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Population Movements in the Presence of Agglomeration and Congestion Effects: Local Policy and the Social Optimum

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ABSTRACT

Population Movements in the Presence of Agglomeration and Congestion Effects: Local Policy and the Social Optimum

We investigate the efficiency properties of population mobility when localities compete in an environment with local amenities and local externalities. Our model is dynamic, incorporating land and labor markets in a context where firms and workers form rational expectations. Concern focuses on whether and under what conditions the substantive conclusions from static models can be reinterpreted to apply in a dynamic context where moving is costly. In the spirit of Tiebout (1956), it can be shown in static models that taxes or subsidies developed by each local jurisdiction representing the interests of landowners can induce an efficient population allocation even in the presence of local externalities. We show that, in a dynamic model, efficiency of mobility requires that localities represent the interests of other local stakeholders, including residents and firms, as well as landowners. Under certain circumstances, the dynamic model resolves problems of indeterminacy implicit in the static model due to multiple equilibria. On the other hand, we also find that there may be multiple sets of equilibrium flows corresponding with alternative expectations. We consider institutional arrangements that may facilitate preferred paths.

JEL Codes: H23, J61, R13, R5

Keywords: Local taxation, Population externalities, Migration

I. Introduction

Following from the early work of Rosen (1979) and Roback (1982), an extended literature has examined how free movement of individuals and firms induces a static equilibrium distribution of wages and rents across locations in the presence of location-specific consumption and production amenities. Although the insights of static models in this tradition are substantial, Sjaastad (1961) noted quite early that migration is an investment, involving a comparison of the costs of mobility with future expected benefits. Yet the development of fully dynamic models of population redistribution with rational decision makers, especially as applied to questions of how the market performs in the face of externalities and political competition, has lagged. The present paper extends the static strand of the literature by explicitly modeling land and labor markets in a dynamic context where mobility is costly and firms and workers form rational expectations about future opportunities. Much of our concern focuses on whether several of the substantive conclusions from the static model can be reinterpreted to apply in a dynamic context. Although unconstrained movement will not generally produce an efficient population allocation in the presence of local agglomeration or congestion effects, it is known that competition between local jurisdictions in the spirit of Tiebout (1956) can induce an efficient population allocation. We examine the conditions under which such a conclusion carries over to a setting with mobility costs. We also consider the institutional arrangements and competition among locations that may develop to select among multiple equilibrium population paths.

In a static model incorporating land and labor markets, free movement by individuals and firms yields a social optimum provided there are no economies of scale or agglomeration, congestion costs, or other local externalities in the production of market goods or resident utility.[1] In contrast, in the presence of these kinds of local externalities, the resulting population distribution need not be Pareto optimal, since the individual migration decision does not consider all effects on fellow residents. Yet, if landowners at each location can vote to establish taxes and subsidies applying to local land and residents so as to maximize their own surplus, they will agree on a set of transfers that induce an efficient allocation of population across locations.[2] Since, in equilibrium, residents and firms are indifferent between locations, they have no interest in the tax policies of any one locality. It therefore appears plausible that landowners—who have direct financial interests—would be granted effective control of local taxes, and so the efficient outcome would prevail.

The primary focus of our analysis is to determine whether parallel conclusions regarding optimal population can obtain in a dynamic model, in which migration and employment adjustment are costly. Here, the optimal choice must be considered in terms of the dynamic path, not merely the desirability of the steady-state equilibrium. We first consider conditions under which levels of population redistribution occurring without

[1] Haurin (1980) finds that free mobility does not yield a socially optimal population distribution. The important factor in producing this conclusion is his assumption that a portion of land rent is returned to tenants, essentially allocating land ownership on the basis of residency. When Haurin considers the case where no rent is returned to tenants, he finds that the resulting population allocation is socially optimal.

[2] This is essentially Henderson's (1985a) result. He considers optimal population in a locale where economies of scale in consumption result from the production of local public goods. See also Wildasin (2006).

local taxes or subsidies will be efficient. As in the static model, necessary conditions for efficiency are met in the absence of local externalities despite the adjustment costs. We then investigate whether local political structures can be established to assure the social optimum even in the presence of local externalities. In contrast to the implication of the static model, in the dynamic case landowners will not necessarily make efficient taxation decisions. In particular, their actions will exhibit a kind of dynamic inconsistency in which the interests of residents and firms already present in a location are undervalued.[3] However, we show that if taxes and subsidies can be determined in each locality by a coalition involving residents, firms, and property holders, the population redistribution among locations can be optimal. Static models admit the possibility of multiple population steady-state equilibria. There is no obvious way to choose between them, but in a dynamic model, we show it may be possible for migrant decisions to induce flows between locations that allow a preferable population steady state to be achieved.

Although there are relatively few studies that address dynamic efficiency issues, there is an extensive literature that considers economies of agglomeration and congestion costs to explain urban development and variation in settlement density. Glaeser (2008, Chapter 4) provides a particularly useful treatment of agglomeration effects that admits the possibility of differences in inherent productivity and consumption amenities across locations and considers both theoretical models and analyses that attempt to empirically identify underlying relationships.

Among studies that examine the dynamics of population redistribution, Krugman (1991) presents a simplified model with agglomeration effects, multiple equilibria, and adjustment costs modeled similar to our approach. He does not, however, address issues of efficiency or the role of local government policy. Rauch (1993) also provides an analysis of the dynamics of firm location choice in a model with multiple equilibria and adjustment costs. His analysis focuses on the role of developers, showing that they can often push the system toward efficient outcomes. His detailed results depend on the particular specification of his model and the structure of adjustment costs, which are specified very differently from those here. Several recent dynamic models do not lend themselves to a full efficiency analysis. For example, the inefficiencies in Devillanova (2001) are the result of regional shocks, effectively removing any power that local governments might have to improve efficiency, as they have no more ability to predict the shocks than the citizens living in the community.[4]

Among the papers examining efficiency in models of dynamic population redistribution, Wildasin and Wilson (1996) present a model that is most similar to ours.[5] Still, there are three important differences between our model and that of Wildasin and Wilson that cause results to differ. First, Wildasin and Wilson use an overlapping-

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[3] Henderson (1980) demonstrates such effects in a two-period model. In a later paper focusing on which political structures can lead to an efficient solution, Henderson (1985b) reiterates the importance of these dynamic issues and poses several questions to be answered in the dynamic context. The model developed in this paper provides answers to some of these questions. For another interesting study that focuses on dynamic inconsistencies in this setting, see Richer (1995), whose model parallels the problem faced by a durable goods monopolist.

[4] Chau (1997) and Glomm and Lagunoff (1999) also develop models with dynamic structures, but neither model is designed to consider the efficiency of the system.

[5] The set-up of Zeng's (2002) model is also quite similar to the set-up in our paper, but Zeng is concerned with equilibrium stability, not efficiency.

generations model, in contrast to the model of infinitely-lived individuals in our paper. Second, they model moving costs in a very different way than they are modeled here, by assuming that moving is costless in the first period and varies across individuals in the second period. Since the inefficiency in their model is the result of landowners extracting rents from the relatively "captive" older residents, this moving costs assumption is important to their results.

Finally, only landowners vote on local policy in the Wildasin and Wilson model. This assumption is motivated by the static model, where a landowner-controlled local government produces efficient results. In our paper, the composition of the local government coalition is allowed to vary, with the result that the efficient solution can arise if a coalition of landowners, current residents, and firms determines policy. Not only is this result substantively interesting, it also provides a more realistic description of local policymaking.

It is useful to place our work within the extensive literature that examines optimal taxation in the presence of potentially mobile factors of production. In large part, the literature considers the case where a given factor is either mobile or immobile, and so the efficiency of mobility in the presence of moving costs is not considered. There are some exceptions. Wildasin (2003), for example, presents a model where taxes on a factor facing moving costs (in his model it is capital) are taken to benefit owners of the immobile factor. His analysis addresses issues of competition and time consistency that are similar to those we address, and his results foreshadow ours. However, the focus of our analysis on the efficiency of population mobility and our investigation of the role of congestion or agglomeration effects in a fully dynamic optimizing model distinguishes our approach from his and others in this literature.

Our paper is organized as follows. We present a benchmark static model in section II in which production and consumption are dependent on local resources. The model also admits the possibility of local economies or diseconomies of agglomeration in population. Sections III and IV consider how the competitive outcome compares with a social optimum, showing that free mobility produces a social optimum even in the presence of population agglomeration effects if landowners are permitted to act as a group to establish taxes or subsidies for residents.

Section V introduces a dynamic model that allows for migration costs for individuals as well as adjustment costs for firms. The subsequent section investigates the welfare properties of the competitive outcome, and section VII shows how a social optimum may prevail if taxes or subsidies are determined by a local coalition. Section VIII examines the issue of multiple equilibria. Section IX concludes.

II. The Static Model

This section presents a static population distribution model that serves as a benchmark for the ultimate objective of analyzing a fully dynamic model. The static benchmark follows Roback (1982) and Blomquist, et al. (1988) in that it assumes utility equalization across locations for residents and cost equalization across locations for firms to define an equilibrium population distribution.

Residents/Migrants

Let $u(c_i, L_i^h, a_i, N_i)$ be utility received by a representative resident in location *i* from consumption c_i (the *numeraire*) and household land use L_i^h, when local consumption amenities are a_i and aggregate local population is N_i. Like Roback, we ignore labor supply decisions and assume each resident or household supplies one unit of labor to the local market. However, in contrast to Roback, we allow for the possibility that individual utility depends directly on the local population N_i. The impact may be positive, reflecting benefits in consumption from public goods or positive agglomeration effects in local service markets that are not explicitly included in our model, or it may be negative, reflecting congestion effects.

Taking the local wage w_i and land rent r_i as given, this resident receives indirect utility

$$U(w_i, r_i, a_i, N_i) = \max_{\{c_i, L_i^h\}} u(c_i, L_i^h, a_i, N_i) \text{ subject to } c_i + r_i L_i^h = w_i. \tag{1}$$

We denote optimal household land demand in location *i* by $L^h(w_i, r_i, a_i, N_i)$.

Firms and Landowners

Assume constant returns in production at the firm level. Then the aggregate output of firm *k* in location *i* can be written

$$y_k = N_k^f h(L_k^f, b_i, N_i), \tag{2}$$

where N_k^f is the quantity of labor input, $h(L_k^f, b_i, N_i)$ is output per worker, a function of the quantity of land input per worker, L_k^f, exogenous local productive resources, indexed by b_i, and population at the location, N_i. We allow for the possibility that aggregate population at the location may influence unit cost but assume firms do not consider the effects of their actions on population (if any) when minimizing cost, so each individual firm faces a constant unit cost. It is important to recall as the analysis proceeds that N_i in the production function *h* is an externality, not labor as a factor of production.

Constant returns implies that equilibrium profits are zero and output y_k is determined by demand rather than by firm choice. Hence, firm *k* chooses the inputs N_k^f and L_k^f to minimize cost $N_k^f \left[w_i + r_i L_k^f \right]$ subject to the production constraint (2), so the firm's problem can be expressed as choosing the land input to minimize cost per unit of output:

$$C(w_i, r_i, b_i, N_i) = \min_{\{L_k^f\}} \frac{w_i + r_i L_k^f}{h(L_k^f, b_i, N_i)}. \tag{3}$$

This optimization is independent of the firm *k*, so we drop the subscript and denote optimal firm land demand per worker in location *i* by $L^f(w_i, r_i, b_i, N_i)$.

The land supply in location *i* is fixed at L_i, all of which is owned by a set of landowners who receive aggregate rent $r_i L_i$, reflecting payments from both firms and residents. Landowners consume the total value of payments received; for simplicity, we do not allow them to consume land. Since they are immobile, any benefits they receive from local amenities are not relevant.

Equilibrium

Free mobility implies that equilibrium utility equals a common level at all locations:

$$U(w_i, r_i, a_i, N_i) = U^*, \ \forall i. \tag{4}$$

Equation (4) defines a horizontal labor supply curve in location *i*. For given local amenities a_i and population N_i and a given land rental rate r_i, a wage that exceeds the level defined by (4) will attract an infinite supply of labor into location *i*, while a wage below the level defined by (4) will drive all labor out of location *i*.

Assuming firms are competitive, the unit cost at each location must equal the equilibrium product price, which we have assumed is the *numeraire*:[6]

$$C(w_i, r_i, b_i, N_i) = 1, \ \forall i. \tag{5}$$

Equation (5) defines a horizontal labor demand curve in location *i*. For given local productive resources b_i and population N_i and a given rent r_i, a wage that exceeds the level defined by (5) will drive labor demand (and output) to zero in location *i*, while a wage below the level defined by (5) will cause firms in location *i* to demand an infinite amount of labor (and produce infinite output).

Equilibrium in the land market requires that demand for land equal the local fixed supply, so:

$$N_i[L^h(w_i, r_i, a_i, N_i) + L^f(w_i, r_i, b_i, N_i)] = L_i, \ \forall i. \tag{6}$$

Equation (6) determines the equilibrium quantity of labor N_i in location *i*, given the wage and land rental rate determined by (4) and (5).

Given U^*, (4) – (6) determine the three equilibrating variables w_i, r_i, and N_i for each occupied location *i*. The values at any one location are tied to those at other locations because the equilibrium system utility level U^* is endogenous, and population across all locations must add to an exogenous total *N*:

$$\sum_i N_i = N, \tag{7}$$

where summation is across all locations. So the system is (4) – (6) in the three unknowns (w_i, r_i, N_i) for each location, and one systemic equation (7) in the one systemic unknown U^*. We assume throughout that production and utility are "well-behaved" in the sense that (4) and (5) define a unique interior competitive equilibrium in (w_i, r_i) for a given N_i. Nonetheless, there still may be multiple equilibria of the entire system because of the effects of N_i through the per capita land endowment and its effects through the consumption and production externalities.

The conditions above assume that all locations are occupied; allowing for some locations to be vacant alters the conditions in expected ways. In particular, unoccupied locations must offer utility to an arriving migrant that is no greater than U^* and must

[6] Although it is common in the literature to describe (5) as a consequence of firm mobility, the same condition obtains if there is price-taking among firms in each locality. We will maintain the latter interpretation because it eases discussion of the dynamic model.

offer per unit costs to firms that are no smaller than unity.[7] Just as in the case where we assume all locations are occupied, there may exist multiple equilibria; a location occupied in one equilibrium may not be occupied in another. Since accommodating empty locations is tedious and adds essentially nothing to the results, the presentation will assume all locations are occupied. Our treatment allows locations to have extremely small numbers of residents in equilibrium, so the formal model we present can be structured to correspond as closely as desired to an environment with truly empty locations.

III. Pareto Efficiency in the Static Model

This section establishes that a competitive equilibrium in the benchmark static model cannot be Pareto optimal unless all locations have the same marginal net population externalities evaluated in terms of migrant utility. This result has several implications. First, it is consistent with the first welfare theorem, and in fact a minor extension verifies the first welfare theorem (i.e., that a competitive equilibrium is efficient when there are no population externalities) for the benchmark static model under some additional assumptions. Second, a competitive equilibrium can be efficient if it is symmetric across locations, even in the presence of externalities. Third, the basic efficiency property leads naturally to the subsequent section, where we show that a tax/subsidy scheme operated by landowners generates an allocation that is consistent with a social optimum even when a competitive equilibrium in the absence of such a scheme would be inefficient.

A Pareto optimum for our purposes maximizes the income available to firms and landowners in one location, while assuring a fixed level of aggregate welfare to residents in all locations and a fixed level of income to firms and landowners in each other location, subject to the land and nationwide population resource constraints. We do not allow the social planner to move consumption across locations. This constraint is imposed in order to assure that individuals cannot consume resources in one area that have been produced in another. Without such a constraint, the meaning of amenities in such a model would be lost.

By stating the utility constraint in the aggregate, this statement of the Pareto problem may appear to omit the constraint that each individual resident or migrant receive a threshold utility level. We ignore individual utility constraints because they are tedious in this model, requiring a full accounting system for each resident rather than only for the representative individual, and because individual utility constraints are unimportant to the conclusions.[8] The important point is that residents "consume" the

[7] Rather than (4)-(6), for any location *i* that is not occupied, in addition to $N_i = 0$, the following conditions hold: $U(w_i, 0, b_i, 0) \leq U^*$ and $C(w_i, 0, b_i, 0) \geq 1$. This reflects the fact that in a vacant location land is not scarce, so r_i=0. These conditions can only be met if the marginal value of land in consumption and production approaches zero sufficiently fast as quantity of land increases. Otherwise, firms and individuals will always find it worthwhile to occupy a location with at least a very small population, taking advantage of the high land-resident ratio corresponding with an infinitesimal population.

