Asymmetric Information and Bank Runs

Chao Gu*

October 12, 2007

Abstract

It is known that sunspots can trigger panic-based bank runs and that the optimal banking contract can tolerate panic-based runs. The existing literature assumes that these sunspots are based on a publicly observed, extrinsic randomizing device. In this paper, I extend the analysis of panic-based runs to include an asymmetric-information, extrinsic randomizing device. Depositors observe different, but correlated, signals on the stability of the bank. I find that if the signals that depositors obtain are highly correlated, there exists a correlated equilibrium for some demand deposit contracts. In this equilibrium, a full bank run, a partial bank run, or non-bank run occurs depending on the realization of the signals. Computed examples indicate that in some economies, a demand-deposit contract that tolerates bank runs and partial bank runs is optimal, whereas in some other economies a run-proof contract is optimal.

JEL Classification Numbers: D82, G21, P11

Keywords: Bank runs, randomizing device, sunspot equilibrium, correlated equilibrium, imperfect information.

1 Introduction

In the classic Diamond and Dybvig (1983) bank runs model, bank runs are triggered by a commonly observed random variable, which is modeled formally by Peck and Shell (2003) as a sunspot variable.

*I would like to thank David Easley, Ani Guerdjikova, Todd Keister, Tapan Mitra, Ted Temzelides and seminar participants at the Cornell/Pennsylvannia State University Macro Workshop, Midwest Economic Theory Meeting 2006, Midwest Macroeconomics Meetings 2006, and Far Eastern Meeting of the Econometric Society 2006 for insightful comments. I am especially grateful to Karl Shell for numerous discussions and helpful guidance. All remaining errors are my own. Financial support from the Center for Analytic Economics at Cornell University is gratefully acknowledged. Correspondence: Department of Economics, University of Missouri, Columbia, MO 65203, USA. Email: guc@missouri.edu.

The sunspot variable does not affect any of the fundamentals, such as endowment, preference, or technology. Depositors perfectly observe the realization of the sunspot, and they make their withdrawal decisions accordingly, given that everyone else makes the same decision. An individual depositor faces no uncertainty when he makes the withdrawal decision, for he knows all other depositors will behave in the same way because they observe the same thing. Thus, the publicly observed sunspot serves as a randomizing device, and the probability of bank runs depends solely on the distribution of the sunspot variable.

In this paper, I consider a more general extrinsic randomizing device in the sense that depositors receive different, but correlated, sunspot signals. Depositors are grouped into networks according to their observation of the signals. Depositors in the same network share sunspot information, but depositors do not share information among networks. Receiving their own signals of the sunspot, depositors try to infer the signals that others observe and the actions they take. In this situation, a depositor faces uncertainty when he makes a withdrawal decision, as other people might observe very different signals and make different decisions.

For simplicity, there are only two networks in the model. Each network observes a sunspot signal that takes a value of either 0 or 1. Signals are imperfectly correlated. This is the minimum structure required for the analysis of imperfect coordination. I find that if the signals that depositors obtain are highly correlated, there exists a correlated equilibrium for some demand deposit contracts. In this equilibrium, a full bank run, a partial bank run, or a non-bank run occurs, depending on the realization of the signals. Depositors are coordinated by the imperfectly correlated sunspot signals in the equilibrium. Thus, the sunspot signals serve as an imperfect randomizing device. The probabilities of bank runs and partial runs are determined by the joint distribution of the sunspot signals.

By assuming a more general extrinsic randomizing device, I intend to capture a more general situation in the economy: Our judgment of the economy is based on different information sources. Even if we have the same information, our interpretation of the information can be different. How does an individual depositor make his decisions, knowing others might have different information and might make different judgments? The extrinsic uncertainty can be understood as the intrinsic uncertainty taken to the limit. By focusing on the extrinsic uncertainty, I explain that there exist multiple equilibrium outcomes, due to the imperfect coordination among the depositors in the absence of any fundamental shocks. Empirical studies show that most banking crises are extrinsic-driven panic runs (Boyd et al., 2001). Specifically, before most banking crises happen, no indicator of the economy serves as a good predictor. Nevertheless, the randomizing device in this paper can be understood in either way: as an intrinsic variable taken to the limit or as a pure randomizing device.

Partial bank runs are the unique result of an imperfect coordination device. A partial run results in a lower level of welfare than a non-bank run, however, it does less damage than a full bank run. In this regard, this paper implies that imperfect coordination due to asymmetric information can be a reason why some banking crises are more serious than others.

Given the possibility that the bank runs happen *ex post*, the demand-deposit contract that admits the first-best allocation is usually not optimal (Postlewaite and Vives (1987)). Cooper and Ross (1998) show that with a perfect randomizing device, if the probability of bank runs is small, then the optimal demand-deposit contract admits a run equilibrium; otherwise, a run-proof contract will be provided. Peck and Shell (2003) illustrate that within a broad class of banking mechanisms including partial suspension of convertibility, the optimal contract can tolerate bank runs if the probability is small. This paper confirms the findings of these authors. In some economies, full bank runs and partial bank runs are tolerated.

This paper focuses on a simple demand-deposit contract. A simple demand-deposit contract is widely used in the banking industry. However, the results in my model obtain in a broad class of banking mechanisms. In the appendix, I consider a contract that allows for partial suspension of convertibility. The results still hold.

There is some literature that is related to this model. Soloman (2003, 2004) considers an imperfectly correlated sunspot randomizing device in a twin-crisis model. By assuming *ex-ante* different types of agents, the banking aspect of the twin crisis is reduced to a traditional sunspot equilibrium model. Goldstein and Pauzner (2005) construct a model by the approach of global games in which the noisy signal about fundamentals determines the decisions of the depositors.

The remainder of the paper is organized as follows: Section 2 introduces the model set-up. Section 3 discusses the equilibrium in the postdeposit game. Section 4 discuss the optimal contract in the predeposit game. Section 5 addresses the conclusions.

2 The Model

There are three periods, $t = 0, 1, 2$, and a measure 1 of depositors in the economy. Each depositor is endowed with 1 unit of consumption good in period 0. There is a measure of α $(0 < \alpha < 1)$ impatient depositors; the rest are patient. Impatient depositors derive utility only from consumption in period 1. Their utility is described by $u(c_1)$, where c_1 is the consumption received at $t = 1$. Patient depositors consume in the last period. If a patient depositor receives consumption at $t = 1$, he can store it costlessly and consume it at $t = 2$. Thus, a patient depositor's utility is described by $u(c_1 + c_2)$, where c_2 is the consumption received at $t = 2$. The coefficient of relative risk aversion of the utility function, $-xu''(x)/u'(x)$, is greater than 1 for $x \geq 1$. The utility function is normalized to 0 at $x = 0$, that is, $u(0) = 0$. Whether a depositor is patient or impatient is his private information and is revealed to the individual depositor at $t = 1$.

The investment technology is as follows: One unit of consumption goods invested in period 0 yields 1 unit in period 1 and R ($R > 1$) units in period 2.

The banking market is competitive. The representative bank offers a demand-deposit contract, which describes the amount of consumption goods paid to the depositors who withdraw deposits in periods 1 (c^1) and 2 (c^2), respectively. The bank pays c^1 to the depositors until it runs out of resources. It distributes the remaining resources equally among the depositors who wait until the last period. Therefore, $c^2 = \max\left\{0, \frac{1-nc^1}{1-n}R\right\}$, where n ($0 \leq n \leq 1$) denotes the measure of depositors who withdraw the deposits in period 1. Depositors are isolated from one another (see Wallace (1988)).

Depositors are grouped into two networks. Network i observes a sunspot variable, θ_i, $i = 1, 2$. $\theta_i = \{0, 1\}$. θ_1 and θ_2 are imperfectly correlated. The joint distribution of θ_1 and θ_2 is denoted by $\Pr(\theta_1,\ \theta_2) = p_{\theta_1\theta_2}$, $\sum_{\theta_2=0}^{1}\sum_{\theta_1=0}^{1} p_{\theta_1\theta_2} = 1$. The marginal probabilities are defined by $p_{\theta_1} = \sum_{\theta_2=0}^{1} p_{\theta_1\theta_2}$ and $p_{\theta_2} = \sum_{\theta_1=0}^{1} p_{\theta_1\theta_2}$.

