might still tailor their own curriculum goals around local needs while ensuring a much greater level of consistency across the states.

Clearly much work and effort has occurred at the state level for setting learning goals for mathematics. The state-level GLE documents present specific learning goals and also describe developmental trajectories for attaining these goals across the elementary years of schooling. For many states, grade-level learning expectations represent a new level of state leadership for curriculum articulation. Although individual documents may provide increased clarity and coherence within their respective state, as a collection they highlight a consistent lack of national consensus regarding common learning goals in mathematics at particular grade levels.

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## ABOUT THE CENTER FOR THE STUDY OF MATHEMATICS CURRICULUM (CSMC)

The Center for the Study of Mathematics Curriculum, funded by the National Science Foundation in 2004, is engaged in a coordinated plan of scholarly inquiry and professional development around mathematics curriculum, examining and characterizing the role of curriculum materials and their influence on both teaching and student learning. The goal is to engage in systemic research to illuminate the essential features and characteristics of curriculum materials and related teacher support that contribute to increased student learning.

Major areas of CSMC work include understanding the influence and potential of mathematics curriculum materials, enabling teacher learning through curriculum material investigation and implementation, and building capacity for developing, implementing, and studying the impact of mathematics curriculum materials.

#### PRINCIPLES THAT GUIDE THE WORK OF CSMC:

A well-articulated, coherent, and comprehensive set of K-12 mathematics learning goals/standards is necessary to large-scale improvement of school mathematics.

Mathematics curriculum materials play a central role in any effort to improve school mathematics and that their development is a scholarly process involving a continual cycle of research-based design, field-testing, evidence gathering, and revision.

Teaching and curriculum materials are highly interdependent and increasing opportunities for student learning rests on better understanding the relationship between curriculum and instruction.

Research addressing mathematics curriculum can inform policy and practice and in so doing narrow the gap between the ideal and the achieved curriculum.

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