[8] Essentially, we endow the social planner with the power to transfer utility between individuals in a given location. One way to interpret this "transferable utility" condition is to think of a social planner who allocates individuals to live fractions of their time in various locations, without changing the population distribution, for time intervals whose lengths are chosen to accomplish any desired transfer.

household land allocation, local amenities, and population externality of the location from which their consumption is drawn.

Taking the location under study to be location 1 for convenience, a social optimum solves:

$$\max_{\{c_i, L_i^h, L_i^f, N_i, \forall i\}} N_1\left[h(L_1^f, b_1, N_1) - c_1\right] \tag{8}$$

subject to

$$\sum_i N_i u(c_i, L_i^h, a_i, N_i) \geq \bar{u}, \tag{9}$$

$$N_i\left[h(L_i^f, b_i, N_i) - c_i\right] \geq R_i, \forall\ i > 1 \tag{10}$$

$$N_i\left[L_i^h + L_i^f\right] = L_i, \forall\ i \tag{11}$$

$$\sum_i N_i = N, \tag{12}$$

where $\bar{u}$ is the aggregate utility that must be provided to residents in all locations and R_i is the aggregate income that must be received by firms and landowners in location *i*. The objective in (8) is the aggregate income of firms and landowners in the location under study, which, when maximized, can be allocated to firms and landowners as necessary to ensure a Pareto improvement for each of them relative to some initial allocation of income, as can any aggregate income satisfying (10) in the other locations. (9) is the residents' aggregate utility constraint. (11) and (12) are the land and population resource constraints, respectively.

One necessary condition for efficiency is that the marginal rate of substitution equal the marginal rate of transformation between the land and non-land commodities in every location. It is straightforward to establish that this occurs in a competitive equilibrium. In particular, the solution to (1) equates the marginal rate of substitution to the price ratio,[9]

$$u_{L_i^h} / u_{c_i} = r_i, \tag{13}$$

and the solution to (3) equates the marginal rate of transformation to the price ratio,

$$C(w_i, r_i, b_i, N_i) h_{L_i^f} = r_i. \tag{14}$$

So, using (5), we see that a competitive equilibrium in the static model efficiently allocates land and consumption for each location:

$$h_{L_i^f} = \frac{u_{L_i^h}}{u_{c_i}}, \forall\ i. \tag{15}$$

In contrast, the competitive equilibrium allocation of population is not generally Pareto optimal. Proposition 1 gives a necessary condition for Pareto optimality of a competitive equilibrium allocation.

[9] Recall that consumption has price one. Throughout, $u_{L_i^h}$, u_{N_i}, and u_{c_i} denote the partials of direct utility with respect to household land use, local population, and consumption, respectively, evaluated at the values for location *i*; and $h_{L_i^f}$ and h_{N_i} denote the partials of per capita production with respect to land per worker and local population, respectively, also evaluated at the values for location *i*.

Proposition 1.[10] If constraint qualification holds for the program (8) - (12), a necessary condition for the competitive equilibrium allocation to be a solution to this maximization program is

$$u_{c_i} N_i \left[h_{N_i} + \frac{u_{N_i}}{u_{c_i}} \right] = u_{c_j} N_j \left[h_{N_j} + \frac{u_{N_j}}{u_{c_j}} \right], \forall\ i, j\,, \tag{16}$$

evaluated at the competitive equilibrium allocation.

Equation (16) is the population distribution condition necessary for a competitive equilibrium to be a social optimum. The aggregate value of the net marginal externality in location *i* associated with one additional resident is $N_i \left[h_{N_i} + \frac{u_{N_i}}{u_{c_i}} \right]$. Multiplying this by the marginal utility of consumption in location *i* gives the value of the net marginal externality in that location in terms of utility in that location. So the result states that a competitive equilibrium cannot be efficient unless the utility values of all marginal net externalities are equated across locations in equilibrium.

A special case in which marginal net population externalities are equal across all locations is when there are no marginal agglomeration or congestion effects at the equilibrium (i.e., $u_{N_i} = h_{N_i} = 0, \forall\ i$). In this case, Proposition 1 is consistent with the first welfare theorem for the static model, and verifies the first welfare theorem when the conditions of Proposition 1 are sufficient.[11] Another special case in which the competitive equilibrium can be efficient is when locations are identical and the competitive equilibrium is symmetric, so that all locations are subject to the same agglomeration or congestion effects.

It is widely argued that the existence of large cities demonstrates the importance of positive agglomeration effects. On the other hand, it seems equally likely that, as population grows, congestion effects become important. If locations differ in terms of inherent attractiveness, the common equilibrium level of utility may well be attained at all locations when location populations are inducing unequal marginal agglomeration externalities. If so, Proposition 1 shows that the competitive equilibrium is inefficient.

IV. Static Efficiency and Local Politics

A subsidy or tax on residents, depending on whether there are economies or diseconomies of agglomeration, can remedy the potential inefficiency caused by externalities. If subsidies are needed to neutralize externalities, the subsidies can be financed via an excise tax on land with no social loss. Similarly, if taxes are needed, the revenues can be given to landowners without creating a distortion. Moreover, if landowners at each location are residual claimants of local resident taxes or subsidies, they will make tax/subsidy decisions that satisfy the necessary conditions for efficiency.

[10] Unless otherwise indicated, formal proofs for all propositions are in the appendix.

[11] Mas-Colell et al. (1995) Theorem M.K.3 gives an exact statement of the curvature conditions that assure sufficiency. It is possible that the equilibrium is a local but not a global welfare maximum. We wish to allow for this latter possibility because we are concerned about the behavior of the system when there are positive population agglomeration effects over some range but negative population congestion effects over some other range.

Perfectly inelastic land supply and the independence of landowner decisions across locations drive this result.

Formally, let x_i be a per-capita net subsidy paid to residents in location i, with this expense covered out of the revenues of location i landowners (they receive payments if x_i is negative). Then the budget constraint for a resident of i is

$$c_i + r_i L_i^h = w_i + x_i , \tag{17}$$

so $w_i + x_i$ replaces w_i in the household land demand and indirect utility functions. Noting that the equilibrium values of w_i, r_i and N_i now depend on x_i, the equilibrium conditions (4) – (6) become

$$U(w_i(x_i) + x_i, r_i(x_i), a_i, N_i(x_i)) = U^*, \tag{18}$$

$$C(w_i(x_i), r_i(x_i), b_i, N_i(x_i)) = 1, \text{ and} \tag{19}$$

$$N_i(x_i)\left[L^h(w_i(x_i) + x_i, r_i(x_i), a_i, N_i(x_i)) + L^f(w_i(x_i), r_i(x_i), b_i, N_i(x_i))\right] = L_i . \tag{20}$$

The efficiency of the land and consumption allocation in a location does not change with the subsidy, so (15) continues to hold in the subsidy-induced equilibrium, but (16) is modified by the marginal utility value of the subsidy.

Corollary to Proposition 1. When x_i is in the model, the necessary condition of Proposition 1 is

$$u_{c_i}\left\{N_i\left[h_{N_i} + \frac{u_{N_i}}{u_{c_i}}\right] - x_i\right\} = u_{c_j}\left\{N_j\left[h_{N_j} + \frac{u_{N_j}}{u_{c_j}}\right] - x_j\right\}, \ \forall i, j , \tag{21}$$

evaluated at the subsidized equilibrium allocation.

Now consider landowners who control the local political process and thereby choose the subsidy level for location *i,* but who also serve as the sink for financing local government. We assume they take as given the utility provided in other locations. Their maximization problem may be written as:

$$\max_{\{x_i\}}[r_i(x_i)L_i - N_i(x_i)x_i]. \tag{22}$$

Proposition 2. Assuming an interior solution, the landowners' optimal value of x_i satisfies

$$x_i = N_i\left[h_{N_i} + \frac{u_{N_i}}{u_{c_i}}\right]. \tag{23}$$

The necessary condition for efficiency (21) holds in an equilibrium with tax/subsidy given by (23) because the landowners' revenue-maximizing plan perfectly offsets the agglomeration effects in their locality that might keep (16) from holding in the no-subsidy, competitive environment. Hence, the landowners' subsidy plan guarantees the competitive equilibrium can be efficient even when there are asymmetric agglomeration or congestion effects across locations in equilibrium.

The substantive conclusion of this exercise is that local economies or diseconomies of agglomeration, whether they affect firm production of goods and/or resident utility, are properly internalized by landowners. Although this general result has been shown in other models,[12] it is worthwhile to stress its importance. The interests of residents and firms are fully protected by the competition among locations because residents are fully mobile and firms earn zero profits regardless of local—or system—policy. In contrast, property owners have a direct common interest in maintaining revenue-maximizing policies, which suggests why they might obtain effective political power.

Several important caveats must be recognized in these strong conclusions. First, the efficiency condition (21) is necessary but not sufficient. This might be viewed as a mostly technical issue that can be solved with additional assumptions (see footnote 11). However, given agglomeration or congestion effects, there may be multiple population distributions that satisfy (21), and there is no obvious way to assure that the competitive outcome achieved is the global maximum. We know only that there is a local tax/subsidy policy that neutralizes the obvious source of inefficiency—the marginal population externalities—and that such a policy satisfies necessary conditions to maximize landowners' rents.

Equally important, there is no obvious way for landowners to choose a particular equilibrium over another. When there are multiple equilibria, the tax scheme associated with the optimum may also be associated with another (suboptimal) equilibrium. Given the structure of the static model, it is difficult to see what actions landowners could take to induce the preferable equilibrium. This issue can be addressed in a dynamic model in which an initial population distribution and future expectations play an explicit role. In particular, when the current population distribution corresponds to an inferior equilibrium, perhaps forward-looking individuals will migrate toward an area in anticipation that population agglomeration effects will ultimately dominate, moving the system toward a preferable distribution.

Finally, these conclusions do not consider the costs individuals might incur in moving between locations, or adjustment costs associated with expanding or reducing employment. If there are such costs, then residents and firms at a location are, to some degree, "captive," which may make designing optimal policies more difficult. It is no surprise that local area political conflicts generally involve those who, because of circumstance or sunk investments, are strongly tied to a location. A model that assumes free mobility ignores this critical element that underlies local area political processes and that may influence the locality's growth policy.

V. A Dynamic Model

This section presents a continuous-time dynamic model with well-behaved marginal migration and employment adjustment costs while retaining the spirit of the static model reviewed in the previous sections. The dynamic model enables investigation of efficient local policies when there are costs associated with population redistribution. It is a rational expectations model in that both firms and workers have perfect foresight concerning all future values.

The dynamic analog to utility equalization across locations assumed in the static models of Roback (1982) and Blomquist et al. (1988) is that net migration be toward more desirable areas, with migration continuing until utility differences disappear. Firms

[12] See Henderson (1985a) and Wildasin (2006).

expand when cumulative future profits are positive and shrink when profits are negative. As in the static model, we assume firms are price-takers in each location but note that, due to employment adjustment costs, they no longer have constant returns to scale; hence equilibrium profits are nonzero in general. The perfect foresight assumption implies that correctly anticipated future utilities and costs are the relevant decision parameters for migration and labor input adjustment decisions.[13] Prices, location attributes, and quantities are all allowed to vary over time in the dynamic model,[14] so the economic actors must now optimize by considering time paths $w_i(t)$, $r_i(t)$, $a_i(t)$, $b_i(t)$, and $N_i(t)$ for all locations *i*.

Residents/Migrants

A representative individual has the same instantaneous maximization problem as in the static model, optimally allocating current income flows between consumption and land use at each point in time. The only difference is that utility is quasilinear,[15] with instantaneous indirect utility for an individual in location *i* at time *t* given as:

$$U(w_i(t), r_i(t), a_i(t), N_i(t)) = \max_{\{c_i(t), L_i^h(t)\}} A_i c_i(t) + \hat{u}(L_i^h(t), a_i(t), N_i(t)) \qquad (24)$$

$$\text{subject to} \qquad w_i(t) = c_i(t) + r_i(t) L_i^h(t),$$

where the function $\hat{u}$ is utility from land consumption, location amenities and the population externality; and A_i is implicitly part of the vector a_i. Instantaneous optimal household land demand in location *i* parallels that of the static case, and is denoted $L^h(r_i(t), a_i(t), N_i(t))$.[16]

Note that each resident's budget constraint is assumed to hold at each point in time. This assumption has no significance within a location because static quasilinear utility, with the location attribute A_i invariant over time, cannot be improved by moving consumption across time within a location. The assumption that economic actors cannot move consumption across locations is significant because otherwise migrants will have incentives to make moves that do not correspond with differences in location lifetime utility. For example, consider two locations that offer different levels of amenities and wage differences such that the locations provide equal levels of lifetime utility. An individual who can move consumption between these locations will be able to increase lifetime utility relative to staying in either location by dividing his time between them. Although such "commuting" may be of importance in some contexts (see Mueser, 1997), the purpose here is to formulate a dynamic model in the spirit of static models such as Roback (1982) and Blomquist et al. (1988) in which migration is driven solely by

[13] Empirical applications based on models with this basic structure include Mueser and Graves (1995) and Rappaport (2004).

[14] We assume for simplicity that the land and total population endowments are fixed over time. Allowing these resources to vary would not change our substantive conclusions.

[15] The quasilinear static utility assumption with constant marginal utility of consumption over time is made in order to avoid issues of consumption smoothing. Quasilinear utility allows us to assume that migrants simply pay moving costs with a lump-sum adjustment to consumption at the point in time when the migration occurs, as there is no benefit of spreading out payment over time.

[16] Note that $w_i(t)$ does not enter this function because of the quasilinear utility structure.

lifetime utility differences across locations. The assumption that migrants cannot move consumption across location is implicit in (24).

Lifetime utility is additively separable over continuous time with discount rate ρ. The utility received by an individual living in location i from time t onward is:

$$V_i(t) = \int_t^{\infty} U(w_i(s), r_i(s), a_i(s), N_i(s)) e^{-\rho(s-t)} ds. \tag{25}$$

Krugman (2006) observes that migration flows respond smoothly to differences in location desirability, suggesting that individual migration costs are increasing in the level of net migration. In order to capture this dynamic, we take moving cost for one individual when moving from location i to j as

$$-k_i(M_i(t)) + k_j(M_j(t)), \tag{26}$$

where $M_i(t)$ is the net flow of migrants into location i at time t, $k_i(0) = 0$, and $k_i' > 0$.[17] The costs of leaving an area increase if there are more individuals leaving, while the costs of moving into a location increase if it attracts more net arrivals. As discussed above, our interest is in a world in which consumption cannot be moved across locations. Hence we assume the cost an individual incurs in leaving location i, $-k_i(M_i(t))$, is paid out of consumption in location i, while the cost of moving into j, $k_j(M_j(t))$, is paid out of consumption in location j. Moving costs are assumed to be paid as a lump sum.

Subject to these moving costs, residents have free choice of movement, in addition to their instantaneous consumption and land use choices. Given homogeneous residents at any one location, free choice of movement implies that each resident is optimized choosing location i at time t if and only if the costs of moving exactly compensate for the difference in realized utility between locations. Given that each resident takes aggregate net migration as given, resident optimality requires[18]

$$V_i(t) = V_j(t) - [A_j k_j(M_j(t)) - A_i k_i(M_i(t))] \;\; \forall\, t,\, \forall\, i, j. \tag{27}$$

The term in brackets identifies the utility cost of moving from *i* to *j*, with costs translated into the utility units of the location in which they are paid (see (24) and (25)).

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[17] These costs are intrinsic migration costs, not government payments designed to influence migration. This formulation implies positive migration costs for any individual who moves in a way consistent with net migration between two locations. For example, if net migration is out of i ($M_i(t) < 0$) and into j ($M_j(t) > 0$) at time t then both terms of (26) are positive. This formal structure implies that moves against net migration incur a negative cost. Although conceptually possible, such moves are of no significance in the model, so it can be assumed without loss of generality the no such moves occur.

[18] As in the static model, it is possible to modify the dynamic model to allow for locations with no population, or no population at certain times. Here it is necessary to keep track of the exact periods in which locations are empty. A location only remains empty for a specified period if, with land at price zero, the flow of utility never exceeds the flow available elsewhere—in the absence of any moving costs associated with arriving in the location—and if firms cannot produce at costs below unity, ignoring adjustment costs. Rather than complicating the formal analysis to allow for empty locations, we will assume all location have nonzero population at all times as we did in the static analysis. Even with this assumption, the model allows for locations to have extremely small populations (at all or at selected times), so there is no substantive loss to this assumption.