Networks do not communicate with each other. Depositors in the same network share the information of the sunspot they obtain and conjecture the sunspot signal that the other network observes. The conditional distribution, derived from the joint distribution, is $\Pr(\theta_2 \mid \theta_1) = \dfrac{p_{\theta_1\theta_2}}{p_{\theta_1}}$.

Network i has a measure of n_i depositors. By law of large numbers, the measure of impatient depositors in network i is αn_i, $i = 1, 2$. Impatient depositors withdraw at $t = 1$ regardless of other people's decisions. Patient depositors make withdrawal decisions given all available information. Depositors know *ex ante* that there are two networks, but they do not know which network they will be in until $t = 1$.

The sequence of timing is as follows:

Bank announces the contract; Depositors make investment decision.	$t = 0$		predeposit game
Consumption types are revealed; Information shock is realized; Depositors make withdrawal decision.	$t = 1$	postdeposit game	predeposit game
Bank allocates the remaining resources to the rest of the depositors.	$t = 2$	postdeposit game	predeposit game

The postdeposit game starts from $t = 1$ and ends at $t = 2$. In the postdeposit game, the depositors are assumed to have already deposited at the bank. Depositors determine whether to withdraw deposits or not, given the banking contract. Knowing how depositors behave in the postdeposit game, at $t = 0$, the bank decides which contract to offer, and depositors decide whether or not to accept the offer. The entire game, played from $t = 0$ to $t = 2$, is called the predeposit game.

3 Postdeposit Game

A demand-deposit banking contract in the postdeposit game $m = (c^1, c^2)$ satisfies

$$c^2 = \max\left\{0, \frac{1 - nc^1}{1 - n} R\right\}, \text{ where } 0 \leq n \leq 1, c^1 \geq 0, \tag{1}$$

where n is the proportion of depositors who withdraw early.

The participation incentive compatibility constraint is defined as

$$u\left(\frac{1 - \alpha c^1}{1 - \alpha} R\right) \geq u(c^1), \tag{2}$$

which means that if all patient depositors wait until period 2, a patient depositor should receive at least the same amount of consumption goods the impatient depositors received in period 1. This is the minimum requirement for a banking contract so that the patient depositors are willing to wait. Given a demand-deposit contract satisfying (2), if all other patient depositors are honest about their consumption type, then an individual depositor would find waiting until $t = 2$ a better choice than withdrawing immediately at $t = 1$.

Let M denote the set that includes all banking contracts satisfying (1) – (2). This is the set that includes all feasible banking contracts in the traditional bank run literature.

A banking contract that satisfies (1) − (2) allows for a run equilibrium if

$$Eu(\text{not withdraw}|\text{all other patient depositors withdraw}) < \\ Eu(\text{withdraw}|\text{all other patient depositors withdraw}), \tag{3}$$

where Eu denotes a patient depositor's expected utility from his own action given the actions of all other depositors. A contract that allows for a run equilibrium is called a run-admitting contract. Given a run-admitting contract, if everyone else withdraws from the bank, the expected utility of a patient depositor if he chooses to wait is strictly lower than the expected utility if he withdraws as well. Let M^{RA} denote the set that contains all run-admitting contracts. It is a subset of M. In this simple model, a contract is run-admitting if $c^1 > 1$. In the event that all people withdraw their deposits from the bank, the bank will be out of resources. Each depositor receives payment with probability $1/c^1$.

A banking contract $m \in M$ is run-proof if it satisfies

$$Eu(\text{not withdraw}|\text{all other patient depositors withdraw}) \geq \\ Eu(\text{withdraw}|\text{all other patient depositors withdraw}). \tag{4}$$

Let M^{RP} denote the set that contains all run-proof contracts. It is a subset of M and is the complement to M^{RA} by definition.

Given a run-admitting contract, let depositors be coordinated by the sunspot variables if possible. In particular, I look for a correlated equilibrium (Aumann (1987)) that is based on the joint distribution of the sunspot variables. Here I focus on the case in which depositors in the same network take the same action, for I want to show how the sunspot signals coordinate people's behavior. I assume pure strategies. Depositor j's strategy set S_j is described by S_j ={withdraw, not withdraw}. Depositors adopt the same action for the same value of θ_i.

Definition 1 *Given a run-admitting banking contract $m \in M^{RA}$, the distribution of the sunspot variables $\Pr(\theta_1, \theta_2)$ is a correlated equilibrium in the postdeposit game if (i) patient depositors in network 1 withdraw their deposits when observing $\theta_1 = 1$ and do not withdraw when observing $\theta_1 = 0$; (ii) patient depositors in network 2 withdraw their deposits when observing $\theta_2 = 1$ and do not withdraw when observing $\theta_2 = 0$.*

This definition is equivalent to the following four conditions.

For a patient depositor in network 1:

$$p_{10}Eu(\text{W}|\text{W, NW}) + p_{11}Eu(\text{W}|\text{W, W}) \geq p_{10}Eu(\text{NW}|\text{W, NW}) + p_{11}Eu(\text{NW}|\text{W, W}), \quad (5)$$

$$p_{01}Eu(\text{NW}|\text{NW, W}) + p_{00}Eu(\text{NW}|\text{NW, NW}) \geq p_{01}Eu(\text{W}|\text{NW, W}) + p_{00}Eu(\text{W}|\text{NW, NW}); \quad (6)$$

For a patient depositor in network 2:

$$p_{11}Eu(\text{W}|\text{W, W}) + p_{01}Eu(\text{W}|\text{NW, W}) \geq p_{11}Eu(\text{NW}|\text{W, W}) + p_{01}Eu(\text{NW}|\text{NW, W}), \quad (7)$$

$$p_{10}Eu(\text{NW}|\text{W, NW}) + p_{00}Eu(\text{NW}|\text{NW, NW}) \geq p_{10}Eu(\text{W}|\text{W, NW}) + p_{00}Eu(\text{W}|\text{NW, NW}), \quad (8)$$

where W stands for withdraw, and NW for not withdraw. The first argument in $Eu(\cdot)$ is the action of an individual depositor. The second argument denotes the action of the depositors in network 1, and the third argument is the action of the depositors in network 2. The expected utility depends on an individual depositor's own action, the actions of his network members, and the actions of depositors in the other network. Other parameters such as c^1, α, n_1, and n_2 are suppressed here.

If a patient depositor in network 1 observes $\theta_1 = 1$, then by conditional probabilities he knows that with probability $\frac{p_{11}}{p_{11}+p_{10}}$ network 2 observes $\theta_2 = 1$ and withdraws, and with probability $\frac{p_{10}}{p_{11}+p_{10}}$ network 2 observes $\theta_2 = 0$ and waits. For a correlated equilibrium, a patient depositor in network 1 should find "withdraw" the best response given the strategies of the members of the other network and his own network. Therefore, (5) holds. If $\theta_1 = 0$, then a patient depositor in network 1 knows that network 2 will run on the bank with probability $\frac{p_{01}}{p_{01}+p_{00}}$ and will not run with probability $\frac{p_{00}}{p_{01}+p_{00}}$. His network members will not withdraw, and he should find "not withdraw" the best response that maximizes his expected utility. Thus, (6) holds. Similarly, we have equations (7) and (8) for a patient depositor in network 2.

Let M^{CE} denote the set of run-admitting banking contracts that satisfy (5) – (8). (5) and (7) can be interpreted as the incentive compatibility constraints for running on the bank given the probability that some, but not all, patient depositors wait until the last period. (6) and (8) are the incentive compatibility constraints for waiting given the probability that some, but not all, patient depositors run on the bank. Because a contract in M^{CE} has to satisfy four additional constraints in addition to that it is feasible and run-admitting, M^{CE} is a subset of M^{RA}. Two noises exist in the randomizing device, p_{01} and p_{10}. If both of them are zero, then the randomizing device is perfect, and we are back to the Peck-Shell (2003) sunspot approach. For a perfect randomizing device, $M^{CE} = M^{RA}$.

Given the information structure, not all contracts permit a correlated equilibrium. Proposition 1 demonstrates that for any feasible run-admitting contract, there are upper bounds of the noises, below which the contract permits a correlated equilibrium. Proposition 2 shows that the set of feasible contracts that allow for a correlated equilibrium diminishes when the noises in signals (p_{01} and p_{10}) increase.