Firms

A representative firm has the same instantaneous production structure as in the static model, except that now firms must consider adjustment costs when they change employment levels. We assume firm k choosing labor force growth $\tilde{N}_k^{f\prime}(t)$ in location i incurs twice differentiable strictly convex adjustment cost $\tilde{\psi}_k(\tilde{N}_k^{f\prime}(t))$, where $\tilde{\psi}_k(0)=\tilde{\psi}_k'(0)=0$ and $\tilde{\psi}_k''>0$.[19] This implies $\tilde{\psi}_k{}'(\tilde{N}_k^{f\prime}(t))$ and $\tilde{N}_k^{f\prime}(t)$ have the same sign. This firm's maximization problem at time t is:

$$\max_{\{\tilde{N}_k^f(s),\tilde{L}_k^f(s)\}} \int_t^\infty \{\tilde{N}_k^f(s)[h(\tilde{L}_k^f(s),b_i(s),N_i(s))-\tilde{L}_k^f(s)r_i(s)-w_i(s)] - \tilde{\psi}_k(\tilde{N}_k^{f\prime}(s))\}e^{-\rho(s-t)}ds. \tag{28}$$

All the terms in (28) correspond to those defined in the static model, except that they reference flows. The term in brackets is production (the *numeraire*) minus land rent and wages, all expressed per employee, whereas the last term inside the braces is the cost of adjusting employment.

The land input has no dynamics in this objective, so land use is optimized by choosing the instantaneous maximum at each point in time. Therefore optimal firm land use per employee $\tilde{L}_k^f$ satisfies:

$$h_{\tilde{L}_k^f}(\tilde{L}_k^f(s),b_i(s),N_i(s))=r_i(s),\ \forall\ s. \tag{29}$$

This condition is independent of k due to constant returns, even if firms have different employment levels at time s, so all firms in location i choose the same level of land use per worker, and instantaneous land demand per worker can be denoted $L^f(r_i(s),b_i(s),N_i(s))$.

Instantaneous cost per unit of output is then:[20]

$$C_i(s)\equiv C(w_i(s),r_i(s),b_i(s),N_i(s))=\frac{w_i(s)+r_i(s)L^f(r_i(s),b_i(s),N_i(s))}{h_i(s)}, \tag{30}$$

where $h_i(s)\equiv h(L^f(r_i(s),b_i(s),N_i(s)),b_i(s),N_i(s))$.

Lemma 1. The path $\tilde{N}_k^{f\prime}(s)$ that solves (28) satisfies:

$$\tilde{\psi}_k'(\tilde{N}_k^{f\prime}(t))=\int_t^\infty h_i(s)[1-C_i(s)]e^{-\rho(s-t)}ds,\ \forall\ t. \tag{31}$$

[19] Although $\tilde{\psi}_k$ may differ across firms, we do not allow it to vary with $\tilde{N}_k^f$. This is consistent with the assumption of constant returns to scale in production, which assures that the value of an additional employee in production does not vary with $\tilde{N}_k^f$.

[20] In contrast to the static model, per unit cost does not equal price at all times. Indeed, aggregate profit can be either positive or negative for individual firms, depending on their initial employment levels and their adjustment cost functions. This is because adjustment costs make it costly for a firm to instantaneously change its scale of operation; losses may occur because firms are not permitted to costlessly shut down. Land use is chosen to equate the marginal revenue product and input price of land at each point in time, as specified by (29), and then the scale of operation at each point in time is determined by choosing the entire employment path to maximize the present value of profit, taking into account adjustment costs.

The left side of (31) is the cost of increasing employment by one additional person at time t. The right side is the aggregate future benefit of having one more employee, who increases output by $h_i(s)$ at each time $s > t$.

As k appears only on the left side of (31), different firms k in a given location i have different employment growth rates only if their adjustment cost functions differ, and individual profit-seeking causes firms to equalize their marginal adjustment costs. Equalization of marginal adjustment costs implies that aggregate adjustment cost in location i incurred by firms can be expressed as a function of aggregate labor adjustment in location $i,$ and the marginal aggregate adjustment cost evaluated at aggregate labor adjustment is the same as the equalized marginal adjustment costs of the individual firms. In particular, let $N_i^{f\prime}(t)$ denote the aggregate labor adjustment in location i at time t,

$$N_i^{f\prime}(t) = \sum_k \tilde{N}_k^{f\prime}(t)\,, \tag{32}$$

where the summation is over all firms k in location $i,$ and let $\psi_i(N_i^{f'}(t))$ be the aggregate adjustment cost when aggregate labor adjustment is $N_i^{f\prime}(t)$ and firms equate marginal adjustment costs as in (31).

Lemma 2. The equilibrium change in aggregate instantaneous labor demand in location i at time t satisfies

$$\psi_i'(N_i^{f\prime}(t)) = \int_t^\infty h_i(s)[1 - C_i(s)]e^{-\rho(s-t)}ds \tag{33}$$

for an aggregate adjustment cost function $\psi_i(N_i^{f'}(t))$ with the same properties as the individual-firm adjustment cost functions (i.e., $\psi_i(0) = \psi_i'(0) = 0$ and $\psi_i'' > 0$).

As noted in footnote 20, firms may have a positive or negative profit flow at any point in time, and the present value of profit may be positive or negative. For simplicity, we assume that firm owners consume all profits—whether positive or negative—and that they do not consume land. At each point in time, their concern is exclusively the present value of profits. This approach abstracts from issues of firm financing and asset accumulation, which are outside the scope of this model.

Landowners and Owners of Moving Resources

As in the static model, landowners are individually passive, merely spending for non-land consumption the payments received from firms and residents for use of land.

Equation (27) specifies that individual migrants pay the marginal cost of their migration decisions. If increasing marginal costs (i.e., k'>0) reflect congestion or other agglomeration effects, each mover would impose an externality on all others. In this case, we would not expect individual decisions to produce the social optimum. Since our focus is on local population-based externalities (as opposed to externalities in the act of migrating), we wish to remove this potential source of inefficiency. We therefore posit that increasing marginal costs result from an upward sloping supply curve of moving services faced by a competitive moving industry.[21]

[21] We note below how our conclusions would differ if migration costs were due to congestion effects.

We may assume either that the moving firms control the supply of all relevant resources or that they purchase the relevant resources in competitive markets.[22] In the former case, moving firms earn positive rents from their resource ownership. In the latter case, free entry of moving firms under constant returns to scale at the firm level assures that moving firms make zero profits, but owners of the resources used in the moving process receive rents. In either case, we refer to those who receive the surplus as owners of moving resources, and assume they consume their surplus directly. They do not consume land.

Like owners of firms, landowners and owners of moving resources are assumed tied to their locations. We have chosen not to distribute ownership across migrants or to have individuals own resources in more than one location for two primary reasons. First, since we wish to focus on local policy decisions, it is useful to unambiguously identify these actors' interests with a particular location. Second, allowing ownership dispersion raises the issue of how to model consumption by a given economic actor at a single point in time of goods produced at more than one location. As noted above, we constrain migrants to consume goods produced in a given location only while they are resident there. It is simplest to merely assign other actors to a single location. Although it would be possible to consider alternative ownership structures, and to adjust consumption accordingly, we see no benefits accruing from the additional complications.[23]

Equilibrium

Instantaneous equilibrium in location i at time t is defined by simultaneous market-clearing in both the land and labor markets. There are no dynamics in land use decisions by either migrants or firms, so in equilibrium the instantaneous aggregate demand for land equals the fixed supply of land at every point in time, just as in the static model:

$$N_i(t)[L^h(r_i(t), a_i(t), N_i(t)) + L^f(r_i(t), b_i(t), N_i(t))] = L_i\,,\ \forall t\,. \tag{34}$$

Equilibrium in the labor market is more complicated because it is dynamic. Migrant choices create a flow of labor into or out of a location which must equal the change in labor demanded by firms. Dynamic labor supply is derived from optimal migration decisions as expressed in (27), which implies there is a common value $V^*(t)$ across locations satisfying

$$V_i(t) - A_i k_i(M_i(t)) = V^*(t)\,,\ \forall\ i\,.$$

$V^*(t)$ plays the same role as U^* in the static model. It is a systemic path that is determined in equilibrium by the population resource constraint. Equilibrium net migration $M_i(t)$ in each location i and at each time t satisfies

$$k_i(M_i(t)) = \frac{V_i(t) - V^*(t)}{A_i}\,. \tag{35}$$

[22] We assume the relevant resources are outside the model in the sense that the land and non-land consumption goods in the model are not used to produce moving services.

[23] If ownership rights were conferred to migrants and permitted to vary on the basis of residency, as in Haurin (1980), this would alter our model substantially. As noted above, such a structure in essence eliminates markets and so induces inefficiency.

The migrant optimality condition (35) implicitly gives the equilibrium change in labor supply $M_i(t)$ in a location at a given time *t*.

Equation (35) implies some properties of dynamic labor supply. Net migration into location i at time t increases when the utility value of location i from time t onward, $V_i(t)$, increases relative to the net system utility of other locations at time t, $V^*(t)$, i.e., $\frac{\partial M_i}{\partial V_i} > 0$ holding $V^*(t)$ constant. Because $k_i(0) = 0$, migrant choice implies positive (negative) net migration for any location providing utility above (below) $V^*(t)$, and zero net migration for any location providing exactly $V^*(t)$. Net migration does not have these properties if, for example, the cost of movement between a pair of locations is a fixed fee that does not vary with changes in the rate of migration. A specification like (26), in which migration costs vary smoothly with the level of net migration, is required for net migration to be responsive to marginal changes in relative desirability.

Instantaneous equilibrium in the labor market requires that dynamic labor supply as specified by (35) equal dynamic labor demand as specified by (33). That is, $M_i(t) = N_i^{f\prime}(t)$, or:

$$k_i^{-1}\left(\frac{V_i(t) - V^*(t)}{A_i}\right) = \psi_i'^{-1}\left(\int_t^{\infty} h_i(s)[1 - C_i(s)]e^{-\rho(s-t)}ds\right), \forall\ t. \quad (36)$$[24]

Instantaneous equilibrium in the labor market can be expressed in the standard supply and demand form (36), in contrast to the static model, because migration costs make changes in labor supply upward sloping as a function of the instantaneous wage, and firm adjustment costs make changes in labor demand downward sloping as a function of the instantaneous wage. This also creates the possibility of nonzero equilibrium profits.

Note that the instantaneous labor market equilibrium condition (36) involves the future paths of population, wages and rents in location *i* because

$$V_i(t) = \int_t^{\infty} U(w_i(s), r_i(s), a_i(s), N_i(s))e^{-\rho(s-t)}ds,$$

$h_i(s) = h(L^f(r_i(s), b_i(s), N_i(s)), b_i(s), N_i(s))$, and $C_i(s) = C(w_i(s), r_i(s), b_i(s), N_i(s))$. The paths of wages and rents adjust in equilibrium to make the instantaneous labor supply and demand choices of firms and migrants consistent, and to make their static land demand choices consistent with the fixed supply of land. Together, (34) and (36) define the wage and rent paths that simultaneously clear the land and labor markets at each instant of time, given a population level $N_i(t)$ and a systemic utility level $V^*(t)$ at that instant.

The assumption that residents' labor supply is inelastic means $N_i'(t) = M_i(t) = N_i^{f\prime}(t)$. This relates the instantaneous labor market equilibrium quantity, which is a flow of population, to the existing stock of population $N_i(t)$. Hence

$$N_i'(t) = k_i^{-1}\left(\frac{V_i(t) - V^*(t)}{A_i}\right) = \psi_i'^{-1}\left(\int_t^{\infty} h_i(s)[1 - C_i(s)]e^{-\rho(s-t)}ds\right), \forall\ t. \quad (37)$$

[24] Both inverse functions in (36) exist because k_i and ψ_i' are strictly increasing.

(37) is the equation of motion that ties the instantaneous equilibria together over time.

Just as in the static model, the systemic population resource constraint

$$N = \sum_i N_i(t) \; \forall \; t \tag{38}$$

completes the definition of equilibrium, where we assume for simplicity that aggregate population is constant over time. So the whole system consists of three instantaneous equilibrium conditions, (34) and the two sides of (37), in the three paths $w_i(s)$, $r_i(s)$, and $N_i'(s)$; and one systemic equation (38) that determines the systemic path $V^*(t)$.

VI. Pareto Efficiency in the Dynamic Model

The dynamic model retains some of the efficiency properties that constitute a hallmark of the static model. We have confirmed that the static model is consistent with the first welfare theorem and, more generally, that a competitive equilibrium can be efficient if there is full symmetry across locations. Moreover, we have shown that a competitive equilibrium *must* be *in*efficient in the static model if the marginal net population externality in equilibrium is not the same across locations. These results carry over to the dynamic model, for essentially the same reasons as in the static model.

In contrast, the result from the literature based on static models that Pareto efficiency can be achieved, even in the presence of externalities and location asymmetries, via landowner-controlled tax/subsidy schemes (Henderson, 1985a; Wildasin, 2006), does not hold in the dynamic setting. The dynamic model efficiency results of the present section are used in the next section to show that fundamentally different local politics and policies are needed to address the competitive equilibrium inefficiency that can occur in an explicitly dynamic asymmetric setting with population externalities. Landowner-controlled local governments will not adopt efficiency-inducing policies in general, but tax/subsidy policies designed by local coalitions of landowners, firms, and residents may indeed overcome the potential inefficiency of a competitive equilibrium.

A Pareto optimum in the dynamic model for our purposes maximizes the income available to firms, landowners, and owners of moving services in one location; while assuring the portion of output and land allocated to residents' consumption in each location at each time delivers fixed aggregate resident/migrant welfare over all locations, and a fixed level of income to firms, landowners, and owners of moving services in each other location; subject to the land and nationwide population resource constraints. This statement of the Pareto problem differs from the statement used for the static model only in that we include owners of moving services. Owners of scarce moving services earn rents in the dynamic economy, so their income must be counted. As there are no capital constraints, maximization is always in terms of present values, calculated at each location.

The social planner has control over the paths of population and the consumption and land allocation paths within each location but cannot avoid the costs of moving residents and adjusting production levels at each location. Corresponding to the constraint faced by migrants and firms, the planner cannot move consumption across locations.

Taking the location under study to be location 1 for convenience, a social optimum beginning at time *t* solves

$$\max_{\{Z_i(s),L_i^h(s),L_i^f(s),N_i(s),\ \forall\ i\}} \int_t^\infty \Bigg[N_1(s)h(L_1^f(s),b_1(s),N_1(s)) - Z_1(s) \\ - \int_0^{N_1'(s)} k(m)dm - \psi_1(N_1'(s)) \Bigg] e^{-\rho(s-t)} ds \tag{39}$$

subject to

$$\sum_i \int_t^\infty [A_i Z_i(s) + N_i(s)\hat{u}(L_i^h(s),a_i(s),N_i(s))] e^{-\rho(s-t)} ds \geq \bar{V} \tag{40}$$

$$\int_t^\infty \Bigg[N_i(s)h(L_i^f(s),b_i(s),N_i(s)) - Z_i(s) \\ - \int_0^{N_i'(s)} k_i(m)dm - \psi_i(N_i'(s)) \Bigg] e^{-\rho(s-t)} ds \geq R_i,\ \forall\ i > 1 \tag{41}$$

$$N_i(s)\left[L_i^h(s) + L_i^f(s)\right] = L_i,\ \forall\ i,\ \forall\ s > t \tag{42}$$

$$\sum_i N_i(s) = N,\ \forall\ s > t \tag{43}$$

$$N_i(t) = N_{i0}(t),\ \forall\ i, \tag{44}$$

where $Z_i(s)$ is the flow of aggregate consumption allocated to residents in location *i* at time *s*. $Z_i(s)$ differs from the aggregate consumption flow $N_i(s)c_i(s)$ received by residents in location *i* in a competitive equilibrium because in a competitive equilibrium migrants who arrive in location *i* must pay aggregate moving costs of $N_i'(s)k_i(N_i'(s))$, whereas a social planner merely chooses an allocation to satisfy a utility threshold (and other constraints). A social planner does not bother with the monetary economy and therefore has no reason to identify moving costs paid by migrants separately from the total resources used in the course of moving. $\bar{V}$ is the present value of aggregate utility that must be provided to residents in all locations from time *t* onward, and R_i is the present value of aggregate income that must be provided to firms, landowners, and owners of moving services in location *i* from time *t* onward. $N_{i0}(t)$ is the initial population in location *i*.

The first term of the integrand in (39) is aggregate production and the second term is the part that is allocated to residents/migrants. Together, these two terms are analogous to the two terms in the static welfare objective (8). The third term (i.e., the integral of k_1) is aggregate resource costs of net migration and the last term is aggregate firm net adjustment costs. So the integrand in (39) is the aggregate production available at time *s* to firms, landowners, and owners of moving services in location 1, net of the amount allocated to residents/migrants and net of migration costs. The present value of this excess production can be allocated to firms, landowners, and owners of moving services as necessary to ensure a Pareto improvement for each of them relative to some initial allocation, as can any aggregate excess production satisfying (41) in the other locations.

As in the static model (see equation (9)), the utility constraint (40) does not explicitly impose the constraint that each individual resident or migrant receive a threshold utility level. Tracking each individual's moves would be exceptionally tedious and would add nothing to the conclusions.[25] (42) and (43) are the land and population resource constraints, respectively, and (44) is the initial condition.

As in the static model, one necessary condition for efficiency is that the marginal rate of substitution equal the marginal rate of transformation between the land and non-land commodities in each location at each point in time. Also as in the static model, it is straightforward to establish that this occurs in a competitive equilibrium. In particular, (29) and the solution to (24) yield

$$h_{L_i^f}\left(L_i^f(s), b_i(s), N_i(s)\right) = \frac{\hat{u}_{L_i^h}\left(L_i^h(s), a_i(s), N_i(s)\right)}{A_i} \tag{45}$$

in equilibrium.

However, again analogous to the static model, the level of migration that obtains in the competitive equilibrium is not necessarily Pareto optimal. Proposition 3 gives a necessary condition for Pareto optimality of a competitive equilibrium allocation.

Proposition 3. If constraint qualification[26] holds for the program (39) – (44), a necessary condition for the competitive equilibrium allocation to be a solution to this maximization program is

$$\begin{aligned} &\int_\tau^\infty [\hat{u}_{N_i}(s) + A_i h_{N_i}(s)] N_i(s) e^{-\rho(s-t)} ds = \\ &\qquad \int_\tau^\infty [\hat{u}_{N_j}(s) + A_j h_{N_j}(s)] N_j(s) e^{-\rho(s-t)} ds, \ \forall\ i, j,\ \forall\ \tau \ge t, \end{aligned} \tag{46}$$

evaluated at the competitive equilibrium allocation.

Equation (46) is the population distribution condition necessary for a dynamic competitive equilibrium to be a social optimum. It is identical to the static condition given in Proposition 1, except that the aggregate value of the net marginal externality in each location is now explicitly a present value, and the marginal utilities of income are the constants A_i due to the assumption of quasilinear utility. The interpretation is identical as well: a competitive equilibrium cannot be efficient unless the marginal net population externalities have equal marginal utility values across locations. It follows immediately, just as in the static model, that Proposition 3 is consistent with the first welfare theorem[27] and that fully symmetric equilibria can be optimal. The dynamic model also reinforces the conclusion that a competitive equilibrium may not be efficient in the presence of externalities when locations are not symmetric.

[25] The social planner has the same ability to transfer utility among individuals as in the static model. This power is consistent with the formal dynamic model, which specifies costs associated with net, as opposed to gross, migration, in effect allowing costless mobility so long as net migration is not affected. Given a location-specific allocation satisfying (40), the social planner can move individuals across locations, thereby manipulating gross migration, to achieve any collection of individual utility thresholds that aggregate to $\bar{V}$.

[26] That is, the constraints are "independent" or "consistent" in the sense discussed by Chiang (1992, section 6.1).

[27] As noted above, if migration costs were derived from congestion effects, we would not attain efficiency even in the absence of population externalities.

VII. Dynamic Efficiency and Local Politics

If local property owners can tax or subsidize local residents in the static model, the landowners' decision will induce a population distribution that is consistent with Pareto efficiency. This is because competition between landowners at different locations removes their monopoly power, so that in setting taxes/subsidies for residents they maximize the aggregate surplus. Residents are protected by their ability to choose freely among locations.

In the dynamic model, residents can no longer move costlessly between locations. Firms and owners of moving services, as well as landowners, earn location-specific rents, as do initial residents. In this setting, if landowners control tax policy, they will have incentives to set policies that extract surplus from residents (who must pay costs to depart), as well as to redistribute other location-specific rents to themselves. These factors distort landowners' decisions even when there are no population externalities, so the political efficiency result in static migration models is not robust to the introduction of explicit dynamics. Instead, tax/subsidy policies will ensure that the necessary conditions for efficiency are satisfied only if the policies are designed by coalitions of landowners, firms, owners of moving services, and residents. The interests of all those who receive location-specific rents must be represented in the local polity.

Formally, let $x_i(s)$ be a per-capita net subsidy flow paid to residents in location *i* at time *s*. Then the budget constraint for the residents' problem is

$$c_i(s) + r_i(s)L_i^h(s) = w_i(s) + x_i(s), \tag{47}$$

so $w_i(s) + x_i(s)$ replaces $w_i(s)$ in the indirect utility function. Among the equilibrium conditions (34), (37), and (38), only the left side of (37) is affected by this change. Substituting for $V_i(t)$, it becomes

$$A_i k_i(N_i'(t)) = \int_t^\infty U(w_i(s) + x_i(s), r_i(s), a_i(s), N_i(s))e^{-\rho(s-t)}ds - V^*(t), \forall\ t. \tag{48}$$

This changes the equilibrium paths $w_i(s)$, $r_i(s)$, $N_i(s)$ and $V^*(s)$ as well as the allocation of land and consumption. The efficiency of the land and consumption allocation in a location does not change with the subsidy, so (45) continues to hold in the subsidy-induced competitive equilibrium, but (46) is modified by the marginal utility value of the subsidy flows. Pareto optimality in the presence of the subsidy requires that the subsidized marginal net population externalities be equal across locations.

Corollary to Proposition 3. When $x_i(s)$ is in the model, the necessary condition of Proposition 3 is

$$\begin{aligned} &\int_\tau^\infty \{[\hat{u}_{N_i}(s) + A_i h_{N_i}(s)]N_i(s) - A_i x_i(s)\}e^{-\rho(s-t)}ds = \\ &\qquad \int_\tau^\infty \{[\hat{u}_{N_j}(s) + A_j h_{N_j}(s)]N_j(s) - A_j x_j(s)\}e^{-\rho(s-t)}ds,\ \forall\ i, j,\ \forall\ \tau \ge t, \end{aligned} \tag{49}$$

evaluated at the subsidized equilibrium allocation.

Now consider landowners who control the local political process and also serve as the sink for financing local government, just as in the static model. If there is a population already in place in the location when landowners make this decision, given the existence of moving costs, landowners have an incentive to exploit residents by imposing taxes on them. This complicating incentive obscures understanding of the relationship between the local polity and Pareto efficiency, so we begin by analyzing landowners' incentives in a context where they do not have this incentive. In particular, assume that all residents are migrants from other locations who have been provided with a binding contract specifying the path of taxes or subsidies they will face in the new location. Landowners are therefore forced to account for the effects of their taxes or subsidies on all earlier migration decisions, there is no pre-existing captive population, and the binding nature of the contract eliminates any potential dynamic inconsistency in landowner behavior.

The total tax/subsidy at time s is $x_i(s)N_i(s)$, so the dynamic analog to the landowners' objective (22) in the static model is:

$$\max_{\{x_i(s)\}} \int_t^{\infty} [L_i r_i(s) - x_i(s)N_i(s)]e^{-\rho(s-t)}ds \tag{50}$$

subject to $N_i(t) = 0$ (all residents are migrants), and subject to the equilibrium conditions (34), (48), and the right side of (37) since $w_i(s)$, $r_i(s)$, and $N_i(s)$ are endogenous paths.

Proposition 4. The subsidy flow $x_i(s)$ that solves (50) and the associated constraints must satisfy[28]

$$\int_{\tau}^{\infty} x_i(s)e^{-\rho(s-\tau)}ds = \int_{\tau}^{\infty} N_i(s)\left[\frac{\hat{u}_{N_i}(s)}{A_i} + h_{N_i}(s)\right]e^{-\rho(s-\tau)}ds \\ - N_i'(\tau)[k_i'(N_i'(\tau)) + \psi_i''(N_i'(\tau))], \ \forall \tau \geq t. \tag{51}$$

The solution to (50) does not generally satisfy the necessary condition for Pareto optimality given by (49) in the Corollary to Proposition 3. Equation (51) shows that the landowners' optimal subsidy path fails to neutralize population externalities, in present value, by an amount that depends on moving costs and employment adjustment costs. The extra term is the amount of migration, $N_i'(\tau)$, multiplied by the change in the marginal cost of migration when the amount of migration changes. This gives the inframarginal value of a change in the "price" of migration, when migration is valued at its marginal cost. The first component, $k_i'(N_i'(\tau))$, is the rental rate of migration to owners of moving services. The second component, $\psi_i''(N_i'(\tau))$, is the rental rate of migration to firms. These are the only two sources of rent, aside from land rent, under the structure considered in Proposition 4. Landowners distort the subsidy away from an externality-neutralizing level in order to capture some of these rents. If we take as given a particular socially optimal growth path of population for a location, the landowners have incentives to set the subsidy below the optimal level when migration is positive and above the optimal level when migration is negative.

As with a no-subsidy competitive equilibrium, it is possible that the landowners' solution to (50) is consistent with a Pareto optimum. This occurs if the subsidized externalities are equal across locations, for every t, when the landowners in each location simultaneously optimize. However, this will happen if and only if the inframarginal

[28] Analogous to our assumption in the static model, we assume that landowners take as given the utility path available to migrants in other locations.

value of a change in the "price" of migration is the same across locations in the subsidized equilibrium, which would be a mere coincidence unless the locations and equilibrium are fully symmetric. Landowner control of local politics does not automatically satisfy the necessary conditions for an efficient outcome in the dynamic model, in contrast to the static model.

This conceptual exercise suggests that taxation policy will offset externalities only if it is designed by coalitions that include the interests of landowners, firms, and owners of moving resources, as well as any initial residents. Assume initially that decisions made by such a coalition at t are fully enforceable at all future times $s \geq t$.[29] As above, the coalition in each location takes as given the utility provided in other locations, so its objective may be written as

$$\max_{\{x_i(s)\}} \int_t^{\infty} \Big\{ L_i r_i(s) - x_i(s) N_i(s) + N_i(s) h_i(s)[1 - C_i(s)] - \psi_i(N_i'(s)) \tag{52}$$

$$+ N_i'(s) k_i(N_i'(s)) - \int_0^{N_i'(s)} k_i(m)\,dm \Big\} e^{-\rho(s-t)} ds \; + \; N_{i0}(t)\left(\frac{V_i(t) - V^*(t)}{A_i} \right).$$

The first term of (52) is income of landowners, as in (50); the next term is the subsidy cost or tax revenue; the next two terms are firms' profits including adjustment costs; the next two terms are the excess of revenues in the "moving services" industry over moving costs generated by net migration into the location when moving services are priced at marginal cost; and the last term is the relative utility benefit (translated into consumption-equivalent units) for existing residents at time t. Here, $N_{i0}(t) = N_i(t)$ is the initial condition assuring that the path chosen by the coalition starts from the existing population.

As before, the paths $w_i(s)$, $r_i(s)$, and $N_i(s)$ are endogenous and the objective (52) is maximized subject to the initial condition, as well as the competitive equilibrium conditions (34), (48), and the right side of (37).

Proposition 5. The subsidy flow $x_i(s)$ that solves (52) and associated constraints satisfies

$$x_i(s) = N_i(s)\left[\frac{\hat{u}_{N_i}(s)}{A_i} + h_{N_i}(s) \right], \; \forall \; s \geq t. \tag{53}$$

This is the dynamic analog of (23). It indicates that the subsidy or tax on each resident at each point in time equals the instantaneous marginal net externality that person imposes on firms and other residents.

The necessary condition for efficiency, given by the Corollary to Proposition 3, holds trivially when all locations implement the tax/subsidy plan (53). The coalition's optimal subsidy exactly offsets the externality that causes migrant decisions to violate the socially optimal migration program. Indeed, the competitive equilibrium and socially optimal population and land use paths are determined by the same set of conditions when

[29] We discuss later how to relax this assumption.

the subsidy (53) is imposed in the competitive system. This is stated formally in Propostion 6.

Proposition 6. In a subsidized competitive equilibrium with subsidies $x_i(s)$, the equilibrium land and population triples $(L_i^h(s), L_i^f(s), N_i(s);\ \forall\ i,\ \forall\ s \geq t)$ satisfy:

$$N_i(s)[L_i^h(s) + L_i^f(s)] = L_i \tag{42}$$

$$\sum_i N_i(s) = N \tag{43}$$

$$h_{L_i^f}(s) = \frac{\hat{u}_{L_i^h}(s)}{A_i} \tag{45}$$

$$\begin{aligned} A_i \int_s^\infty [Q_i(L_i^h(m), L_i^f(m), N_i(m)) + x_i(m)] e^{-\rho(m-s)} dm \\ - A_i[k_i(N_i'(s)) + \psi_i'(N_i'(s))] = \Gamma(s) \end{aligned} \tag{54}$$

where

$$\begin{aligned} Q_i = h(L_i^f(m), b_i(m), N_i(m)) \\ - h_{L_i^f}(m) \frac{L_i}{N_i(m)} + \frac{\hat{u}(L_i^h(m), a_i(m), N_i(m))}{A_i} \end{aligned}$$

and $\Gamma(s) = V^*(s)$. Moreover, the triple $(L_i^h(s), L_i^f(s), N_i(s);\ \forall\ i,\ \forall\ s \geq t)$ in a solution to the Pareto problem (39) – (44) also satisfies these four conditions when x_i is set equal to the value specified by (53) and $\Gamma(s) = A_1 e^{\rho(s-t)} \int_s^\infty \mu(m) dm$, where μ is the Lagrange multiplier associated with the population constraint (43).[30]

(54) is the key migration equation in the model. $Q_i + x_i$ is the flow of benefits from introducing an additional worker into location *i* at time *m*. So the left side of (54) is the present value at time *s* of such benefits less the instantaneous utility value of the adjustment costs from introducing the worker at time *s*. This net value of the extra worker must equal the utility alternative $V^*(s)$ in a competitive equilibrium, or the utility value of the shadow price $A_1 e^{\rho(s-t)} \int_s^\infty \mu(m) dm$ of an additional worker in location 1 in the Pareto problem. Since firm adjustment costs and migration costs enter symmetrically into (54), the dynamic character of the system, which we examine in the next section, relies only on the presence of at least one of these costs.

The optimally subsidized competitive equilibrium and Pareto optimal population and land paths are identical if the system in Proposition 6 has a unique solution. If there is not a unique solution then there can be optimally subsidized competitive equilibria that are not the global Pareto optimum. This possibility is not a mere curiosity. We show in the next section that it can indeed occur, and we discuss the implications for local taxation policy.

The substantive conclusion is that political structures at each location can produce allocations consistent with social optimality even where congestion or agglomeration effects are important and locations are asymmetric, but that the decision-making process

[30] As in the solution to the Pareto problem, we assume that constraint qualification holds. See footnote 26.

must consider the interests of all those who may earn rents in the location. Landowners are not the only recipients of economic rents when migration and employment adjustment are costly. Owners of firms, owners of resources in the mobility industry, and initial residents earn rents as well. The result in static models that landowners will choose an efficient tax structure is a by-product of the feature that landowners are the only recipients of local economic rent in static models.[31] On the other hand, the interests of future migrants to the location need not be explicitly represented, since their ability to choose among competing locations protects their interests, provided the tax/subsidy contract adopted at a particular time is a binding commitment. If the process begins with zero population, new migrants' interests are fully protected by a binding commitment without any explicit representation.

This latter result may be taken as a repudiation of the claim that, given costs of mobility, it is necessary for future residents to have a say in the taxation policies of localities. This conclusion, however, rests heavily on the assumption that taxation policies are set in accord with an enforceable contract that migrants know when they arrive. Although it may be reasonable to assume that such commitments can be made for short periods, the problems of writing contingent contracts to deal with future uncertainties may prohibit long-term contracts.

It is beyond the scope of our treatment to extend the model to an uncertain world, but we want to at least consider the degree to which our conclusions survive if the locality is unable to commit itself to a long-term tax policy. Assume that the locality can commit itself to abide by some tax/subsidy policy only for the period from t to t', after which the decision will be made anew by a similarly constituted coalition, consisting of stakeholders and residents at time t'. The time t coalition only chooses $x_i(s)$ for $t<s\leq t'$, and must take the value of $x_i(s)$ for $s\geq t'$ as given. The coalition formed at t' makes choices of $x_i(s)$ for $t'<s\leq t''$, and so on.

If the choice of $x_i(s)$ for $s\geq t'$ corresponds to the optimum that would have been chosen by the initial coalition at t, the coalition can obtain its global optimum by choosing $x_i(s)$ for $t<s\leq t'$ exactly as it would if it had control after t'. Under these circumstances, the solution is still (53), but with the time period limited to $t<s\leq t'$. As (53) holds for any t, later coalitions will in fact choose the same time path for $x_i(s)$, provided the future path is assured for them as well. If coalitions at future times t', t'', etc., are constituted like those at t, the solution for $s>t'$ will correspond to that which would be chosen by the coalition at t.