Proposition 1 *Given any feasible demand-deposit contract* $m \in M$ *and* p_{11}*, there exist* $\varepsilon\left(p_{11}, c^1\right) \geq 0$ *and* $\delta\left(p_{10}, p_{11}, c^1\right)$ *such that if* $p_{01} \leq \varepsilon\left(p_{11}, c^1\right)$ *and* $p_{10} \leq \delta\left(p_{10}, p_{11}, c^1\right)$*, the contract allows for a correlated equilibrium in the postdeposit game.* $\varepsilon\left(p_{11}, c^1\right) = 0$ *and/or* $\delta\left(p_{10}, p_{11}, c^1\right) = 0$ *if and only if* $c^1 = \frac{R}{1-\alpha+\alpha R}$.

Proof. See appendix. ■

With noises in the sunspot information, depositors face uncertainty when they make withdrawals. $(5)-(8)$ are the conditions for individual depositors to follow their signals given the probability that the other network runs on the bank. When the participation incentive compatibility constraint is binding, only the minimum requirement for a patient depositor to wait is satisfied. Any increase in the measure of depositors running on the bank, or any increase in the probability of more than α measure of depositors running on the bank, breaks down the participation incentive compatibility constraint. Therefore, if $c^1 = \frac{R}{1-\alpha+\alpha R}$, the contract does not allow for a correlated equilibrium unless $p_{10} = p_{01} = 0$. On the other hand, if the participation incentive compatibility constraint is unbinding, then there is room for the possible increase in the measure of depositors running on the bank, and in an individual depositor's own interest, he still would prefer to wait. Hence, all other feasible contracts allow a correlated equilibrium if noises are small enough.

Given the joint probability of θ_1 and θ_2, denote the set of contracts that satisfies $(5)-(8)$ by $M^{CE}\left(p_{11}, p_{01}, p_{10}\right)$. The following proposition illustrates that the set $M^{CE}\left(p_{11}, p_{01}, p_{10}\right)$ shrinks when the noises (p_{01} and p_{10}) get larger.

Proposition 2 *If* $\left(-p'_{11}, p'_{01}, p'_{10}\right) \geq \left(-p_{11}, p_{01}, p_{10}\right)$, $\left(p'_{01}, p'_{10}\right) \geq \left(p_{01}, p_{10}\right)$ *and* $M^{CE}\left(p'_{11}, p'_{01}, p'_{10}\right) \neq \varnothing$*, then* $M^{CE}\left(p'_{11}, p'_{01}, p'_{10}\right) \subset M^{CE}\left(p_{11}, p_{01}, p_{10}\right)$.

Proof. See appendix. ■

The strategies in the postdeposit game are complementary. If more people run on the bank or the probability of bank runs is increased, then an individual depositor's incentive to wait falls as the expected payoff in the last period is lowered. Similarly, if more people wait or the probability of non-run is increased, then a patient depositor's expected payoff at $t = 2$ is raised, and he is more willing to wait. With a perfect randomizing device, that is, $p_{01} = 0$ and $p_{10} = 0$, an individual depositor knows that all other depositors take the same action as he does. With the decrease in p_{11} and the increase in p_{01} and p_{10}, the conditional probabilities $\Pr(\theta_i = 0|\theta_j = 1)$ and $\Pr(\theta_i = 1|\theta_j = 0)$ are increased. Receiving a signal, an individual depositor knows that the other network is more likely to take a different action. Given the contract, due to the strategic complementarity, it is better for an individual depositor to switch to the other network's action rather than follow her own signal for p_{01} and p_{10} are above the thresholds. Hence, as p_{10} and p_{01} increase, there are fewer contracts consistent with the definition of the correlated equilibrium. In an extreme case, for example, if $p_{01} = 1 - p_{11}$ and p_{11} is small enough, there is no contract allowing for a correlated equilibrium. When a banking contract m falls in the subset of $M^{RA} \backslash M^{CE}$, it neither allows for a coordinating equilibrium nor is run-proof.[1] In this situation, we are back to the original Diamond-Dybvig world in which a contract has a run as well as a non-run equilibrium. According to the traditional Diamond-Dybvig interpretation, if such a contract is offered, at $t = 0$, depositors would either accept it, believing non-run equilibrium will take place, or not accept it, believing the run equilibrium will occur.

The following example is provided to explain the partitions of feasible banking contracts.

A Numerical Example:

Let $u(c) = \dfrac{(c+b)^\gamma - b^\gamma}{\gamma}$, $\gamma = -1$, $b = 0.5$. $R = 1.5$, $\alpha = 0.4$, $n_1 = n_2 = 0.5$, $p_{11} = p_{01} = p_{10} = 0.001$.

In this example, M, M^{RP}, M^{CE} and $M^{RA} \backslash M^{CE}$ are as follows (summarized by c^1):

Banking contract	c^1
M	$[0, 1.2500]$
M^{RP}	$[0, 1]$
M^{CE}	$(1, 1.2495]$
$M^{RA} \backslash M^{CE}$	$(1.2495, 1.2500]$

In a correlated equilibrium, there are three possible outcomes. If $\theta_1 = \theta_2 = 1$, all depositors withdraw deposits. If $\theta_1 = 0$ and $\theta_2 = 1$, or if $\theta_1 = 1$ and $\theta_2 = 0$, only a fraction of patient

[1] Soloman (2003, 2004) does not have this problem due to the assumptions that foreigners are paid in nominal asset and they are risk neutural.

depositors run on the bank. No patient depositor runs on the bank when $\theta_1 = \theta_2 = 0$. I define a full bank run, partial bank run, and non-bank run as follows:

Definition 2 *(Full Bank Run) In the postdeposit game, if all depositors withdraw deposits, then a full bank run occurs.*

Definition 3 *(Partial Bank Run) In the postdeposit game, if some, not all, patient depositors withdraw deposits, then a partial bank run occurs.*

Definition 4 *(Non-Bank Run) In the postdeposit game, if all patient depositors do not withdraw deposits in period 1, then a non-bank run occurs.*

By definition, in a correlated equilibrium, depositors in both networks interpret a signal of value 1 as the sign to withdraw and 0 as the sign to wait. Generally speaking, because sunspots do not affect the fundamentals, people can interpret the signals in any way they please. For example, one network views 1 as the signal to wait, and the other network treats 0 as the signal to run. Thus, an imperfect information structure can allow for more than one type of correlated equilibrium. In this paper, the interpretations of signals are assumed to be exogenously given. Instead of varying the interpretations, the joint probability distribution of θ_1 and θ_2 can be changed to achieve the same results. If the exogenous uncertainty is understood as the uncertainty in fundamentals taken to the limit, then it can be understood that people usually have common views on which signal is good and which is bad.

Given the imperfect randomizing device, not every run-admitting contract allows for a correlated equilibrium. Before I start the welfare analysis, let me clarify the strategies of an individual depositor in the postdeposit game. To start with a banking contract $m \in M$,

1. If $m \in M^{CE}$, that is, m allows for a correlated equilibrium, then patient depositors are coordinated by the sunspots. Patient depositors withdraw the deposits when $\theta_i = 1$, $i = 1, 2$ is observed, and wait otherwise.

2. If $m \in M^{RP}$, then patient depositors do not run regardless of the realization of the sunspot variable.

3. If $m \in M^{RA} \backslash M^{CE}$, then the contract neither allows for a correlated equilibrium nor is run-proof. The randomizing device fails. Depositors either accept the offer *ex ante* and do not run *ex post,* or reject the offer *ex ante* and anticipate that the run equilibrium always occurs.

In the last two scenarios, sunspots do not matter in the postdeposit game, because the strategies of depositors are independent of the realization of the sunspot signals.

4 Predeposit Game

Knowing the strategies of depositors in the postdeposit game given the information structure, the presentative bank chooses the optimal contract to offer at $t = 0$. As a result of a competitive market, the bank offers a contract that maximizes depositors' expected utility. If the contract yields an *ex-ante* expected utility level higher than that under autarky, depositors will accept it and the postdeposit game will be played. In all other cases, depositors prefer to stay in autarky.

The same notation is used in this section to denote the banking contracts in the predeposit game. The bank can choose from three types of contracts, corresponding to the partitions of M in the postdeposit game. I will first calculate the *ex-ante* expected utility obtained given a contract $m \in M^{RP}$ and a contract $m \in M^{CE}$, assuming the depositors always accept the banking contract. I also calculate the *ex-ante* expected utility if a non-run occurs given a run-admitting contract that does not allow for a correlated equilibrium. A sufficient condition for an optimal contract to tolerate a correlated equilibrium in the postdeposit game is that the best contract in M^{CE} is better than the best contract in M^{RP} and than the best outcome (non-run) given a best contract in $M^{RA} \backslash M^{CE}$. If the *ex-ante* utility is higher than that under the autarky, the depositor will accept the contract *ex ante*.

A run-proof contract is a contract such that a patient depositor weakly prefers to wait even though everyone else withdraws the deposits. In the demand-deposit contract framework, a banking contract is run-proof if and only if $c^1 \leq 1$. It is equivalent to the autarky when $c^1 = 1$. Because the coefficient of relative risk aversion is greater than 1, the banking contract that depositors are willing to accept *ex ante* should satisfy $c^1 \geq 1$. Thus, the only *ex-ante* acceptable run-proof contract requires $c^1 = 1$, which results in the same allocation as under autarky. I impose an assumption that if a contract yields expected utility equal to that under autarky, people still deposit in the bank. With this assumption, the bank can at least offer the run-proof contract to the depositors.

The expected utility under the run-proof contract in the predeposit game is:

$$W^{RP}(m) = \alpha u(1) + (1 - \alpha)u(R) \tag{9}$$

Next, I discuss the expected utility given a contract that allows for a correlated equilibrium in the postdeposit game. I define the correlated equilibrium in the predeposit game as follows.

Definition 5 *Given a feasible contract $m \in M$, the predeposit game has a correlated equilibrium if there is a subgame perfect Nash-Aumann equilibrium in which (i) depositors are willing to deposit and (ii) the postdeposit game has a correlated equilibrium.*

I use partial run 1 and partial run 2 to distinguish the partial runs conducted by networks 1 and 2, respectively. If the contract admits a correlated equilibrium, then the probabilities of full bank runs, partial bank runs, and non-bank run are determined by the information structure. The probabilities of having a full run and a non-run are p_{11} and p_{00}, respectively. The probabilities of having partial runs driven by networks 1 and 2 are p_{11} and p_{01}, respectively. I assume the social welfare is the aggregated expected utilities of individual depositors and all depositors are weighted equally. In the following context, welfare and *ex-ante* expected utility are used interchangeably.

When a non-bank run occurs, the welfare, denoted by $W^{non-run}(m)$, is

$$W^{non-run}(m) = \alpha u(c^1) + (1-\alpha) u\left(\frac{1-\alpha c^1}{1-\alpha} R\right).$$

When partial run i occurs, the welfare, denoted by $W^{p-run-i}(m)$, $i = 1, 2$, is

$$W^{p-run-i}(m) = \begin{cases} \frac{1}{c^1} u(c^1), & \text{if } (n_i + \alpha n_{-i})c^1 > 1; \\ (n_i + \alpha n_{-i}) u(c^1) + (1-\alpha) n_{-i} u\left(\frac{1-(n_i+\alpha n_{-i})c^1}{(1-\alpha)n_{-i}} R\right), & \text{otherwise.} \end{cases}$$

Note that $W^{p-run-i}(m)$ is continuous in c^1. Also note that given $c^1 \geq 1$, the welfare under a partial run is strictly less than that under a non-run.

When a full bank run occurs, the welfare, denoted by $W^{run}(m)$, is

$$W^{run}(m) = \frac{1}{c^1} u(c^1).$$

Given the probabilities, the best contract offered by the bank, which allows for a correlated equilibrium, is

$$\hat{W}(m) = \max_{c^1} p_{11} W^{run}(m) + p_{10} W^{p-run-1}(m) + p_{01} W^{p-run-2}(m) + p_{00} W^{non-run}(m) \quad \text{(PCE)}$$

$$s.t. \; m \in M^{CE}.$$

Given p_{11}, the value function of problem PCE is strictly decreasing in noises for two reasons: First, the set of contracts that allows for correlated equilibrium shrinks when noises increase. Hence, the choice set is smaller. Second, because the welfare under partial runs is lower than that under non-run, the same contract yields the lower expected utility if the probabilities of partial runs are larger and the probability of non-run is smaller.

Lemma 1 *Given p_{11}, the value function of problem PCE is strictly decreasing in p_{01} and p_{10} for $M^{CE}(p_{11}, p_{01}, p_{10}) \neq \varnothing$.*

Proof. By proposition 2, given p_{11}, the set of M^{CE} diminishes when p_{01} and/or p_{10} increase. Let $(p'_{01}, p'_{10}) \geq (p_{01}, p_{10})$ and $(p'_{01}, p'_{10}) \neq (p_{01}, p_{10})$. The solution to PCE given $(p_{11}, p'_{01}, p'_{10})$, $m' = (c^{1\prime}, c^{2\prime})$, is in $M^{CE}(p_{11}, p_{01}, p_{10})$. Plug $m' = (c^{1\prime}, c^{2\prime})$ into the objective function of PCE given (p_{11}, p_{01}, p_{10}). Denote the welfare achieved by $W(m'; p_{11}, p_{01}, p_{10})$. The welfare under partial runs is lower than non-run, so $W(m'; p_{11}, p_{01}, p_{10}) > \hat{W}(m'; p_{11}, p'_{01}, p'_{10})$. Because $\hat{W}(m; p_{11}, p_{01}, p_{10})$ is at least as high as $W(m'; p_{11}, p_{01}, p_{10})$, $\hat{W}(m; p_{11}, p_{01}, p_{10}) > \hat{W}(m'; p_{11}, p'_{01}, p'_{10})$. ∎

Proposition 3 and its corollary demonstrate that there exist upper bounds of the probabilities of full runs, partial runs, and non-run, below which a contract has a correlated equilibrium in the predeposit game.

Proposition 3 *There exist $\bar{p}_{11} > 0$, $\bar{p}_{01}(p_{11}) \geq 0$, and $\bar{p}_{10}(p_{11}, p_{01}) \geq 0$ such that for $p_{11} \leq \bar{p}_{11}$, $p_{01} \leq \bar{p}_{01}(p_{11})$, and $p_{10} \leq \bar{p}_{10}(p_{11}, p_{01})$, there exist at least one feasible demand-deposit contract $m = (c^1, c^2)$ allowing for a correlated equilibrium and is strictly better than the run-proof contract. $\bar{p}_{01}(\bar{p}_{11}) = 0$, and $\bar{p}_{10}(p_{11}, \bar{p}_{01}) = 0$.*

Proof. The optimal contract allowing for a correlated equilibrium solves problem PCE. Given p_{11}, if $p_{01} = p_{10} = 0$, the problem is the same as the traditional symmetric sunspot equilibrium problem. The conditions for a correlated equilibrium are always satisfied. According to Cooper and Ross (1998), there is a unique cutoff level of p_{11}, above which a run-proof contract is better and below which the optimal contract is run-admitting. Denote this cutoff level by $\bar{p}_{11}$.

Given p_{11} and $p_{10} = 0$, the value function of PCE is strictly decreasing in p_{01} by lemma 1. If $p_{11} = \bar{p}_{11}$, only $p_{01} = 0$ can make a run-admitting contract as good as a run-proof contract. Hence, $\bar{p}_{01}(\bar{p}_{11}) = \bar{p}_{11}$.

If $p_{11} < \bar{p}_{11}$ and $p_{01} = p_{10} = 0$, the value of PCE is strictly higher than that of a run-proof contract. Holding $p_{11} < \bar{p}_{11}$ and $p_{10} = 0$, p_{01} can be increased a little bit and the value of PCE is still higher than the value of a run-proof contract. Note that the set of M^{CE} is diminishing in p_{01}. Let $p_{01}^{\varnothing}(p_{11})$ denote the cutoff of value of p_{01}, below which the set of M^{CE} is not empty and above which it is empty. If the value of PCE at $p_{01}^{\varnothing}(p_{11})$ is lower than W^{RP}, then by the monotonicity of the value function, a cutoff p_{01} depending on p_{11} can be found, below which the contract allowing for a correlated equilibrium is better than the run-proof contract, and above which the run-proof contract is better. Denote such p_{01} by $p_{01}^{V}(p_{11})$. Let $\bar{p}_{01}(p_{11}) = \min\left\{p_{01}^{\varnothing}(p_{11}), p_{01}^{V}(p_{11})\right\}$. It is the cutoff value of p_{01}, below which the contract allowing for a correlated equilibrium is better than the run-proof contract, above which the run-proof contract is better, or there is no contract that allows for a correlated equilibrium.