In short, even if the period in which the commitment for taxation or subsidy is extremely short, the necessary conditions for efficiency will be satisfied provided the decision is made at each point in time by such a coalition. Hence, new migrants need not have an assurance concerning the specific tax policy to be followed in the future. It is sufficient merely that they be included in future coalitions that make taxation decisions.

[31] If increasing costs of moving are due to congestion, there will be no profits in the moving industry. Omitting the moving industry from (52) alters the subsidy or tax chosen, presented in (53), but the coalition of residents, landowners and firm owners will choose a subsidy that satisfies the relevant efficiency condition (corresponding to (49), modified as appropriate). In short, the decision process outlined here satisfies the necessary conditions for optimal subsidies even when moving costs are due to congestion effects.

We have left open the question of how the coalition operates to maximize the present value of its aggregate wealth. If we accept Wittman's (1989) argument, we may merely assume that the political process of a democracy is efficient. The mechanism we have described suggests that so long as efficient decisions are made at a given point in time and the efficiency of future decision processes is assured, there may be little need to establish long-term commitments.

VIII. Expectations, Local Taxation Policy, and Multiple Equilibrium Paths

As noted above, when local agglomeration effects are important, there may be multiple dynamic paths satisfying the equilibrium and efficiency conditions presented in Proposition 6. In particular, for a given initial population distribution, there may be a large number of equilibrium migration paths, each associated with a set of internally consistent (rational) expectations. Some of these paths may involve "cycling," in which population at a location oscillates, even when exogenous factors ($a_i(s)$ and $b_i(s)$ in our model) do not vary over time.[32]

The possibility of multiple equilibrium paths raises important questions about whether one equilibrium path Pareto dominates others and, if so, whether there is a mechanism that can select the welfare-superior path. In particular, will local policy-makers design tax/subsidy policies that induce a global Pareto optimum? A full investigation of the question would require extended study, but we can present an example here that reveals some of the possibilities.

We begin by specifying reasonable model primitives for which the migration equation (54) is relatively simple but has multiple solutions. We will focus on a particular location, suppressing subscript *i* and holding the systemic parameter $\Gamma(s)$ (or $V^*(s)$) constant. Assume production and utility have Cobb-Douglas functional forms plus additively separable per capita population externalities *g* and *f*, respectively. Local amenities are taken to be time invariant and the land endowment and output price are normalized to unity. The per capita functions are:

Production: $h(L^f, b, N) = (L^f)^{0.5} + g(N)$

Utility: $A=1,\ \hat{u}(L^h, a, N) = (L^h)^{0.5} + f(N)$

Land Endowment: $L = 1$

Using the unit land endowment and these functional forms for production and utility in (42) and (45) of Proposition 6 gives $L^h = L^f = \dfrac{1}{2N}$ as the land allocation at each instant of time in every competitive equilibrium and in every Pareto efficient allocation. Substituting this land allocation into the definition of *Q* following (54) yields

$$Q(N) = f(N) + g(N) + (2N)^{-0.5}, \tag{55}$$

which depends on time *s* only through the population level $N(s)$ and therefore may be written $Q(N(s))$ or $Q(N)$ as circumstances warrant.

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[32] The character of such multiple paths is investigated in some detail by Krugman (2006) and Matsuyama (1991). In their models, movement between only two locations is examined, and they do not consider subsidy or taxation policies, but the multiple dynamic paths in the model considered here are qualitatively similar to those analyzed by Krugman and Matsuyama.

The externalities appear in Q (and thus in (54)) exclusively as a sum $f + g$, so we need only specify these as an aggregate. We will take

$$f(N) + g(N) = -4 + 4N - N^2 . \qquad (56)$$

Agglomeration effects dominate with this specification up to the maximum at $N = 2$, beyond which congestion effects dominate. Now we have

$$Q(N) = -4 + 4N - N^2 + (2N)^{-0.5} . \qquad (57)$$

The optimal tax/subsidy policy (53) for the tax/subsidy world in this example is

$$x(N) = 4N - 2N^2 . \qquad (58)$$

Again, this function depends on time s only through $N(s)$, so we may write $x(N(s))$ or $x(N)$.

Finally, assume conditions in the other locations peg the equilibrium systemic utility at $V^*(t) = 0 \ \forall \ t$, so (54) becomes

$$\psi'(N'(t)) + k(N'(t)) = \int_t^\infty [Q(N(s)) + x(N(s))]e^{-\rho(s-t)}ds . \qquad (59)$$

The left side is zero if and only if $N'(t) = 0$, so the roots of the integrands $Q(N)$ and $Q(N) + x(N)$ are the steady-state population levels for the no-tax and optimal tax worlds, respectively. These integrands are displayed in figure 1.[33] Each has three roots. As $\psi'(N'(t)) + k(N'(t))$ is strictly increasing, the outer two steady states are locally stable, while the middle steady state is unstable.

The steady state's roots (derived by numerical solution) are

$Q(N)$: $N = 0.033$, $N = 1.196$, $N = 2.659$

$Q(N) + x(N)$: $N = 0.036$, $N = 0.439$, $N = 2.112$.

It is straightforward to verify from (56) that the aggregate population externality $f(N) + g(N)$ is increasing at $N = 0.033$ and is decreasing at $N = 2.659$, consistent with the observation above. The optimal tax/subsidy policy increases population at the low-population stable steady state to garner agglomeration benefits, and decreases population at the high-population stable steady state to reduce congestion costs.

There are multiple equilibrium migration paths in both the no-tax and optimal-tax worlds, one leading to each stable steady state and potentially different path(s) leading away from the unstable steady state. The growth paths can be illustrated explicitly by specifying migration/adjustment costs $\psi'(N') + k(N')$ and the discount rate ρ in the migration equation (59) and then numerically solving the differential equation for $N(t)$. Assume

$$\psi'(N') = k(N') = N'/2 \qquad (60)$$

and $\rho = 0.1$. Growth paths with and without a taxation/subsidy policy are qualitatively similar in this example. As our primary concern is with the impact of local intervention, we focus on the paths in the optimal tax/subsidy world. Substituting (57), (58), and (60) into (59) gives the migration equation

$$N'(t) = \int_t^\infty [-4 + 8N(s) - 3N(s)^2 + (2N(s))^{-0.5}]e^{-0.1(s-t)}ds . \qquad (59')$$

The paths that numerically solve this differential equation are illustrated in figure 2.

[33] Note that the horizontal scale of both figure 1 and figure 2 has been modified to show detail at lower population levels.

The heavy lines show N' as a function of N with the time parameter t eliminated but varying along the illustrated paths. Where the line is above the horizontal axis, population is growing, and where it is below the axis, population is declining. Starting near zero population, we observe two equilibrium paths. The lower one leads to the low-population stable steady state, whereas the higher one leads to the high-population stable steady state. In each case, as the population approaches the stable point, migration declines, and the stable population is approached but not reached in finite time.[34]

The path that leads to the low-population stable steady state from the right has a spiral that circles around the unstable steady state an infinite number of times (the full spiral cannot be illustrated in the figure) before approaching the low-population stable steady state. Movement is clockwise around the spiral, so, if the system begins with zero net migration and population just below the unstable equilibrium, population grows and then declines, oscillations becoming progressively greater, until population finally approaches the lower stable population level. It is important to recall that actors perfectly anticipate the entire future in all equilibrium paths, and Proposition 6 ensures that the path satisfies the conditions for a social optimum. Nonetheless, the existence of multiple paths at a given initial population level and multiple steady states that can be reached from that population level shows that there is little assurance a particular path is a global optimum. Expectations matter for placing the system on a globally optimal path.

Comparing the two possible paths at a population level below the smallest root provides the easiest interpretation of the relative benefits associated with alternative paths. The present value of future benefit flows from adding a worker, $\int_t^\infty [Q(m)+x(m)]e^{-\rho(m-s)}dm$, is much higher on the higher path, which draws many more migrants from other locations so that the marginal adjustment cost of adding population is driven to a correspondingly high level. Equation (58) shows that the flows of per-resident subsidies are identical at a given population level on the two paths, as a consequence of the assumption in this example that exogenous factors do not change over time. But the high-migration path attracts more migrants because they expect that future population growth will ultimately make the location more desirable due to returns to agglomeration (recall that agglomeration effects dominate congestion effects for $N \le 2$). The lower migration path reflects expectations that the location will never grow sufficiently for such large economies of agglomeration to take hold—expectations that are self-confirming.

Numerically calculating the present value of aggregate benefits received by individuals in the coalition as specified in (52) shows that these benefits are larger on the upper path than on any of the other paths at any common level of population. The difference is largely due to higher rent earned by landowners. Unlike the flows of benefits earned by other stakeholders, which approach zero as population nears a stable steady state, rent payments to landowners continue indefinitely.

Although this example clearly illustrates the possibility that one equilibrium path may Pareto dominate another, the Pareto dominance is only in location i since we are not considering welfare in other locations and varying population in location i will generally affect welfare in other locations. However, it is easy to see that the Pareto-dominance

[34] Regarding the behavior of the system as it approaches a steady state, see Fukao and Benabou (1993), who provide a correction to Krugman (2006).

can be global across all locations. For example, if stakeholders in other locations are not influenced by the population distribution, then the high-population path in this example Pareto dominates all other equilibrium paths.[35]

It may appear that a higher level of population growth is always preferred by local policy makers, but this is not necessarily true. If we look at the spiral path, and consider a population level close to the unstable steady state level of 0.439, there are a large number of possible points on the path, some with positive and some with negative net migration. The measure of local stakeholders' net benefits in (52) is highest on the path where net *out-migration* is as large as possible, that is, at the point on the bottom of the outermost spiral. In particular, positive growth is not preferred to other points on the same path. In this example, if ultimate population decline will occur in the future, it is best that it occur as quickly as possible. Oscillations in population along the spiral path merely waste resources.

This example shows that the encouraging result of Proposition 6 must be accompanied by important caveats when inferring its substantive import in the presence of multiple equilibrium paths. Although Proposition 6 shows that any optimally subsidized equilibrium path satisfies the necessary conditions for efficiency, the example shows that multiple paths meeting these conditions are possible and that some of the alternatives may Pareto dominate others, both from the perspective of one locality and globally as well.

On the positive side, Proposition 6 assures us that at least *one* of the subsidized competitive equilibrium paths is a global Pareto optimum. Furthermore, for certain initial population levels, even in a system like this one with multiple steady-state populations, the growth path is fully determined and it is therefore optimal.

In those cases where there are multiple possible equilibrium paths, there are internally consistent expectations associated with each. The differences in these expectations across alternative competitive equilibria suggest the kinds of strategies that may be pursued by a locality to induce one path rather than another. Consider efforts to achieve the high-growth path. Insofar as advertising can convince employers and potential migrants that others believe this possibility will occur, advertising may create expectations of high growth that become self-fulfilling. Hence, even advertising that provides little information about the location may be successful if it focuses communal expectations.

There may be other methods that can be used by localities to achieve a more desirable growth path if local government powers are expanded beyond the imposition of an optimal tax/subsidy scheme. If a locality guarantees future levels of production costs and residents' utilities by specifying a contingent subsidy/tax, with residents and employers insured against the possibility that anticipated growth may not occur, the higher growth path can be assured. Although such guarantees may be unusual, a number of institutions exist that may approximate these types of policies. Municipalities provide

[35] This will occur, for example, if the systemic utility $V^*=0$ is merely the utility available in one alternative location j which produces output with no congestion or agglomeration effects, has no migration or adjustment costs for N_j in the relevant range, and in which per capita land use does not affect production or consumption. Then migration between i and j has no impact on the welfare of any resident initially living in j, nor does it affect the level of revenues received by those with resources in j. Clearly, the high-migration path in location i then globally Pareto dominates the other paths.

subsidies to new firms and to those expanding employment. Where the local government floats bonds to subsidize economic development, failure of growth to occur may force absentee bondholders to absorb some of the risk associated with future growth rates.

Although the example is suggestive of how efforts by a locality may improve efficiency for that locality, it does not provide a framework for global efficiency analysis because the welfare in other locations is not specified. Moreover, global efficiency analysis requires that we compare equilibria for the overall system taking account of reactions in one location to policies chosen by another location. There may be many optimal equilibria, with dramatic differences in the benefits that accrue at alternative locations. In contrast to the case considered above, the most important problem may not be how actors in one location can focus expectations on the best path in that locality but rather the possibility of a struggle between locations to determine which will prevail among alternative migration flows favoring different locations.

Consider a system consisting of two locations, each with the same land, consumption and production amenities as the location in the example above. Using (54) and taking total system population between 0.27 and 2.59, we can show that there are three population distributions with zero net migration, just as in the example above. An equal population distribution is unstable, whereas the two asymmetric distributions are stable. For many initial population distributions, there are multiple paths, leading either one or the other of the locations to attract the lion's share of the population. In such cases, the highest growth path in a location is welfare-superior for initial residents and owners of resources at that location, as in the previous example, so the interests of stakeholders across locations are at odds. The location that succeeds in convincing migrants that it will attain high growth will achieve the preferred path, while the other location will be forced onto the inferior population growth path. This illustrates the point that expectations may play a decisive role in determining the winners and losers.

Of course, each location has incentives to induce its preferred path. If such attempts involve costly activities, such as advertising, this could result in wasteful competition between localities. One may be tempted to consider whether the services of a system planner would be valuable. If simply identifying those cities where growth was "expected" to occur succeeded in focusing expectations, this might provide a useful coordinating function. The market may provide an alternative source of coordination. If all localities attempt to refocus expectations by floating bonds, as suggested above, rational investors may be less likely to invest where competition for migrants makes a high growth path less certain. If the process of making such investments is sequential, the first localities to elicit investments may then be successful. Alternatively, competition in the bond market may focus expectations on a subset of locations. Providing a full description of such a process is well beyond the scope of the current model.

IX. Conclusion

In the static model, if landowners in a location can set taxes or subsidies for residents, their income-maximizing choices will compensate for any consumption or production externalities resulting from population congestion or agglomeration affects. Such local decisions can support a globally efficient allocation: Any globally efficient population distribution corresponds to a regime in which local landowners at each location choose policies that maximize their income. Our formal model both confirms this "conventional wisdom" and shows that it depends critically on the absence of migration costs. In a dynamic model with migration costs, landowners with political power will not set taxes to induce efficient migration. However, if power is shared by a set of local stakeholders who maximize their aggregate benefits, a result comparable to that of the static model holds. Any set of efficient *migration paths* corresponds to a subsidized competitive equilibrium in which policy at each location maximizes the relevant coalition's aggregate income. These results have particular power when there is a unique population distribution (static model) or a unique set of migration paths (dynamic model) corresponding to the set of locally optimal policies. In these cases, system-wide efficiency is ensured by local maximizing decisions.

Where there are multiple equilibria, it no longer is clear that local decisions will induce the social optimum. Consider the static model implicit in the structure underlying figure 2.[36] In the static model, the only stakeholders are landowners, and their incomes are much greater at the highest static population level, which is the rightmost steady state. This steady state is also socially optimal for the location under study. However, there is another static population level (ignoring the middle—unstable—steady state) and it is not clear how landowners can induce the preferred one. At the higher population level, the optimal plan imposes a tax on residents, reflecting negative net congestion effects, whereas the optimal plan associated with the lower population level provides a subsidy. The model provides no leverage for local landowners to obtain the more desirable outcome, nor does it provide a basis for predicting which outcome will occur.

Notwithstanding these negative results, the dynamic model does provide a mechanism on which efforts may build to achieve the social optimum. Whereas the static model was silent about how or whether migration might allow for movements to a more desirable steady-state population, the dynamic model demonstrates that this is clearly possible. Figure 2 shows that if a location finds itself at the inferior (lower) population level, rational forward-looking migrants may be induced to move into the location, yielding redistribution that is preferable for stakeholders in the location (even considering migration and adjustment costs) and that is possibly globally Pareto preferred.

For certain initial populations, the dynamic model may provide a unique path to a particular steady state even when there are multiple steady-state populations. Figure 2 illustrates this possibility. For all population levels above the rightmost point on the spiral path there is a single subsidized migration path, and it leads to the upper steady-state population. If the initial population level similarly corresponds to a single dynamic path at each location, a globally efficient pattern of migration follows directly from optimal local decisions by the properly constituted coalitions at each location.