The value function of PCE is not necessarily continuous, because the choice set can be nonconvex. We need to prove that $\bar{p}_{01}(p_{11})$ is not equal to 0 for $p_{11} < \bar{p}_{11}$. Let c^{1**} denote the solution to problem PCE with $p_{01} = p_{10} = 0$ and $p_{11} < \bar{p}_{11}$. According to Ennis and Keister (2004), c^{1**} can not be $\frac{R}{1-\alpha+\alpha R}$. By proposition 1, given p_{11}, c^{1**} is a feasible contract that allows for a correlated equilibrium for $p_{01} \leq \varepsilon\left(p_{11}, c^{1**}\right)$, where $\varepsilon\left(p_{11}, c^{1**}\right) > 0$. The welfare at c^{1**} on $p_{01} \leq \varepsilon\left(p_{11}, c^{1**}\right)$ is continuous in p_{01}. So p_{01} can be increased at least to $\min\{\bar{p}_{11} - p_{11}, \varepsilon\left(p_{11}, c^{1**}\right)\}$ and c^{1**} can still be better than the run-proof contract. Thus, when $p_{11} < \bar{p}_{11}$, the cutoff level of $p_{01} > 0$.

Let $p_{11} \leq \bar{p}_{11}$ and $p_{01} \leq \bar{p}_{01}(p_{11})$, the same process can be repeated to prove there exists $\bar{p}_{10}(p_{11}, p_{01}) \geq 0$. Therefore, if the probabilities of partial runs and full runs are small, then the contract that allows for a correlated equilibrium is better than the run-proof contract. ■

Because the best run-proof contract is equivalent to autarky, we have the following corollary:

Corollary 1 *For $p_{11} \leq \bar{p}_{11}$, $p_{01} \leq \bar{p}_{01}(p_{11})$ and $p_{10} \leq \bar{p}_{10}(p_{11}, p_{01})$, there exists at least one feasible banking contract such that the predeposit game has a correlated equilibrium.*

If $m \in M^{RA} \backslash M^{CE}$, the best outcome one can hope for is that all depositors anticipate the non-run equilibrium in the postdeposit game, and that depositors deposit at the bank *ex ante* and they do not run at $t = 1$. The welfare of the best outcome is given by:

$$\hat{W}(m) = \max_{c^1} \alpha u(c^1) + (1-\alpha) u\left(\frac{1-\alpha c^1}{1-\alpha} R\right) \qquad \text{(PDD)}$$
$$s.t.\ m \in M^{RA} \backslash M^{CE}.$$

A sufficient condition for allowing for a correlated equilibrium is that the welfare of the best $m \in M^{CE}$ is higher than that of the best run-proof contract and that of the best outcome of the best m in $M^{RA} \backslash M^{CE}$. Similarly, a run-proof contract will be the best if the welfare is higher than that of the best m in M^{CE} and than the best outcome of the optimal contract in $M^{RA} \backslash M^{CE}$.

The objective function of PDD is not affected by p_{11}, p_{01} or p_{10}, but the choice set increases in p_{01} and p_{10} given p_{11}. Thus, the value function of PDD is increasing in p_{01} and p_{10}. However, because the choice set is not necessarily convex, the value function can be discontinuous at $p_{01} = 0$ and/or $p_{10} = 0$.

Some computed examples indicate that in some economies the optimal contract allows for a correlated equilibrium and in other economies, the run-proof contract is optimal.

Proposition 4 *In some economies, the optimal demand-deposit banking contract allows for a correlated equilibrium.*

Proof. Prove by example. All parameters are the same as in the previous example. The expected utilities of the optimal contracts in M^{CE}, M^{RP}, and the best outcome in $M^{RA} \backslash M^{CE}$ are as follows:

m in	c^{1**}	$\hat{W}(m)$
M^{RP}	1	1.4333
M^{CE}	1.0707**	1.4341
$M^{RA} \backslash M^{CE}$	$\rightarrow$ 1.2495	$\rightarrow$ 1.4286

The optimal contract in this example is $c^{1**} = 1.0707$, which yields a welfare level of 1.4341. It is better than the best run-proof contract and autarky. If m in $M^{RA} \backslash M^{CE}$, in the best situation, that is, depositors accept the contract and do not run *ex post*, the highest welfare level is 1.4286, which is still lower than that under M^{CE} and also lower than that under autarky. Thus, depositors will not accept the contract in the first place. Hence, the optimal m is in M^{CE} in this example. ■

By Proposition 4, the asymmetric randomizing device can be part of the equilibrium in an economy. The full bank runs and partial bank runs are equilibrium phenomena. Runs are tolerated because the gain from liquidity smoothing is large.

Corollary 2 *In some economies, the optimal demand-deposit banking contract is run-proof.*

Proof. Prove by example. The example in the previous section is used here, but p_{11}, p_{01}, and p_{10} are varied. Let $p_{11} = p_{01} = p_{10} = 0.005$. The expected utilities of the optimal contracts in M^{CE},

M^{RP}, and the best outcome in $M^{RA} \backslash M^{CE}$ are as follows:

m in	c^{1**}	$\hat{W}(m)$
M^{RP}	1^{**}	1.4333
M^{CE}	1.0610	1.4330
$M^{RA} \backslash M^{CE}$	$\to 1.2476$	$\to 1.4287$

The run-proof contract is the best one. ■

The best outcome in $M^{RA} \backslash M^{CE}$ can achieve higher welfare than the best run-proof contract and the best contract that allows for a correlated equilibrium. Let us continue the example, but let $p_{11} = 0.1$, $p_{01} = 0.2$ and $p_{10} = 0.4$. The welfare is as follows:

m in	c^{1**}	$\hat{W}(m)$
M^{RP}	1	1.4333
M^{CE}	$\to 1$	$\to 1.3933$
$M^{RA} \backslash M^{CE}$	$\to 1.1407^{**}$	$\to 1.4335$

In this economy, it is hard to tell which contract is optimal. A run-proof contract is better than a contract that allows for a coordinating equilibrium. But, if this randomizing device is not used, then a higher welfare level may be achieved.

So far the analysis is based on a simple demand-deposit contract. However, the results hold in a broad class of banking mechanism. In the appendix, I consider a banking contract that allows for partial suspension of convertibility. A correlated equilibrium exists in some economies. Bank runs and partial runs are tolerated in some economies.

5 Conclusions

In this paper, I extend the analysis of panic-based runs to include an asymmetric-information, extrinsic randomizing device. I show that in an economy with asymmetric sunspot information structure, there exists a correlated equilibrium for some demand-deposit contracts. In this equilibrium, a full bank run, a partial bank run, or non-bank run occurs, depending on the realization of the sunspot signals. In some economies, the optimal banking contract tolerates full runs and partial runs. The run-proof banking contract is not the best, because it sacrifices too much welfare.

Interestingly, there are banking contracts that are neither run-proof nor consistent with correlated equilibrium if sunspots are imperfectly observed. It is hard to describe the equilibrium without further discussion of game theory or further assumptions on the preference of depositors. Therefore, the analysis provides a necessary condition for using a run-proof banking contract or a contract allowing for a correlated equilibrium. Sufficient condition also is discussed, assuming the best outcome of a run-admitting contract that does not allow for a correlated equilibrium. These results hold in a broad class of banking mechanisms, including partial suspension of convertibility.

The exogenously given interpretation of the signal is assumed in this paper. However, taking the sunspot seriously, depositors can interpret the signals in any way they prefer. When people are allowed to choose networks, there also can be multiple equilibria in the predeposit game. Which equilibrium is mostly likely to occur? In the extension of this paper, I will consider the refinement of the equilibria and aim to provide a better answer to these remaining questions.

References

[1] Aumann, Robert, "Correlated Equilibrium as an Expression of Bayesian Rationality," *Econometrica*, Vol. 55, No.1, (January 1987): 1-18.

[2] Boyd, John, Pedro Gomis, Sungkyu Kwak, and Bruce Smith, "A User's Guide to Banking Crises," *working paper* (2001).

[3] Cooper, Russell, and Thomas Ross, "Bank Runs: Liquidity Costs and Investment Distortions," *Journal of Monetary Economics,* Vol. 41, No. 1, (February 1998): 27-38.

[4] Diamond, Douglas, and Philip Dybvig, "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy,* Vol. 91, No. 3, (June 1983): 401-419.

[5] Easley, David, and Maureen O'Hara, "Regulation and Return: the Role of Ambiguity," *working paper* (2005).

[6] Ennis, Huberto, and Todd Keister, "Bank Runs and Investment Decisions Revisited," *Journal of Monetary Economics,* Vol. 53, No. 2, (March 2006): 217-232.

[7] Goldstein, Itay, and Ady Pauzner, "Demand-Deposit Contracts and the Probability of Bank Runs," *Journal of Finance*, Vol. 60, No.3, (June 2005): 1293-1327.

[8] Peck, James, and Karl Shell, "Equilibrium Bank Runs," *Journal of Political Economy,* Vol. 111, No.1, (February 2003): 103-123.

[9] Postlewaite, Andrew, and Xavier Vives, "Bank Runs as an Equilibrium Phenomenon," *Journal of Political Economy,* Vol. 95, No. 3, (June 1987): 485-491.

[10] Soloman, Raphael, "Anatomy of a Twin Crisis," *Bank of Canada working paper* (2003).

[11] ——, "When Bad Things Happen to Good Banks: Contagious Bank Runs and Currency Crises," *Bank of Canada working paper* (2004).

[12] Wallace, Neil, "Another Attempt to Explain an Illiquid Banking System: The Diamond and Dybvig Model with Sequential Service Taken Seriously," *Federal Reserve Bank Minneapolis Quarterly Review* 12 (Fall 1988): 3-16.

6 Appendix

6.1 Proofs of Propositions

Proposition 1 Given any feasible demand-deposit contract $m \in M$ and p_{11}, there exist $\varepsilon\left(p_{11}, c^1\right) \geq 0$ and $\delta\left(p_{10}, p_{11}, c^1\right)$ such that if $p_{01} \leq \varepsilon\left(p_{11}, c^1\right)$ and $p_{10} \leq \delta\left(p_{10}, p_{11}, c^1\right)$ the contract allows for a correlated equilibrium in the postdeposit game. $\varepsilon\left(p_{11}, c^1\right) = 0$ and/or $\delta\left(p_{10}, p_{11}, c^1\right) = 0$ if and only if $c^1 = \frac{R}{1-\alpha+\alpha R}$.

Proof. When $p_{01} = 0$, $(6) - (7)$ are satisfied for any feasible demand-deposit contract. p_{01} does not affect (5) and (8). Given p_{11} and c^1, p_{10} can achieve its upper bound when p_{01} is 0.

Suppose $(\alpha n_1 + n_2)\, c^1 \leq 1$ and $(n_1 + \alpha n_2)\, c^1 \leq 1$. Let $p_{01} = 0$, rewrite equations (5) and (8) explicitly, we get

$$p_{00}\left[u\left(\tfrac{1-\alpha c^1}{1-\alpha}R\right) - u\left(c^1\right)\right] \geq p_{10}\left[u\left(c^1\right) - u\left(\tfrac{1-(n_1+\alpha n_2)c^1}{n_2(1-\alpha)}R\right)\right] \geq -p_{11}\frac{1}{c^1}u\left(c^1\right)$$

By simple algebra, we get $\varepsilon\left(p_{11}, c^1\right)$, the upper bound of p_{10}, $\varepsilon\left(p_{11}, c^1\right)$, as follows:

$$\varepsilon = \begin{cases} \min\left\{1 - p_{11}, \frac{p_{11}\frac{1}{c^1}u\left(c^1\right)}{u\left(\frac{1-(n_1+\alpha n_2)c^1}{n_2(1-\alpha)}R\right) - u\left(c^1\right)}\right\}, & \text{if } u\left(c^1\right) < u\left(\frac{1-(n_1+\alpha n_2)c^1}{n_2(1-\alpha)}R\right); \\ \frac{(1-p_{11})\left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u\left(c^1\right)\right]}{u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u\left(\frac{1-(n_1+\alpha n_2)c^1}{n_2(1-\alpha)}R\right)}, & \text{if } u\left(c^1\right) \geq u\left(\frac{1-(n_1+\alpha n_2)c^1}{n_2(1-\alpha)}R\right). \end{cases}$$

It is easy to see that $\varepsilon = 0$ if and only if $\frac{1-\alpha c^1}{1-\alpha}R = c^1$.

Given $p_{10} < \varepsilon\left(p_{11}, c^1\right)$, the upper bound of p_{01},

$$\delta\left(p_{11}, p_{01}, c^1\right) = \min\left\{\begin{array}{c} \frac{(1-p_{11}-p_{10})\left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right)-u\left(c^1\right)\right]}{u\left(\frac{1-\alpha c^1}{1-\alpha}R\right)-u\left(\frac{1-(\alpha n_1+n_2)c^1}{n_1(1-\alpha)}R\right)}, \\ \frac{p_{11}\frac{1}{c^1}u\left(c^1\right)}{u\left(\frac{1-(\alpha n_1+n_2)c^1}{n_1(1-\alpha)}R\right)-u(c^1)}, 1-p_{11}-\frac{p_{10}\left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right)-u\left(\frac{1-(n_1+\alpha n_2)c^1}{n_2(1-\alpha)}R\right)\right]}{u\left(\frac{1-\alpha c^1}{1-\alpha}R\right)-u(c^1)} \end{array}\right\}$$

if $u\left(c^1\right) < u\left(\frac{1-(\alpha n_1+n_2)c^1}{n_1(1-\alpha)}R\right)$;

$$\delta\left(p_{11}, p_{01}, c^1\right) = \min\left\{\begin{array}{c} \frac{(1-p_{11}-p_{10})\left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right)-u\left(c^1\right)\right]}{u\left(\frac{1-\alpha c^1}{1-\alpha}R\right)-u\left(\frac{1-(\alpha n_1+n_2)c^1}{n_1(1-\alpha)}R\right)}, \\ 1-p_{11}-\frac{p_{10}\left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right)-u\left(\frac{1-(n_1+\alpha n_2)c^1}{n_2(1-\alpha)}R\right)\right]}{u\left(\frac{1-\alpha c^1}{1-\alpha}R\right)-u(c^1)} \end{array}\right\}$$

if $u\left(c^1\right) \geq u\left(\frac{1-(\alpha n_1+n_2)c^1}{n_1(1-\alpha)}R\right)$.

With $\frac{1-\alpha c^1}{1-\alpha}R = c^1$, we have $\varepsilon = 0$, the only possible value of p_{10} is 0. It is easy to see that $\delta = 0$. Also, the only way to make $\delta = 0$ is to have $\frac{1-\alpha c^1}{1-\alpha}R = c^1$ regardless of the value of p_{10}.

In the same way, we can get the upper bound of p_{01} and p_{10} for the other three cases in which (1) $(\alpha n_1 + n_2)\, c^1 > 1$ and $(n_1 + \alpha n_2)\, c^1 \leq 1$, or (2) $(\alpha n_1 + n_2)\, c^1 \leq 1$ and $(n_1 + \alpha n_2)\, c^1 > 1$, or (3) $(\alpha n_1 + n_2)\, c^1 < 1$ and $(n_1 + \alpha n_2)\, c^1 < 1$. ■

Proposition 2 *If* $(-p'_{11}, p'_{01}, p'_{10}) \geq (-p_{11}, p_{01}, p_{10})$, $(p'_{01}, p'_{10}) \geq (p_{01}, p_{10})$ *and* $M^{CE}(p_{11}, p_{01}, p_{10}) \neq \varnothing$, *then* $M^{CE}(p'_{11}, p'_{01}, p'_{10}) \subset M^{CE}(p_{11}, p_{01}, p_{10})$.

Proof. Prove the proposition in two steps. First, I illustrate that for any m in $M^{CE}(p'_{11}, p'_{01}, p'_{10})$, it is also in $M^{CE}(p_{11}, p_{01}, p_{10})$. Second, I determine that there exists some m in $M^{CE}(p_{11}, p_{01}, p_{10})$ but not in $M^{CE}(p'_{11}, p'_{01}, p'_{10})$.