On the other hand, figure 2 shows that such an outcome need not always occur, so the intuition that explicit dynamics solve the problem is incorrect in general. There may be multiple migration paths at a given initial population distribution that tend toward dramatically different steady-state population distributions. The optimal subsidy or tax policy at a location may be consistent with multiple paths, even given an initial population distribution. In our example, there are multiple paths consistent with optimal

[36] Interpreted within the static structure, $Q + x$ is the benefits for residents in the location in question. Although the structure underlying this graph represents flows, these may be easily reinterpreted as static values.

local policies at initial population levels below the rightmost point on the spiral path, but the path leading to the largest steady-state population can Pareto dominate all others.[37]

In applying these ideas, it would be valuable to understand how widespread such indeterminacies are. If large urban areas experience growth corresponding to the portion of a growth path where it is unique, we might conclude that growth in large urban areas is largely explained by the economic environment and initial conditions, with expectations playing no independent role. If smaller locations can be represented on the same graph but at population levels where there is not a unique growth path, expectations would be of independent importance for these locations. Investigating these issues might provide a fruitful area for future research.

[37] Sufficient conditions for the high growth path to Pareto dominate the others in the example are provided in footnote 35.

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Figure 1

Figure 1: Benefit Flows (Q) and Subsidized Benefit Flows (Q+x)

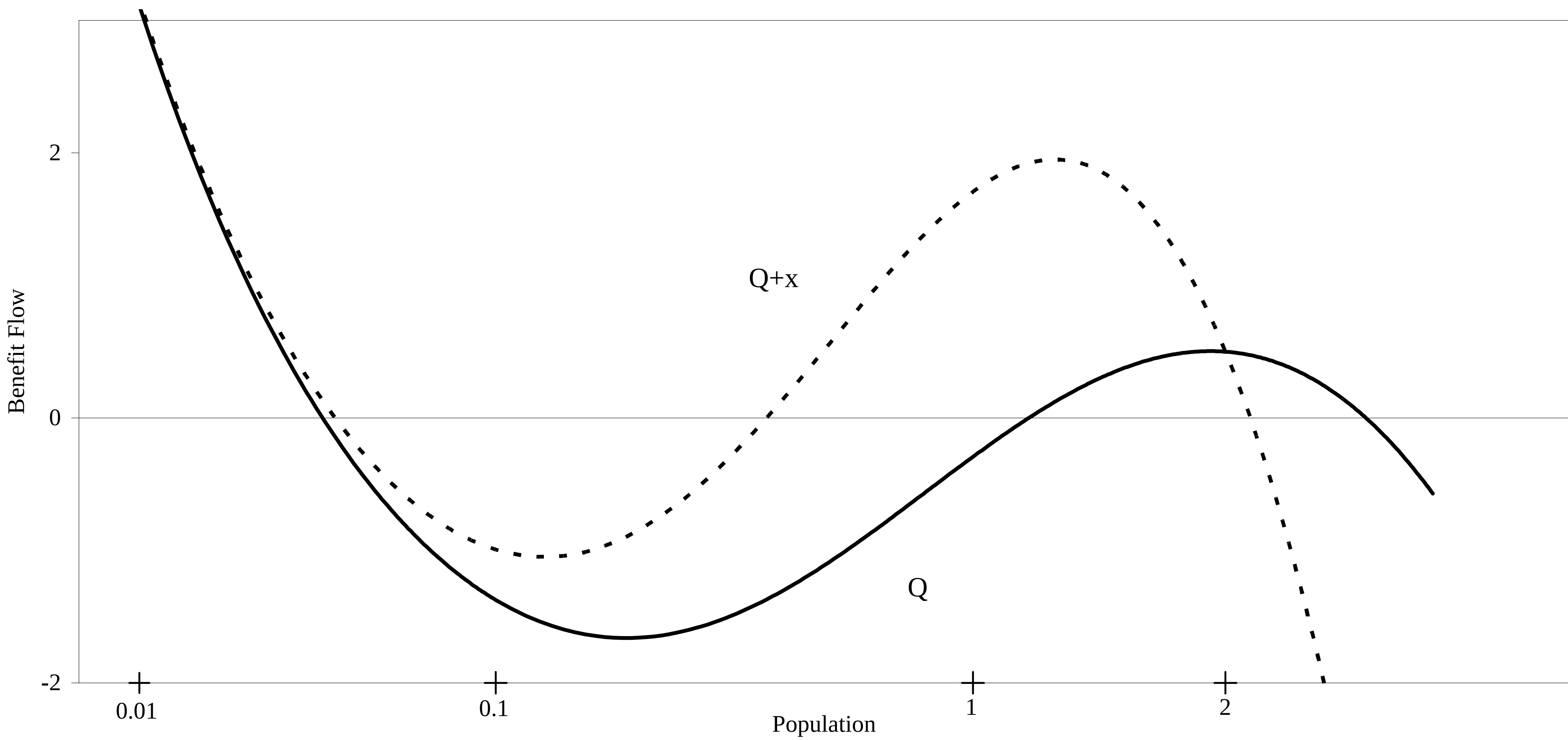
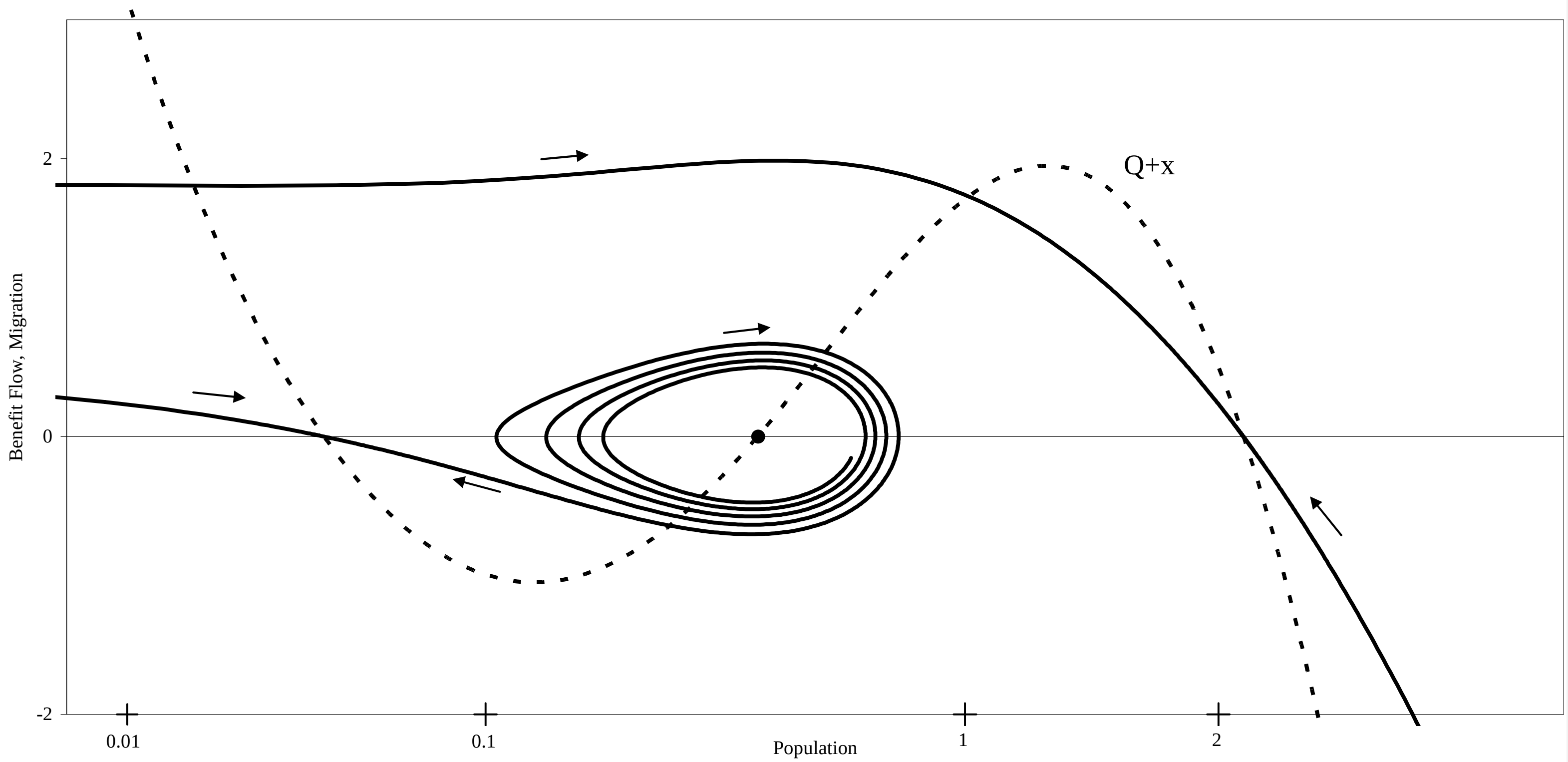


Figure 2

Figure 2: Subsidized Migration Paths



Appendix

Proof of Proposition 1. A competitive equilibrium allocation[1] (c_i, L_i^h, L_i^f, N_i), $\forall i$ (and U^*) is defined by (4), (6), (7), (15), and Walras' Law:

$$u(c_i, L_i^h, a_i, N_i) = U^* \tag{A1}$$

$$N_i[L_i^f + L_i^h] = L_i \tag{A2}$$

$$\sum_i N_i = N \tag{A3}$$

$$P^* h_{L_i^f} = \frac{u_{L_i^h}}{u_{c_i}} \tag{A4}$$

$$P^* h(L_i^f, b_i, N_i) = P^* h_{L_i^f} \frac{L_i}{N_i} + c_i. \tag{A5}$$

(A5) is Walras' Law, obtained by noting that the zero profit condition on firms and the residents' budget constraints together imply that the value of output per capita in location i, $P^* h(L^f(w_i, r_i, b_i, N_i), b_i, N_i)$, in equilibrium equals the value of residents' and landowners' consumption per capita in location i, $r_i L_i / N_i + c_i$, and then substituting for r_i from (14) and C_i from (5).[2] Each of these equations, except (A3), applies for all locations i. (A3) can be used to eliminate U^*, but it is convenient to retain U^* for the efficiency discussion.

The Kuhn-Tucker conditions for the program (8) – (12) are necessary for a solution, given constraint qualification.[3] Using (11) to eliminate L_i^h, the Lagrangian function is

$$\mathcal{L} = N_1 \left[P^* h(L_1^f, b_1, N_1) - c_1 \right] + \lambda \left[\sum_i N_i u(c_i, L_i/N_i - L_i^f, a_i, N_i) - \bar{u} \right]$$
$$+ \sum_{i>1} \gamma_i \left[N_i \left[P^* h(L_i^f, b_i, N_i) - c_i \right] - R_i \right] + \mu \left[N - \sum_i N_i \right],$$

where λ, γ_i, and μ are the Lagrange multipliers on (9), (10), and (12), respectively. Setting $\gamma_1 = 1$ for notational convenience, the Kuhn-Tucker conditions are:

$$\frac{\partial \mathcal{L}}{\partial L_i^f} = -\lambda N_i u_{L_i^h} + \gamma_i N_i P^* h_{L_i^f} = 0, \ \forall i \tag{A6}$$

[1]Note that we are defining the allocation here, not the equilibrium prices, so we require the equations that define the equilibrium quantities.

[2]Although we have no reason to distinguish between quantity consumed and the value of consumption, Walras' Law implies equilibrium in the output market, even if the price paid by residents and landowners is not implicitly normalized to one.

[3]There is also a nonnegativity constraint on each of the choice variables. By assumption, these constraints do not bind at competitive equilibrium allocations, which are the only allocations studied in this proposition, so we ignore the nonnegativity constraints.

Typeset by $\mathcal{AMS}$-TEX

$$\frac{\partial \mathcal{L}}{\partial c_i} = \lambda N_i u_{c_i} - \gamma_i N_i = 0, \ \forall i \tag{A7}$$

$$\frac{\partial \mathcal{L}}{\partial N_i} = \lambda N_i \left[u_{N_i} - u_{L_i^h} \frac{L_i}{N_i^2} \right] + \lambda u(c_i, L_i/N_i - L_i^f, a_i, N_i)$$
$$+\gamma_i \left[P^* h(L_i^f, b_i, N_i) - c_i + N_i P^* h_{N_i} \right] - \mu = 0, \ \forall i \tag{A8}$$

$$\lambda \geq 0 \tag{A9}$$

$$\gamma_i \geq 0, \text{ for } i > 1 \tag{A10}$$

$$\lambda \left[\sum_i N_i u(c_i, L_i/N_i - L_i^f, a_i, N_i) - \bar{u} \right] = 0 \tag{A11}$$

$$\gamma_i \left[N_i \left[P^* h(L_i^f, b_i, N_i) - c_i \right] - R_i \right] = 0, \text{ for } i > 1 \tag{A12}$$

$$\sum_i N_i = N. \tag{A13}$$

Consider an allocation that satisfies the competitive equilibrium conditions (A1) – (A5), and set utility $\bar{u}$ and income R_i in the Pareto problem at their competitive equilibrium levels. That is, select

$$\bar{u} = NU^* \tag{A14}$$

and, since profits are zero in a competitive equilibrium, select R_i to be landowners' income $r_i L_i$, where $r_i = P^* h_{L_i^f}$ is the competitive equilibrium price. By (A5),

$$R_i = N_i \left[P^* h(L_i^f, b_i, N_i) - c_i \right], \ \forall i > 1. \tag{A15}$$

Taking note of (A1) and (A3), it is immediate that (A11) – (A13) are satisfied by a competitive equilibrium allocation when the Pareto problem is evaluated at the corresponding competitive equilibrium levels of $\bar{u}$ and R_i.

Now use (A7) for $i = 1$ (recalling that $\gamma_1 = 1$) to define λ in terms of the competitive equilibrium allocation as $\lambda = 1/u_{c_1}$, and note that this value of λ is positive so this definition satisfies (A9) as well as (A7) for $i = 1$. Use this definition of λ and (A7) for $i > 1$ to define γ_i in terms of the competitive equilibrium allocation as $\gamma_i = \frac{u_{c_i}}{u_{c_1}}$, and note that this value of γ_i is positive so this definition satisfies (A10) as well as (A7) for $i > 1$. Moreover, (A4) implies (A6) for these definitions of λ and γ_i. This leaves only (A8) for the competitive equilibrium allocation to satisfy. Substitute these definitions of λ and γ_i into (A8); and use (A1), (A4), and (A5); to show that (A8) evaluated at the competitive equilibrium allocation is

$$u_{c_i} N_i \left[P^* h_{N_i} + \frac{u_{N_i}}{u_{c_i}} \right] = u_{c_1} \mu - U^*, \ \forall i. \tag{A8'}$$

Since (A8') is implied by (A6) – (A13), there is no set of Lagrange multipliers that satisfies the Kuhn-Tucker conditions when the conditions are evaluated at an allocation satisfying (A1) – (A5) unless there is a real number μ such that (A8') is satisfied. By the Kuhn-Tucker theorem (for example, Mas-Colell et al. (1995) Theorem M.K.2), this means an allocation satisfying (A1) – (A5) is not optimal when the threshold welfare levels $\bar{u}$ and R_i are set according to (A14) and (A15) unless there is a real number μ satisfying (A8'). If one of these thresholds is set at some other level in the Pareto problem then (A11) or (A12) implies that the corresponding multiplier must be zero if the allocation satisfying (A1) – (A5) is to satisfy (A6) – (A13). This violates either $\lambda > 0$ (i.e., (A7) for $i = 1$) or $\gamma_i > 0$ (i.e., (A7) for $i > 1$). Thus, a necessary condition for a competitive equilibrium allocation to be a solution is that the left side of (A8') be the same for all values of i. □

Proof of Corollary to Proposition 1. Landowners' consumption is diminished by the subsidy x_i they pay to consumers, so Walras' Law (A5) becomes

$$P^* h(L_i^f, b_i, N_i) = P^* h_{L_i^f} \frac{L_i}{N_i} + c_i - x_i. \tag{A5'}$$

Landowners pay the subsidy, so their equilibrium income is $R_i = r_i L_i - N_i x_i$. Using this in the Pareto problem along with the new version of (A5) leaves (A15) unchanged. Using (A5') in the derivation of (A8') yields

$$u_{c_i} \left[N_i \left[P^* h_{N_i} + \frac{u_{N_i}}{u_{c_i}} \right] - x_i \right] = u_{c_1} \mu - U^*, \ \forall i. \tag{A8''}$$

The remainder of the proof is identical to the proof of Proposition 1. □

Proof of Proposition 2. Location subscripts, i, are suppressed throughout the proof. Begin by noting some envelope theorem properties of the resident and firm optimization problems.

From (1), indirect utility is

$$U(w + x, r, a, N) = \max_{\{c, L^h\}} u(c, L^h, a, N) \text{ subject to } c + rL^h = w + x. \tag{A16}$$

Roy's Identity for land demand is

$$L^h = -\frac{U_r}{U_w}. \tag{A17}$$

As consumption c is expressed in monetary units, optimal consumption satisfies

$$U_w = u_c. \tag{A18}$$

The envelope theorem also yields

$$U_N = u_N \tag{A19}$$

at the optimum.