Let $m \in M^{CE}(p'_{11}, p'_{01}, p'_{10})$. Discuss cases by parameters. If $(\alpha n_1 + n_2)c^1 \leq 1$ and $(n_1 + \alpha n_2)c^1 \leq 1$, rewrite (5) – (8) as

$$p'_{10}\left[u(c^1) - u\left(\frac{1-(n_1+\alpha n_2)c^1}{(1-\alpha)n_2}R\right)\right] \geq -p'_{11}\frac{1}{c^1}u(c^1) \quad (5')$$

$$-p'_{01}\left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u\left(\frac{1-(\alpha n_1+n_2)c^1}{(1-\alpha)n_1}R\right)\right] - p'_{10}\left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u(c^1)\right] \geq -(1-p'_{11})\left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u(c^1)\right] \quad (6')$$

$$p'_{01}\left[u(c^1) - u\left(\frac{1-(\alpha n_1+n_2)c^1}{(1-\alpha)n_1}R\right)\right] \geq -p'_{11}\frac{1}{c^1}u(c^1) \quad (7')$$

$$-p'_{10}\left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u\left(\frac{1-(n_1+\alpha n_2)c^1}{(1-\alpha)n_2}R\right)\right] - p'_{01}\left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u(c^1)\right] \geq -(1-p'_{11})\left[u\left(\frac{1-\alpha c^1}{1-\alpha}R\right) - u(c^1)\right] \quad (8')$$

Note that $(5') - (6')$ and $(7') - (8')$ are symmetric in terms of p_{01} and p_{10}. Therefore, the analysis can be focused on $(5') - (6')$. The RHS of $(5')$ is negative. An increase in p'_{11} to p_{11} decreases the RHS. If $u(c^1) - u(\frac{1-(n_1+\alpha n_2)c^1}{(1-\alpha)n_2}R) \geq 0$, any change in p'_{10} does not change the sign of $(5')$. If $u(c^1) - u(\frac{1-(n_1+\alpha n_2)c^1}{(1-\alpha)n_2}R) < 0$, a decrease in p'_{10} to p_{10} rises the LHS. Therefore, the inequality sign in $(5')$ still holds for $(-p'_{11}, p'_{01}, p'_{10}) \geq (-p_{11}, p_{01}, p_{10})$ .In $(6')$, each term in the brackets is positive. The LHS is decreasing in p'_{01} and p'_{10}, and the RHS is increasing in p'_{11}. Therefore, the contract m satisfies $(6')$ for $(-p'_{11}, p'_{01}, p'_{10}) \geq (-p_{11}, p_{01}, p_{10})$. Same reasoning for $(7')$ and $(8')$.

In the second step, I show that for $(-p'_{11}, p'_{01}, p'_{10}) \geq (-p_{11}, p_{01}, p_{10})$, there exists a contract m that is in $M^{CE}(p_{11}, p_{01}, p_{10})$ but not in $M^{CE}(p'_{11}, p'_{01}, p'_{10})$. Suppose that there is $m \in M^{CE}(p_{11}, p_{01}, p_{10})$ such that at least one of $(5) - (8)$ is binding. Change in the probabilities from $(-p_{11}, p_{01}, p_{10})$ to $(-p'_{11}, p'_{01}, p'_{10})$ breaks down the inequality sign, such that at least one of $(5) - (8)$ is no longer satisfied. Such a contract is not in $M^{CE}(p'_{11}, p'_{01}, p'_{10})$. Next, I prove that not every feasible contract allows for a coordinating equilibrium, and some inequalities must be binding for some m. Let $p_{10}, p_{01} \geq 0$. Because at least one of p'_{10} and p'_{01} is strictly greater than zero, $c^1 = \frac{R}{1-\alpha+\alpha R}$ is no longer in $M^{CE}(p'_{11}, p'_{01}, p'_{10})$ by proposition 1, which means at least one of the inequalities does not hold at $c^1 = \frac{R}{1-\alpha+\alpha R}$. By the continuity of the utility function, the inequality must be binding at some c^1 because $M^{CE}(p_{11}, p_{01}, p_{10})$ is not empty.

The other three cases can be proved in a similar way.

Note, if p_{11} is small enough and p_{10} and p_{01} are large enough, $M^{CE}(p_{11}, p_{10}, p_{01}) = \varnothing$. ■

6.2 A Banking Contract Allowing Partial Suspension of Convertibility

6.2.1 Model Set Up

Banking contracts will be generalized in this section. I discuss an economy that bears aggregate uncertainty and let the contract be contingent on the positions of the depositors in the queue. To keep the illustration simple, a discrete case will be analyzed here. I follow the notations and the definitions in Peck and Shell (2003) as much as possible. There are N depositors in the economy, among whom there are α number of impatient depositors, where $\alpha \leq N$ is a random number with probability density function $f(\alpha)$. Each depositor is endowed with 1 unit of consumption good at $t = 0$. The utility function of the impatient depositors is denoted by $u(c^1)$, and the utility function of the patient depositors is by $v(c^1 + c^2)$. u and v are strictly increasing, strictly concave, and twice continuously differentiable. The coefficients of relative risk aversion of u and v are greater than 1.

The specification of the information structure is the same as in the previous framework. Depositors do not know which network they will be in *ex ante*. Network i has N_i number of depositors, where $i = 1, 2$, and $N_1 + N_2 = N_1$. N_1 and N_2 are known *ex ante*. Depositors have probability N_1/N to be in network 1, and probability N_2/N to be in network 2. For each α, let α_i $(0 \leq \alpha_i \leq \min\{N_i, \alpha\})$ be the number of impatient depositors in network i, $i = 1, 2$, and $\alpha_1 + \alpha_2 = \alpha$. Denote the *ex-ante* conditional probability of having α_1 impatient depositors in group 1, and α_2 in group 2 conditional on α by $g(\alpha_1, \alpha_2|\alpha)$. The *ex-ante* probability that there are α number of impatient depositors, α_1 of them in group 1 and α_2 of them in group 2 is:

$$h(\alpha_1, \alpha_2, \alpha) = f(\alpha)g(\alpha_1, \alpha_2|\alpha).$$

After the consumption shock and information shock are realized, the patient depositors update the probability of α by Bayes' rule conditional on their consumption type and information type (which group he is in). The *ex-post* probability of α, contingent on a depositor being patient is denoted by $f_p(\alpha)$. The ex-post probability that there are α_i number of patient depositors in network i contingent on α and on a patient depositor is in network i is denoted by $g_p^i(\alpha_1, \alpha_2|\alpha)$. Hence, the *ex-post* probability that there are α number of impatient depositors, and among them α_1 are in network 1 and α_2 are in network 2 for a patient depositor in network i is:

$$h_p^i(\alpha_1, \alpha_2, \alpha) = f_p(\alpha)g_p^i(\alpha_1, \alpha_2|\alpha)$$

The technology is the same as in the demand-deposit case. Following Peck and Shell (2003), let $c^1(z)$ denote the period 1 withdrawal of consumption by the depositor in arrival position z. The resource constraint is

$$c^2(\alpha^1) = \frac{N - \Sigma_{z=1}^{\alpha^1} c^1(z)}{N - \alpha^1} R, \ c^1(N) = N - \sum_{z=1}^{N-1} c^1(z). \tag{10}$$

Depositors do not know their positions in the line when they make withdrawals. They have equal chance to be in any position in the line. If there are α^1 depositors withdrawing the deposits, then the probability of getting $c^1(z)$ will be $\frac{1}{\alpha^1}$ for $z = 1, 2, .., \alpha^1$. Therefore, the expected utility for a patient depositor if he withdraws the deposit in period 1 is $\frac{1}{\alpha^1} \sum_{z=1}^{\alpha^1} v(c^1(z))$.