From (3), cost per unit of output is

$$C(w,r,b,N) = \min_{\{L^f\}} \frac{w + rL^f}{h(L^f,b,N)}. \tag{A20}$$

Shephard's Lemma for land and labor demand is

$$C_r = \frac{L^f}{h} \text{ and } C_w = \frac{1}{h}, \tag{A21}$$

so

$$\frac{C_r}{C_w} = L^f. \tag{A22}$$

The envelope theorem also yields

$$C_N = -\frac{w + rL^f}{h^2} h_N = -\frac{C}{h} h_N \tag{A23}$$

at the optimum, so

$$\frac{C_N}{C_w} = -Ch_N. \tag{A24}$$

Now consider the first order condition for the landowners' optimization problem (22), which is

$$x = \frac{N\left[\frac{L}{N}\frac{\partial r}{\partial x} - 1\right]}{\frac{\partial N}{\partial x}}. \tag{A25}$$

This condition involves changes in the endogenous variables r and N with respect to the landowners' choice variable x. The endogenous variables (w, r, N) are determined by (18) – (20). Differentiating the first two equations of this system with respect to x yields:

$$U_w \frac{\partial w}{\partial x} + U_r \frac{\partial r}{\partial x} + U_N \frac{\partial N}{\partial x} = -U_w \tag{A26}$$

$$C_w \frac{\partial w}{\partial x} + C_r \frac{\partial r}{\partial x} + C_N \frac{\partial N}{\partial x} = 0. \tag{A27}$$

Using these two equations to eliminate $\frac{\partial w}{\partial x}$ yields:

$$\left[\frac{C_r}{C_w} - \frac{U_r}{U_w}\right]\frac{\partial r}{\partial x} - 1 = \left[\frac{U_N}{U_w} - \frac{C_N}{C_w}\right]\frac{\partial N}{\partial x}. \tag{A28}$$

Using (A17), (A22), and (20), this is:

$$\frac{L}{N}\frac{\partial r}{\partial x} - 1 = \left[\frac{U_N}{U_w} - \frac{C_N}{C_w}\right]\frac{\partial N}{\partial x}. \tag{A29}$$

So the first order condition (A25) becomes

$$x = N\left[\frac{U_N}{U_w} - \frac{C_N}{C_w}\right]. \tag{A30}$$

Now substitute (A18), (A19), and (A24) to obtain

$$x = N\left[Ch_N + \frac{u_N}{u_c}\right]. \tag{A31}$$

Finally, using (19) yields

$$x = N\left[P^* h_N + \frac{u_N}{u_c}\right]. \quad \square \tag{A32}$$

Proof of Lemma 1. Substituting (30) into (28) yields

$$\max_{\{\tilde{N}_k^f(s)\}} \int_t^\infty \left[\tilde{N}_k^f(s)h_i(s)\left[P^*(s) - C_i(s)\right] - \tilde{\psi}_k\left(\tilde{N}_k^{f\prime}(s)\right)\right] e^{-\rho(s-t)} ds.$$

The Euler condition for this problem is

$$h_i(s)\left[P^*(s) - C_i(s)\right] e^{-\rho(s-t)} = \frac{d}{ds}\left[-\tilde{\psi}_k'\left(\tilde{N}_k^{f\prime}(s)\right) e^{-\rho(s-t)}\right], \ \forall s \geq t.$$

Integrating the Euler condition from t to ∞ yields (31), provided $\lim_{s\to\infty} \tilde{\psi}_k'\left(\tilde{N}_k^{f\prime}(s)\right) e^{-\rho(s-t)} = 0$.

There are also transversality conditions that are necessary for an optimum. We assume throughout the paper that transversality conditions hold, and state them here once for completeness. Denoting the integrand in (28) by $F\left(s, \tilde{N}_k^f(s), \tilde{N}_k^{f\prime}(s)\right)$ and its partial derivative with respect to the third argument by F_3, the transversality conditions are

$$\lim_{s\to\infty}\left[F\left(s, \tilde{N}_k^f(s), \tilde{N}_k^{f\prime}(s)\right) - \tilde{N}_k^{f\prime}(s)F_3\left(s, \tilde{N}_k^f(s), \tilde{N}_k^{f\prime}(s)\right)\right] = 0, \text{ and}$$

$$\lim_{s\to\infty} \tilde{N}_k^f(s) = \bar{N}_k^f \text{ or } \lim_{s\to\infty} F_3\left(s, \tilde{N}_k^f(s), \tilde{N}_k^{f\prime}(s)\right) = 0,$$

where $\bar{N}_k^f$ is a constant. $\square$

Proof of Lemma 2. Define the aggregate adjustment cost function by

$$\psi_i(N_i^{f'}(t)) = \sum_k \tilde{\psi}_k(B_k(N_i^{f'}(t))),$$

where $B_k(N_i^{f'}(t))$ is the labor adjustment for firm k consistent with aggregate adjustment $N_i^{f'}(t)$ according to both (32) and the equalized marginal adjustment cost rule. That is, the B_k functions are implicitly defined by the set of equations

$$y = \sum_k B_k(y)$$
$$\tilde{\psi}'_k(B_k(y)) = \tilde{\psi}'_1(B_1(y)), \ \forall k.$$

The first of these equations implies $1 = \sum_k B'_k(y)$. Differentiating the definition of ψ_i yields

$$\begin{aligned} \psi'_i(y) &= \sum_k \tilde{\psi}'_k(B_k(y))B'_k(y) \\ &= \tilde{\psi}'_1(B_1(y)) \sum_k B'_k(y) \\ &= \tilde{\psi}'_1(B_1(y)), \end{aligned}$$

which establishes that the marginal aggregate adjustment cost equals the common adjustment cost of each firm. Given $\tilde{\psi}_k(0) = \tilde{\psi}'_k(0) = 0$ and $\tilde{\psi}''_k(0) > 0$, $B_k(0) = 0$ is clearly the simultaneous solution to the equations defining $B_k(y)$ for $y = 0$. Substituting this into the definition of ψ_i and using $\psi'_i(y) = \tilde{\psi}'_1(B_1(y))$ and $\tilde{\psi}_k(0) = 0, \ \forall k$ establishes that the total and marginal aggregate adjustment cost are zero at $y = 0$. Differentiating the second equation in the definition of B_k yields

$$\tilde{\psi}''_k(B_k(y))B'_k(y) = \tilde{\psi}''_1(B_1(y))B'_1(y), \ \forall k.$$

Using $\tilde{\psi}''_k > 0$, this implies $B'_k(y)$ has the same sign as $B'_1(y)$ for every k. $\sum_k B'_k(y) = 1$ then implies that this sign is positive. Differentiating the equality between marginal aggregate and marginal firm adjustment cost then yields $\psi''_i(y) = \tilde{\psi}''_k(B_k(y))B'_k(y) > 0$, which shows that the aggregate adjustment cost is strictly convex. □

Proof of Proposition 3. A competitive equilibrium allocation[4] $(c_i(s), L_i^h(s), L_i^f(s), N_i(s)), \ \forall i$ (and $V^*(s)$ and $C_i(s)$) is defined by (34), both sides of (37), (38), (45), and a condition analogous to Walras' Law (along

[4] See footnote 1.

with the initial conditions):

$$N_i(s)\left[L_i^h(s)+L_i^f(s)\right]=L_i \tag{A33}$$

$$\int_s^\infty\left[c_i(m)+\frac{\hat{u}(L_i^h(m),a_i(m),N_i(m))}{A_i}\right]e^{-\rho(m-s)}dm-\frac{V^*(s)}{A_i}=k_i(N_i'(s)) \tag{A34}$$

$$\int_s^\infty h(L_i^f(m),b_i(m),N_i(m))\left[P^*(m)-C_i(m)\right]e^{-\rho(m-s)}dm=\psi_i'(N_i'(s)) \tag{A35}$$

$$\sum_i N_i(s)=N \tag{A36}$$

$$P^*(s)h_{L_i^f}(s)=\frac{\hat{u}_{L_i^h}(s)}{A_i} \tag{A37}$$

$$h(L_i^f(s),b_i(s),N_i(s))C_i(s)=P^*(s)h_{L_i^f}(s)\frac{L_i}{N_i(s)}+c_i(s). \tag{A38}$$

(A38) is analogous to Walras' Law in the static model, obtained by noting that the definition of cost (30) and the residents' budget constraints together imply that, in equilibrium, the cost of output per capita in location i at time s, $h_i(s)C_i(s)$, equals the value of residents' and landowners' consumption per capita in location i at time s, $r_i(s)L_i(s)/N_i(s)+c_i(s)$, and then substituting for $r_i(s)$ from (29).[5] Each of these, except (A36), applies for all locations i and all times s. (A36) and (A38) can be used to eliminate $C_i(s)$ and $V^*(s)$, but it is convenient to retain $C_i(s)$ and $V^*(s)$ for the efficiency discussion.

The Euler-Lagrange conditions (see Chiang 1992, section 6.1) for the program (39) – (44) are necessary for a solution, given constraint qualification. Using (42) to eliminate $L_i^h(s)$, the Lagrangian function is

$$\begin{aligned}\mathcal{L}=&\left[N_1(s)P^*(s)h(L_1^f(s),b_1(s),N_1(s))-Z_1(s)-\int_0^{N_1'(s)}k_1(m)dm-\psi_1(N_1'(s))\right]e^{-\rho(s-t)}\\&+\lambda\sum_i\left[A_iZ_i(s)+N_i(s)\hat{u}(L_i/N_i(s)-L_i^f(s),a_i(s),N_i(s))\right]e^{-\rho(s-t)}\\&+\sum_{i>1}\gamma_i\left[N_i(s)P^*(s)h(L_i^f(s),b_i(s),N_i(s))-Z_i(s)-\int_0^{N_i'(s)}k_i(m)dm-\psi_i(N_i'(s))\right]e^{-\rho(s-t)}\\&-\mu(s)\sum_i N_i(s),\end{aligned}$$

where λ, γ_i, and $\mu(s)$ are the Lagrange multipliers on (40), (41), and (43), respectively. Setting $\gamma_1=1$ for

[5]This is "analogous to" but not exactly Walras' Law because $h_i(s)C_i(s)$ is cost of output per capita but may not be market value of output per capita since $P^*(s)\neq C_i(s)$ is possible in the dynamic model. Walras' Law need not hold at each instant of time in the dynamic model because firms' instantaneous profit margins can be either positive or negative, so aggregate instantaneous consumption need not equal instantaneous production in a location or even aggregated across locations. Indeed, production of output need not equal consumption even over the entire time horizon. A simple assumption that renders this possible "disequilibrium" unimportant to our analysis is that firms' "deep pockets" can be drawn upon to finance consumption in excess of production. Alternatively, since our results hold for any particular path $P^*(t)$ provided that the path is exogenous to each location, we can select a price path that equates production and consumption at some appropriate level of aggregation.

notational convenience, the Euler-Lagrange conditions are:

$$\frac{\partial \mathcal{L}}{\partial L_i^f(s)} = \left[-\lambda N_i(s)\hat{u}_{L_i^h}(s) + \gamma_i N_i(s) P^*(s) h_{L_i^f}(s) \right] e^{-\rho(s-t)} = 0, \ \forall i, \ \forall s \geq t \tag{A39}$$

$$\frac{\partial \mathcal{L}}{\partial Z_i(s)} = [\lambda A_i - \gamma_i] \, e^{-\rho(s-t)} = 0, \ \forall i, \ \forall s \geq t \tag{A40}$$

$$\int_\tau^\infty \frac{\partial \mathcal{L}}{\partial N_i(s)}(s) ds + \frac{\partial \mathcal{L}}{\partial N_i'(s)}(\tau) = 0, \ \forall i, \ \forall \tau \geq t \tag{A41}$$

$$\lambda \geq 0 \tag{A42}$$

$$\gamma_i \geq 0, \ \text{for } i > 1 \tag{A43}$$

$$\lambda \left[\sum_i \int_t^\infty \left[A_i Z_i(s) + N_i(s) \hat{u}(L_i^h(s), a_i(s), N_i(s)) \right] e^{-\rho(s-t)} ds - \bar{V} \right] = 0 \tag{A44}$$

$$\begin{aligned} \gamma_i \Bigg[\int_t^\infty \Bigg[& N_i(s) P^*(s) h(L_i^f(s), b_i(s), N_i(s)) - Z_i(s) \\ & - \int_0^{N_i'(s)} k_i(m) dm - \psi_i(N_i'(s)) \Bigg] e^{-\rho(s-t)} ds - R_i \Bigg] = 0, \ \text{for } i > 1 \end{aligned} \tag{A45}$$

$$\sum_i N_i(s) = N, \ \forall s \geq t, \tag{A46}$$

where (A41) is expressed in integral form and the derivatives in (A41) are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial N_i(s)}(s) = & \lambda \left[\hat{u}(L_i^h(s), a_i(s), N_i(s)) - \hat{u}_{L_i^h}(s) \frac{L_i}{N_i(s)} + N_i(s) \hat{u}_{N_i}(s) \right] e^{-\rho(s-t)} \\ & + \gamma_i \left[P^*(s) h(L_i^f(s), b_i(s), N_i(s)) + N_i(s) P^*(s) h_{N_i}(s) \right] e^{-\rho(s-t)} - \mu(s) \end{aligned} \tag{A47}$$

$$\frac{\partial \mathcal{L}}{\partial N_i'(s)}(\tau) = \gamma_i \left[-k_i(N_i'(\tau)) - \psi_i'(N_i'(\tau)) \right] e^{-\rho(\tau-t)}. \tag{A48}$$

Consider an allocation that satisfies the competitive equilibrium conditions (A33) – (A38), and set aggregate utility $\bar{V}$, income R_i, and the resident/migrant aggregate money allocation $Z_i(s)$ in the Pareto problem at their competitive equilibrium levels. In particular,

1. Set $\bar{V}$ as

$$\bar{V} = NV^*(t) + \sum_i A_i N_i(t) k_i(N_i'(t)); \tag{A49}$$

2. Since instantaneous firm income is $[P^*(s) - C_i(s)]h_i(s)N_i(s) - \psi_i(N_i'(s))$, landowner income is $r_i(s)L_i$, and moving service income is $N_i'(s)k_i(N_i'(s)) - \int_0^{N_i'(s)} k_i(m)dm$, set R_i as

$$\begin{aligned} R_i = \int_t^\infty \Bigg[& [P^*(s) - C_i(s)] h(L_i^f(s), b_i(s), N_i(s)) N_i(s) - \psi_i(N_i'(s)) \\ & + r_i(s) L_i + N_i'(s) k_i(N_i'(s)) - \int_0^{N_i'(s)} k_i(m) dm \Bigg] e^{-\rho(s-t)} ds, \end{aligned} \tag{A50}$$

where $C_i(s)$ is the competitive equilibrium production cost per unit of output and $r_i(s) = P^*(s)h_{L_i^f}(s)$ is the competitive equilibrium price of land; and

3. Set[6]

$$Z_i(s) = N_i(s)c_i(s) - N_i'(s)k_i(N_i'(s)). \tag{A51}$$

Substituting for $c_i(s)$ from (A51) in (A38) and then substituting for $C_i(s)$ and $r_i(s)$ in (A50) yields

$$R_i = \int_t^\infty \left[N_i(s)P^*(s)h(L_i^f(s), b_i(s), N_i(s)) - Z_i(s) - \int_0^{N_i'(s)} k_i(m)dm - \psi_i(N_i'(s)) \right] e^{-\rho(s-t)}ds. \tag{A52}$$

Taking note of (A36), it is immediate that (A45) – (A46) are satisfied by the competitive equilibrium allocation when the Pareto problem is evaluated at the corresponding equilibrium levels of $\bar{V}$, R_i, and $Z_i(s)$.