6.2.2 Postdeposit Game

A banking contract m that allows for partial suspension in the postdeposit game is described by the vector

$$\begin{aligned} m &= (c^1(1),...,c^1(z),...,c^1(N),c^2(0),...,c^2(N-1)) \text{ and} \\ &\quad (c^1(1),...,c^1(z),...,c^1(N),c^2(0),...,c^2(N-1)) \text{ satisfies (10).} \end{aligned}$$

The participation incentive compatibility constraint requires

$$\sum_{\alpha=0}^{N-1} f_p(\alpha) v\left(\frac{N-\Sigma_{z=1}^{\alpha}c^1(z)}{N-\alpha}R\right) \geq \sum_{\alpha=0}^{N-1} f_p(\alpha)\left[\frac{1}{\alpha+1}\sum_{z=1}^{\alpha+1} v(c^1(z))\right]. \tag{11}$$

The set of feasible banking contracts M is defined as:

$$M = \left\{m \in R_+^{2N} : (10)-(11) \text{ hold for all } \alpha\right\}.$$

A run-proof contract requires

$$v\left((N-\Sigma_{z=1}^{N-1}c^1(z))R\right) \geq \frac{1}{N}\sum_{z=1}^{N} v(c^1(z)). \tag{12}$$

The set of run-proof banking contracts M^{RP} is defined as:

$$M^{RP} = \{m \in M : (12) \text{ hold for all } \alpha\}.$$

I continue to use the definition of correlated equilibrium as in the previous section. The corresponding restrictions on the banking contract that allows for a correlated equilibrium are:

For a patient depositor in network 1:

$$\begin{aligned} & p_{10}\sum_{\alpha=0}^{N-1}\sum_{\alpha_1=0}^{\min\{N_1-1,\alpha\}} h_p^1(\alpha_1,\alpha_2,\alpha)\frac{\sum_{z=1}^{N_1+\alpha_2} v(c^1(z))}{N_1+\alpha_2} + p_{11}\frac{1}{N}\sum_{z=1}^{N} v(c^1(z)) \geq \\ & p_{10}\sum_{\alpha=0}^{N-1}\sum_{\alpha_1=0}^{\min\{N_1-1,\alpha\}} h_p^1(\alpha_1,\alpha_2,\alpha) v\left(\frac{N-\Sigma_{z=1}^{N_1+\alpha_2-1}c^1(z)}{\alpha_2+1}R\right) + p_{11} v\left(\left[N-\Sigma_{z=1}^{N-1}c^1(z)\right]R\right) \end{aligned} \tag{13}$$

$$\begin{aligned} & p_{01}\sum_{\alpha=0}^{N-1}\sum_{\alpha_1=0}^{\min\{N_1-1,\alpha\}} h_p^1(\alpha_1,\alpha_2,\alpha) v\left(\frac{N-\Sigma_{z=1}^{N_2+\alpha_1}c^1(z)}{N_1-\alpha_1}R\right) + p_{00}\sum_{\alpha=0}^{N-1} f_p(\alpha) v\left(\frac{Ny-\Sigma_{z=1}^{\alpha}c^1(z)}{N-\alpha}R\right) \geq \\ & p_{01}\sum_{\alpha=0}^{N-1}\sum_{\alpha_1=0}^{\min\{N_1-1,\alpha\}} h_p^1(\alpha_1,\alpha_2,\alpha)\frac{\sum_{z=1}^{N_2+\alpha_1+1} v(c^1(z))}{N_2+\alpha_1+1} + p_{00}\sum_{\alpha=0}^{N-1} f_p(\alpha)\frac{\sum_{z=1}^{\alpha+1} v(c^1(z))}{\alpha+1} \end{aligned} \tag{14}$$

For a patient depositor in network 2:

$$p_{11}\frac{1}{N}\sum_{z=1}^{N-1} v(c^1(z)) + p_{01}\sum_{\alpha=0}^{N-1}\sum_{\alpha_2=0}^{\min\{N_2-1,\alpha\}} h_p^2(\alpha_1,\alpha_2,\alpha)\frac{\sum_{z=1}^{N_2+\alpha_1} v(c^1(z))}{N_2+\alpha_1} \geq$$
$$p_{11}v\left(\left[N-\Sigma_{z=1}^{N-1}c^1(z)\right]R\right) + p_{01}\sum_{\alpha=0}^{N-1}\sum_{\alpha_2=0}^{\min\{N_2-1,\alpha\}} h_p^2(\alpha_1,\alpha_2,\alpha)v\left(\frac{N-\Sigma_{z=1}^{N-\alpha_1-1}c^1(z)}{\alpha_1+1}R\right) \tag{15}$$

$$p_{10}\sum_{\alpha=0}^{N-1}\sum_{\alpha_2=0}^{\min\{N_2-1,\alpha\}} h_p^2(\alpha_1,\alpha_2,\alpha)v\left(\frac{N-\Sigma_{z=1}^{N_1+\alpha_2}c^1(z)}{N_2-\alpha_2}R\right) + p_{00}\sum_{\alpha=0}^{N-1} f_p(\alpha)v\left(\frac{Ny-\Sigma_{z=1}^{\alpha}c^1(z)}{N-\alpha}R)\right) \geq$$
$$p_{10}\sum_{\alpha=0}^{N-1}\sum_{\alpha_2=0}^{\min\{N_2-1,\alpha\}} h_p^2(\alpha_1,\alpha_2,\alpha)\frac{\sum_{z=1}^{N_1+\alpha_2+1} v(c^1(z))}{N_1+\alpha_2+1} + p_{00}\sum_{\alpha=0}^{N-1} f_p(\alpha)\frac{\sum_{z=1}^{\alpha+1} v(c^1(z))}{\alpha+1} \tag{16}$$

The set of banking contracts that are consistent with a correlated equilibrium is defined as:

$$M^{CE} = \{m \in M : (13) - (16) \text{ hold for all } \alpha\}.$$

An Example

The parameters in the following example are similar to that in Peck and Shell (2003). There are two depositors; one in each network. The probability of being in either group is equal for both of them *ex ante*. Let $u(x) = \frac{Ax^{1-a}}{1-a}$, $v(x) = \frac{x^{1-b}}{1-b}$, $A = 7$, $a = b = 1.01$, $R = 1.1$, $y = 3$. A depositor is impatient with probability p, $p = 0.4$. In this simple example, there is only one choice variable, which is $c^1(1)$.

Let $p_{11} = 0.001$, $p_{01} = 0.009$, $p_{10} = 0$. Sets of banking contracts are described as follows:

m in	$c^1(1)$
M	$[0, 3.2937]$
M^{RP}	$[0, 3.2852]$
M^{CE}	$[3.2928, 3.2936]$
$M^{RA}\backslash M^{CE}$	$(3.2852, 3.2928) \cup (3.2936, 3.2937]$

6.2.3 Predeposit Game

In the predeposit game, the bank decides the best contract to offer. The depositors compare the welfare under autarky with the *ex-ante* welfare the contract yields. The optimal $m \in M^{CE}$

that allows for a correlated equilibrium can be calculated in the same way as in the previous section. p_{11}, p_{10}, p_{01}, and p_{00} are the probabilities of full runs, partial runs driven by network 1, partial runs driven by network 2, and non-run, respectively. The examples below show that in some economies, the optimal banking contract with partial suspension of convertibility tolerates a correlated equilibrium. In some economies, the optimal banking contract with partial suspension of convertibility is run-proof.

Examples

The economy has three depositors: network 1 has 1 depositor and network 2 has 2 depositors. $\alpha = 0.5$. $u(x) = \dfrac{Ax^{1-a}}{1-a}$, $v(x) = \dfrac{x^{1-b}}{1-b}$, $A = 10$, $a = b = 1.01$, $R = 2$. There are two choice variables here: $c^1(1)$ and $c^1(2)$. Welfare is normalized to be $W + 1646$. In autarky, $W^{aut} = -1.9473$.

(1) $p_{11} = 0.0001$, $p_{01} = 0.0009$

The highest *ex-ante* welfare levels that the best contracts in each subset can achieve are:

m in	$c^{1*}(1)$	$c^{1*}(2)$	$\hat{W}(m)$
M^{RP}	1.5780	0.9826	0.3366
M^{CE}	1.6226*	1.0368*	0.4023*
$M^{RA} \backslash M^{CE}$	1.6545	1.0289	0.4005

(2) $p_{11} = 0.0008$, $p_{01} = 0.0002$

The highest *ex-ante* welfare levels that the best contracts in each subset can achieve are:

m in	$c^{1*}(1)$	$c^{1*}(2)$	$\hat{W}(m)$
M^{RP}	4.7340	2.9479	0.3280
M^{CE}	4.8678	3.1097	0.3911
$M^{RA} \backslash M^{CE}$	4.9643*	3.0865*	0.3912*

(3) The example in the postdeposit game continued. The welfare in each partition of M is calculated as follows. The welfare is normalized to be $W + 673$.

m in	$c^{1*}(1)$	$\hat{W}(m)$
M^{RP}	3.2852	0.787
M^{CE}	3.2936	0.791
$M^{RA} \backslash M^{CE}$	3.2937	0.793

The welfare under autarky is $W = 0.5427$.

In this example, the best run-proof contract and the best contract that allows for a correlated equilibrium will be accepted *ex ante*. However, a run-proof contract is inferior to a contract that allows for a correlated equilibrium.