To verify that (A44) is satisfied by the competitive equilibrium allocation when the Pareto problem is evaluated at the corresponding equilibrium levels of $Z_i(s)$ and $\bar{V}$, differentiate (A34) with respect to s to obtain:

$$A_i c_i(s) + \hat{u}(L_i^h(s), a_i(s), N_i(s)) = \rho[V^*(s) + A_i k_i(N_i'(s))] - \frac{d}{ds}[V^*(s) + A_i k_i(N_i'(s))]. \tag{A53}$$

Multiplying by $N_i(s)$, subtracting $A_i N_i'(s)k_i(N_i'(s))$, and multiplying by $e^{-\rho(s-t)}$ yields

$$\begin{gathered}\Big[N_i(s)[A_i c_i(s) + \hat{u}(L_i^h(s), a_i(s), N_i(s))] - A_i N_i'(s)k_i(N_i'(s))\Big]e^{-\rho(s-t)} = \\ \left[N_i(s)\Big[\rho[V^*(s) + A_i k_i(N_i'(s))] - \frac{d}{ds}[V^*(s) + A_i k_i(N_i'(s))]\Big] - A_i N_i'(s)k_i(N_i'(s))\right]e^{-\rho(s-t)}.\end{gathered} \tag{A54}$$

It is straightforward to verify that the right side of (A54) is

$$-\frac{d}{ds}\Big[N_i(s)[V^*(s) + A_i k_i(N_i'(s))]e^{-\rho(s-t)}\Big] + N_i'(s)V^*(s)e^{-\rho(s-t)}. \tag{A55}$$

Substituting for $N_i(s)c_i(s)$ from (A51), (A54) becomes

$$\begin{aligned}&[A_i Z_i(s) + N_i(s)\hat{u}(L_i^h(s), a_i(s), N_i(s))]e^{-\rho(s-t)} = \\ &\qquad -\frac{d}{ds}\Big[N_i(s)[V^*(s) + A_i k_i(N_i'(s))]e^{-\rho(s-t)}\Big] + N_i'(s)V^*(s)e^{-\rho(s-t)}.\end{aligned} \tag{A56}$$

Integrating yields

$$\begin{aligned}&\int_t^\infty [A_i Z_i(s) + N_i(s)\hat{u}(L_i^h(s), a_i(s), N_i(s))]e^{-\rho(s-t)}ds = \\ &\qquad N_i(t)[V^*(t) + A_i k_i(N_i'(t))] + \int_t^\infty N_i'(s)V^*(s)e^{-\rho(s-t)}ds.\end{aligned} \tag{A57}$$

[6]This redefines the choice variable in the problem to be $c_i(s)$ rather than $Z_i(s)$.

Therefore, using (A36) and (A49),

$$\sum_i \int_t^{\infty} [A_i Z_i(s) + N_i(s)\hat{u}(L_i^h(s), a_i(s), N_i(s))]e^{-\rho(s-t)}ds = \bar{V} + \int_t^{\infty} \sum_i N_i'(s)V^*(s)e^{-\rho(s-t)}ds = \bar{V}, \quad \text{(A58)}$$

where the last equality follows because $\sum_i N_i'(s) = 0$ from (A36). This is (A44).

Now use (A40) for $i = 1$ (recalling that $\gamma_1 = 1$) to define λ in terms of the competitive equilibrium allocation as $\lambda = \frac{1}{A_1}$, and note that this value of λ is positive so this definition satisfies (A42) as well as (A40) for $i = 1$. Use this definition of λ and (A40) for $i > 1$ to define γ_i in terms of the competitive equilibrium allocation as $\gamma_i = \frac{A_i}{A_1}$, and note that this value of γ_i is positive so this definition satisfies (A43) as well as (A40) for $i > 1$. Moreover, (A37) implies (A39) for these definitions of λ and γ_i. This leaves only (A41) for the competitive equilibrium allocation to satisfy.

Substitute these definitions of λ and γ_i into (A47) and (A48), and substitute the results into (A41) to show that (A41) evaluated at the competitive equilibrium allocation is

$$\int_{\tau}^{\infty} \Bigg[\hat{u}(L_i^h(s), a_i(s), N_i(s)) - \hat{u}_{L_i^h}(s)\frac{L_i}{N_i(s)} + N_i(s)\hat{u}_{N_i}(s) + A_i P^*(s)[h(L_i^f(s), b_i(s), N_i(s)) + N_i(s)h_{N_i}(s)]\Bigg]e^{-\rho(s-t)}ds - A_i[k_i(N_i'(\tau)) + \psi_i'(N_i'(\tau))]e^{-\rho(\tau-t)} = A_1 \int_{\tau}^{\infty} \mu(s)ds, \ \forall i, \ \forall \tau \geq t. \quad \text{(A41')}$$

Substituting from (A37) and (A38), and from (A34) and (A35) evaluated at $s = \tau$, yields

$$\int_{\tau}^{\infty} [\hat{u}_{N_i}(s) + A_i P^*(s)h_{N_i}(s)]\, N_i(s)e^{-\rho(s-t)}ds = A_1 \int_{\tau}^{\infty} \mu(s)ds - V^*(\tau), \ \forall i, \ \forall \tau \geq t. \quad \text{(A41")}$$

Since (A41") is implied by (A39) – (A46), there is no set of Lagrange multipliers that satisfies the Euler-Lagrange conditions when the conditions are evaluated at an allocation satisfying (A33) – (A38) unless there is a path $\mu(s)$ satisfying (A41"). By necessity of the Euler-Lagrange conditions for a solution, this means an allocation satisfying (A33) – (A38) is not optimal when the threshold welfare levels $\bar{V}$, R_i, and $Z_i(s)$ are set according to (A49) – (A51) unless there is a path $\mu(s)$ satisfying (A41"). If one of these thresholds is set at some other level in the Pareto problem then (A44) or (A45) implies that a multiplier must be zero if the allocation satisfying (A33) – (A38) is to satisfy (A39) – (A48). This violates either $\lambda > 0$ (i.e., (A40) for $i = 1$) or $\gamma_i > 0$ (i.e., (A40) for $i > 1$). Thus, a necessary condition for a competitive equilibrium allocation to be a solution is that the left side of (A41") be the same for all values of i. □

Proof of Corollary to Proposition 3. Landowners' consumption is diminished by the subsidy $x_i(s)$ they pay to consumers, so (A38) becomes

$$h_i(L_i^f(s), b_i(s), N_i(s))C_i(s) = P^*(s)h_{L_i^f}(s)\frac{L_i}{N_i(s)} + c_i(s) - x_i(s),\ \forall i,\ \forall s. \tag{A38'}$$

Landowners pay the subsidy, so (A50) becomes

$$R_i = \int_t^\infty \Bigg[[P^*(s) - C_i(s)]h(L_i^f(s), b_i(s), N_i(s))N_i(s) - \psi_i(N_i'(s)) + [r_i(s)L_i - x_i(s)N_i(s)] + N_i'(s)k_i(N_i'(s)) - \int_0^{N_i'(s)} k_i(m)dm \Bigg] e^{-\rho(s-t)}ds. \tag{A50'}$$

Using this in the Pareto problem along with (A38') leaves (A52) unchanged. Using (A38') in the derivation of (A41") yields

$$\int_\tau^\infty [\hat{u}_{N_i}(s) + A_iP^*(s)h_{N_i}(s)]\,N_i(s)e^{-\rho(s-t)}ds - A_i\int_\tau^\infty x_i(s)e^{-\rho(s-t)}ds = A_1\int_\tau^\infty \mu(s)ds - V^*(t),\ \forall i,\ \forall\tau \geq t. \tag{A41"'}$$

The remainder of the proof is identical to the proof of Proposition 3. □

Proof of Proposition 4. The location subscript i is suppressed throughout the proof. From (24) and (47), and (30), respectively,

$$\frac{U(s)}{A} = x(s) + w(s) - r(s)L^h(s) + \frac{\hat{u}(s)}{A}$$
$$h(s)[P^*(s) - C(s)] = h(s)P^*(s) - w(s) - r(s)L^f(s),$$

where $U(s)$ is shorthand for $U(w(s) + x(s), r(s), a(s), N(s))$. Adding these equations, using (36), and rearranging yields an alternative expression for the integrand in (50):

$$Lr(s) - x(s)N(s) = N(s)\left[\frac{\hat{u}(s)}{A} + h(s)P^*(s)\right] - N(s)\left[\frac{U(s)}{A} + h(s)[P^*(s) - C(s)]\right]. \tag{A59}$$

Integration by parts of the second expression in the integrand yields:

$$\int_t^\infty N(s)U(s)e^{-\rho(s-t)}ds = N(t)V(t) + \int_t^\infty N'(s)V(s)e^{-\rho(s-t)}ds \tag{A60}$$

$$\int_t^\infty N(s)h(s)[P^*(s) - C(s)]e^{-\rho(s-t)}ds = N(t)\int_t^\infty h(s)[P^*(s) - C(s)]e^{-\rho(s-t)}ds + \int_t^\infty N'(s)\left[\int_s^\infty h(m)[P^*(m) - C(m)]e^{-\rho(m-s)}dm\right]e^{-\rho(s-t)}ds, \tag{A61}$$

where $V(s) = \int_t^\infty U(s)e^{-\rho(s-t)}ds$. Using the right side of (37), (48), (A60), (A61), and $N(t) = 0$ in (A59), the objective in (50) can be written

$$\int_t^\infty \left[N(s)\left[\frac{\hat{u}(s)}{A} + h(s)P^*(s)\right] - N'(s)\left[\frac{V^*(s)}{A} + k(N'(s)) + \psi'(N'(s))\right]\right] e^{-\rho(s-t)}ds.$$

These manipulations have yielded an objective that does not depend on the endogenous variables $x(s)$ and $w(s)$ ($r(s)$ is still present as an argument of $\hat{u}(s)$ and $h(s)$). Essentially, (37) and (48) have been used to eliminate $x(s)$ and $w(s)$. The objective is now written in standard form for a variational problem, with an integrand G that depends only on s, $N(s)$, and $N'(s)$; provided we regard $r(s)$ as determined from $N(s)$ by (34). The Euler condition is $G_N = \frac{d}{ds}G_{N'}$.

We have

$$G_N = \left[\frac{\hat{u}(s)}{A} + h(s)P^*(s) + N(s)\frac{\partial}{\partial N}\left(\frac{\hat{u}(s)}{A} + h(s)P^*(s)\right)\right] e^{-\rho(s-t)}.$$

By (34),

$$\frac{\hat{u}(s)}{A} + h(s)P^*(s) = \frac{1}{A}\left[\hat{u}(L^h(s), a(s), N(s)) - Ar(s)L^h(s)\right] + \left[P^*(s)h(L^f(s), b(s), N(s)) - r(s)L^f(s)\right] + r(s)\frac{L}{N(s)}.$$

The first bracketed term is maximized over L^h by the residents' actions, and the second bracketed term is maximized over L^f by the firms' actions. Hence, by the envelope theorem, L^h is ignored in the first term and L^f is ignored in the second term when differentiating with respect to N. So

$$\frac{\partial}{\partial N}\left(\frac{\hat{u}(s)}{A} + h(s)P^*(s)\right) = \frac{\hat{u}_N(s)}{A} + h_N(s)P^*(s) - r(s)\frac{L}{N(s)^2} - \left[L^h(s) + L^f(s) - \frac{L}{N(s)}\right]\frac{\partial r}{\partial N}.$$

Using (34) again, and substituting into G_N, yields

$$G_N = \left[\frac{\hat{u}(s)}{A} + h(s)P^*(s) + N(s)\left[\frac{\hat{u}_N(s)}{A} + h_N(s)P^*(s)\right] - r(s)\frac{L}{N(s)}\right] e^{-\rho(s-t)}. \quad \text{(A62)}$$

Turning to the right side of the Euler condition, we have

$$\begin{aligned} G_{N'} &= -\left[\frac{V^*(s)}{A} + k(N'(s)) + \psi'(N'(s)) + N'(s)\left[k'(N'(s)) + \psi''(N'(s))\right]\right] e^{-\rho(s-t)} \\ &= -\left[\frac{V(s)}{A} + \int_s^\infty h(m)[P^*(m) - C(m)]e^{-\rho(m-s)}dm + N'(s)\left[k'(N'(s)) + \psi''(N'(s))\right]\right] e^{-\rho(s-t)} \\ &\qquad \text{by (37), and} \\ &= -\left[\int_s^\infty \left[x(m) + \frac{\hat{u}(m)}{A} + h(m)P^*(m) - r(m)\frac{L}{N(m)}\right] e^{-\rho(m-s)}dm \right. \\ &\qquad \left. + N'(s)\left[k'(N'(s)) + \psi''(N'(s))\right]\right] e^{-\rho(s-t)} \text{ by (25), (24), (30), and (34).} \end{aligned}$$

So

$$\frac{d}{ds}G_{N'} = \left[x(s) + \frac{\hat{u}(s)}{A} + h(s)P^*(s) - r(s)\frac{L}{N(s)}\right]e^{-\rho(s-t)} - \frac{d}{ds}\left(N'(s)\left[k'(N'(s)) + \psi''(N'(s))\right]e^{-\rho(s-t)}\right). \tag{A63}$$

Equating (A62) and (A63), the Euler condition is

$$\left[N(s)\left[\frac{\hat{u}_N(s)}{A} + h_N(s)P^*(s)\right] - x(s)\right]e^{-\rho(s-t)} = -\frac{d}{ds}\left(N'(s)\left[k'(N'(s)) + \psi''(N'(s))\right]e^{-\rho(s-t)}\right).$$

Integrating over s from τ to infinity yields (51). □

Proof of Proposition 5. The location subscript i is suppressed throughout the proof. Use (A59) to write the objective (52) as

$$\int_t^\infty \left[-N(s)\frac{U(s)}{A} + N(s)\left[\frac{\hat{u}(s)}{A} + h(s)P^*(s)\right] - \psi(N'(s)) + N'(s)k(N'(s)) - \int_0^{N'(s)} k(m)dm\right]e^{-\rho(s-t)}ds$$
$$+ N_0(t)\frac{V(t) - V^*(t)}{A}.$$

Now use (A60), $N(t) = N_0(t)$, (48) and (25) (noting that the wage argument of instantaneous indirect utility is $w(s) + x(s)$ when there is a tax/subsidy policy) to write the objective as

$$\int_t^\infty \left[-N'(s)\frac{V^*(s)}{A} + N(s)\left[\frac{\hat{u}(s)}{A} + h(s)P^*(s)\right] - \psi(N'(s)) - \int_0^{N'(s)} k(m)dm\right]e^{-\rho(s-t)}ds - N_0(t)\frac{V^*(t)}{A}.$$

As in the proof of Proposition 4, these manipulations have yielded an objective that does not depend on the endogenous variables $x(s)$ and $w(s)$ ($r(s)$ is still present as an argument of $\hat{u}(s)$ and $h(s)$). Essentially, (48) has been used to eliminate $x(s)$ and $w(s)$. Only this one constraint is needed to eliminate both of these endogenous variables because $x(s)$ and $w(s)$ enter both the objective and the constraint exclusively in the form $x(s) + w(s)$. Moreover, the objective is now written in standard form for a variational problem, with an integrand G that depends only on s, $N(s)$, and $N'(s)$, provided we regard $r(s)$ as determined from $N(s)$ by (34) (the constant outside of the integral is ignored for the purpose of finding a maximum). The Euler condition is $G_N = \frac{d}{ds}G_{N'}$.

G_N is identical to G_N in the proof of Proposition 4, and is therefore given by (A62).

Consider the right side of the Euler condition and note

$$
\begin{aligned}
G_{N'} &= -\left[\frac{V^*(s)}{A} + \psi'(N'(s)) + k(N'(s))\right] e^{-\rho(s-t)} \\
&= -\left[\frac{V(s)}{A} + \int_s^\infty h(m)[P^*(m) - C(m)]e^{-\rho(m-s)}dm\right] e^{-\rho(s-t)} \text{ by (25), (48) and the right side of (37)} \\
&= -\int_s^\infty \left[x(m) + \frac{\hat{u}(m)}{A} + h(m)P^*(m) - r(m)\frac{L}{N(m)}\right] e^{-\rho(m-t)}dm \text{ by (25), (24), (30), and (34).}
\end{aligned}
$$

So

$$
\frac{d}{ds}G_{N'} = \left[x(s) + \frac{\hat{u}(s)}{A} + h(s)P^*(s) - r(s)\frac{L}{N(s)}\right] e^{-\rho(s-t)}. \tag{A64}
$$

Equating (A62) and (A64) yields (53). □

Proof of Proposition 6. From the proof of Proposition 3 and its corollary, a subsidized competitive equilibrium allocation satisfies (A33) – (A37) and (A38'). (A33) is (42), (A36) is (43), and (A37) is (45). This leaves only (54) to establish for a subsidized competitive equilibrium. Substituting for $h_i(m)C_i(m)$ in (A35) from (A38'), and then for $\int_s^\infty c_i(m)e^{-\rho(m-s)}dm$ from (A34) in the resulting expression, yields (54) with $\Gamma(s) = V^*(s)$.

From the proof of Proposition 3, a solution to the Pareto problem (39) – (44) satisfies (A39) – (A46) and (42). (A46) is (43). That proof also establishes $\lambda = 1/A_1$ and $\gamma_i = A_i/A_1$. Substituting these values into (A39) yields (45). This leaves only (54) to be established as a property of a solution to (39) – (44). The proof of Proposition 3 establishes (A41') for a solution to (39) – (44). Substituting for $\hat{u}_{L_i^h}(s)$ in (A41') from (45) and multiplying through by $e^{\rho(\tau-t)}$ yields (54) when $x_i(m)$ is given by (53) and $\Gamma(s) = A_1 e^{\rho(s-t)} \int_s^\infty \mu(m)dm$. □